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Applications of simulation and optimization techniques in optimizing room and pillar mining systems

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APPLICATIONS OF SIMULATION AND OPTIMIZATION TECHNIQUES IN OPTIMIZING ROOM AND PILLAR MINING SYSTEMS

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ABSTRACT

The goal of this research was to apply simulation and optimization techniques in solving mine design and production sequencing problems in room and pillar mines (R&P). The specific objectives were to: (1) apply Discrete Event Simulation (DES) to determine the optimal width of coal R&P panels under specific mining conditions; (2) investigate if the shuttle car fleet size used to mine a particular panel width is optimal in different segments of the panel; (3) test the hypothesis that binary integer linear programming (BILP) can be used to account for mining risk in R&P long range mine production sequencing; and (4) test the hypothesis that heuristic pre-processing can be used to increase the computational efficiency of branch and cut solutions to the BILP problem of R&P mine sequencing.

A DES model of an existing R&P mine was built, that is capable of evaluating the effect of variable panel width on the unit cost and productivity of the mining system. For the system and operating conditions evaluated, the result showed that a 17-entry panel is optimal. The result also showed that, for the 17-entry panel studied, four shuttle cars per continuous miner is optimal for 80% of the defined mining segments with three shuttle cars optimal for the other 20%. The research successfully incorporated risk management into the R&P production sequencing problem, modeling the problem as BILP with block aggregation to minimize computational complexity. Three pre-processing algorithms based on generating problem-specific cutting planes were developed and used to investigate whether heuristic pre-processing can increase computational efficiency. Although, in some instances, the implemented pre-processing algorithms improved computational efficiency, the overall computational times were higher due to the high cost of generating the cutting planes.

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NOMENCLATURE

Symbol	Description
Indices	
$t \in \{1, \ldots\ldots, T)$	Period index
$j \in \{1, \ldots\ldots, J)$	Section[1] index
$i \in \{1, \ldots\ldots, I)$	Block (in room) index
$k \in \{1, \ldots\ldots, K)$	Pillar index
$\eta \in \{1, \ldots\ldots, N\}$	Risk attribute index
$\alpha \in \{1, \ldots\ldots, A\}$	Resource index
$\mu \in \{1, \ldots\ldots, U\}$	Grade or quality index
Decision Variables	
$x_{it} \in \{0,1\}$	Binary integer variable; equals one if block *i* is to be mined in period *t*, otherwise zero
$y_{kt} \in \{0,1\}$	Binary integer variable; equals one if pillar *k* is to be mined in period *t*, otherwise zero
$z_{jt} \in \{0,1\}$	Binary integer variable; equals one if section *j* is to be mined in period *t*, otherwise zero
Sets	
Oi	Set of blocks that should precede block *i*, with N_i members

[1] Section refers to a region of the mine that contains more than one block or room. It represents an aggregation of blocks.

Oj	Set of sections that should precede section *j*, with N_j members
Oik	Set of blocks and pillars that should precede block *i*, with N_k members
O_j^s	Set of blocks and pillars that are contained in section *j*

Parameters

c_t	Unit cost of mining block *i*
c_k^η, c_i^η	Unit cost penalty of block *i* or pillar *k* for risk η
p_t	Unit price of commodity
d	Project discount rate
d_r^η	Discount rate of risk for risk η
MR_t^l	Lower bound on production in period *t*
MR_t^u	Upper bound on production in period *t*
w_e, w_r^η	Effective ratio of NPV and risk, respectively, in objective function (priority levels)
$R_t^{\alpha,l}$	Lower bound on resource α in period *t*
$R_t^{\alpha,u}$	Upper bound on resource α in period *t*
r_i	Effective recovery of block *i*
r_k	Effective recovery of pillar *k*
$q_t^{\mu,l}, q_t^{\mu,u}$	Lower and upper bound on product quality (or grade) of constituent μ[2] in period *t*, respectively.

[2] The grade (or quality) is indexed so the framework is applicable to multi-product deposits (e.g. polymetallic deposits).

q_i^{μ}	Average grade of constituent μ in block i
q_k^{μ}	Average grade of constituent μ in pillar k
r_{it}^{α}	Amount of resource α required to mine block i in period t
r_{kt}^{α}	Amount of resource α required to mine pillar k in period t
t_i	Tonnage in block i
t_k	Tonnage in pillar k
t_j	Tonnage in section j mined
R_k^{η}	Risk η of pillar k
R_i^{η}	Risk η of block i

1. INTRODUCTION

1.1. BACKGROUND

The room and pillar (R&P) method is one of the oldest underground mining methods used to mine deposits in both hard (mainly metalliferous ores) and soft (e.g. coal, potash, salt) rock. In hard rock mining, the method is viable for near horizontal deposits (< 30°) at moderate depths. It is capable of handling ore and host rock formations with high strength properties and can achieve mining recoveries as high as 85%. Generally, the R&P method is applicable to soft rock usually tabular and fairly horizontal (< 15%). The depth of the deposit is preferably less than 2,000 ft deep (Harraz 2014). Due to the flexibility of this mining method, over 60% of non-coal and 90% of coal underground mines in the United States of America (USA) use the R&P method (Tien 2011). Room and pillar's contribution to society is most evident in coal production. Coal is the leading source of energy in the world. It contributes to approximately 39% of the total electricity generated in the USA and 40% of electricity generated globally (EIA 2014).

Room and pillar is a self-supported mining method in which stopes (rooms) are driven into near horizontal ore bodies. The objective of the method is to implement a design that ensures maximum extraction of ore in the safest possible manner. The key design parameters include dimensions of the pillar, roof span, entry width, and panel width. The production plan should also maximize value (based on management's goals) while meeting all the constraints placed on the production system. To meet this objective, the extraction process should take into account the inherent risks (such as geotechnical, grade and environmental) associated with room and pillar mines. It is also necessary to select optimal design parameters that maximize productivity and minimize cost.

Generally, mine planning involves maximizing the value of mineral resource by optimizing ore and waste production sequences, as well as mine design. A good mine plan evaluates the impact of alterative designs and extraction sequences on the value of the mine. In R&P mines, the choice of design parameters (including panel dimensions) and extraction sequence affects recovery, productivity, equipment type, ventilation, ground control effectiveness, and other variables. In metalliferous R&P mines, uncertainties associated with metal prices, grade, metallurgical properties, and mining costs affect the optimal sequence in which geologic blocks should be extracted. This is particularly so in multi-element deposits. Also, the mining methods used in metalliferous R&P mines can accomplish more flexible production sequences since mining in a particular section does not require the production team to build out of infrastructure (e.g. conveyor belts), as required in coal mines. Hence, the number of feasible production sequences for metalliferous R&P mines tend to be higher than those for coal R&P mines. The optimal production sequence should maximize the value of the mine and account for uncertainty in market prices, geologic properties and other operational constraints dominant with metal deposits. The relevant geologic properties of coal deposits, such as the energy content, are less erratic (compared to a metal deposits). Therefore, the effect of uncertainties in geologic properties on the optimal production sequence is marginal.

In deposits that result in contiguous reserves (such as coal), paneling is useful for minimizing geotechnical risk for room and pillar mines. The choice of panel dimensions affects the recovery (because it affects the number and size of barrier pillars), the complexity of coal cutting sequences within a panel, the equipment fleet, productivity, unit costs and ground control strategies. Hence, the panel width[3] is one of the key design aspects of coal R&P mining. The rate of extraction and the extraction method is primarily affected by the dimensions of the panel.

In coal mining, the pillars are usually square or rectangular in shape and arranged in a regular pattern (Figure 1-1). To maximize the recovery of ore, pillars are made as small as possible. There are two basic operations in R&P coal mining: entry development and coal production. Development openings (entries) and production entries (rooms) are very similar, with both openings driven parallel to one another and connected by crosscuts. The optimal number of entries is often a function of geotechnical concerns, coal production and characteristics, and size of the production fleet. Room and pillar coal mines are divided into rectangular arrays called panels. The width of a panel with regular pillars and rooms is measured by the number of entries. The panels are separated by barrier pillars which prevent the progressive collapse of the roof, if a panel's pillar fails. The panel design affects coal recovery, material haulage and mining sequence, which in turn affects the overall mining cost and productivity. A smaller panel width may cause congestion and under-utilization of equipment even with a faster advance. However, too large a panel width will result in a slower advance and longer haulage distances, even though coal recovery may increase significantly. Therefore, it is essential to identify the optimal panel width that maximizes productivity.

Typical production equipment used in R&P coal mines includes the continuous miner (CM) and shuttle car. The CM cutting, loading and tramming capabilities, as well

[3] Panel width, in regular room and pillar mines (equal sizes of rooms and pillars on regular grid), is synonymous with the number of entries in the panel.

as coal haulage, makes up a significant part of the production cycle. Material handling in R&P mining still makes up over 40% of the operating cost (Chugh et al. 2002). Mine managers and engineers implement continuous technological improvements, such as high voltage CMs and electric shuttle cars, to meet production demands and minimize cost. The benefits of such technologies cannot be fully realized without optimizing the actual use of the haulage system. It is crucial to match the CM to an efficient haulage system to harvest its full potential.

