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Dynamic Response of Flexible Foundations on Multilayered Medium

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SYNOPSIS: The dynamic response of flexible strip-foundations placed on a viscoelastic multi-layered medium under conditions of plane strian and subjected to harmonic excitation is studied in this paper. A semi-analytic model is developed to determine the dynamic flexibility matrix of the medium, and the spline finite element method is employed to treat the foundation. The two methods are appropriately combined through equilibrium and compatibility considerations at the soil-foundation interface. Numerical results describing the influence of the non-homogeneity of the soil medium on the compliance are presented, and the effects of the relative stiffness K_r on the foundation response are brifly discussed.

INTRODUCTION

The dynamic response of foundations placed on an elastic half-space and subjected to harmonic excitation has received considerable attention in recent years. In the great majority of previous investigations, the assumption is made that the medium is elastic, isotropic and homogeneous, and the analytical solutions are based on the classic work by Lamb [1] who formulated and solved the problem of harmonically varying point force acting on the surface of an elastic half-space. Typically, the force-displacement relationship for the foundation is obtained by assuming that the foundation can be represented as a rigid body.

While the analysis of the static response of a flexible foundation on an elastic multi-layered medium has drawn the attention of many researchers, the analysis of the corresponding dynamic problem has not been as extentive. Non-homogeneity of the medium is one of the factors which further complicate the modelling process. Whereas rigorous mathematical treatment appears impossible at the present time. Although the finite element method has been used extensively because of its ability to treat the complex properties of the soil, there is a drawback in the FEM analysis that the semi-infinite medium is normally represented by a finite-size modal the boundaries of which trap energy radiating away from the foundation. Also, the discretization causes a filtering effect on the waves in the higher frequency range. These problems may be somewhat mitigated by placing the boundaries far from the foundation and by keeping the size of the elements sufficiently small, but such action leads to a system with a relative large number of degree of freedom causing a severe problem on computer time and storage.

This paper studies the dynamic response of the flexible strip-foundations resting on a viscoelastic multi-layered medium. A sime-analynic model is developed which is applicable to determine the dynamic flexibility matrix of the medium, where the displacement functions are expressed by the cubic splines and Fourier's series, and the flexibility coefficients can be evaluated by solving Laprange's equation. The foundation and the soil-foundation interface are discretized into a number of spline elements, and the assumption is made that the contact stresses are uniformly distributed within each subregion. Then, the dynamic stiffness matrix of the ground is obtained by inversion of the dynamic flexibility matrix. Combining the stiffness matrices of the foundation and the ground leads to a set of complex liner equations for the soil-foundation system in terms of the spline nodal parameters. Once the nodal displacements are obtained, the contact stresses for each subregion can be easily determined. In the present analysis, only out-of-plane deformation of the foundation is considered and the shearing contact stresses are neglected. Also, although slipage between the foundation and the soil is allowed, it is assumed that the foundation remains in contact with the ground. The analytical procedure is applicable to both the multi-layered soil deposits and the mediums continuously varying elastic modulus with depth which are approximately regarded as consisting of a number of distinct layers.

The main advantage of the present approach is that the two-dimensional problem is simplified to a one-dimensional discretization. Due to the orthogonal properties of Fourier's series, the computational effort and storage requirement have been greatly saved, thus the boundaries of the finite-size model may be placed sufficiently far from the foundation, and the interaction analysis can be carried out on a micro-comnputer. Excellent accuracy is shown in numerical examples.

FUNDAMENTALS AND GOVERNING EQUATION

A flexible massless strip- footing supported on a viscoelastic multi-layered medium and subjected to harmonic vertical load is shown in Fig. 1. The dynamic response of the foundation will be analysed. Since the steady-state vibration is considered here, the time factor $e^{i\omega t}$ will be omitted in what follows.

