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Kensuke Baba

Osaka University, Japan

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Dynamical Analysis of Soil-Piles Interaction Systems Under Earthquake Type Loadings

Kensuke Baba
Architectural Engineering, Osaka University, Japan

SYNOPSIS In order to investigate and discuss a theoretical prediction on the seismic response of pile groups in soil-structure interaction systems, the dynamic characteristics of piles surrounded by soil ground due to earthquake type disturbances in addition to those excited by the loadings at the head of piles.
This paper is concerned with a theoretical analysis which is based on the three-dimensional wave propagation theory to find dynamic interaction characteristics of elastic pile groups embedded in the viscoelastic soil stratum on a rigid basis, under external forces concentrated at the head of piles and uniformly distributed bedrock motion. In dealing with this complicated boundary configuration and exciting condition, the technique of superposition principle associated with the auxiliary problems is effectively applied. And, the governing equations in the domain of frequency and wave numbers reduce to the Fredholm type integral equations.

INTRODUCTION

There has recently been a growing interest in the dynamic behavior of the interaction system composed of pile groups and soil ground during wind, ocean wave and earthquake excitations.
The present study is concerned with a method of theoretical analysis of elastic pile groups and the surrounding viscoelastic soil stratum on a rigid basis due to the external loadings concentrated at the head of piles and uniformly distributed bedrock motion.
General approaches to such dynamic interaction problems based on the three-dimensional wave propagation theory are to be related to solve a class of mixed boundary value problems with complex boundary configurations and exciting conditions.
In dealing with these complicated boundary value problems, the motion of piles is affiliated with one-dimensional fields along their symmetric axes, while the interaction field of the surrounding soil layer is separated into the incident motion associated with the bedrock excitations without piles and the interacted field due to the presence of pile groups.
The latter is further separated into two sets of fields, corresponding to the following subproblems:
(I) one related to the original soil stratum rested on a rigid bedrock and having the surfaces extended flat in lateral direction, and
(II) the other corresponding to a viscoelastic soil layer having the circumferential boundaries around piles enlarged symmetrically in respect of the bottom of the layer for the fixed condition at the tips of piles, or anti-symmetrically for the pinned one.
Then, the respective stress and displacement components of auxiliary problems are combined to satisfy the original boundary conditions.
By applying the integral transforms in respect of time and spatial variables and performing the transformation in cylindrical polar coordinate systems, the Fredholm type integral equations are derived in the domain of frequency and wave numbers. By making use of numerical method in solving the above equations, the solutions in frequency domain are expressed in terms of multiple summations and integrals.

FORMULATION OF THE PROBLEM

The displacement vector u of the soil stratum is required to satisfy the following mixed boundary value forms in frequency domain,

$$\begin{aligned} {}_G L(u) &= 0 & x \in G \\ \beta_0(u) &= 0 & x \in \Gamma_0 \\ \beta_B(u) &= u_G & x \in \Gamma_B \\ u = u_{pj},\quad \beta_j(u) &= {}_p L_j(u_{pj}) - {}_G P_j(u) = f_j & \\ & & x \in \Gamma_j \;:\; j=1,2,\cdots J \end{aligned} \tag{1}$$

in which x is the position vector in the layered soil medium G and
(i) the three-dimensional wave equation of the soil medium given by the vector differential operator ${}_G L$,
(ii) the stress-condition associated with the operator β_0 at the surface Γ_0,
(iii) the displacement-condition in welded contact with the rigid basis related to the operator β_B at the interface Γ_B,
(iv) the welded contact condition between the displacement of piles u_{pj} and that of the soil stratum, which is accompanied with the condition of dynamic equilibrium of piles, the boundary conditions at the tips of piles and non-deformability in respect of the circular cross-sections of piles associated with the operator β_j at the interface Γ_j, where the pile groups are counted to J.
In addition, the radiation condition in the infinitely far field is required to be satisfied.