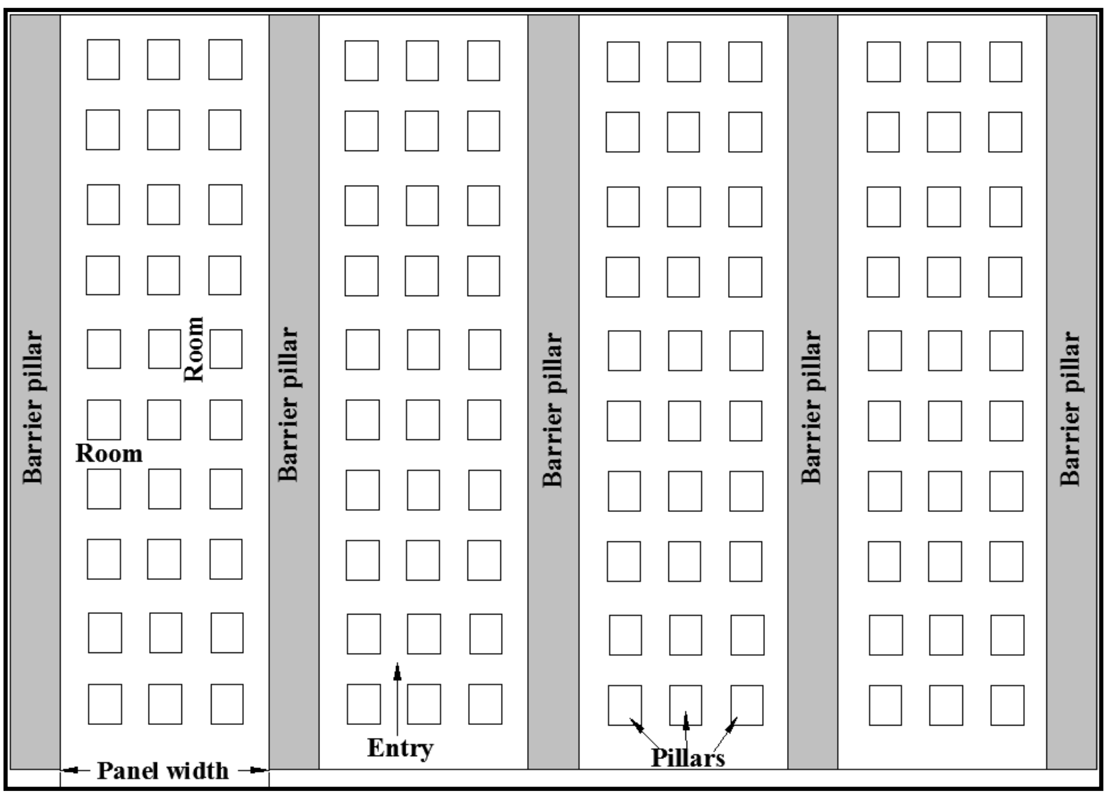

Figure 1-1 Room-and-Pillar layout with four-entry panels

An efficient room and pillar mine design relies heavily on the dimensions of the mining panel, the rooms and pillars that make up the panel, and the underlying production sequence. Some of the challenges of coal R&P mine planning and design that still need to be addressed in detail are: (i) how to determine the optimal number of entries to use in designing and producing from panels based on unit cost and productivity; and (ii) account for the constant changes in duty cycles in matching an optimal fleet size to the continuous miner. For hard rock metal mining, a key issue that remains to be addressed is how to determine the optimal production/extraction sequence that integrates comprehensive risk management into long term mine planning.

1.2. STATEMENT OF RESEARCH PROBLEM

There are two broad problems addressed in this research: (i) determining the optimal panel width for coal R&P mines and the associated optimal equipment fleet, which is simply referred to as "panel width optimization" in this dissertation; and (ii) accounting for risk in determination of the optimal mining sequence for R&P metal mines.

The design parameters in coal room and pillar mining depend on several factors including production recovery, strength of the coal, depth of mining, and stability of the hanging wall (Farmer 1992). A key aspect of room and pillar mine design is panel design, which depends on the strength and dimensions of the panel's pillars, coal recovery and mine production requirements. The size of a panel affects mining (cut) sequence, with larger panels resulting in more complicated cut sequences and more tramming by the continuous miners. Usually, greater emphasis is placed on panel design in retreat mining methods, where the rooms are mined first and the pillars recovered afterwards. Although pillar recovery is not common in US coal mining, there is still a great need to design panels that are optimal. Recent advances in electric haulers have spurred a move towards wider panels, to take full advantage of hauler capabilities. However, the effect of wider panels on productivity and unit operating costs has not been investigated fully. This means most mine managers and engineers make panel width design decisions based solely on past experiences. The need for an advanced R&P design decision making tool is imperative, and one aim of this research is to fill the gap, which is currently filled with heuristic decision making with regard to panel width selection.

In R&P mining the operating cost of a continuous miner and shuttle car is typically over $100 and $70 per hour, respectively (InfoMine 2013). To minimize the cost per ton resulting from running the loading and hauling equipment, it is essential to efficiently utilize them as much as possible. Utilization is a function of equipment matching. CM-shuttle car matching depends on the balance between the cutting and loading rate, as well as the cycle times. Since the CM has to move from one cut to another to allow for roof bolting and other operations, such as ventilation, which have to be completed while the CM is mining elsewhere, cut sequences have to be pre-planned to ensure efficient production. The cut sequence in each panel can require excessive tramming of the CM and shuttle cars from one cut to the other. As mining progresses through the panel, the duty cycles[4] of the CM and shuttle cars change as different cuts are mined each time. The changing duty cycles of the CM and shuttle cars influence the fleet size necessary at each stage of mining in the panel. To avoid under-utilization of equipment at different stages of mining, the changing duty cycles should be considered when matching an optimal number of shuttle cars to the CM. The challenges associated with accounting for changing duty cycles includes: (i) the choice of the size and number

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[4] The duty cycle is the cycle of operation of a cyclical piece of equipment. “Varying duty cycles” here mean particular aspects of the duty cycle (e.g. travel times for shuttle cars or tramming times for CMs) take longer or shorter times to complete.

of segments in the panel for analysis (i.e. a reasonable discretization of the process), and (ii) computational time and cost needed to model and determine the optimal fleet size in each segment of the panel.

An important aspect of exploiting mineral resources is implementing a feasible and optimal mining sequence. Production sequences in underground room and pillar mines depend primarily on the stability of the bearing rock mass, ventilation, and production requirements (Tien 2011). As discussed earlier, optimization of such production sequences is of particular importance for metal R&P mines. The risk associated with the input parameters makes sequencing in room and pillar mines a challenge. The main challenges for modeling R&P mine sequencing include modeling several processes in the production cycle, managing mining risk (such as quality, production and geotechnical risk) (Alford et al. 2007) and very strict sequencing requirements (Newman et al. 2010). In hard-rock mining, the primary factor that affects production sequencing is ore grade control (Farmer 1992). To mine high grade ore that meets production demands, pillar design may be irregular (in both spacing and shape) with low grade material left behind as pillars for roof support. Inability to fully characterize the risk as part of the production sequence can result in abandoned mining zones. Adequate planning can be done by engineers if the multiple risks inherent in room and pillar mine sequencing are accounted for in the initial production sequencing process.

Research in the past decade has focused extensively on the use of advance mathematical optimization programs that can model the complex nature of production sequencing (Askari-Nasab et al. 2010, Bienstock and Zuckerberg 2010). While most of these avoid the heuristic approach used in commercial software, the computation challenges of solving large mathematical problems are eminent. Common mathematical optimization programs used in mine production sequencing are binary integer linear programming (BILP) and mixed integer linear programming (MILP).

Integer linear programs (ILP) are known to be non-deterministic polynomial (NP) time hard[5] problems (Schrijver 1998). The relationship between computational times for these problems and number of decision variables, in the best case, is polynomial. Mine production systems consists of millions of jobs scheduled over long periods of time. Modeling mine production sequencing problems as integer linear programming problems result in large precedence constraints and decision variables with very high computational complexity. There is a persistent need to develop methodologies that allow engineers to solve a full size problem with reasonable computational power. The majority of these problems are solved with commercial algorithms such as CPLEX ® (Ramazan et al. 2005, Boland et al. 2009) which use the branch and cut method to solve integer problems. These algorithms define general policies efficient for all ILP problems, thus eliminating customized techniques which may be necessary for computational efficiency.

1.3. OBJECTIVES AND SCOPE OF STUDY

The objective of this research is to apply advance simulation and optimization tools to optimize room and pillar mining systems. In accordance with the overall goal of this study, the specific objectives are to:

1. Apply discrete event simulation (DES) to determine the optimal width of coal R&P panels under specific mining conditions;

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[5] NP-hard – A problem is NP-hard if an algorithm for solving it can be converted into one for solving any NP-problem (nondeterministic polynomial time) problem (Weisstein 2009).

2. Investigate whether the shuttle car fleet size used to mine a particular panel width is optimal in different segments of the panel;

3. Test the hypothesis that binary integer linear programming (BILP) can be used to account for mining risk in R&P long range mine production sequencing; and

4. Test the hypothesis that heuristic pre-processing can be used to increase the computational efficiency of branch and cut solutions to the BILP problem of R&P mine sequencing.

The first two objectives relate to panel width optimization in coal R&P mines. The first objective is to investigate the effect of panel width on the unit cost and productivity of an operation. Furthermore, the second objective is to investigate the effect of ignoring changing duty cycles on the productivity, cycle times and the duration of mining. The third and fourth objective relate to accounting for risks in optimization of production sequencing in metal R&P mines. In the third objective, this study seeks to develop a deterministic framework that incorporates multiple mining risks in optimizing a room and pillar production sequence. It is important to note that the developed model is only valid if the objective is to minimize risk and maximize the net present value. Finally, the work investigates using heuristics to generate cutting planes that could potentially speed up the solution.

1.4. RESEARCH METHODOLOGY

Figure 1-2 shows the research methodology used to accomplish the set objectives.