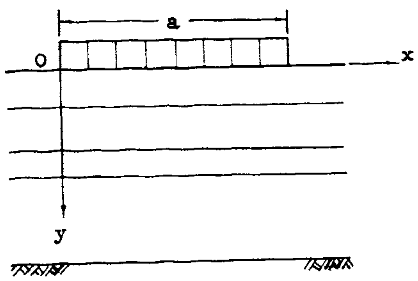

Fig.1. Decription of the Soil-Foundation System

Denoting the vertical displacement of the foundation by $w(x)$, the normal contact stress distributed at the soil-foundation interface by $p(x)$, and the vertical excitation on the foundation by $q(x)$, the governing equation of the plate strip-foundation can be expressed as

$$\int_0^a [D\frac{d^2w(x)}{dx^2}\frac{d^2w(x)}{dx^2} - q(x)w(x) + \frac{1}{2}p(x)w(x)]dx = 0 \qquad (1)$$

where $D=E_f t_f/12(1-u_f)$ represents the flexural rigidity of the plate characterized by Yong's modulus E_f, Poisson's ratio u_f and thickness t_f

By dividing the strip-foundation and the soil-foundation interface into n elements of equal length h, the vertical displacement of the foundation is expressed as the cubic spline function

$$w(x) = \sum_{i=-1}^{n+1} r_i\phi_i(x) = [\phi(x)]\{r\} \qquad (2)$$

where

$$\phi_i(x) = \psi_3(\frac{x}{h} - i)$$

ψ_3 represents the cubic B spline given in explicit form in reference [4].

The last term of equation (1) can be approximately calculated from

$$\int_0^a p(x)w(x)dx = \sum_{i=0}^{n} w(x_i)R_i = \{W\}^T\{R\} \qquad (3)$$

in which $w(x_i)$ is the nodal displacement of the foundation and R_i is the resultant force of the contact stress on the corresponding subregion.

By assuming uniform contact stress within each subregion, $w(x_i)$ and R_i can be related through

$$\sum_{j=0}^{n} f_{ij} R_j = w_i \quad (i=0,1,\dots\dots,n) \qquad (4)$$

where f_{ij} represents the flexibility coefficient of the ground, which relates the normal displacement at point i on the surface to a normal harmonic force of unit amplitude applied at point j. The points of interest are those which are considered to be coincident with the node points of the foundation. The unit load is considered to be uniformly distributed on the surface of the medium over a space equal to that of the individual foundation elements. Chopra [2] and Lou Menglin [3] obtained the flexibility coefficients of homogeneous viscoelastic medium under the conditions of plane strain. However, the solution of layered medium appears to be rather poor. A semi-analytic approach to the problem is developed in this paper and it will be described in next section.

Using matrix notations, equation (4) may be expressed in the form

$$[F]\{R\} = \{W\} \qquad (5)$$

and the nodal force vector {R} can be derived from

$$\{R\} = [F]^{-1}\{W\} \qquad (6)$$

where [F] represents the flexibility matrix of the medium, which is symmetric of order $(n+1)\times(n+1)$, complex and frequency-dependent.

Equation (1) can be discretized by the procedure described. Substitution from equations (2), (3) and (6) into equation (1) leads to a set of complex algebraic equations for the soil-foundation system:

$$([K]_f + [K]_m)\{r\} = \{q\} \qquad (7)$$

in which [K] is the stiffness matrix of the foundation, and takes the form

$$[K]_f = \int_0^a [\phi'']^T[\phi'']dx = D[Ax] \qquad (8)$$

$[K]_m$ represents the dynamic stiffness matrix of the ground given by

$$[K]_m = [\phi(x_i)]^T [F]^{-1}[\phi(x_i)] \qquad (9)$$

where the matrix $[\phi(x_i)]$ is of order $(n+1)\times(n+3)$. The load vector of external forces {q} can be expressed in the form

$$\{q\} = \int_0^a [\phi]^T q\, dx \qquad (10)$$

Once the vector {r} is obtained from equation (7), the displacements and the contact stressess can be easily evaluated from equations (1) and (6), respectively.