It is convenient to write the interaction field in the absolute coordinate system,

$$u = u^i + u^s \tag{2}$$

where u^i is the incident-field motion of the soil stratum in the absence of pile groups and u^s is the interacted-field due to the presence of the pile foundations. The incident-field motion is required to satisfy the conditions:

$$\begin{aligned} {}_G L(u^i) &= 0 & x &\in G \\ \beta_0(u^i) &= 0 & x &\in \Gamma_0 \\ \beta_B(u^i) &= u_G & x &\in \Gamma_B \end{aligned} \tag{3}$$

In cylindrical polar coordinate system (r,θ,z), the incident motion excited in the direction $\theta=0$ is obtained as:

$$u^i = u_G \frac{\cos(kz)}{\cos(kH)} \begin{bmatrix} \cos\theta \\ -\sin\theta \\ 0 \end{bmatrix} \tag{4}$$

$$h = \omega/C_p, \quad k = \omega/C_s$$

where u_G and ω are the displacement amplitude and the circular frequency of harmonic disturbances, c_p, c_s and H are the phase velocities of dilatational and distortional waves and the thickness of the viscoelastic soil stratum. Therefore the equations requested for the interacted-field u^s have the similar form as in the case of the absolute-field, namely Eq.(1) but the inhomogeneous term in the condition set up on the surface Γ_B is vanished.

$$\begin{aligned} {}_G L(u^s) &= 0 & x &\in G \\ \beta_0(u^s) &= 0 & x &\in \Gamma_0 \\ \beta_B(u^s) &= 0 & x &\in \Gamma_B \end{aligned} \tag{5}$$

$$u^s = u_{pj} - u^i, \quad \beta_j(u^s) = {}_p L_j(u_{pj}) - {}_G P_j(u^s) = f_j$$
$$x \in \Gamma_j \quad : j=1,2,\cdots J$$

The interacted displacement field in cylindrical polar coordinates can be expressed in terms of potentials of dilatational and distortional components as follows:

$$u^s = \nabla\varphi + \nabla\times(\psi e) + \nabla\times\{\nabla\times(\chi e)\} \tag{6}$$

$$(\nabla^2+h^2)\varphi = (\nabla^2+k^2)\psi = (\nabla^2+k^2)\chi = 0$$

where ∇ and e denote the gradient operator and the unit base vector along the z-axis, φ, ψ and χ are particular solutions of the associated scalar Helmholtz equations. This interacted field is separated into the two sets of displacement fields which correspond to the subproblems mentioned previously, and given in the following potential forms:

$$\begin{bmatrix} \varphi \\ \psi \\ \chi \end{bmatrix} = \sum_{\nu=-\infty}^{\infty} e^{i\nu\theta} \int_0^\infty dq\, q J_\nu(qr) \begin{bmatrix} A_1 \mathrm{sh}\alpha z + A_2 \mathrm{ch}\alpha z \\ B_1 \mathrm{sh}\beta z + B_2 \mathrm{ch}\beta z \\ C_1 \mathrm{sh}\beta z + C_2 \mathrm{ch}\beta z \end{bmatrix}_\nu$$

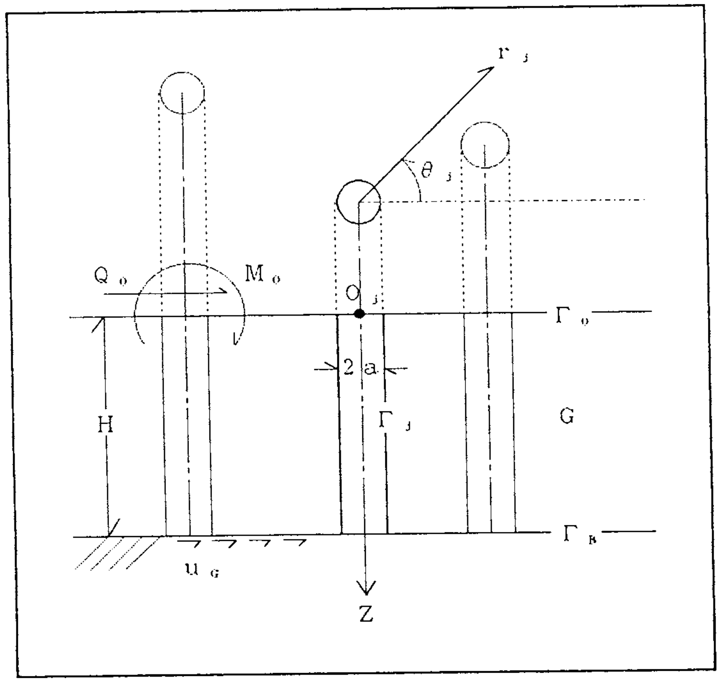

Fig. 1. System cofiguration and coordinate systems.