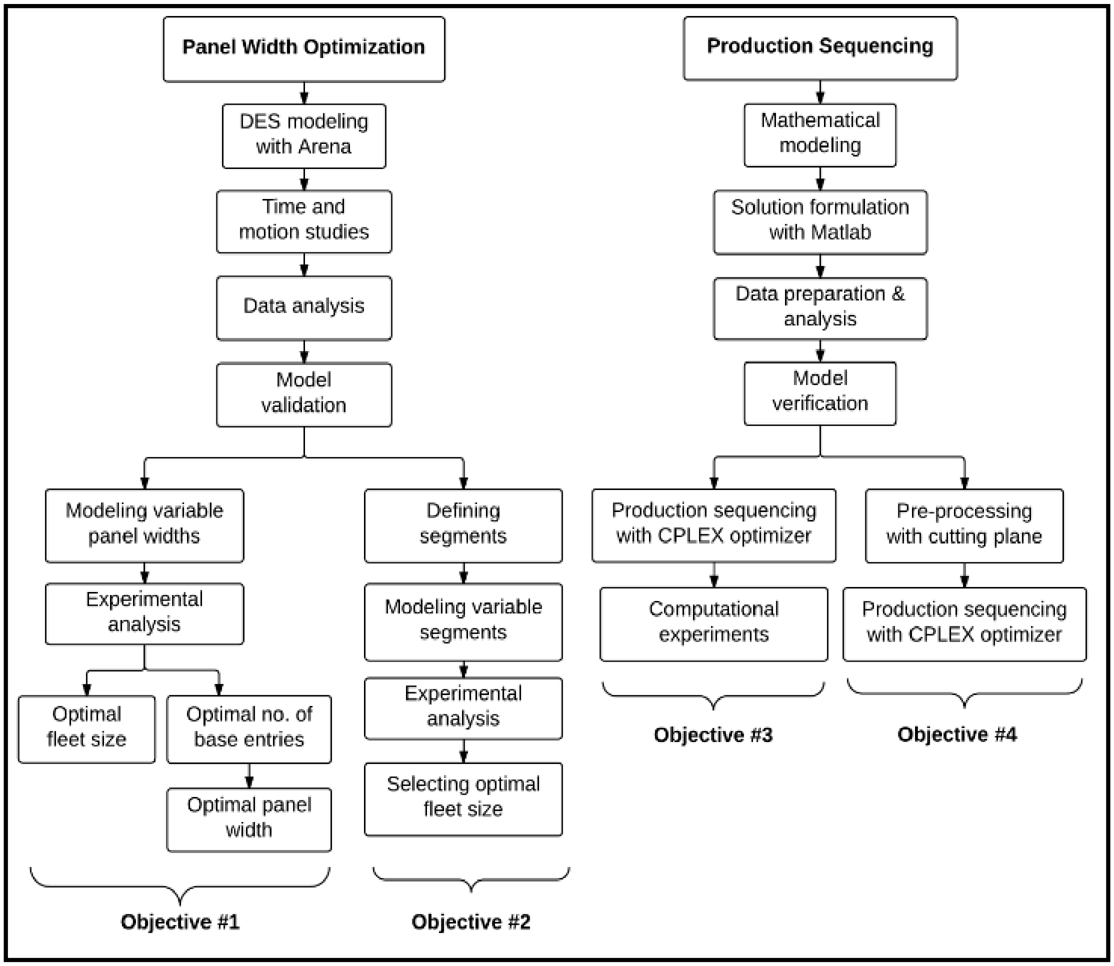

Figure 1-2 Methodology used in this research

To meet the objectives in Section 1.3, a simulation optimization framework based on discrete event simulation is proposed to optimize panel widths. A DES model of an existing room and pillar mine was built as a case study to investigate the effect of variable panel width, as well as fleet size on the unit cost and productivity of the mine. The model was developed in Arena® simulation software, which is based on the SIMAN language. The DES model was validated by comparing the simulated production to the actual mine production. Arena® experimental frame work (Process Analyzer software) was used to investigate the effect of panel width and fleet matching on cost and productivity. For the first objective, 36 experiments were done to investigate optimal width of the coal panel, as well as the sensitivity of the fleet size to panel width. To achieve the second objective, the optimal panel width obtained was used to investigate the effects of changing duty cycles in determining an optimal fleet size. The panel is divided into segments that captures the changes in equipment cycle times. Experiments were conducted to determine the optimal fleet size for each segment.

To achieve the third objective, the room and pillar operation is modeled as a binary integer linear program (BILP). A dual objective function is modeled that maximizes the overall net present value of the operation while minimizing mining risk. To obtain a feasible mine sequence, the model is subject to resource, quality, precedence, reserve, and mining rate constraints. The resulting BILP problem is solved using CPLEX® optimization software. The last objective includes developing cutting plane constraints that minimizes the number of enumerations required to obtain a feasible solution of the BILP problem. This includes solving the linear programming (LP) relaxation of the problem using the Matlab® linear programming function (LINPROG) to determine valid cutting planes.

1.5. SCIENTIFIC AND INDUSTRIAL CONTRIBUTION

This research contributes significantly to both the literature and industrial applications. The acquired knowledge is applicable to areas of engineering design, equipment dispatch and allocation, as well as underground production sequencing. The research uses multiple operations research techniques such as DES, optimization and the cutting plane method to optimize R&P systems.

1.5.1. Contribution to Literature.

1.5.1.1. Panel width optimization. As far as this author can tell, no previous work can be found in the literature that optimizes productivity and cost (maximizes the productivity and minimizes unit mining cost) as a function of coal panel width. Currently, mine design parameters are optimized primarily based on ground control requirements. The width of a coal panel affects the tramming of the CM and shuttle cars, cut sequence and fleet requirements (Segopolo 2015). This research introduces a modeling framework that incorporates the dynamics of the loading, hauling and dumping cycles in the panel width selection. The framework includes how to incorporate the variable cut sequences for each individual panel width, as well as how to optimize sections of the panel width (with distinct duty cycles for material handling equipment) independent of the remaining panel. Optimizing panel width is an optimization problem where the objective function could reflect the desire to maximize productivity and minimize unit operating costs. The productivity and unit costs of coal cutting, loading and hauling operation as a function of the panel width, equipment fleet, and cut sequence is nonlinear and implicit. Very few techniques (simulation being one) can solve such problems (Zou 2012). This research offers a means to estimate the unit cost and productivity for a given panel width using DES, which makes it possible to optimize the unit costs and productivity using panel width.

1.5.1.2. Effect of changing duty cycle on CM-shuttle car matching. Most mining operations experience changing duty cycles although the nature of such changes may vary from operation to operation. In R&P operations, the CM and the shuttle cars are constantly tramming. The CM cycle times continue to change as mining progresses. In most cases, the overall traveling distance changes from one instance to another. The distance from the dumping site (usually a conveyer belt feeder) varies as the mining face moves from cut to cut. Changes in cycle time results in under-utilization of either the CM or the shuttle cars. Therefore, it is important to assign an optimal number of shuttle cars to the CM for each set of duty cycles. Very few studies in the literature incorporate the changing duty cycles in equipment matching (Awuah-Offei et al. 2003, Dong and Song 2012). The most common examples can be found in surface mining, where changes in duty cycle are comparatively less frequent. A major challenge to incorporating duty cycles in R&P mining, where the duty cycle is changing almost continually, is how to discretize the operation into reasonable periods of operation (segments) to facilitate realistic solutions. This research introduces an approach for the selection of segments, which balances the need to optimize for changing duty cycles with realistic and reasonable operating periods. It also introduces an experimental approach that investigates the sensitivity of productivity, cycle times, utilization, and duration of mining to changing duty cycle with minimum computational effort.

1.5.1.3. Production sequencing. Incorporating risk and uncertainty into optimization models and solutions can be challenging. Doing so can result in stochastic optimization problems, which are much more computationally expensive than their deterministic counterparts (Ramazan et al. 2005). Although one can easily conduct sensitivity analysis for pure LP problems, most sequencing problems include binary or integer variables leading to BILP or mixed integer linear programming (MILP) problems for which such information is only available for the LP relaxations of the problems. Hence, past attempts to incorporate uncertainty into the open pit problem, for instance, have resulted in longer solution times. Even then, the approaches have mostly incorporated only grade uncertainty (Dimitrakopoulos 1998, Sarin, and West-Hansen 2005, Ramazan, Dagdelen, and Johnson 2005, Bienstock and Zuckerberg 2010, Askari-

Nasab et al. 2011). However, most mine engineers and mine planners are aware of the level of risk associated with different mining zones that go beyond grade uncertainty. Uncertainty in ground control design parameters, drainage parameters and geologic risks (grade, deleterious elements, etc.) fully describe the risk inherent in mine planning. This research presents a deterministic framework of modeling multiple mining risk as BILP. The model includes constraints specific to underground mining and the usefulness of the approach is verified using a case study. Most researchers tend to use commercial software such as CPLEX to solve sequencing problems. Commercial optimization solvers like CPLEX are designed to solve all the diverse problems that users will possibly want to solve. Using commercial solvers alone misses the opportunity to take advantage of the unique characteristics of the problem to customize the solution algorithms. This research develops problem specific pre-processing techniques using the cutting plane method to minimize computational complexity.

1.5.2. Contribution to the Mining Industry. This research involved closely working with industry to investigate the optimal panel width that maximizes productivity. The result of this study was recommended to the collaborating mine for implementation. The results were also described in a project report for the funding agency, which was distributed via the website to other companies, and presented to a meeting of the industry advisory board of the funding agency, which is made up of leaders from industry. The use of DES eliminated the high cost associated with practical experiments with different panel width that was currently practiced at the mine. Due to limited use of telemetry in most underground mines, there is limited production monitoring data necessary for equipment matching. Engineers rely on trial and error that significantly affects operation costs and equipment utilization. By providing a discrete event simulator of the mining system that accounts for changing duty cycles, experimentation with different fleet sizes is plausible without loss in productivity or increased cost. Although a few studies have incorporated changing equipment cycle times, they do not provide a comprehensive approach that can easily be adopted by the mining industry. There has been no work done specifically in R&P mines to incorporate changing duty cycles in equipment matching. This research presents a modeling approach that accounts for changing duty cycles, as well as providing information needed for equipment dispatch. By disseminating the results in relevant forums, the research results can influence industry practices and improve mining engineering practice for coal R&P mines.

The limited application of advanced mathematical modeling tools in sequencing can be attributed to the complex nature of underground mines. All the commercial mine planning software that deal with optimization of production sequences use heuristics or meta-heuristics to produce *optimal* sequences and do not incorporate mining risks. Using the deterministic approach developed in the research, engineers can develop in-house algorithms specific to a mining operation.

The findings from this research have been properly disseminated through journal and conference publications. So far, three journal papers have been submitted for peer review and publication. The journal papers cover work done in Chapters 3-5 to meet the first three objectives. These include: panel width optimization using DES; a deterministic modeling framework that incorporates multiple mining risk in R&P production sequencing; and accounting for changing duty cycles in CM-shuttle car matching. More peer review journal publications are expected from this research. Two conference papers

(Anani and Awuah-Offei 2013, Anani and Awuah-Offei 2015) have been presented at conferences and published in proceedings. They focus on modeling mining risk and R&P production sequencing. Disseminating these findings will provide advance simulation and optimization tools for engineers to evaluate the impact of panel width design and production sequencing on R&P operations.