SEMI-ANALYTIC MODEL FOR MULTI-LAYERED MEDIUM

Lou [3] presented in 1984 that the homogeneous viscoelastic half-plane may be represented by a finite-size dodel with boundaries placed sufficiently far from the srtuctures. This conclusion is employed to treat the semi-infinite multi-layered medium. The model considered in this paper with finite length L and depth H is shown in Fig. 2.

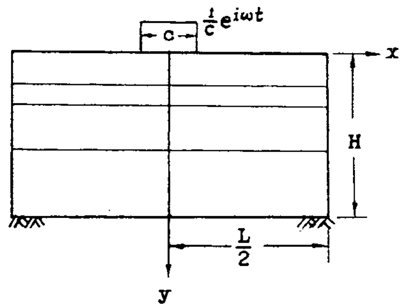

Fig. 2. Description of Model

The dynamic flexibility coefficients of the viscoelastic material idealized as a constant hysteretic solid, according to the complex-damping principle, can be determined from

$$[-\omega^2[M]+(1+i\eta)[K]]\{r\} = \{P\} \tag{11}$$

where [K] and [M] represent the stiffness and mass matrices, respectively. ω is the excitation frequency, and η denotes the energy loss coefficient.

Dividing the soil medium into N horizontal strips, the displacement functions can be given as a product of the cubic splines and Fourier's series in the form of

$$u(x,y)=\sum_{m=1}^{M}[\phi]X_m\{a\}_m \tag{12}$$

$$v(x,y)=\sum_{m=1}^{M}[\phi]Y_m\{b\}_m \tag{13}$$

where

$$[\phi]=[\phi_{-1}\ \ \phi_0\ \ \phi_1\ \ldots\ldots\ \phi_{N-1}]$$

$$\phi_i=\psi_3\left(\frac{y}{h}-i\right),\ i=-1,\ 0,\ 1,\ \ldots\ldots\ N-2$$

$$\phi_{N-1}=\psi_3\left(\frac{y}{h}-N+1\right)-\frac{1}{2}\psi_3\left(\frac{y}{h}-N\right)+\psi_3\left(\frac{y}{h}-N-1\right)$$

X_m and Y_m represent Fourier's series,

$$X_m=\cos\frac{m\pi\ (x+L/2)}{L}, \quad Y_m=\sin\frac{m\pi\ (x+L/2)}{L}$$

Equation (12) and (13) can be expressed in matrix form

$$\{\delta\}=[u\ \ v]^T=\sum_{m=1}^{M}[N]_m\{r\}_m=[N]\{r\} \tag{14}$$

where

$$[N]_m=\begin{bmatrix}[\phi]X_m & 0\\ 0 & [\phi]Y_m\end{bmatrix} \tag{15}$$

$$\{r\}_m=[\ \{a\}^T\ \ \{b\}^T\ ]^T \tag{16}$$

The strain components are given in the form

$$\{\varepsilon\}=[\varepsilon_x\ \varepsilon_y\ \varepsilon_{xy}]^T=\sum_{m=1}^{M}[B]_m\{r\}_m=[B]\{r\} \tag{17}$$

where

$$[B]_m=\begin{bmatrix}[\phi]X'_m & 0\\ 0 & [\phi']Y_m\\ [\phi']X_m & [\phi]Y_m\end{bmatrix} \tag{18}$$

Similar to the procedures of the standard finite element method, the stiffness matrix [K] and mass matrix [M] can be obtained from

$$[K]=\sum_{k=1}^{N}\int_{S_k}[B]^T\ [D_k]\ [B]\ dxdy$$

$$=\begin{bmatrix}[K]_{11} & [K]_{12} & \ldots & \ldots & \ldots & [K]_{1M}\\ [K]_{21} & [K]_{22} & \ldots & \ldots & \ldots & [K]_{2M}\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ [K]_{M1} & [K]_{M2} & \ldots & \ldots & \ldots & [K]_{MM}\end{bmatrix} \tag{19}$$

