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SYNOPSIS: A finite element algorithm is presented to analyze the dynamic response of rectangular foundations resting on an elastic medium. The foundation is modeled by rectangular thin plate elements and the supporting elastic medium is represented by the two parameter model proposed by Vlasov. The natural frequencies of the foundation-elastic medium are investigated. A parametric study is conducted to examine the effects of the edge and corner forces, that are accounted for in the two parameter model, on the dynamic response of a plate subjected to a moving vertical force.

INTRODUCTION

The vibration of rectangular foundations resting on an elastic medium is important to several areas of geotechnical engineering such as vibrating machine foundations and highway and airport pavements. The design of machine foundations and pavements requires an accurate prediction of the dynamic response in order to obtain a safe and reliable structure. Several analytical and numerical methods for vibration analysis are available in the literature to analyze dynamic response of foundations. In most existing methods, the foundation is idealized as beams or plates resting on an elastic halfspace, representing the supporting soil medium. Although the elastic halfspace idealization is more appropriate, the Winkler model is widely used to represent the soil medium due to the complexities involved in the dynamic analysis.

The vibration of beams resting on a Winkler foundation has been studied by several researchers in the past (Timoshenko et. al.(1974), Gorman(1975)). Doyle and Pavlovic(1982) and Eisenberger et. al. (1985) studied the vibration of beams fully or partially supported on elastic foundations for different end conditions. The dynamic analysis of beams and plates resting on an elastic foundation to moving force has received significant attention in recent years due to its importance in the reliable pavement design. The literature available in this area is mostly related to the analytical solutions applicable to infinitely long beams (Kenny (1954), Achenbach and Sun(1965)) or infinitely long plates (Thompson (1963)) resting on an elastic foundation. Some recent studies (e.g. Taheri and Ting (1990)) have considered the dynamic analysis of discontinuous plates to moving force based on the finite element procedure.

In the aforementioned studies, the soil medium was idealized by Winkler type foundation model which assumes that the beam or the plate is supported by a series of isolated vertical springs. Such an idealization does not have any mechanism to provide shear interaction between individual spring elements. This shortcoming can be removed by idealizing the soil medium by a two parameter model. A number of two parameter models are available in the literature, among which the models proposed by Filonenko-Borodich (1940), Hetenyi(1946), Pasternak (1954) and Vlasov (1966) are particularly notable. In these models, except Vlasov model, the shear interaction between the individual spring elements is provided by introducing either elastic membranes, elastic plates or elastic layers capable of undergoing purely shear deformation. Vlasov model (1966), on the other hand, achieves the shear interaction by imposing restrictions on the distribution of displacements and stresses in the elastic halfspace.

In the present study, an analysis procedure based on the finite element technique is developed for the dynamic analysis of rectangular foundations resting on an isotropic elastic medium. The foundation plate is modeled by thin rectangular plate elements. The supporting soil medium is modeled by the two parameter model proposed by Vlasov. The natural frequencies of the plate are obtained by solving the resulting eigen value problem. The dynamic response of a plate to moving force is obtained for different material properties of the plate-foundation system.

ALGORITHM FOR DYNAMIC ANALYSIS OF A RECTANGULAR FOUNDATION

Governing Equations

The governing differential equation for the bending of a rectangular foundation plate resting on a two parameter elastic medium can be expressed as (Vlasov (1966))

$$D\nabla^4 w(x,y) - 2t\nabla^2 w(x,y) + kw(x,y) = p(x,y) \qquad (1)$$

where D is the plate rigidity, w is the vertical plate deflection, t and k are the parameters describing the two parameter model and p(x,y) is the dynamic force acting on the plate and ∇^2 denotes the Laplace's operator in the rectangular cartesian coordinate system (x,y).

For a concentrated vertical force of magnitude F moving on the plate, the dynamic force p(x,y) can be expressed as (Taheri and Ting(1990))

$$p(x,y) = F\delta(x-\xi,y-\eta) + (mg-m\ddot{w}-c_s\dot{w}) \quad (2)$$

where δ is the dirac-delta function, (ξ,η) is the position of the moving force, mg is the plate weight and c_s is the damping ratio of the supporting elastic medium. In case of undamped free vibration of the plate, all terms on the right hand side of Eq. (2) are dropped out, except the inertia force $(-m\ddot{w})$.