1.6. STRUCTURE OF DISSERTATION

This dissertation comprises seven chapters, including this introductory chapter. Chapter 2 covers a detailed description of all relevant literature. It covers simulation optimization, in particular, the use of DES in optimizing productivity as a function of mine design, as well as accounting for changing duty cycles in equipment matching. It also covers the application of optimization and solution algorithms in mine production sequencing. Chapter 3 focuses on a framework for panel width optimization using DES and a case study to illustrate the approach. Chapter 4 discusses the approach used to incorporate changing equipment duty cycles in determining the optimal allocation of shuttle cars to continuous miners. Chapter 5 covers the mathematical modeling of R&P production sequencing as BILP and solution formulation. Chapter 6 deals with an exploration of whether the use of heuristics to pre-process the R&P sequencing BILP problem, prior to solving with the branch and cut method, reduces the solution complexity. Chapter 6 covers the conclusions of this study and recommendations for future work.

2. LITERATURE REVIEW

This section covers a comprehensive review of the relevant literature on mine design and production sequence optimization. The review takes a closer look at simulation optimization, coal panel width optimization, equipment fleet sizing, production sequencing, mathematical optimization, and exact algorithms.

Optimization is defined as the method of finding the best solution (or alternative) in a set under given constraints (Ruszczynski 2006). Mineral extraction methods consist of millions of activities within a mining system that needs to be optimized in order to operate an efficient and sustainable mine. The main aspects of mining system optimization include mine design, production sequencing and equipment selection and dispatch (Govinda et al. 2009). Most of the early tools used in mine system optimization, were based on trial and error. For the past decades, numerous methods have been developed that makes mining system optimization more efficient. One of the main techniques used today is operations research (OR), which was developed by the military during the Second World War. Since its development, the technique has been continuously improved (Dantzig 1948) and adopted by business and industry. Operations research is a discipline that applies advanced analytical methods such as statistical analysis, mathematical modeling, and mathematical optimization to help make better decisions (iBernis 2013). Scientific methods are applied systematically to obtain optimal levels of operation based on the current state of the system (Sharma 2009). Operations research encompasses methods such as simulation, queuing theory, Markov's decision process, mathematical optimization, expert systems, econometric methods, data envelopment analysis, neural networks, analytic hierarchy process, and decision analysis.

The application of OR techniques in mine planning and sequencing dates back to the early 1960s (Lerchs and Grossmann 1965). Since then, simulation and mathematical optimization in particular have been used in both underground and surface mining (Johnson 1968, Barbaro and Ramani 1986, Dowd and Onur 1993, Oraee and Asi 2004, Boland et al. 2009, Bley et al. 2010, Tarshizi et al. 2015). Operations research techniques are used in many areas of mining including meeting quality targets (Samanta et al. 2005), maximizing net present value (Akaike and Dagdelen 1999), equipment dispatch (White and Olson 1992), and fleet sizing (Burt et al. 2005). This chapter takes a closer look at the application of decision models (specifically simulation, mathematical optimization and exact algorithms) in optimizing mine design parameters and production sequencing.

2.1. SIMULATION OPTIMIZATION

Simulation is an applied technique that describes or imitates real-world system behavior using a symbolic or mathematical model (Sokolowski and Banks 2010). Simulation has always been a part of problem solving and optimization in all aspects of life (including transportation, energy and natural resources, health, public, and military systems). Simulation involves a system and a model of the system. Computer simulation has become the most advanced modeling tool used today, because of its ability to model highly complex systems. Many simulation techniques exist, which include computational fluid dynamics, kinematics and dynamics simulation of mechanisms and robots, and discrete event simulation.

A good simulation model is one that closely resembles and is representative of the actual system. It should be capable of providing feasible answers to questions about the system. To develop an efficient model representative of the system, the system’s state variables should be defined such that all information needed for complete evaluation is available. The variables are defined as discrete or continuous, static or dynamic, deterministic or stochastic depending on the nature of the system (Kelton et al. 2010). The state variables in discrete event models change in discrete time steps and intervals. That is, the values remain the same over the time intervals between events and changes at discrete points in time, when an event occurs. On the other hand, the state variables continuously change over time in continuous models.

The main advantages of simulation include gaining understanding in the operation of a system, testing new systems or concept before implementation and obtaining important information without disturbing the actual system. In doing so, experimentation of system alternatives can be done in a much shorter time frame. Using computers, analysts can study a system with minimum analytical effort using valid models. Simulation is flexible and can easily handle complex features of a system such as stochastic variables and time delays, which are difficult to treat analytically. Problems that require both qualitative and quantitative solutions that cannot be solved using qualitative methods, can be solved by simulation (Meerschaert 2013). However, simulation also has certain disadvantages including the inability to determine the optimal solution (out of all possible solutions) for the problem by itself without input from the user. Also, simulation will not give accurate results if the input data used is inaccurate, regardless of how well the model is designed (Chung 2003). Furthermore, the only way to test sensitivity to specific system parameter is to run the simulation repetitively and then interpolate.

This research applies discrete event simulation (DES) in optimizing mining systems. The following sections define discrete event simulation, discuss applications of

DES in mining and simulation optimization using DES for determining optimal design parameters.

2.1.1. Discrete Event Simulation. DES is a computer-based approach that facilitates modeling, simulation, and analysis of the behavior of complex systems as a sequence of discrete events. DES is simulation in which state variables change at discrete points in time at which certain events occur (Banks 1998). The basis of DES includes the system studied, the representative model, activities and delays, state variables, processes, resources, entities and their attributes. In DES, the entities are explicitly defined as objects with attributes needed for one or more investigations. Entities can be modeled such that they move through a system with time (dynamic) or serve other entities (static). Resources are static entities that provide services to dynamic entities.[6] Activities in a system are initiated and terminated by the occurrence of events and are responsible for changing the state of a system over time. A process is, therefore, a sequence of activities scheduled on time (Banks 1998).

To develop a DES model, analysts are guided by four main conceptual frameworks (also known as world views), which have been extensively used since their development in the 1960's (Gordon 1961, Markowitz et al. 1962, Dahl et al. 1967). These frameworks include: (i) event scheduling; (ii) activity scanning; (iii) three-phase approach[7]; and (iv) process interaction. The analyst must select the framework that meets

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[6] Dynamic entities are usually referred to as entities and static entities are usually referred to as resources. This dissertation uses this convention to refer to entities and resources.
[7] Often in the literature, the three-phase approach is not discussed as a distinct framework because it is a combination of the event and activity frameworks.

the system characteristics and specific model objectives (Balci 1988). These frameworks are defined as follows:

- *Event scheduling***.** In this framework, the main focus of modeling the system depends on the occurrence of an event. The entities, attributes and events are defined based on the objective of the study. The events include scheduling activities that reallocate entities and release resources for specific activities. This framework requires the specification at the event level instead of the activity level. To capture system behavior, the analyst is required to define a set of future events. The changes in the system are recorded by the analyst once the defined event occurs (Pegden 2010).
- *Activity scanning***.** This framework was first used by Buxton and Laski (1962) in a simulation language. In this framework, the analyst describes two constructs: conditions and actions. Conditions refer to the states of the model at which an activity can take place. Actions refer to the operations of the activity undertaken when the set conditions are satisfied. When using this framework, all conditions are prioritized and tested repeatedly (i.e. scanning) to determine when they are met in order to execute the appropriate actions. The scanning is done at fixed time intervals to determine the occurrence of an event. The state of the system is updated when an event occurs. This framework leads to longer simulation runs in most cases. However, in cases where the analyst desires ease of maintaining and implementing of the model, the activity scanning framework is the optimum choice.

- *Three-phase approach*. To remediate the execution inefficiencies associated with the activity scanning framework, Tocher (1963) introduced a three-phase approach. The first phase advances time until there is a change in the system state or an event occurs. In the second phase, scheduled resources are released at the end of their activities. The third phase involves initiating activities once resources are available to perform them. The method is a combination of the event scheduling and activity scanning frameworks. In this approach, events are defined as activities with a duration of zero. The activities are classified into conditional and unconditional activities that change the state of the system.
- *Process interaction.* This approach entails describing the life cycle of an object as it moves and interacts with processes involved in the system under study. The entity moves through the system until it is stopped by a delay, activity or exist a system. Time is then advance to the point where the entity starts moving again.

Most simulation models are dynamic, which allows analysts to evaluate systems over time, as compared to static models (e.g. mathematical and statistical models). The advantage of DES lies in its ability to model complex systems with relative ease. DES allows engineers and scientists to evaluate new designs and methods without interfering with the real-life system. It also helps answer the question of why certain phenomena occur (Asplund and Jakobsson 2011). Moreover, DES has the ability to capture random behavior (uncertainty) caused by a large number of factors that impact the system, using statistical sampling techniques (e.g. Monte-Carlo sampling).

DES software has been continuously improved over the past four decades leading to more advanced simulation languages (Pegden et al. 1995, Nance 1995, Rice et al. 2005). Simulation languages are symbols/codes recognized by computers or computer programs as issued commands a programmer wishes to perform (Kiviat 1968). Common simulation languages currently used for DES include SIMAN, GPSS, and SLAM.

SIMAN, which is used in this work, is a SIMulation ANalysis program generally used to model either discrete, continuous, or a combination of discrete and continuous systems (Pegden et al. 1995). SIMAN allows process-oriented, event-oriented, and continuous components to be integrated into a single system. A unique characteristic of a SIMAN program is the distinct decomposition of model and experimental frames. The static or dynamic nature of a system can be defined in the system model. Different experiments can be done in the experimental framework resulting in multiple sets of output (McHaney 1991). However, the close link between its arithmetic and list processes on the one hand and its demand-resource concepts on the other restrict its capability to model demand-driven systems (Fishman 2001). This research uses Arena®, which is based on the SIMAN language for DES modeling and simulation.

