01 Jan 1993

Simplified Strength Prediction For Cold-formed C- And Z-shapes

Roger A. LaBoube
Missouri University of Science and Technology, laboube@mst.edu

Simplified Strength Prediction for Cold-Formed C- and Z-Shapes

By R. A. LaBoube,[1] Member, ASCE

Abstract: Although cold-formed steel C- and Z-shaped members possess a very favorable strength-to-weight ratio, are easily fabricated and erected, and offer a cost-effective structural design solution, they can be highly susceptible to local buckling. To adequately address the impact of the local buckling limit state, a complex, time-consuming design office effort may be required. Therefore, to facilitate the design of such shapes, a study was conducted that focused on developing relatively simple, easy-to-apply design guidelines. The design guidelines address the behavior of cold-formed steel C- and Z-shaped members when used as either a flexural member or a compression member. Both edge-stiffened and unstiffened compression elements are considered by the design guidelines. The design guidelines are unique in that they employ the concept of applying strength-reduction factors to either the bending strength or the axial compression strength. All strength computations use the full cross-section areas.

Introduction

The design of a cold-formed steel structural member is complicated by the necessity to consider the negative effect of local buckling on the overall behavior of the section. Routine design calculations for cold-formed steel members are performed under the auspices of the *Specification for the Design of Cold-Formed Steel Structural Members* (1989). This allowable stress specification accounts for the effect of local buckling by using an effective-width approach (Yu 1991). Although the effective-width approach more technically models the local buckling behavior of the thin flat compression plates that form a section geometry, it is a complex approach that is not readily understood by practicing engineers.

To facilitate the evaluation of a cold-formed steel C- or Z-shaped member, simplified strength expressions have been developed and presented herein. The concept adopted for the design expressions is to use only full section properties. The negative influence that local buckling has on the load capacity is accounted for by strength-reduction factors. Although the use of full-section properties is contrary to the American Iron and steel Institute (AISI) specifications on design philosophy, it is consistent with the use of full-section properties for hot-rolled design, a concept that all structural engineers are familiar with.

Strength equations are presented for computing the load capacity of members subjected to either bending or axial load. These strength equations have been developed and calibrated to generally produce a conservative load capacity when compared with the allowable stress version of the specification. The strength equations also serve as the foundation of the AISI *Preliminary Design Guide for Cold-Formed C- and Z-Members* (*Preliminary* 1991).

---

[1]Assoc. Prof. of Civ. Engrg., Univ. of Missouri-Rolla, Rolla, MO 65401.

Note. Discussion open until November 1, 1993. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on October 28, 1992. This paper is part of the ***Journal of Structural Engineering***, Vol. 119, No. 6, June, 1993. ©ASCE, ISSN 0733-9410/93/0006-1733/$1.00 + $.15 per page. Paper No. 4463.

J. Struct. Eng., 1993, 119(6): 1733-1739

Strength for Bending

Both local and overall instability may influence the strength of a flexural member. The study reported herein was limited to flexural members with full lateral support; therefore, overall instability of the member is precluded. Any decrease in load capacity as a result of local instability is accounted for by using strength-reduction factors applied to the nominal yield moment

$$M_n = R_f R_w F_y S_x \quad (1)$$

where R_f = flange-reduction factor; R_w = web reduction factor; F_y = yield point; S_x = elastic section modulus for the gross cross section, with respect to a centroidial axis perpendicular to the web.

Web Local Buckling

Web local buckling effects R_w can be accounted for by applying the following strength-reduction factor:

$$R_w = 1.21 - 0.00034 \frac{d'}{t} \sqrt{F_y} \quad (2)$$

where $d' = d - 2t$; d = overall depth of the member; and t = base thickness of steel sheet. This reduction reflects the decrease in compression-flange capacity resulting from the interaction of flange and local web buckling, and is based on research conducted at the University of Missouri-Rolla (LaBoube and Yu 1982).