The Vlasov foundation model parameters k and t are given by (Vlasov (1966)

$$k=\frac{E_o\gamma}{2m(1-\nu_o^2)}\cdot\frac{\sinh(\gamma H/m)\cosh(\gamma H/m)+\gamma H/m}{\sinh^2(\gamma H/m)}, \quad (3a)$$

$$t=\frac{E_o m}{8\gamma(1+\nu_o)}\cdot\frac{\sinh(\gamma H/m)\cosh(\gamma H/m)-\gamma H/m}{\sinh^2(\gamma H/m)}, \quad (3b)$$

$$E_o=E_s/(1-\nu_s^2) \quad \text{and} \quad (3c)$$

$$\nu_o=\nu_s/(1-\nu_s) \quad (3d)$$

In the above expressions, E_s and ν_s are the Young's modulus and the Poisson's ratio of the elastic foundation, respectively, H is the foundation thickness, and m is the short half length of the plate. The constant γ determines the rate of decrease of displacements with depth.

Boundary Conditions

The solution to the moving force problem can be obtained by solving the governing differential equation given in Eq. (1) by applying appropriate boundary conditions. For an unrestrained plate resting on a Winkler medium, the boundary conditions are identical to that of Krichoff boundary conditions for an unsupported plate. However, for the case of two parameter medium, it is necessary to apply distributed edge and the concentrated corner forces along the plate boundary to account for the deformation of the supporting elastic medium

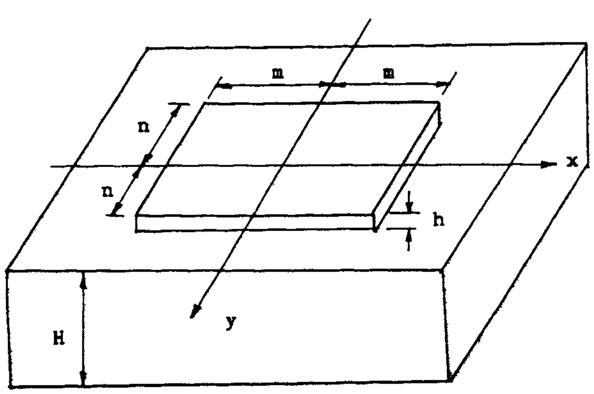

Fig. 1 A Rectangular Plate Resting on a Two Parameter Elastic Medium

beyond the plate edges. Using the virtual work principle, the distributed edge forces Q_n and Q_m acting along the edges $x=\pm m$ and $y=\pm n$ respectively, can be obtained as (Vlasov and Leontiev (1966))

$$Q_n=2t\left[\alpha w_n+\left[\frac{\partial w}{\partial x}\right]_n-\frac{1}{2\alpha}\left[\frac{\partial^2 w}{\partial^2 y}\right]_n\right] \quad (4a)$$

$$Q_m=2t\left[\alpha w_m+\left[\frac{\partial w}{\partial y}\right]_m-\frac{1}{2\alpha}\left[\frac{\partial^2 w}{\partial^2 y}\right]_m\right] \quad (4b)$$

where w_n and w_m are the deflections along $x=\pm m$ and $y=\pm n$, respectively, and $\alpha=\sqrt{k/2t}$. The corner reaction R at each corner is given by

$$R=\frac{3}{2}tw_c \quad (5)$$

where w_c is the corner deflection.

Finite Element Formulation

The four noded rectangular non-conforming thin plate elements are used here to discretize the plate. Following the necessary steps of the finite element formulation, the governing differential equation (Eq.(1)) for the plate-two parameter elastic medium can be expressed in the form

$$[k]\{e\} + [M]\frac{d^2}{dt^2}\{e\} = 0 \quad (6)$$

where

$$[k] = \sum\{[k_o]-[k_1]+[k_2]\} \quad (7)$$

$$[M] = \sum [m] \quad (8)$$

In the above expressions, $\{e\}$ is the system displacement vector, $[k_o]$ is the plate stiffness matrix, $[k_1]$ and $[k_2]$ are the stiffness matricess of the elastic medium corresponding to two parameters t and k, respectively, and [M] and [m] are the system and element mass matrices, respectively. The summation sign $\sum$ refers the assembly of the individual element matrices. If a periodic response $\{e\}=\{e\}\sin\omega t$ is assumed, Eq. (6) can be transformed to the form

$$\{[k] - \omega^2[M]\}\{e_o\} = 0 \quad (9)$$

The natural frequencies and mode shapes of the plate-two parameter elastic medium system can be obtained by finding the eigen values and eigen vectors of the matrix $\{[k] - \omega^2 [M]\}$.