GPSS/H (General Purpose Simulation System) is one of the oldest simulation languages used for discrete event simulation. It is a process-oriented language, which is independently controlled either by activity-type processes or event scheduling. One advantage of process-oriented language is the ability to reduce the amount of overhead statements a programmer has to write by combining multiple events in a single process

(Kiviat 1968). GPSS is well suited for queuing models. Compared to SIMAN, GPSS/H lacks significant flexibly and power for modifying the state of the system (Krasnow and Merikallio 1964).

SLAM (Simulation Language for Alternative Modeling) is a simulation language known for its ability to allow a system to be modeled using any of three (process, event and activity) frameworks (world view) or a combination of any three. The framework takes advantage of the process-oriented approach and its able to extend to discrete event simulation constructs if the approach becomes restrictive. SLAM is the first language to model systems using any of the world views or a combination of them. A major advantage of SLAM is the ability to build combined process-oriented-discrete event continuous models with interactions between each orientation (Pritsker 1995).

DES can be used to perform “bottleneck” evaluations to discover where work in process in a system is delayed and which variables are responsible. Identifying problems and gaining understanding into the importance of these variables increases awareness of their importance relative to the performance of the overall system. DES allows an analyst to vary the system operating periods, cheaply and easily (Schriber 1977). On the other hand, even though DES provides a way to analyze and understand the changing behavior of the system, it only provides an estimate of the model output.

Building DES models can be costly and time consuming. It requires special training and experience over time. The use of random variables can make it difficult to determine if observed results are due to system interactions or randomness. DES is not always the best alternative for evaluating specific objectives. In some cases, analytical solutions are preferable or possible. Therefore, it should be used on an as-needed basis where benefits outweigh costs (Asplund and Jakobsson 2011). DES can also be used, along with optimization, to find the optimal configuration of a system. This approach is often referred to as simulation optimization and is useful for optimizing design parameters.

2.1.2. DES Application in Mining. Applications of discrete event simulation as a decision making tool for improving mining systems are vast and increasing (Vagenas 1999, Basu and Baafi 1999, Awuah-Offei et al. 2003, Michalakopoulos et al. 2005, Yuriy and Vagenas 2008, Ben-Awuah et al. 2010). In surface mining simulation, open pit operations are the most common. The studied systems include shovel-truck, dragline, and bucket wheel excavator systems, among others. DES has been used to optimize production scheduling (Ben-Awuah et al. 2010), processing plant operation (González et al. 2012), fleet size (Ataeepour and Baafi 1999), fuel efficiency (Awuah-Offei et al. 2012), and design parameters (Que et al. 2015) in mining systems. For underground mining systems, the application of DES can mainly be found in stope operations (Potter et al. 1988, Sturgul 1989) and material handling (Topuz et al. 1982, Runciman et al. 1997, McNearny and Nie 2000). The first application of DES in mining was by Rist (1961) for an underground haulage system in molybdenum mine. After the first successful application, many studies were found in literature including the first application of GPSS simulation language (Harvey 1964), Monte Carlo simulation (Achttien and Stine 1964), first conveyor belt simulation model and the simulation of a R&P system (Suboleski and Lucas 1969). However, DES application specifically in R&P mining is limited to a few examples (Suboleski and Lucas 1969, Hanson and Selim 1974, Suglo and Szymanski 1995, Szymanski and Suglo 2004, Pereira et al. 2012).

Suglo and Szymanski (1995) used SLAM to model a CM-shuttle car R&P mining system as DES. The system modeled includes the cutting, loading and haulage operations. The objective of the model was to determine the optimal equipment combination and duration of mining. The output included the total production, material stockpiled, duration of mining, queue length at feeder breaker, serve utilization at feeder breaker, and waiting time at feeder breaker.

Szymanski J, and Suglo (2004) continued their work by using SLAM to determine the best equipment allocation to meet production targets. They modeled three mining systems including a continuous miner-shuttle car system in an underground room and pillar coal mine. The output parameters included the production, duration of mining, equipment combination and number of servers at the conveyer belt feeder breaker. The experimental analysis included varying the number of CMs and shuttle cars, as well as the number of servers at the feeder breaker to determine an optimal value.

Also, Pereira et al. (2012) used simulation to evaluate the impact of a new scheme on the productivity of a coal room and pillar mining system. The production system consisted of drilling, blasting, loading, and hauling by a loader and shuttle car. They evaluated the benefit of a cut sequence that advances the panel center ahead of the panel flank as compared to mining all the entries simultaneous along its entire expansion. The experiment included evaluating the impact of equipment placement and variable cut sequences on the productivity. The input for the model include equipment cycle times and characteristics. The output parameters included the number of cycles in a shift and the daily production. The results demonstrated that a more organized cut sequence can maintain the maximum productivity compared to the traditional trial and error approach used by the mine.

Most of the examples found in literature that use DES to optimize R&P mining systems and underground mines in general (Runciman et al. 1997, Yuriy and Vagenas 2008, Salama et al. 2014) are limited to optimizing equipment allocation and placement that maximizes productivity. This research introduces a new area of study that optimizes mine design parameters as a function of unit cost and productivity using DES. The research adopts several techniques from DES application in surface mining which includes accounting for changing equipment duty cycle in optimizing equipment combinations.

2.1.3. DES for Optimizing Design Parameters. DES can be used as a decision making tool in determining optimum design parameters. It can be used to simulate system performance at varying operating conditions and design parameters. Thus, what-if analysis can be performed quickly and cheaply with a valid model. Through such experiments, *optimum* design parameters can be determined that meet design goals and respect all constraints of the design problem. For instance, to design a greenhouse crop system for maximum production and quality of labor, van't Ooster et al. (2013) successfully used DES to perform sensitivity analysis in identifying design parameters that influence labor performance and the effect of uncertainty on the performance indicator. Similarly, Petering (2009) investigated the optimal width of storage blocks in a terminal container and its effect on gross crane rate. Reichardt and Wiechert (2007) used DES to design a new grinding chamber in a ball mill that maximizes the collision velocity. The design parameters optimized included the height and number of buckles.

Optimization problems are generally in the form:

$$\mu^* = \min_{x \in \Psi}(f(x)) \tag{2-1}$$

Where $f\ (.)$ is the simulation model with input parameters x of length m which may be implicit or explicitly defined. The output variable is also usually a random variable (Y) with the aim of finding the minimum expectation ($E[Y]$). Since $f\ (.)$ is a stochastic simulation model, observations are made only based on experiments. The optimal input parameter is obtained by comparing the deterministic output values (mean and standard deviation) of all experiments conducted (Buchholz 2009).

When DES is used for simulation optimization, the objective function of the optimization problem is defined as a function of the DES output variables, subject to system constraints (Fu 2002). There are several simulation based meta-heuristics and meta-modeling techniques adopted in research to optimize the performance of a system. These include deterministic and stochastic methods tailored to both continuous and discrete input parameters. The most common analytical techniques for solving continuous simulation optimization problems are gradient-based techniques (*e.g.* finite differences, perturbation analysis and likelihood ratio), stochastic approximation methods, and response surface methodology (RSM) and sample path method (Carson and Maria 1997). Discrete parameter simulation optimization techniques include random search and exact algorithms (*e.g.* branch and bound methods). The most common simulation optimization environment implemented in simulation software is AutoStat and OptQuest. These environments include the statistical, mathematical and design of experiment tools necessary for system optimization. The main comparison between the analytical algorithms and those implemented in current DES software are based on: (i) continuous versus discrete parameters, (ii) statistical, and convergence validity, (iii) stochastic analysis, (iv) single point versus family of solutions, and (v) the use of memory. Although current research has modified several techniques such as RSM to solve stochastic problems, these methods are developed based on deterministic frameworks. The deterministic characteristic makes it almost impossible to analytically solve large-scale, real-life, stochastic problems. Table 2-1 is a summary of the characteristics of each simulation optimization approach (Fu 2002, Buchholz 2009).

Table 2-1 Characteristics of analytical and software-based simulation optimization methods

Traditional analytical algorithms	**Software algorithms**
• Deterministic optimization • Memory needed for best current solution • Usually iterates to a single point • Records number of iterations • Form of convergence include probability convergence, distribution convergence, and convergence to true optimum	• Stochastic optimization • May or may not require some form of memory • Usually iterates on family of solutions • Uses current state not past solutions • Imitates nonlinear programming • Larger replications bring stochastic settings close to deterministic domains • Based on discrete search strategies

The limitations of the traditional deterministic approach for simulation optimization include the inability to guarantee the optimality of a solution. They are also specifically not tailored to solve stochastic optimization problems (Buchholz 2009).

To optimize the unit cost and productivity as a function of panel width in R&P coal mining, the input parameters such as the cut sequences, number of shuttle cars and entries are discrete in nature with stochastic variables. The system is therefore simulated as a stochastic discrete event model with stochastic input variables that characterize the uncertainty inherent in R&P systems.

2.2. EQUIPMENT FLEET SIZING

2.2.1. Techniques for Fleet Size Optimization. Material handling in mining operations contributes significantly to mining cost. Generally, a fleet of equipment is needed to transport extracted material from the mining face to the dumping site. The operating costs associated with haulage systems includes labor, maintenance, fuel, and wear and tear. The equipment fleet size affects the mine's productivity. The amount of material produced in a unit time differs depending on the number of equipment in the system. Using less/more than the optimal number fleet size results in under-utilization of the loading or haulage equipment. In order to maximize the productivity and minimize the cost per ton of material produced, it is essential to determine the optimal number of equipment for each operation.