$$+\sum_{j=1}^{J} \sum_{\nu=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} e^{i\nu\theta_j + ip_n z} \begin{bmatrix} K_\nu(\alpha_n r_j) E \\ K_\nu(\beta_n r_j) F \\ K_\nu(\beta_n r_j) G \end{bmatrix}_{j\nu} W_a^\infty(r_j) \tag{7}$$

$$\mathrm{sh}(z) = \sinh(z), \quad \mathrm{ch}(z) = \cosh(z).$$

where the symmetrical axis of the pile named (j) coincides with the z-direction in cylindrical polar coordinates (r_j, θ_j, z) $: j=1,2,\cdots J$, p_n and q are the discrete and continuous parameters of wave number, and their associate parameters are:

$$p_n = n\pi/H, \ (2n+1)\pi/2H.$$

$$\alpha_n = \sqrt{p_n^2 - h^2}, \ \beta_n = \sqrt{p_n^2 - k^2}, \ \alpha = \sqrt{q^2 - h^2}, \ \beta = \sqrt{q^2 - k^2}$$

and $J_\nu(x)$, $K_\nu(x)$ are the Bessel and modified Bessel functions of integer order ν, both of which are close to zero at infinity. In addition, the following cutoff operator is defined to avoid the region inside the pile with a radius of 'a'.

$$W_a^\infty(r) = \begin{matrix} 1 & : r>a \\ 0 & : a>r \end{matrix}$$

The components of these potential functions presented in the finite Fourier expansions along the z-direction are selected symmetrically in respect of the plane surface z=H for the fixed condition at the tips of piles, or on the contrary anti-symmetrically for the pinned one. In accordance with the expanded forms along the axes of piles, the external loadings are enlarged symmetrically or anti-symmetrically in respect of the surface z=H and expressed in the finite Fourier series as follows:

i) in the case of the fixed condition at the tips of piles,

$$Q_0\delta(z)=\frac{Q_0}{2H}\sum_{n=0}^{\infty}\varepsilon_n\cos(p_n z) \qquad :p_n=n\pi/H$$

$$M_0[\delta(z-\varepsilon)-\delta(z)]/\varepsilon=\frac{M_0}{H}\sum_{n=0}^{\infty}p_n\sin(p_n z) \qquad :p_n=(2n+1)\pi/2H$$

$$u_G[\cos(kz)W_0^H(z)+\cos k(2H-z)W_H^{2H}(z)]/\cos(kH)$$

$$=-\frac{u_G}{H}k\tan(kH)\sum_{n=0}^{\infty}\varepsilon_n\cos(p_n H)\cos(p_n z)/\beta_n^2 \qquad :p_n=n\pi/H$$

$$:2H>z>0 \qquad (8\text{-}1)$$

ii) in the case of the pinned condition at the tips of piles,

$$Q_0\delta(z)=\frac{Q_0}{H}\sum_{n=0}^{\infty}\cos(p_n z) \qquad :p_n=(2n+1)\pi/2H$$

$$M_0[\delta(z-\varepsilon)-\delta(z)]/\varepsilon=\frac{M_0}{H}\sum_{n=0}^{\infty}p_n\sin(p_n z) \qquad :p_n=n\pi/H$$

$$u_G[\cos(kz)W_0^H(z)-\cos k(2H-z)W_H^{2H}(z)]/\cos(kH)$$

$$=\frac{2u_G}{H}\sum_{n=0}^{\infty}p_n\sin(p_n H)\cos(p_n z)/\beta_n^2 \qquad :p_n=(2n+1)\pi/2H$$

$$:2H>z>0 \qquad (8\text{-}2)$$

where Q_0 and M_0 are the amplitudes of the concentrated lateral force and bending moment at the head of piles, $\delta(z)$, ε_n and ε are the Dirac's delta function, the Neumann's factor and an insignificant length along the axes of piles.

$$\varepsilon_n=\begin{matrix}1 & :n=0\\ 2 & :n\neq 0,\end{matrix} \qquad 1\gg\varepsilon>0$$

In order to derive the boundary equations standing on the plane surfaces of the soil layer parallel to the lateral direction in the domain of wave numbers, it is necessary for all terms of the potentials to arrange their components along the radial and circumferential directions expanded in the same style, to which the following two steps of operation are applied, 1) the Hankel transforms of modified Bessel functions in respect of the radial directions:

$$\tilde K_\nu(\alpha_n)$$

$$=\int_0^\infty dr\,rJ_\nu(qr)K_\nu(\alpha_n r)W_a^\infty(r) \qquad (9)$$

$$=[\alpha_n aJ_\nu(qa)K_{\nu+1}(\alpha_n a)-qaJ_{\nu+1}(qa)K_\nu(\alpha_n a)]/(q^2+\alpha_n^2)$$