Solution to the Moving Force Problem

In view of Eq. (2) and Eq. (6), the equation of equilibrium for the total system can be expressed as

$$\left\{[k]+\left[\frac{1}{h^2\beta}v^2+\frac{\alpha}{h\beta}a\right][M]+\frac{\alpha}{h\beta}v[c]\right\}\{e\}_j=$$

$$v^2[M]\left[\frac{1}{h^2\beta}\{e\}+\frac{1}{h\beta}\{\dot{e}\}+(\frac{1}{2\beta}-1)\{\ddot{e}\}\right]_{j-1}$$

$$+(a[M]+v[c])\left[\frac{\alpha}{h\beta}\{e\}+(\frac{\alpha}{\beta}-1)\{\dot{e}\}+\frac{h}{2}(\frac{\alpha}{\beta}-2)\{\ddot{e}\}\right]_{j-1}$$

$$+\{W\}+[\tilde{N}(\xi,\eta)]^TF \qquad (10)$$

where [c] is the damping matrix, {W} is the plate weight vector, $[\tilde{N}(\xi,\eta)]$ is the interpolation matrix evaluated for a specific element where the moving force is located, v and a are the velocity and acceleration of the moving force, respectively, α and β are constants associated with the Newmark-Beta integration scheme, h is the increment in the position of moving force and j refers the time step. Note that the concept of pseudo time variable (Taheri and Ting (1990)) and the the Newmark-Beta method are utilized to obtain Eq. (10). Eq. (10) is solved using a stepwise procedure to obtain the solution to the moving force problem. It may be noted that the direct solution to Eq. (10) is possible only for the Winkler idealization of the elastic soil medium. For two parameter idealization, the edge and corner forces have to be determined and included before solving Eq. (10). However, those forces cannot be directly determined at time step j since the edge and the corner forces are dependent upon the deflection of the plate. On the other hand, the plate deflections cannot be determined until the edge and the corner forces are known. To overcome this difficulty, an iterative scheme proposed by Yang (1972) is used here. Further details of the finite element algorithm are given by Zaman et. al.(1990).

NUMERICAL RESULTS AND DISCUSSIONS

The accuracy of the finite element algorithm developed in this study is verified by comparing the results with the numerical solution given by Yang (1972) for a static problem of a rectangular plate resting on a two parameter foundation and subjected to a vertical concentrated load. The displacements as well as the edge and the corner forces associated with the two parameter model show an excellent agreement when compared with the results reported by Yang (1972). Due to space limitation, the details of the comparison is not given here.

Free vibration of a Rectangular Plate

The natural frequencies of a rectangular plate for two different cases are obtained by solving the eigen value problem and the results are compared with the exact values given by Gorman (1982). In the first case, a completely free rectangular plate is considered. In the second case, a rectangular plate resting on an elastic foundation is analyzed. In this case, the elastic foundation is modeled by Winkler springs. An aluminum plate of 0.125 in. thickness and edge dimensions of 2n = 16 in. and 2m = 8 in. are assumed. The modulus of elasticity and the Poisson's ratio of the plate material are taken as 10.0×10^6 psi and 0.33, respectively. The density of the plate and the modulus of subgrade reaction of the elastic medium are considered as 0.0975 pci and 40 pci, respectively. In the finite element analysis, the plate is discretized into 18, 32 and 60 elements and the corresponding results for the first four frequencies are tabulated in Tables 1 and 2. In these tables, the first and the second mode of vibrations correspond to fully symmetric and fully antisymmetric vibrations, respectively. The third and the fourth are the symmetric-antisymmetric modes of vibration.

TABLE 1. Natural frequencies of a completely free plate (rad/sec.)

Mode	Exact	Finite Element solution		
		18 ele.	32 ele.	60 ele.
1	632	637	633	633
2	772	776	774	774
3	1705	1714	1711	1709
4	1756	1790	1772	1766

TABLE 2. Natural frequencies of a plate resting on an elastic foundation (rad/sec.)