Since the late 1960s, researchers have solved fleet size optimization problems using operations research techniques. The first application of such techniques to fleet size optimization was implemented by O'Shea et al. (1964) and Griffis (1968). The authors determined the optimum number of earth moving equipment based on cost analysis using mathematical models and the queuing theory. Since then, many advance techniques have been used to determine the optimal fleet size needed to perform a particular operation. Amongst these techniques is queueing theory, which has been widely applied by researchers. Typically, the cycle queue in the mine is divided into different phases depending on equipment loading, loaded travel, dumping, and empty travel operation. The average cycle time in each period is predicted by calculating the utilization; the number of trucks serviced at each phase in a unit time, and expected time at each phase. The production and unit cost are calculated using equations such as are in Equations (2-2) and (2-3), respectively. The number of haulage units is varied to determine the fleet size that optimizes cost and productivity (Parikh 1977, Carmichael 1986, Ercelebi and Bascetin 2009).

$$\text{Productivity} = \frac{P}{average\ cycle\ time} \bullet N \bullet haulage\ unit\ capacity \tag{2-2}$$

$$\text{Cost} = \frac{C_1 + C_2 \bullet N}{Unit\ production \bullet truck\ capacity} \tag{2-3}$$

Where:

P	time period of interest
N	number of hauling equipment
C_1	unit operation cost of haulage unit
C_2	unit operating cost of loading unit

For example, Fanti et al. (2014) used closed queuing networks to determine the optimal fleet size of electric car sharing systems. The electric car sharing stations form a closed network to provide service to customers. The authors define three phases: car rental, travelling and recharge operations. The model predicts the queue length and waiting times in each of these phases. The optimal fleet size of the car sharing system is determined based on the total revenue.

Queueing theory can be used in conjunction with exact algorithms such as linear programming (LP) to optimize fleet size. The approach includes initially defining a network of different phases (nodes) of the operation and predicting the queue length, waiting time and utilization at each phase. The problem is then defined as a discrete (LP) optimization problem, usually with a dual objective function that maximizes productivity and reduces cost (Fanti et al. 2014). Other than cost and productivity, the objective function can be defined as the equipment utilization (Choobineh et al. 2012). The set constraint for the LP optimization problem is such that the limit of throughput in each phase is met.

Another method used for fleet size optimization is dynamic programming. Dynamic programming, unlike most implicit approaches (Kirby 1959, Wyatt 1961), is able to incorporate policies that capture changes in the system with time. These include changes in labor and equipment cost, as well as product demand. Mole (1975) optimized equipment fleet size using a dynamic programming model, based on a regeneration sequence. Murotsu and Taguchi (1975) combined dynamic programming with nonlinear programming to determine the optimal ship fleet size that minimizes cost subject to geometrical and technological constraints. The problem was modeled as a nonlinear program. The modeled constraints were based on drought limit, ship type and capacity.

Based on dynamic programming concepts, the number of ships in the system is defined as control variables to be optimized since the number of ships is unknown.

Besides the above mentioned methods, several other methods can be found in the literature to determine the optimum fleet size of an operation. Examples of these methods include genetic algorithms (Chakroborty et al. 2001, Liu et al. 2009, Yao 2012), demand Pivot method (Li et al. 2010), inventory theory (Fedorčáková, and Šebo 2012), heuristics (Fu and Ishkhanov 2004), simulated annealing (Tavakkoli-Moghaddam et al. 2009), linear programming (Gould 1969, Beamon and Deshpande 1998, Li and Tao 2010) and other analytical techniques (Tanchoco, Egbelu and Taghaboni 1987, Egbelu 1987, Sinriech and Tanchoco 1992, Mahadevan and Narendran 1993).

2.2.2. DES for Fleet Size Optimization. DES can be used to model an operation, the resources needed, haulage fleet type and availability, as well as the current performance of the system (Vis et al. 2005, Boyd et al. 2006, Chen 2009). The defined resources usually include loading and off-loading equipment, maintenance personnel, and other equipment. The typical input data needed for fleet size optimization models include cycle time and equipment speed, loading, dumping/delivery and production rates, and travel distances. The output parameters are dependent on the set objectives. Typical output variables include queue length, waiting time, resource utilization, duration of mining, unit cost, and productivity. Longer queue lengths indicate excess equipment fleet capacity in the system, and vice versa. Similarly, under-utilization of loading and dumping resources indicate the fleet size is less than optimal. Experiments are conducted, which typically include varying the size of the fleet and evaluating the impact on the output variables. Determining the optimal fleet size is dependent on a balance between these variables.

For example, Lesyna (1999) used DES to optimize the size of rail car fleet needed to deliver final goods to customers in each production period. The author addresses the optimal route to be used by the rail cars, as well as the optimal number of auxiliary equipment (such as trucks). Input parameters necessary to model these objectives included customer demand rates, plant production rates, travel times and waiting times at the customer site. Marlow and Novak (2013) also used DES to determine the minimum fleet size that meets the minimum daily embarked requirements and the number of flying hours. An important aspect of this system is how to assign maintenance resources and use both scheduled and unscheduled maintenance. Input data needed to evaluate the optimal fleet size include maintenance data (duration and frequency), flying hours, and number of embarked aircrafts. The output data included time spent waiting and in maintenance, percentage time the minimum requirement for embarked air craft is met and the annual embarked and ashore hours achieved by the fleet.

Shyshou et al. (2010) also modeled an anchor handling operation as discrete event simulation that optimizes the vessel fleet size. The input data for the model vessel speeds, sailing, demobilizing, mobilizing, and towing times. The output variables used to validate the simulation model includes the number of spot hire days. The optimal number of vessels is selected based on the effect of fleet size on vessel hiring cost. Godwin et al. (2008) used DES to optimize the number of locomotives in a rail network. Assignment of locatives were done on a daily basis, therefore, it was essential to optimize the fleet size. The optimal fleet size was determined based on a balance between rake waiting time and locomotive utilization. The authors demonstrate that a large fleet size causes congestion in the system. Compared to other methods, DES allows for the evaluation of large sets of scenarios. Most analytical methods used in the literature rarely capture delays in the system and therefore, underestimates the optimal fleet size (Srinivasan et al. 1994).

The literature also contains several examples of work that uses DES to estimate optimal mining fleet sizes or composition (Baafi and Ataeepour 1996, Awuah-Offei et al. 2003, Askari-Nasab et al. 2012, Salama et al 2014, Fioroni et al. 2014, Dindarloo et al. 2015). For example, Askari-Nasab et al. (2012) used DES to model an open pit truck-shovel mining system. The aim was to optimize productivity and minimize cost as a function of truck fleet size and allocation. The model was unique because it used output data from a short-term production sequence optimized using a mixed integer linear programming as input. Geologic blocks were modeled as entities that become available for mining based on the production sequence. The output data included the amount of each ore type produced, mill feed and stockpile material, tonnage recovery and resource utilization. Fioroni et al. (2014) also determined the optimal truck fleet size at the end of each year for an underground gold mine. They used total transportation capacity as the performance measure since different truck types were used. Similarly, Dindarloo et al. (2015) modelled an iron ore mining operation using DES to determine the optimum number of dump trucks and cable shovels needed to maximize productivity. They implemented a dispatch algorithm that assigns empty trucks to the idlest shovel. An optimum number of trucks is matched to an optimum number of shovels to meet production targets.

2.2.3. Incorporating Duty Cycles in Fleet Sizing. In most operations, the duty cycle (and associated cycle times) vary at different instances during the operation (this is different from variability in cycle times that are attributed to *randomness*). (Duty cycle is generally defined as the cycle of operation of a cyclical piece of equipment. Aspects of duty cycles for haulage equipment usually include travel times, loading and off-loading an equipment.) The most common cause of such changes in mining is as result of changes in travel distances. Ignoring varying duty cycles of mine equipment in fleet size optimization can result in under/over estimating the optimal fleet size needed for an operation. Selecting sub-optimal fleet size affects the overall mine productivity (Ronen 1988, Callow 2006).

For example, Ataeepour and Baafi (1999) optimize the number of trucks assigned to five different shovels based on queue length, productivity and equipment utilization in an open pit mine. The model does not account for the effect of changing duty cycles resulting from relocation of the shovels. As the shovels move closer or farther away from the crusher, waste dump and stockpile, the traveling distance changes leading to changes in duty cycle and cycle times.

Very few examples can be found in literature that incorporate changing duty cycles in fleet sizing using DES (Ronen 1988, Alarie and Gamache 2002, Awuah-Offei, et al. 2003, Dong and Song 2012). Nevertheless, none of these applications can be found in room and pillar mining, where changes in duty cycle can be documented more frequently. Awuah-Offei et al. (2003) evaluated the changing haul routes, as the mine progressed, as a factor that influences the optimal fleet size. The optimal fleet size (number of trucks assigned to a shovel) was found to increase from six to eight as mining progressed and haul routes got longer. Dong and Song (2012) evaluates the effect of inland transport times and their variability impact on the container fleet sizing. In both cases, the authors demonstrate that changing duty cycles affect the optimal fleet sizes.

This PhD research evaluates the impact of changing duty cycles on fleet size optimization for underground R&P systems.

2.3. PRODUCTION SEQUENCE OPTIMIZATION IN MINING

Production sequence optimization in mining is determining a feasible extraction sequence that maximizes the stated objectives (e.g. net present value) over the mine planning period. The definition of production sequencing depends on the planning horizon (Hartman and Mutmansky 2002). A sequence is said to be feasible if it meets all constraints.

The need for an optimal sequence has spurred extensive research in optimizing mine production sequencing. The extraction sequence in mining is optimized over the life of the mine (or planning period) as more and more geologic data becomes available through the exploration, development and mining process. The production sequencing relies on the geologic properties of the deposit, mining method, economic parameters and technology. Production sequencing is a decision making process, which entails, determining which blocks to be extracted, when they should be extracted and what to use the blocks for once they are extracted (Lambert et al. 2014). Mine production sequencing has come a long way since Lerchs and Grossman (1965) who proposed the basis for modern production sequencing optimization by proposing a method for determining the boundaries of a surface mine. The most common problem that persists with time is the rate and availability of computational power capable of large data manipulation (Johnson 1968).

The production sequencing problem has been modeled and solved using various methods. The next sub-section will focus primarily on how some of these algorithms have been used to formulate production sequencing problems.

2.3.1. Mine Production Sequencing Models. Some of the common algorithms used to develop production sequencing problems include dynamic programming, genetic algorithms, simulated annealing, Markov decision process, and linear programming (and extensions). Perhaps, models based on LP and extensions are the most widely used to model production sequencing problems.