2) the transformation of Bessel functions in a number of cylindrical polar coordinate systems by means of the Bessel's addition theorem as shown in Fig. 2,

$$J_m(qr_j)e^{im\theta_j}$$

$$=(-1)^m\sum_{\nu=-\infty}^{\infty}J_{\nu-m}(q\ell_j)J_\nu(qr)e^{i[\nu\theta-(\nu-m)\gamma_j]} \qquad (10)$$

After the arrangements mentioned above, the resulting equations in the domain of wave numbers are concerned with the surface Rayleigh waves travelling along the horizontal axes.

For the cylindrical boundary surfaces along the symmetrical axes of piles, the finite Fourier transform in respect of z is applied to the potentials of the subproblems and the inhomogeneous terms in consideration of the boundary condition at the tips of piles. And in succession, the transformation of the Bessel and

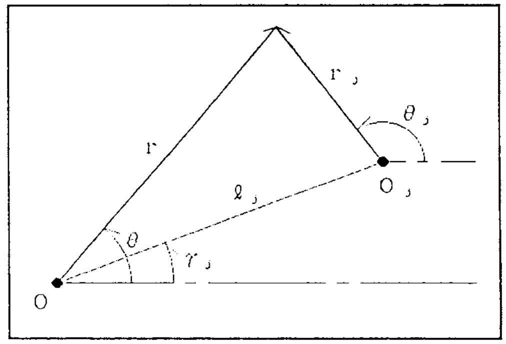

Fig. 2. Relation between cylindrical polar coordinate systems

modofied Bessel functions in cylindrical polar coordinates is performed so as to have the same forms of any component of the subproblems expanded in the direction of r and θ parameters.

$$\begin{bmatrix}\varphi & u & \tau_{rr}\\ \psi & v & \tau_{r\theta}\\ \chi & w & \tau_{rz}\end{bmatrix}^s=\sum_{\nu n}\sum e^{i\nu\theta+ip_n z}\begin{bmatrix}\tilde\varphi & \tilde u & \tilde\tau_{rr}\\ \tilde\psi & \tilde v & \tilde\tau_{r\theta}\\ \tilde\chi & \tilde w & \tilde\tau_{rz}\end{bmatrix}^s_{\nu n} \qquad (11)$$

in which the transformed displacement and stress components of the interacted-field are composed of the transformed potentials with the Lame's constants λ, μ of the soil layer.

$$\begin{bmatrix}\tilde u+i\tilde v\\ \tilde u-i\tilde v\\ \tilde w\end{bmatrix}^s_{\nu n}=\begin{bmatrix}\partial/\partial r-\nu/r & & \\ & \partial/\partial r+\nu/r & \\ & & 1\end{bmatrix}\begin{bmatrix}1 & -i & -p_n\\ 1 & i & -p_n\\ -p_n & 0 & -\beta_n^2\end{bmatrix}\begin{bmatrix}\tilde\varphi\\ \tilde\psi\\ \tilde\chi\end{bmatrix}_{\nu n}$$

$$\begin{bmatrix}\tilde\tau_{rr}+i\tilde\tau_{r\theta}\\ \tilde\tau_{rr}-i\tilde\tau_{r\theta}\\ \tilde\tau_{rz}\end{bmatrix}_{\nu n}=\mu\begin{bmatrix}2\Delta_{-\nu}+2h^2-k^2 & -2i\Delta_{-\nu}+i\beta_n^2 & 2p_n\Delta_{-\nu}\\ 2\Delta_\nu+2h^2-k^2 & 2i\Delta_\nu-i\beta_n^2 & 2p_n\Delta_\nu\\ -2p_n\partial/\partial r & -i\nu p_n/r & -(2p_n^2-k^2)\partial/\partial r\end{bmatrix}\begin{bmatrix}\tilde\varphi\\ \tilde\psi\\ \tilde\chi\end{bmatrix}_{\nu n} \qquad (12)$$

$$\Delta_\nu=\partial^2/\partial r^2+\nu\partial/r\partial r-\nu/r^2$$

Similarly, the displacements of the incident loadings and pile foundations are transformed to obtain the presentation accompanied by the potential forms by making use of the Kronecker's symbol in the domain of circumferential wave number.