Mode	Exact	Finite Element solution		
		18 ele.	32 ele.	60 ele.
1	1290	1294	1293	1293
2	1364	1369	1368	1368
3	2043	2050	2048	2047
4	2086	2110	2099	2094

It is observed from Tables 1 and 2 that the finite element results agree well with the exact solution. Further, the accuracy of results increases with the refinement of the finite element mesh. It is found that the finite element discretization with 32 elements is adequate to obtain reasonable results. The natural frequencies of a plate resting on a two parameter foundation cannot be directly determined by solving the eigen value problem given in Eq. (9). This is because the stiffness matrix [k] does not include the stiffness of the soil medium lying outside the plate area. This problem does not arise in Winkler model since it does not account for the soil deformation outside the plate area. Due to this difficulty, the natural frequencies obtained from the present study show substantial difference in comparison with the exact solution given by Vlasov and Leontev (1966). However, the approximate iterative method involving incorporation of the edge and the corner forces while solving the governing differential equation can be employed effectively in solving the forced vibration problems. This is because, unlike free vibrations, the computed displacements can be corrected to account for the effect of forces by the iterative technique.

Static and Moving Force Solution

A parametric study is conducted to determine the effects of various parameters on the dynamic response of a rectangular plate resting on a two parameter elastic medium and subjected to a static and a moving force. The results are presented for three different foundation idealizations: (i) Winkler idealization, (ii) two parameter idealization without the edge and corner forces, and (iii) two parameter idealization with the edge and the corner forces. The following geometric and material properties are assumed for the plate and the elastic medium.

Length (2n) = 300 in., Width (2m) = 150 in.,
plate thickness = 12 in.,
mass density of the plate material = 0.0002174 $lb.s^2/in^4$,
$E = 3.6\times10^6$ psi ; $\nu = 0.15$
Poisson's ratio of the elastic medium = 0.3,
thickness of the elastic medium = ∞
γ = the parameter determining the rate of decrease of displacement within the soil layer = 1.5, and foundation damping = 5.0 %

The nondimensional static displacements ($\overline{w}=\dfrac{wE_0m}{P(1-\nu_0^2)}$) along the center line of the plate are presented in Figs. 2,3 and 4 for different flexibility index ($r=\dfrac{\pi E_0 n^2 m}{D(1-\nu_0^2)}$) values of the plate. Only half of the plate response is plotted due to its symmetry. Fig. 2 presents the solution for r = 20. It is observed that the central deflection for the Winkler model is larger than the other two foundation models considered. The results obtained for the Winkler model and the two parameter model without edge and corner reactions are close to each other. When the edge and the corner forces are included in the analysis, the two parameter elastic medium reduces the deflection significantly. For the problem considered here, a maximum reduction of 19% is observed at the center of the plate. When the plate flexibility index is increased to 50 and 100, the percentage of reduction in the central deflection reduces to 18 and 12 % , respectively (Figs. 3 and 4).

Figures 5 through 7 present the dynamic response of the plate to a moving force. The solutions are obtained for the situation when the moving force is located at the center of the plate and traveling from the left transverse edge to the right along the center line. The velocity ratio (v/v_{cr}) is taken as 0.5 where the critical velocity v_{cr} is defined by

$$v_{cr}=\left[\frac{4kD}{(\rho h)^2}\right]^{1/4} \tag{11}$$

Fig. 5 illustrates the moving force solution for r = 20. The deflections obtained with the Winkler model and the two parameter model including the edge and corner forces show maximum difference in the central region

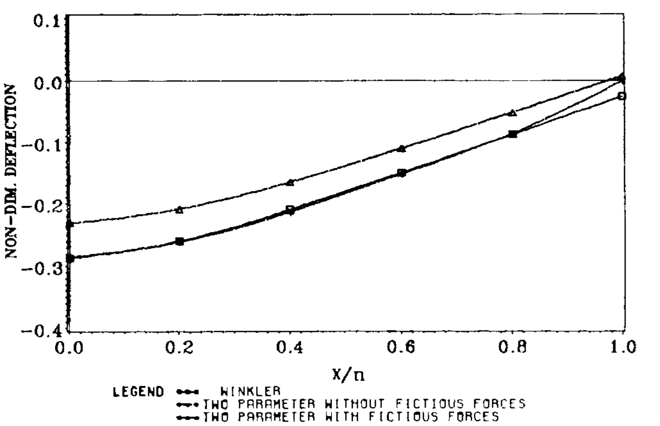

Fig. 2 Static Deflection of a Rectangular Plate Resting on a Two Parameter Elastic Medium (r = 20)