2.3.1.1. Linear programming (LP) models. The ability to model complex systems with a variety of constraints makes LP versatile compared to other mathematical models. LP is used to solve the production sequencing problem in this research. The application of LP for mine production sequencing optimization dates back several decades. In the late 1960s, Johnson (1968) modeled the open pit production sequencing problem as an LP problem using the block modeling concept.[8] Current work done in mine sequence optimization is based on the LP modeling framework by Johnson (1968).

Generally, LP is a class of constrained optimization that seeks to find a set of values for continuous decision variables ($x_1, x_2, \ldots, x_n$) that minimizes or maximizes a linear objective function z, while satisfying a set of linear constraints (a set of simultaneous linear inequalities and/or equations). There are different forms of LP, which includes integer linear programming (ILP) and mixed integer linear programming

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[8] In order to facilitate mine planning, the deposit is usually divided into blocks of mineable units which are commonly known in the literature as geologic blocks or simply blocks (Axelson 1964).

(MILP), depending on whether additional constraints are placed on the decision variables. In ILP, all decision variables are strictly restricted to integer values whereas only some of the decision variables are restricted to integer values for MILP, (Chen et al. 2010). An LP model is generally expressed mathematically as:

$$\text{Maximize } z = \sum c_j x_j \tag{2-4}$$

$$\text{Subject to } \sum a_{ij} x_j \leq b_i \quad (i = 1, 2, ..., m) \tag{2-5}$$

$$x_j \geq 0 \quad (j = 1, 2, ..., n) \tag{2-6}$$

Where:

x_j	Decision variable
c_j	Cost or profit coefficient
a_{ij}	Constraint
b_i	Limit (upper or lower bound) for constraint
z	Objective function value
m	Number of constraints
n	Number of decision variables

Typical objectives in mine sequencing optimization include maximizing the net present value, minimizing cost, minimizing quality (or production) target deviations in each period, or a combination of these. Over the past decade, researchers have introduced unique objectives as a function of the mine production sequence. In mine production sequencing, the decision variables x_j in Equation (2-4), represent the fraction of a block mined in a particular period. A generalize version of Johnson's (1968) model was later presented by Chicoisne et al. (2012). A detailed description of the generalized modeling approach is presented below:

Indices:

x_{bdt}	Binary decision variable with a value of 1 if a block is scheduled to be mined and 0 otherwise
b	Block index
d	Destination index
t	Period index
r	Resource index

Objective function: The objective was to maximize the overall (discounted) profit for a planning period, subject to processing plant capacity (for material and treatment), refinery capacity, maintenance facilities, blasting, stockpile capacity, production rate constraint and other constraints. Although, this objective function is very common for long range mine planning, it is not the only objective function used in mine planning. However, there is no loss of generality by assuming this particular objective function in the discussion here.

$$M\text{ax} \sum_{b\in\beta}\sum_{d=1}^{D}\sum_{t=1}^{T} P_{bdt} \bullet (x_{bdt} - x_{b,d,t-1}) \tag{2-7}$$

Where P_{bdt} is the (discounted) profit generated per unit of block b taken to destination d in period t. x_{bdt} is the fraction of block b taken to destination d in period t.

*Subject to***:**

Resource/equipment constraints: These ensure the amount of resource (equipment) r needed to mine a unit of the material in block b sent to destination d in time period t (a_{bdtr}) does not exceed the available resource (c_{rt}).

$$\sum_{d=1}^{D}\sum_{b\in\beta} a_{bdtr}(x_{bdt} - x_{b,d,t-1}) \le c_{rt} \quad \forall r \in R, \forall t = 1,.....,T \tag{2-8}$$

Reserve constraints: This constraint ensures that the amount of material scheduled to be mined in period t does not exceed the amount available.

$$\sum_{t=1}^{T}\sum_{d=1}^{D}(x_{bdt} - x_{b,d,t-1}) \le 1 \quad \forall b \in \beta \tag{2-9}$$

Since x_{bdt} takes on a binary value (0 or 1), the amount of material sent to destination d from block b mined in period $t-1$ should be less or equal to that in period t.

$$x_{b,d,t-1} \le x_{bdt} \quad \forall b \in \beta, \quad \forall t = 1,...,T, \quad \forall d \in D \tag{2-10}$$

The amount of material sent to destination d from block b at $t = 0$ is equal to zero.

$$x_{bd0} = 0 \quad \forall b \in \beta, \quad \forall d \in D \tag{2-11}$$

Variable/Non-negativity constraints**:** These ensure that the decision variables (x_{bdt}) are non-negative and cannot exceed one.

$$0 \le x_{bdt} \le 1 \quad \forall b \in \beta, \quad \forall d \in D, \quad \forall t = 1, \ldots, T \tag{2-12}$$

Precedence constraints**:** These ensure that if block a precedes block b laterally (or vertically), then block b can only be mined if block a has been mined. The fraction of block b to be mined in period t and delivered to destination d can never exceed that of block a.

$$x_{bdt} \le x_{adt} \quad \forall (a,b) \in \mathrm{A}, \quad \forall t = 1, \ldots, T \tag{2-13}$$

In current versions of the LP model, the destination of the block is defined (x_{bt} instead of x_{bdt}) prior to sequencing (Askari-Nasab et al. 2011, Martinez and Newman 2011, Gholamnejad and Moosavi 2012, Chicoisne et al. 2012). Also, with the increasing use of mixed integer linear programming (MILP) for modeling production sequencing, alternate models of precedence constraints have been formulated (Equation (2-14) and (2-15)). The model requires constraints for both the binary and continuous decision variables (x_{bt} and $y_{bt'}$, respectively).

$$x_{b,t} \leq \sum_{t'=1}^{t} y_{a,t'}, \quad \forall (a,b) \in \mathrm{A}, \quad \forall t = 1,....,T \tag{2-14}$$

$$\sum_{t'=1}^{t} y_{b,t'} \leq x_{b,t}, \qquad \forall t = 1,....,T \tag{2-15}$$

Since the introduction of Johnson (1968)'s research, numerous LP models have being built specifically for different mining operations. Such models now include ventilation constraints (Jawed and Sinha 1985, Brickey 2015), grade blending constraints (Dimitrakopoulos and Ramazan 2004), development constraints (O'Sullivan et al. 2015), space and equipment routing constraint (Nehring et al. 2010), and early start and late start algorithm constraints that restrict the period in which each block is to be mined (Trout 1995, Newman and Kuchta 2007, Topal 2008).

2.3.1.2. Other models. Many other modeling approaches have been used to model the mining production sequencing problem. Dynamic programming, genetic algorithms, simulated annealing, and Markov decision process are common in the literature.

Dynamic programming is known as a simple technique that allows the analyst to solve the optimization problems in stages. At each stage a sub-problem is solved giving the best solution so far to solve the next problem (Bellman 1953). For example, Tolwinski (1998) used dynamic programming to determine the long term production sequence in an open pit mine.

Genetic algorithm (GA) is a metaheuristic random search (based on natural selection) approach used to model and solve combinatorial optimization and scheduling problems. The algorithm uses previous data to determine new search regions where the probability of finding the optimal solution is higher. A detailed description of the application of genetic algorithms to solve production sequencing problems can be found in Knosala and Wal (2001). Samanta et al. (2005), for example, used genetic algorithms to determine the optimal sequence that minimizes quality deviations for a bauxite mine.

Simulated annealing is a probabilistic metaheuristic stochastic optimization algorithm capable of providing good approximations of the optimal solutions to optimization problems. Similar to GA, this algorithm requires an initial (state of the system) value and step size to ensure that the optimal solution is obtained quickly (Eglese 1990). Kumral and Dowd (2005) used simulated annealing to improve the solution to an open pit mine production scheduling problem. Leite and Dimitrakopoulos (2007) also used simulation annealing to optimize the mine production sequence for a copper deposit as a function ore and waste target deviations.

Markov’s decision process (MDP) is a discrete stochastic process used to solve optimization problems. Archambeault (2007), for example, used MDP to optimize an open pit mine production sequence subject to grade and price uncertainty.

2.3.2. Production Sequencing in Underground Mines. Research in production sequencing in underground mines has not been as widespread as that in surface mining (O'Sullivan and Newman 2015). The process of determining the optimum production sequence of an underground operation depends on the mining method, which results in different optimization objectives. Many important questions still remain, including: (1) what are the best constraints necessary to obtain a feasible production schedule?; (2) how can models capture the stochastic nature of mine production variables in the so-called best model and solution algorithms?; and (3) how do we formulate models of practical problems that can be solved with reasonable computational power and within reasonable time without compromising the usefulness of the solution? The mining constraints defined for each production sequencing problem, depends on the deposit, mining method and the objective of the optimization. In polymetallic deposits, heavily controlling ore quality to minimize target deviations, plant capacity and processing requirement may be necessary. On the other hand, coal deposits with relatively uniform quality over the ore domain may prioritize the maximization of production per period as a function of the production sequence.

In recent studies, significant strides have been made to account for uncertainties associated with the ore extraction in optimizing the production sequence. Linear programming specifically does not have the ability to characterize the stochastic nature of systems. Thus, LP approach has been combined with techniques such as simulation and extended as a stochastic integer programming technique (SIP) (Carpentier et al. 2015). The extended versions of LP techniques still have had limited application in underground mine sequencing (Grieco and Dimitrakopoulos 2007, Carpentier et al. 2015). The initial applications of LP to solve underground production sequencing problems were also limited to a short term planning period (Jawed 1993, Winkler 1996) due to the lack of computational power. As a decision making process, researchers have also realized the benefit of using some integer variables (Barbaro and Ramani 1986). By adding integer variables to the traditional continuous variables, the solution time has increased at an exponential rate.

Although advances in computer capabilities have been significant, the long standing problem of solving large scale problems for a long range mine plan without simplifying the problem (and, therefore, sacrificing some of the usefulness of the solution) still persist today (Koushavand et al. 2014, O'Sullivan et al. 2015). In an attempt to overcome this, some researchers have used block aggregation, solving multiple short term problems at the same time, heuristic decomposition approaches or implementing constraints that minimizes problem enumeration can be used to minimize the computational complexity without increasing computer power (Almgren 1994, Sarin and West-Hansen 2005, Newman and Kuchta 2007, Boland et al. 2009).