$$\begin{bmatrix}u\\ v\\ w\end{bmatrix}^i=\sum_n e^{ip_n z}\begin{bmatrix}\cos\theta\\ -\sin\theta\\ 0\end{bmatrix}\tilde u_n^i \qquad (13\text{-}1)$$

$$=\frac{1}{2}\sum_{\nu n}\sum e^{i\nu\theta+ip_n z}\begin{bmatrix}\delta_{\nu,1}+\delta_{\nu,-1}\\ i(\delta_{\nu,1}-\delta_{\nu,-1})\\ 0\end{bmatrix}\tilde u_n^i$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}_p = \sum_n e^{ip_n z} \begin{bmatrix} \cos(\theta-\zeta) \\ -\sin(\theta-\zeta) \\ 0 \end{bmatrix} U_n^p \quad (13\text{-}2)$$

$$= \frac{1}{2}\sum_\nu\sum_n e^{i\nu\theta+ip_n z} \begin{bmatrix} e^{-i\zeta}\delta_{\nu,1}+e^{i\zeta}\delta_{\nu,-1} \\ i(e^{-i\zeta}\delta_{\nu,1}-e^{i\zeta}\delta_{\nu,-1}) \\ 0 \end{bmatrix} U_n^p$$

where U_n^p, ζ are the displacement amplitude of piles and its phase angle measured by the difference between the direction of excitations and that of responded motions.

Around the circumferemtial boundaries of piles, the welded contact condition between the displacements of piles and its surrounding soil layer with the non-deformability in respect of the circular cross-sections and the dynamic equilibrium of piles are required in the domain of wave numbers,

$$\begin{bmatrix} \tilde{u}+i\tilde{v} \\ \tilde{u}-i\tilde{v} \\ \tilde{w} \end{bmatrix}^s_{\nu n} = \begin{bmatrix} (U_n^p e^{i\zeta}-\tilde{u}_n^i)\delta_{\nu,-1} \\ (U_n^p e^{-i\zeta}-\tilde{u}_n^i)\delta_{\nu,1} \\ 0 \end{bmatrix}$$

$$(E_pI_pp_n^4-\pi a^2\rho_p\omega^2)U_n^p e^{i\zeta} -2\pi a\sum_\nu \delta_{\nu,-1}(\tilde{\tau}_{rr}+i\tilde{\tau}_{r\theta})^s_{\nu n}=\tilde{f}_n$$

$$(E_pI_pp_n^4-\pi a^2\rho_p\omega^2)U_n^p e^{-i\zeta}-2\pi a\sum_\nu \delta_{\nu,1}(\tilde{\tau}_{rr}-i\tilde{\tau}_{r\theta})^s_{\nu n}=\tilde{f}_n$$

$$: r=a \quad (14)$$

where E_pI_p, ρ_p are the bending stiffness and the density of piles and $\tilde{f}_n$ is the inhomogeneous term in relation to the excitations transformed in the z-direction.

By eliminating the displacement parameters U_n^p and ζ of piles in the above equations, the equations in respect of the unknown functions of the interacted potentials are given in the domain of wave numbers, the solutions of which are meaningful only on the parameters of wave number ν=1, -1 and it is not prevented from analyzing this soil-piles system to take the other parameters out of consideration.

In consequence, the mixed equations composed of the Fredholm type simultaneous integral equations determining the unknown coefficients of potentials are obtained in combination with the boundary equations on the lateral surfaces.

$$A_\nu(q)X_\nu(q) + \sum_{j=1}^{J}\sum_n B_\nu^j(q,p_n)Z_\nu^j(p_n) = 0$$

$$\int dq C_\nu^j(p_n,q)X_\nu(q) + \sum_{i=1}^{J} D_\nu^{ij}(p_n)Z_\nu^i(p_n) = g^j(p_n) \quad (15)$$

$$: j=1,2,\cdots J, \quad \nu=1,-1.$$

$$X_\nu(q) = [A_1\ A_2\ B_1\ B_2\ C_1\ C_2]^T_\nu$$

$$Z_\nu^j(p_n)= [E_1\ E_2\ F_1\ F_2\ G_1\ G_2]^T_{\nu j}$$

In the numerical analysis of the above series integral equations, any frequency response of this interaction system is expressed in terms of multiple summations and integrals as follows:

$$u(x) = u_0(x) + \sum_\nu \int dq U_\nu(q,x)X_\nu(q) \quad (16)$$

where x is the position vector of observation stations.