Flange Local Buckling

The influence of flange local buckling is quantified by the relationship R_f, for both edge-stiffened and unstiffened compression flanges:

For edge-stiffened compression flange $w/t \leq 60$

$$R_f = R_1 R_2 \quad (3)$$

where $R_1 = [1.227 - 0.284(w/t)/S] \leq 1.0$; w = flat width of compression flange (in.); $S = 1.28\sqrt{E/F_y}$; $R_2 = 1.5\ D/b + 0.55$, when $R_w = 1$; $0.2 \leq D/b < 0.3$; $R_2 = 1.0$, when $R_w = 1$ and $0.3 \leq D/b \leq 0.4$; $R_2 = 1.0$, when $R_2 < 1$; D = length of edge stiffener where $0.2b \leq D \leq 0.4b$ (in.); and b = width of compression flange (flat width plus radii and thickness) (in.).

For unstiffened compression flange $w/t \leq 30$

$$R_f = 1.190 + 0.190 \left(\frac{\frac{w}{t}}{S} \right) \leq 1.0 \quad (4)$$

in which $S = 0.42\sqrt{E/F_y}$. Eqs. (3) and (4) recognize the degree of slenderness of the compression element as quantified by $(w/t)/S$. The parameter S represents the maximum width-to-thickness ratio $(w/t)_{\lim}$, for which the compression element is fully effective, i.e., no local buckling. This concept has been carried forward from a previous AISI specification (*Specification* 1980).

For edge-stiffened compression elements, an additional modification R_2 is given to account for the stiffening influence of the edge stiffener.

The preceding equations produce conservatively acceptable strength predictions for typical industry standard sections, as demonstrated by the values

J. Struct. Eng., 1993, 119(6): 1733-1739

obtained for the ratio of M_n/M_{AISI} given in Tables 1 and 2. The value of M_{AISI} is computed by using the effective-width equations of the 1989 AISI specification.

Concentrically Loaded Compression Members

Because of the inherent complexity involved in evaluating the load capacity for axially loaded compression members subject to torsion or torsional-flexural buckling, the strength equation was limited to the overall buckling limit state of flexural buckling. This was accomplished by imposing a limit on the unbraced length L_{min}. The following equation for L_{min} is based on Fig. 1, which is taken from the research of Chajes et al. (1966). For $b/d \leq 0.4$, L_{min} has no lower limit; for $b/d > 0.4$

$$L_{min} = \frac{d^2}{t}\left[3.72 - 19.53\frac{b}{d} + 25.91\left(\frac{b}{d}\right)^2\right] \quad (5)$$

The nominal axial strength P_n is evaluated by the following:

$$P_n = F_n A_g R_c \quad (6)$$

TABLE 1. AISI Specification versus Simplified Strength Equations for Sections with Edge-Stiffened Compression Flange

Section (1)	Moment Capacity kN·m: M_{AISI} (2)	M_n (3)	M_n/M_{AISI} (4)
(*a*) Channel Sections F_y = 227.5 MPa			
8 × 2.5 × .04	3.70	3.31	0.895
8 × 2.0 × .04	3.59	3.22	0.897
8 × 1.625 × .04	3.58	3.10	0.864
6 × 1.625 × .04	2.48	2.30	0.929
6 × 2.0 × .04	2.54	2.42	0.955
6 × 2.5 × .04	2.68	2.53	0.945
4 × 2.0 × .04	1.47	1.43	0.976
8 × 1.625 × .058	5.37	5.07	0.944
8 × 2.0 × .058	5.83	5.27	0.904
8 × 2.5 × .085	5.97	5.31	0.890
(*b*) Z-Sections F_y = 227.5 MPa			
8 × 2.625 × .06	7.11	5.91	0.943
8 × 2.625 × .08	10.0	10.00	1.000
8 × 2.65 × .107	13.48	13.48	1.000
9.5 × 2.75 × .06	8.98	8.45	0.940
9.5 × 2.75 × .071	11.64	11.09	0.952
(*c*) Z-Sections F_y = 379 MPa			
9.5 × 2.75 × .061	12.86	11.70	0.910
9.5 × 2.75 × .061	17.41	16.06	0.922
9.5 × 2.75 × .079	19.68	19.03	0.967
9.5 × 2.75 × .084	21.96	21.13	0.963
9.5 × 2.75 × .098	26.79	26.00	0.971

Note: Depth × flange width × thickness = 8 × 2.5 × 0.04.