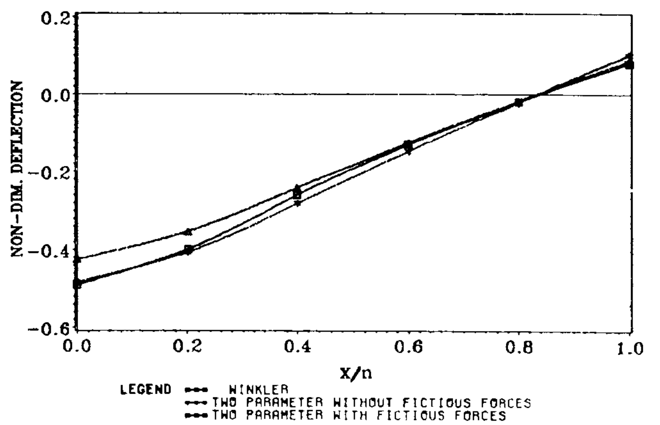

Fig. 4 Static Deflection of a Rectangular Plate Resting on a Two Parameter Elastic Medium (r = 100)

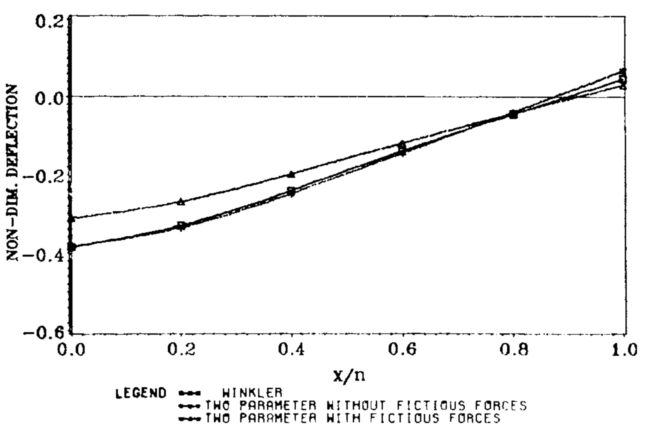

Fig. 3 Static Deflection of a Rectangular Plate Resting on a Two Parameter Elastic Medium (r = 50)

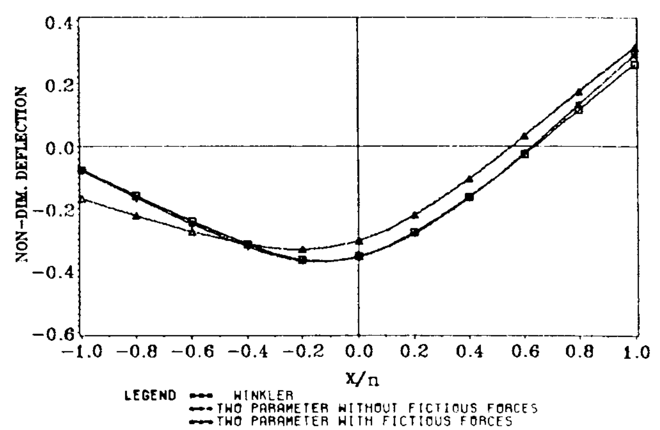

Fig. 5 Moving Force Solution of a Rectangular Plate Resting on a Two Parameter Elastic Medium (r = 20)

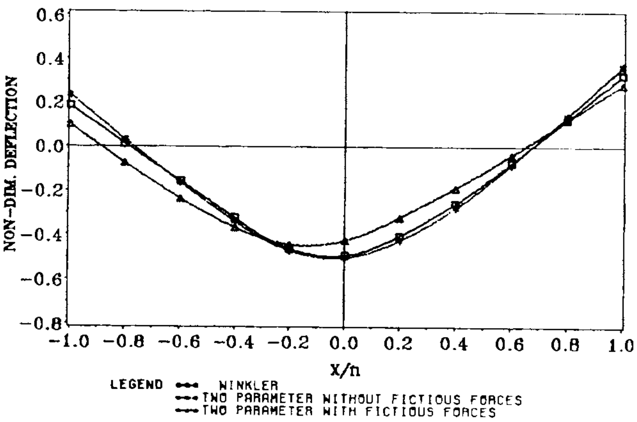

Fig. 6 Moving Force Solution of a Rectangular Plate Resting on a Two Parameter Elastic Medium (r = 50, v/v_{cr} = 0.5)