In this study, the following systems are able to be analyzed under consideration of the tip conditions of piles:

(1) one due to the shearing force Q_0 along a horizontal axis or the bending moment M_0 about a lateral axis concentrated at the head of piles,

(2) the other subjected to the lateral motion u_G uniformly distributed on the bedrock.

In order to constract the model of soil-piles-structure systems and to obtain the dynamic responses of the piles and their surrounding soil ground, the degrees of freedom at the head of each pile are to be at least two in translational and rotational directions.

For instance, in the case of the systems composed of pile groups and the surrounding soil medium due to both external loadings concentrated at the head of piles and uniformly distributed bedrock motions, the displacement responses of the horizontal and ratational motions, u_0 and θ_0 at the head of piles are expressed through the multiple compliance matrices associated with the lateral and rotational forces, and the displacement transfer vector under the bedrock motion as follows:

$$\begin{bmatrix} u_0 \\ \theta_0 \end{bmatrix} = \sum_{j=1}^{J} \begin{bmatrix} C^{HH} & C^{HR} \\ C^{RH} & C^{RR} \end{bmatrix}_j \begin{bmatrix} Q_0 \\ M_0 \end{bmatrix}_j + \begin{bmatrix} S^H \\ S^R \end{bmatrix} u_G \quad (17)$$

CLASSIFICATION OF SOIL-PILES SYSTEMS

As mentioned previously in the formulation, it is easy to analyze the dynamic responses of pile groups and their surrounding soil layer in the classification of the exciting condition and the boundary one at the ends of piles.

For the interaction systems with multiple excitations, the total field of the system is constracted on the multiple superposition of the auxiliary fields which are subjected to a part of excitations individually. And, by accompanyin the type of the boundary condition at the ends o piles, the components of the potentials presente in the expansion along the z-direction have different style of wave numbers p_n.

In order to examine the fundamental constitution of the soil-piles systems, one of the auxiliary problems is introduced in detail, which is composed of a soil stratum around only two piles with the fixed condition at their tips and subjected to the shearing forces concentrated at the heads of piles or uniformly distributed bedrock motions along the lateral direction.

The present system is naturally enlarged symmetric in respect of the plane surface z=H, and by associating with the presentation of the external loadings expanded in the z-direction with the discrete wave number $p_n=n\pi/H$, the forms of the interacted potentials are restricted as follows:

$$\begin{bmatrix} \varphi \\ \psi \\ \chi \end{bmatrix} = \sum_{\nu=-\infty}^{\infty} e^{i\nu\theta_1}\int_0^\infty dq q J_\nu(qr_1) \begin{bmatrix} A_1 sh\alpha z+A_2 ch\alpha z \\ B_1 sh\beta z+B_2 ch\beta z \\ C_1 sh\beta z+C_2 ch\beta z \end{bmatrix}_\nu$$

$$+\sum_{j=1}^{2}\sum_{\nu=-\infty}^{\infty} e^{i\nu\theta_j}\sum_{n=0}^{\infty}\begin{bmatrix}\cos(p_n z) & & \\ & \cos(p_n z) & \\ & & \sin(p_n z)\end{bmatrix}\begin{bmatrix}K_\nu(\alpha_n r_j)E\\ K_\nu(\beta_n r_j)F\\ K_\nu(\beta_n r_j)G\end{bmatrix}_{\nu j} W_a^\infty(r_j) \quad (18)$$

where in concerning with piles having the pinned condition at their tips and being subjected to lateral excitations at the heads of piles, the system is extended anti-symmetric about the bottom of the soil layer and the discrete wave number $p_n=(2n+1)\pi/2H$ is applied to the components of potentials. When subjected to the bending moment concentrated at the heads of piles, the potentials expanded in the z-direction adopt the counter form of the trigonometric functions to those due to the lateral excitation, to which the wave number along the axis of piles is applied in reverse to the procedure used in the case under the shearing forces, as shown in Table 1.
In consequence, the boundary condition standing on the horizontal surfaxes are divided into the following equations associated with the potentials expanded parallel to the lateral direction.