J. Struct. Eng., 1993, 119(6): 1733-1739

TABLE 2. AISI Specification versus Simplified Strength Equation for Sections with Unstiffened Compression Flange

Section (1)	Moment Capacity kN·m M_{AISI} (2)	M_n (3)	M_n/M_{AISI} (4)
(*a*) Channel Sections F_y = 227.5 MPa			
2.5 × 1.375 × .04	0.45	0.45	1.008
3.5 × 1.0 × .04	0.67	0.59	0.887
4.0 × 1.0 × .04	0.82	0.81	0.988
5.0 × 1.0 × .04	1.15	1.09	0.946
6.0 × 1.0 × .04	1.54	1.37	0.889
7.0 × 1.0 × .04	1.97	1.65	0.835
8.0 × 1.0 × .04	2.26	1.92	0.851
8.0 × 1.375 × .04	2.27	1.88	0.830
4.0 × 1.0 × .07	1.55	1.55	1.000
(*b*) Channel Sections F_y = 379 MPa			
4.0 × 1.0 × .07	2.51	2.51	1.001
4.0 × 1.5 × .07	2.83	2.77	0.980

where A_g = gross cross-section area of the member (sq in.); and F_n is determined as follows:

For $F_e > F_y/2$

$$F_n = F_y\left(1 - \frac{F_y}{4F_e}\right) \quad (7a)$$

For $F_e \leq F_y/2$

$$F_n = F_e \quad (7b)$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \quad (7c)$$

in which E = modulus of elasticity (ksi); K = effective length factor; L = unbraced length of member (in.); and r = radius of gyration of the gross cross section (in.).

R_c is a column reduction factor defined as follows:

$$R_c = R_1 R_2 \quad (8)$$

where

$$R_1 = 1.50 - 0.21\left[\frac{\left(\frac{\sum w}{t}\right)}{S}\right] + 0.01\left[\frac{\left(\frac{\sum w}{t}\right)}{S}\right]^2 \leq 1.0 \quad (9)$$

$$R_2 = 0.175\left[\frac{\frac{L}{r}}{C_c}\right] + 0.825 \leq 1.0 \quad (10)$$

J. Struct. Eng., 1993, 119(6): 1733-1739

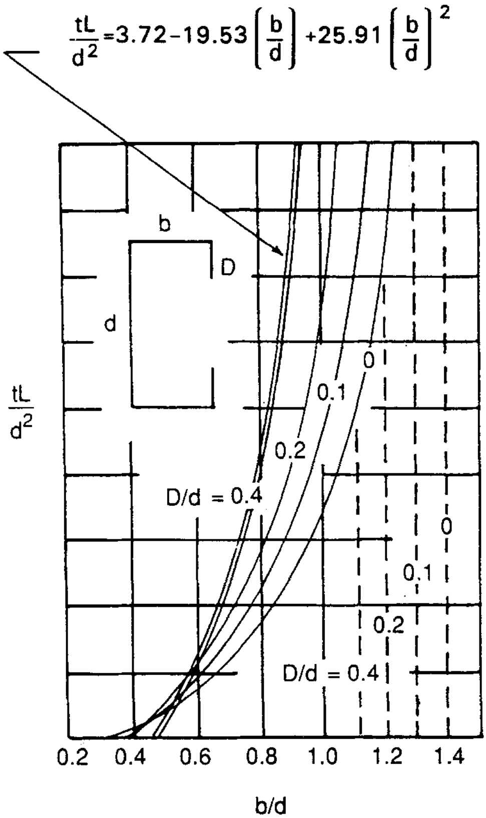

FIG. 1. Influence of Section Geometry on Flexural Buckling of Axially Loaded Columns

$$S = 1.28\sqrt{\frac{E}{F_y}} \qquad (11)$$

$$C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \qquad (12)$$

and $\Sigma\, w$ = the sum of all flat widths of the cross section (i.e., flanges plus web plus stiffeners) given in inches.

The influence of local buckling on the nominal axial load capacity is accounted for by a strength reduction factor R_c that recognizes the degree of slenderness of the compression elements $(\Sigma\, w/t)/S$, and the overall slenderness of the member $(L/r_y)/C_c$.

The element slenderness factor R_1 accounts for the instability of all compression elements by using for w the sum of the flat widths of all compression elements in the cross section $\Sigma\, w$. This quantity is normalized with respect to the limiting width-to-thickness ratio S for an edge-stiffened compression element.