where

$$[K]_{mn}=\sum_{k=1}^{N}\int_{S_k}[B]_m^T\ [D_k]\ [B]_n\ dxdy \tag{20}$$

and

$$[M]=\sum_{k=1}^{N}\int_{S_k}\rho_k\ [N]^T\ [N]\ dxdy$$

$$=\begin{bmatrix}[M]_{11} & [M]_{12} & \ldots & \ldots & \ldots & [M]_{1M}\\ [M]_{21} & [M]_{22} & \ldots & \ldots & \ldots & [M]_{2M}\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ [M]_{M1} & [M]_{M2} & \ldots & \ldots & \ldots & [M]_{MM}\end{bmatrix} \tag{21}$$

where

$$[M]_{mn}=\sum_{k=1}^{N}\int_{S_k}\rho_k[N]_m^T\ [N]_n\ dxdy \tag{22}$$

$$[D_k]=\frac{2(1-\mu_k)G_k}{1-2\mu_k}\begin{bmatrix}1 & \frac{\mu_k}{1-\mu_k} & 0\\ \frac{\mu_k}{1-\mu_k} & 1 & 0\\ 0 & 0 & \frac{1-2\mu_k}{2(1-\mu_k)}\end{bmatrix} \tag{23}$$

S_k donates the area of kth strip of the soil characterized by the shear modulus G_k, the density ρ_k and Possion's ratio μ_k.

Because of the orthogonalities of Fourier's series, all the terms do not coupled together, and each term can be analysed separately. therefore equation (11) may be expressed in the form

$$(-\omega^2[M]_m + (1+i\eta)[K]_m)\{r\}_m = \{p\}_m \qquad m=1,\ldots,M \qquad (24)$$

where

$$[K]_m = \sum_{k=1}^{N} \int_{S_k} [B]_m^T [D_k][B]_m \, dxdy = \begin{bmatrix} [K]uu & [K]uv \\ [K]vu & [K]vv \end{bmatrix} \qquad (25)$$

$$[M]_m = \sum_{k=1}^{N} \int_{S_k} \rho_k [N]_m^T [N]_m \, dxdy$$

$$\begin{bmatrix} [M]uu & 0 \\ 0 & [M]vv \end{bmatrix} \qquad (26)$$

in which

$$[K]uu = \sum_{k=1}^{N} \frac{2(1-\mu_k)G_k}{1-2\mu_k}\left[C_x[F_y] + \frac{1-2\mu_k}{2(1-\mu_k)} F_x[C_y]\right]$$

$$[K]uv = \sum_{k=1}^{N} \frac{2(1-\mu_k)G_k}{1-2\mu_k}\left[\frac{\mu_k}{1-\mu_k} H_{yx}[H_y] + \frac{1-2\mu_k}{2(1-\mu_k)} H_{xy}[H_y]^T\right]$$

$$[K]vu = [K]_{uv}^T$$

$$[K]vv = \sum_{k=1}^{N} \frac{2(1-\mu_k)G_k}{1-2\mu_k}\left[[F_y][C_y] + \frac{1-2\mu_k}{2(1-\mu_k)} C_y[F_y]\right]$$

$$[M]uu = \sum_{k=1}^{N} \rho_k F_x [F_y]$$

$$[M]vv = \sum_{k=1}^{N} \rho_k F_y [F_y]$$

Cx,Cy,Fx,Fy,Hxy and Hyx are following constants

$$C_x = \int_{-L/2}^{L/2} X'_m X'_m \, dx \qquad C_y = \int_{-L/2}^{L/2} Y'_m Y'_m \, dx$$

$$F_x = \int_{-L/2}^{L/2} X_m X_m \, dx \qquad F_y = \int_{-L/2}^{L/2} Y_m Y_m \, dx$$

$$H_{xy} = \int_{-L/2}^{L/2} X_m Y'_m \, dx \qquad H_{yx} = \int_{-L/2}^{L/2} X'_m Y_m \, dx$$

and [Cy], [Fy] and [Hy] are matrices given by

$$[C_y] = \int_{h_k} [\phi']^T [\phi'] dy$$

$$[F_y] = \int_{h_k} [\phi]^T [\phi] dy$$

$$[H_y] = \int_{h_k} [\phi]^T [\phi'] dy$$

Once the generalized coordiante vectors $\{r\}_m$ ($m=1, \ldots\ldots, M$) are calculated, the compliance coefficients can be easily determined from equation (14).