$$\begin{bmatrix}1 & \partial/\partial z\\ \partial/\partial z & \partial^2/\partial z^2+k^2\end{bmatrix}\begin{bmatrix}\Phi\\ X\end{bmatrix}_\nu=0,\quad \Psi_\nu=0 \quad :z=H$$

$$\begin{bmatrix}2\partial/\partial z & 2\partial^2/\partial z^2+k^2\\ 2\partial^2/\partial z^2+2h^2-k^2 & 2\partial/\partial z(\partial^2/\partial z^2+k^2)\end{bmatrix}\begin{bmatrix}\Phi\\ X\end{bmatrix}_\nu=0, \quad (19)$$

$$\partial/\partial z\Psi_\nu=0 \quad :z=0$$

$$\begin{bmatrix}\varphi\\ \psi\\ \chi\end{bmatrix} = \sum_\nu e^{i\nu\theta_1}\int_0^\infty dqqJ_\nu(qr_1)\begin{bmatrix}\Phi\\ \Psi\\ X\end{bmatrix}_\nu$$

The resulting equations in the domain of wave numbers are formed homogeneous in respect of the unknown coefficients of potentials and attended with the Rayleigh function which is related to the surface waves travelling along the horizontal direction.

$$\begin{bmatrix}ch\alpha H & \beta ch\beta H\\ 2q^2-k^2 & 2\beta q^2\end{bmatrix}\begin{bmatrix}A_2\\ C_1\end{bmatrix}_\nu+\begin{bmatrix}sh\alpha H & \beta sh\beta H\\ 0 & 0\end{bmatrix}\begin{bmatrix}A_1\\ C_2\end{bmatrix}_\nu$$

$$+\sum_{n=0}^{\infty}\begin{bmatrix}(-1)^n & p_n(-1)^n\\ -(2\alpha_n^2+k^2) & -2p_n\beta_n^2\end{bmatrix}\Bigg[\begin{bmatrix}\tilde{K}_\nu(\alpha_n)E_1\\ \tilde{K}_\nu(\beta_n)G_1\end{bmatrix}_\nu$$

$$+\sum_{m=-\infty}^{\infty}(-1)^m J_{\nu-m}(q\ell)e^{-i(\nu-m)\gamma}\begin{bmatrix}\tilde{K}_m(\alpha_n)E_2\\ \tilde{K}_m(\beta_n)G_2\end{bmatrix}_\nu\Bigg]=0 \quad (20\text{-}1)$$

$$\begin{bmatrix}\alpha sh\alpha H & q^2 sh\beta H\\ 0 & 0\end{bmatrix}\begin{bmatrix}A_2\\ C_1\end{bmatrix}_\nu+\begin{bmatrix}\alpha ch\alpha H & q^2 ch\beta H\\ 2\alpha & 2q^2-k^2\end{bmatrix}\begin{bmatrix}A_1\\ C_2\end{bmatrix}_\nu=0 \quad (20\text{-}2)$$

$$B_{2\nu}ch\beta H+\sum_{n=0}^{\infty}(-1)^n[\tilde{K}_\nu(\beta_n)F_1$$
$$+\sum_m(-1)^m J_{\nu-m}(q\ell)e^{-i(\nu-m)\gamma}\tilde{K}_m(\beta_n)F_2]_\nu=0 \quad (20\text{-}3)$$

$$B_{1\nu}=0 \quad (20\text{-}4)$$

Table 1.
Classification of the soil-piles systems by the exciting condition at the heads of piles and the boundary condition at their tips, in which the discrete wave number p_n along the z-axis is used variously.

at the tips	at the heads: shearing force	at the heads: bending moment
fixed	$p_n=n\pi/H$	$p_n=(2n+1)\pi/2H$
pinned	$p_n=(2n+1)\pi/2H$	$p_n=n\pi/H$

Rayleigh function:

$$R(q)=4\alpha\beta q^2(2q^2-k^2) -\alpha\beta[(2q^2-k^2)^2+4q^4]ch(\alpha H)ch(\beta H) +q^2[(2q^2-k^2)^2+4\alpha^2\beta^2]sh(\alpha H)sh(\beta H) \quad (21)$$

On the other hand, for the boundary condition around the circumferential surfaces of piles the interacted potentials are arranged in the expanded form around the surface of each pile, in which the following finite Fourier transform of the hyperbolic functions along the z-axis is used.

$$\tilde{S}_n^c(\alpha)=\frac{1}{2H}\int_0^{2H}dze^{-ip_n z}[sh(\alpha z)W_0^H(z)+sh\alpha(2H-z)W_H^{2H}(z)]$$
$$=\frac{1}{H}\int_0^H dz\cos(p_n z)sh(\alpha z)$$
$$\tilde{S}_n^s(\alpha)=\frac{1}{2H}\int_0^{2H}dze^{-ip_n z}[sh(\alpha z)W_0^H(z)-sh\alpha(2H-z)W_H^{2H}(z)]$$
$$=\frac{1}{H}\int_0^H dz\sin(p_n z)sh(\alpha z) \quad (22)$$