The element slenderness factor R_1 was developed assuming that the cross

J. Struct. Eng., 1993, 119(6): 1733-1739

TABLE 3. Comparison between AISI Specification and Design Guide for Compression Members

Section (1)	Axial Load (kN) Unbraced length (cm) (2)	P_{AISI} (3)	P_n (4)	P_n/P_{AISI} (5)
		(*a*) F_y = 227.5 MPa		
8.0C.06	244	77.65	65.42	0.842
	366	64.39	52.60	0.817
9.5C.06	244	77.91	64.70	0.831
	366	64.26	51.49	0.801
8.0C.12	122	233.54	225.48	0.965
	183	224.81	219.65	0.977
	244	209.29	207.59	0.992
	366	163.23	164.03	1.005
9.5C.12	122	237.63	234.74	0.988
	244	212.27	214.80	1.012
	366	164.56	167.50	0.018
2.5C.05	244	16.38	16.38	1.000
6.0C.05	244	29.19	25.99	0.890
8.0C.05	244	39.16	34.67	0.885
10.0C.05	244	38.67	33.42	0.865
		(*b*) F_y = 379 MPa		
8.0C.06	244	98.81	77.47	0.776
9.5C.06	244	99.64	76.85	0.771
	366	66.53	44.95	0.676
8.0C.07	244	124.51	104.04	0.839
8.0C.12	122	331.84	324.98	0.979
	244	275.32	270.92	0.984
	366	172.62	156.95	0.909
9.5C.12	244	276.75	274.34	0.991
	366	171.77	155.31	0.900
11.0C.12	244	276.30	275.05	0.996

section had yielded. However, as the overall column slenderness increases, the effective area will increase also. Thus, the overall slenderness factor R_2 accounts for the increase in the effective area of the cross section as the overall column slenderness increases.

The accuracy of (6), and the tendency of this simplified expression to yield conservative strength estimates, is demonstrated by the ratio of P_n/P_{AISI} given in Table 3. The expression P_{AISI} is the axial-load capacity evaluated by the effective width approach given in the specification.

Conclusion

Local buckling of a cross section with slender compression elements can have a detrimental effect on the behavior and performance of the section when used as either a flexural member or a compression member. To quantify the strength of a structural member susceptible to local buckling often requires extensive design calculations that challenge the design engineer.

To facilitate design, a simplified set of equations have been presented that use reduction factors to account for local buckling, in conjunction with the full section properties. These equations enable the design engineer to easily estimate the strength of a C- or Z-shaped member. The simplified design equations were developed to provide conservative predictions for the strength of a member when subjected to either flexure or compression.

Appendix I. References

Chajes, A., Fang, P. J., and Winter, G. (1966). "Torsional flexural buckling, elastic and inelastic, of cold formed thin walled columns." *Cornell Engrg. Res. Bull. 66-1*, Cornell Univ., Ithaca, N.Y.

Cold-formed steel design manual. (1986). Am. Iron and Steel Inst., Washington, D.C.

LaBoube, R. A., and Yu, W. W. (1982). "Bending strength of webs of cold-formed steel beams." *J. Struct. Engrg.*, ASCE, 108(7), 1589–1604.

"Preliminary design guide for cold-formed C- and Z-members." (1991). *Rep. CF 91-1*, Am. Iron and Steel Inst., Washington, D.C.

"Specification for the design of cold-formed steel structural members. (1980). Am. Iron and Steel Inst., Washington, D.C.

Specification for the design of cold-formed steel structural members with 1989 addendum. (1989). Am. Iron and Steel Inst., Washington, D.C.

Appendix II. Notation

The following symbols are used in this paper:

A_g = gross cross-section area of member;
b = width of compression flange (flat width plus radii and thickness);
D = length of edge stiffener;
d = overall depth of section;
d' = $d - 2t$;
E = modulus of elasticity of steel;
F_y = yield point used for design;
K = effective length factor;
L = unbraced length of compression member;
M_{AISI} = nominal moment capacity using AISI specification;
M_n = nominal moment capacity;
P_{AISI} = nominal axial capacity using AISI specification;
P_n = nominal axial capacity;
R_c = axial load reduction factor;
R_f = moment reduction factor;
r = inside bend radius;
r = radius of gyration of gross cross section;
S_x = elastic section modulus for gross cross section;
t = base steel thickness; and
w = flat width of compression flange.

J. Struct. Eng., 1993, 119(6): 1733-1739