NUMERICAL RESULTS

1. Dynamic Flexibility Coefficients

Analytical expressions and numerical results have been presented in reference [2] for the dynamic flexibility influence coefficients for a homogeneous viscoelastic half-plane. as an extension, the solutions for a semi-infinite medium with a finite depth were obtained by Lou [3] in 1984. These influence coefficientshare complex-valued and depends on the dimensionless frequency parameter

$$a_o = \frac{\omega b}{c}$$

where c denotes the shear wave velocity of the medium.

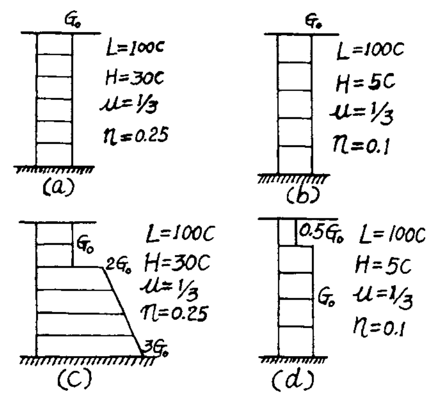

Fig. 3. Models of homogeneous and Non-Homogeneous Medium

Two examples for homogeneous medium, illustrated by Model (a) and (b) in Fig.3.,are analysed by the present approach, and the results are compared with the available solutions. The homogeneous viscoelastic half-plane investigated in reference [2] is represented by Model (a) with a finite length L=60b and a depth H=30b, and a semi-infinite single layered medium from reference [3] is shown in Model (b). It is observed from Tab. 1 that the results given by the present semi-analytic model are quite close to the analytical solutions.

The dynamic flexibility coefficients for the non-homogeneous mediums illustrated by Model (c) and (d) in Fig.3. are calculated and the results are listed in Tab.II. Compared the results with the solutions shown in Tab.I for the corresponding homogeneous mediums,it is indicated that the non-homogeneity of the medium has considerable influence on its compliance coefficients.

TABLE I. Dynamic Flexibity Coefficients for Homogeneous Viscoelastic Medium

x/b		0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
ReFyy	Present Method	0.3621	0.2546	0.0595	-0.0493	-0.0843	-0.1213	-0.1457	-0.1348	-0.1190	-0.1062
	Reference [2]		0.2385		-0.0473		-0.1320		-0.1452		-0.1115
ImFyy	Present Method	-0.3911	-0.3560	-0.2840	-0.2220	-0.1704	-0.1132	-0.0573	-0.0114	0.0294	0.0650
	Reference [2]		-0.3553		-0.2259		-0.1142		-0.0129		0.0620
ReFyy	Present Method		0.4652		0.1511		0.0756		0.0289		0.0046
	Reference [3]		0.4601		0.1732		0.0773		0.0288		-0.0027
ImFyy	Present Method		-0.0496		-0.0174		-0.0090		-0.0036		-0.0006
	Reference [3]		-0.0493		-0.0198		-0.0093		-0.0037		-0.0008

TABLE II. Dynamic Flexibility Coefficients for Non-Homogeneous Viscoelastic Medium

x/b		0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
Model (c)	ReFyy	0.3371	0.2303	0.0355	-0.0729	-0.1073	-0.1436	-0.1672	-0.1554	-0.1387	-0.1250
a_o=0.5	ImFyy	-0.3847	-0.3496	-0.2778	-0.2161	-0.1649	-0.1083	-0.0531	-0.0081	0.0317	0.0662
Model (d)	ReFyy		0.6286		0.1301		0.0796		0.0313		-0.0067
a_o=0.2	ImFyy		-0.0668		-0.0157		-0.0096		-0.0040		0.0004