$p_n=n\pi/H$

And, in the case of the expansion parallel with the cylindrical polar coordinates (r_1, θ_1, z) they are presented as follows:

$$\begin{bmatrix}\varphi\\ \psi\\ \chi\end{bmatrix} = \sum_{\nu n}\sum e^{i\nu\theta_1}\begin{bmatrix}\cos(p_n z) & & \\ & \cos(p_n z) & \\ & & \sin(p_n z)\end{bmatrix}\begin{bmatrix}\Phi_n\\ \Psi_n\\ X_n\end{bmatrix}_\nu^{(1)} \quad (23)$$

$$\begin{bmatrix}\Phi_n\\ \Psi_n\\ X_n\end{bmatrix}_\nu^{(1)}=\varepsilon_n\int dqqJ_\nu(qr_1)\begin{bmatrix}A_1\tilde{S}^c(\alpha)+A_2\tilde{C}^c(\alpha)\\ B_1\tilde{S}^c(\beta)+B_2\tilde{C}^c(\beta)\\ C_1\tilde{S}^s(\beta)+C_2\tilde{C}^s(\beta)\end{bmatrix}_{\nu n}$$
$$+\begin{bmatrix}K_\nu(\alpha_n r_1)E_1\\ K_\nu(\beta_n r_1)F_1\\ K_\nu(\beta_n r_1)G_1\end{bmatrix}_\nu W_a^\infty(r_1)$$
$$+\sum_m(-1)^m e^{-i(\nu-m)\gamma}\begin{bmatrix}K_{\nu-m}(\alpha_n\ell)J_\nu(\alpha_n r_1)E_2\\ K_{\nu-m}(\beta_n\ell)J_\nu(\beta_n r_1)F_2\\ K_{\nu-m}(\beta_n\ell)J_\nu(\beta_n r_1)G_2\end{bmatrix}_\nu$$

Through the analysis in the contact condition between piles and their surrounding soil stratum and the moving condition of piles presented in Eq. (14), the inhomogeneous equations in respect of the coefficients of potentials are derived in the domain of wave numbers so as to include the integral operator accompanied by the Hankel transform along the r-axis.

When the dominant equations are constracted on the coefficients $X_\nu(q)$ of potentials expanded along the horizontal surface by eliminating the other ones among Eq. (15), the integral equations of the Fredholm type are obtained in which the number of the unknown functions is not increased whether the pile groups are crowded.

$$\tilde{A}(q)X(q) + \int_0^\infty dq'\tilde{B}(q, q')X(q') = \tilde{f}(q) \qquad (24)$$

$$X(q) = [X_1(q)\ X_{-1}(q)]^T$$

CONCLUDING REMARKS

In the present method concerning with the dynamic characteristics of soil-piles systems, the following remarks can be made:

(1) By carrying out the theoretical analysis on the interaction systems composed of pile groups and their surrounding soil layer on a rigid basis due to the excitations at the heads of piles and uniformly distributed bedrock motions, it is based only on the three-dimensional and linear wave propagation theory in the domain of frequency and any other assumption or approximation is not necessary.

2) In the integral equations dominating the unknown coefficients in the laterally expanded parts of the potentials, the number of the functions to be obtained is constant regardless of the number of piles. And, the equations have singular properties such as the integration along the infinite axis, the Rayleigh pole associated with the surface waves in the kernel components and the branch points accompanied by the phase velocities of dilatational and distortional waves.

In spite of the singular properties presented in the integral equations, there are not the poles and branch points on the real axis set in the complex plane because of the viscosity in in the soil medium. So that, the integrand evaluated along the real axes of wave numbers does not encounter the violent discontinuity.

3) About the support condition at the tips of piles, it may be possible to introduce in the case of the fixed or the pinned condition by arranging the type of the finite Fourier expansion of the interacted components along the z-direction and applying the technique of symmetric or anti-symmetric extension in respect of the plane bottom of a soil layer to the region of the soil-piles layered medium.

4) As to the multiple condition around the multiple circumferential surfaces of pile groups, it is followed by the superposition of the auxiliary problems which correspond individually to a sigle pile embedded in the surrounding soil ground and the multiple transformation of Bessel and modified Bessel functions in the cylindrical polar coordinate systems along the axes of piles by the Bessel's addition theorem.

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