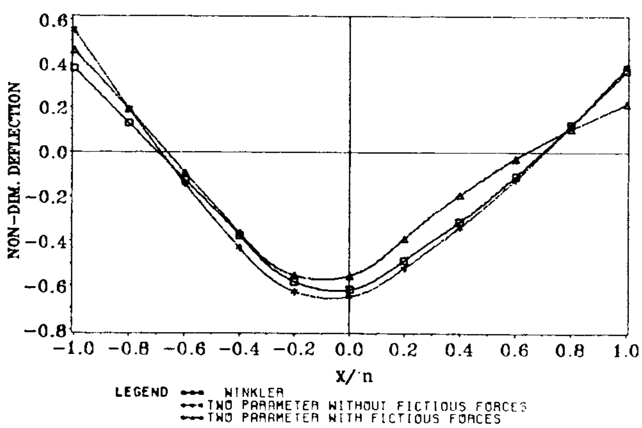

Fig. 7 Moving Force Solution of a Rectangular Plate Resting on a Two Parameter Elastic Medium (r = 100, v/v_{cr} = 0.5)

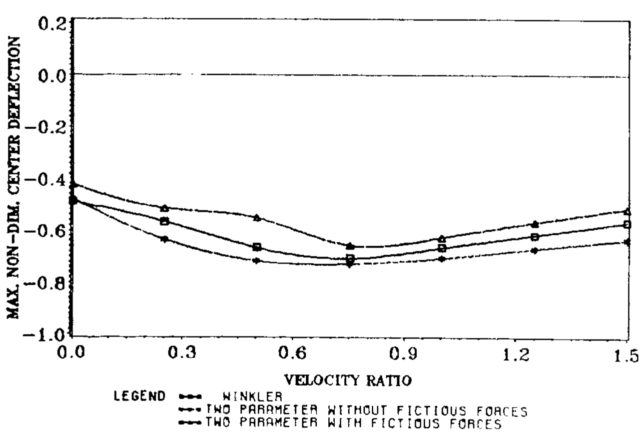

Fig. 8 Effect of Velocity Ratio on the Maximum Central Plate Deflection (r=100)

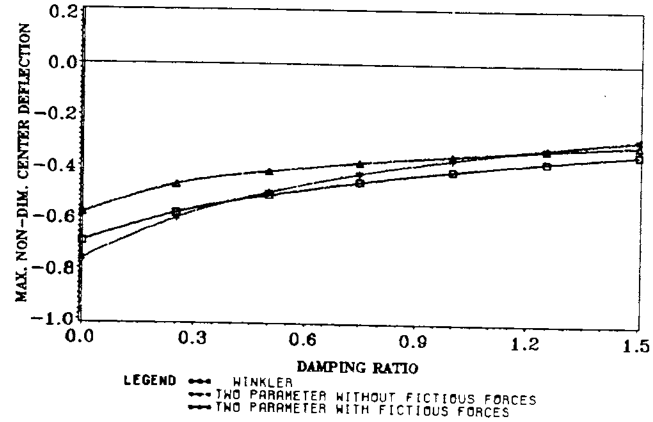

Fig. 9 Effect of Damping Ratio on the Maximum Central Plate Deflection (r=100, v/v_{cr} = 0.5)

of the plate. When the edge and the corner forces are disregarded, the Winkler and the two parameter idealizations yield very close results, particularly in the central plate region. It can also be noted that the transverse plate edge ahead of the moving force tends to uplift, but the other edge does not exhibit this behavior. However, when the value of r increases to 50 and 100 (Figs. 6 and 7), both transverse edges show uplift characteristics due to increasing soil stiffness. In all the cases, the application of edge and corner forces to two parameter model gives the least central deflection, as expected.

The effect of velocity ratio and damping of the elastic medium on the maximum center plate deflection is depicted in Figs. 8 and 9, respectively. A critical velocity ratio of 0.75 is observed from Fig. 8. As expected, the increase in foundation damping decreases the magnitude of the plate response for all three cases considered.

CONCLUDING REMARKS

A finite element algorithm is presented to analyze the vibration of rectangular plates resting on an elastic medium. The plate is modeled by rectangular thin plate elements and the elastic foundation is represented by a two parameter model which accounts for the shear interaction present in the soil medium. The natural frequencies are determined for a completely free plate and a plate resting on an elastic foundation and compared with the exact solutions. The moving force solutions presented for different foundation idealizations show the importance of edge and corner forces that occur due to the deformation of the soil medium beyond the plate edges. It is observed that the influence of these forces reduces with the increasing stiffness of the supporting elastic medium.

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