TABLE III. Vertical Harmonic Point Force Response of Foundation
On Non-Homogeneous viscoelastic Medium (Model (c))

		a_o= 0.1		a_o= 0.3		a_o= 0.5	
		Real	Imagnary	Real	Imagnary	Real	Imagnary
Kr=0.00003	Centre	0.8424	-0.2772	0.4520	-0.4411	0.3377	-0.3846
	Edge	0.2045	-0.1098	-0.1622	-0.0584	-0.0941	0.0866
Kr=0.003	Centre	0.7503	-0.2408	0.3573	-0.4003	0.2405	-0.3340
	Edge	0.2048	-0.1096	-0.1623	-0.0594	-0.0976	0.0869
Kr=0.03	Centre	0.6156	-0.2038	0.2224	-0.3453	0.1104	-0.2607
	Edge	0.2009	-0.1097	-0.1658	-0.0697	-0.1195	0.068
Kr=0.3	Centre	0.4723	-0.1636	0.1134	-0.2382	0.0553	-0.1642
	Edge	0.2460	-0.1307	-0.1040	-0.1277	-0.1137	-0.0290
Kr=3.0	Centre	0.3604	-0.1441	0.0260	-0.1633	0.0005	-0.0954
	Edge	0.3162	-0.1425	-0.0151	-0.1516	-0.0345	-0.0848
Kr=300	Centre	0.3327	-0.1429	0.0056	-0.1519	-0.0145	-0.0875
	Edge	0.3324	-0.1429	0.0056	-0.1514	-0.0148	-0.0874

2. Dynamic Response of the Foundation

The technique described above is employed to determine the dynamic response of a flexible strip-foundation subjected to harmonic vertical loading and in smooth contact with the non-homogeneous viscoelastic medium illustrated by Model (C).

For a rigid surface massless strip-foundation, the response depends only on the elastic constants of the medium and the frequency of the exciting dynamic disturbance. However, the dynamic behaviour of a flexible footing is additionally affected by the material properties of the elastic plate. The main paremeter characterizing the flexibility of the soil-foundation system is the relative stiffness Kr defined by

$$Kr = D_f / D_m \tag{28}$$

where the flexural rigidities D_f and D_m of the foundation and the medium, respectively, are given by

$$D_f = \frac{E_f t_f}{1-\mu_f} \tag{29}$$

and

$$D_m = \frac{E_m a^3}{2(1+\mu_m)} \tag{30}$$

The response of the foundation excited by a series of harmonic point loads applied along the centre line is obtained and the vertical displacements at the centre and at one edge of the fundation are listed in Tab.Ⅲ for several representative values of the relative stiffness ranging from Kr = 0.00003 to Kr = 300. The smallest value of Kr corresponds to a non-existing foundation for which the response is identical to the free field motion, and the largest value of Kr corresponds to a "rigid" plate.

It is observed from Tab.Ⅲ that the displacements decrease as the frequencies increase and the centre displacements seem more sensitive than the edge displacements to changes in relative stiffness. When the relative stiffness Kr increases, the displacements at the edge either decrease or increase depending on the frequency of the applied force.

CONCLUSIONS

The spline function methods (spline finite element and spline semi-analytic method) are employed to analysis the dynamic response of a flexible strip-foundation subjected to harmonic excitation and supported at the surface of a non-homogeneous viscoelastic medium. The spline semi-analytic approach is developed to determine the dynamic flexibility matrix for the layered medium, which simplifies the two-dimensional problem to a one-dimensional discretization.

The numerical computations were performed on a IBM personal computer. The dynamic flexibility coefficients for homogeneous and non-homogeneous viscoelastic mediums are presented. It is shown that the non-homogeneity of the medium considerably affects its compliance. Paremetric studies examining the effect of the relative stiffness Kr on the foundation response are conducted. It is shown that the response at the centre of the footing decreases as the relative stiffness Kr increases.

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