LP and LP extensions (BILP, ILP, and MILP) have been applied to model and solve underground production sequencing. Early work on modeling and solving underground mine production sequencing with computers dates back to the late 1960s (Mathias 1967). The author determines the long range mine sequence for panel caving molybdenum mine using the critical path method. The constraints modeled include tonnage, grade, and ore reserve.

Gentry (1967) developed the first application of linear programming in underground production sequencing. Gershon (1983) built on the Gentry (1967) approach, solving mine production sequencing, mill blending and processing problems. He built on Johnson's (1968) model by introducing the MIP model that allows for the partial mining of blocks. The optimal sequence obtained did not result in fractional block extraction. Jawed and Sinha (1985) developed an LP model that minimizes cost subject to sub-system (such as ventilation and evacuation capacity) constraints. Almgren (1994) introduces an approach that solves multiple short term production sequencing problems to obtain a solution to long range production scheduling. The author solves a five-year problem by running the model one month at a time. The author compared the solution to this "simplified problem" to the traditional optimization approach and found the simplified problem to produce sub-optimal results. Trout (1995) took a different approach by introducing one of the early works that define decision variables as both continuous and integer. He developed a new mixed integer programming model for both stope extraction and backfill sequencing in maximizing the pre-tax net present value (or simply the net present value as most other researchers refer to it). However, the model was limited in its use by computational requirements.

Following this, several other researchers proposed models of increasing complexity to address different issues. For instance, Winkler (1996) also uses MILP to account for fixed cost in production scheduling. He proposed a novel MILP model that is capable of incorporating fixed costs in sequencing. However, the model could not be developed for multiple periods due to time constraints.

There are more recent examples mainly address various MILP modeling strategies to model the underground sequencing problem with a view to reduce the computational time (Smith et al. 2003, Kuchta et al. 2004, Newman and Kuchta 2007). Smith et al. (2003) presented a new approach to defining the precedence constraint by using hard coded dates to determine stope precedence instead of the traditional graph theories. Kuchta et al. (2004) and Newman and Kuchta (2007) used MILP to maximize the monthly production requirement at the Kiruna mine. The authors applied block aggregation techniques to minimize the computational complexity of the problem. Little et al. (2008) introduced a new MIP approach to minimize the solution time by reducing the number of binary integer variables. The authors introduced the concept of natural sequencing and natural commencement. The natural sequencing approach combines consecutive predetermined activities that are scheduled in a period using a single variable. The natural commencement approach also minimizes the number of constraints for activities that always have the same variable values in each period. The work successfully minimized the solution time by reducing five sets of binary variables to one.

Sarin and West-Hansen (2005) presented one of the few examples of LP-based production scheduling in R&P mines. They developed a mixed integer programming model for an underground coal mine with multiple mining methods capable of meeting the desired quality while maximizes the NPV. They also used a benders decomposition technique (generates constraints when needed) to minimize the search space for a feasible solution. The pre-processing technique was successful in minimizing computational time.

Topal (2008) presented an early start and late start algorithm that defines the precedence restrictions for each mining unit in their MILP model of the Kiruna Mine. The authors implemented a machine placement technique based on the block aggregation method to minimize computational complexity. To minimize the solution time, O'Sullivan et al. (2015) developed an optimization-based decomposition heuristic approach to solve an underground production sequencing problem.

Bienstock and Zuckerberg (2009) presents a novel approach based on an iterative Lagrangian-based algorithm that shift the dynamics of solving large precedence constraint production scheduling LP problems. Although the verification of this approach was done using an open pit operation, the algorithm has gained popularity in recent underground mining research (Brickey 2015). Brickey (2015) solves her LP model that accounts for ventilation requirement in underground mines using an Orthogonal Matching Pursuit (OMP) solver which utilizes the Bienstock-Zuckerberg algorithm. The author demonstrates the ability to solve large scale problems with heuristics to induce integer solutions.

2.3.3. Accounting for Risk in Production Sequencing. Ignoring the effect of geologic and operational uncertainties (such as grade, geotechnical and environmental uncertainties) in mine production sequencing can result in a vast difference between the planned and actual profits. Researchers in the past have solve production sequencing problems that seemingly improved the net present value without accounting for uncertainties inherent in the mine. The impact of ignoring uncertainties on profit has been demonstrated by current research (Dimitrakopoulos 2004) and has spurred improvements and extensions of LP in the past decade. The main focus in current literature is how to characterize risk associated with such uncertainties in LP production sequencing problems. To answer this question, there is a shift from deterministic mathematical modeling to stochastic modeling. The risks associated with estimates of underground mine parameters (*e.g.* due to equipment reliability, geotechnical, market, legal, and environmental risks) are vast and add significantly to the complexity of the model. To harvest the benefit of LP in solving production sequencing problems, researchers cannot ignore the effect of risk on the present value (or other objective) of the mine. Therefore, these risks have to be accounted for in production sequencing to ensure the solution is optimal under known uncertainties.

One such method of incorporating uncertainty in traditional LP based production scheduling optimization is a stochastic optimization approach investigated by a number of researchers (Menabde et al. 2005, Boland et al. 2008, Dimitrakopoulos 2011, Goodfellow and Dimitrakopoulos 2013). In this approach, parameters in the optimization model are deemed to be stochastic and thus can yield various realizations. Hence, the objective function and constraints (which are a function of these parameters) also become stochastic. Hence, the problem changes from maximizing (or minimizing) the objective function, to maximizing (or minimizing) the expected value of the objective function or some other measure of the stochastic objective function (e.g. minimizing the variance of the deviation).

Smith and Dimitrakopoulos (1999) present one of the early attempts to incorporate uncertainty in mathematical modeling of production sequencing problems. The authors account for thickness and grade uncertainty in optimizing the production sequence. They evaluated the effect of incorporating uncertainty on the production schedule by using eight simulated realizations of the deposit. The results from the analysis indicate that, the grade target deviation are greater in six out of the seven periods simulated when uncertainty is not accounted for in production sequencing. Godoy (2003) proposed a new algorithm based on simulated annealing that determines the optimal sequence of extraction under grade uncertainty. The authors use equally probable realizations of block grades instead of a single estimate for each block. The algorithm first generates optimal solutions to the production sequencing problem, using LP, for each realization of the orebody. Simulated annealing is then used to generate an "optimal" solution which minimizes the average deviation from the production targets (obtained from the LP solutions) for a given mining sequence over a series of simulated orebody grade models. The solution will also meet the production targets established by all the LP solutions. This approach makes the production sequence more complex. Dimitrakopoulos (2004), though using the same approach of equally probable grade models to account for grade uncertainty, introduced the concept of discounted risk cost penalties that forces the solution to defer mining blocks with high uncertainties to later production periods. This approach has been used by many other researchers (Menabde et al. 2004, Dimitrakopoulos 2004, Ramazan and Dimitrakopoulos 2013). Dimitrakopoulos (2011) introduced a new approach of integrating orebody uncertainty in production scheduling by combining stochastic simulation (simulated annealing) with stochastic optimization (stochastic integer programming).

Gholamnejad and Moosavi (2012) did a comparative study of deterministic and uncertainty-based approaches for optimizing the long term production schedule in an open pit iron ore mine. They determined the optimal production schedule based on tonnage uncertainty. They defined uncertainty as the probability that a block is an ore block using indicator kriging. Their results indicate that traditional algorithms overestimate the NPV when risk is ignored. By accounting for geologic risk, the algorithm maximizes the NPV while minimizing risk, therefore, higher valued blocks with high risk are deferred to later periods. This conclusion was drawn based only on blocks with probability higher than 0.5. The deviation between both approaches might vary depending on the selected cut off probability. Alonso-Ayuso et al. (2014) also used stochastic integer programming model to account for the uncertainty in copper prices over time. The model maximizes the profit over time using a multistage scenario tree to account for price uncertainty. The authors converted the stochastic model to a deterministic mixed binary integer programming equivalent that incorporates uncertainty.

The research also included a comparative study of the use of risk neutral strategy and risk averse measures in accounting for uncertainty.

Koushavand et al. (2014) solved an MILP production sequencing problem that accounts for grade uncertainty and identifies factors that control the importance of uncertainty. The risk-related cost uncertainty was modeled as the cost of under-and-over producing. The work determines the trade-off between minimizing risk and maximizing the NPV. The author compares a deterministic solution to the stochastic model developed. The results indicate that the cost of risk was insignificant for the case studied, therefore, the difference in NPV was insignificant as well. Carpentier et al. (2015) also presents a MILP model that incorporates geologic and cost uncertainty in underground mine production sequencing. Their objective function maximizes profit, while minimizing the cost of deviation from development, production, opening and closure of mines, keeping the mine in operation, handling backfill material, and geological risk.

The main focus of characterizing uncertainty in production scheduling has been to account for the effect of geologic uncertainty on the production sequence. One can see from the literature that accounting for uncertainty in production sequencing in surface mining has advanced more significantly as compared to underground mining. Accounting for risk in underground mining is based on techniques developed for surface mining method (Li et al. 2004, Grieco and Dimitrakopoulos 2007, Carpentier et al. 2015). The risks associated with mine production in underground mining are more complex and differ in some significant respects from surface mining. Mine engineers are aware of these risks (e.g. geotechnical, environmental, legal, geological, resource availability) and their effect on production sequencing. It is therefore essential to advance research frontiers in incorporating multiple risks inherent in underground mining.

Although stochastic (integer or mixed) linear programming is a viable option, this approach can be very complicated and computationally difficult to apply to large scale real-life problems. Although the deterministic equivalent of the stochastic LP problem can be used to reduce the computational time, it is still computationally difficult to solve as the deterministic equivalent increases the size of the problem (Smith and Dimitrakopoulos 1999, Anastassiou 2000). The challenge is to develop modeling frameworks that are flexible (allowing as many as necessary risks to be modeled) and yet efficient (computationally tractable). This research focuses on incorporating multiple risks in a deterministic modeling framework for an underground R&P production sequencing problem.

2.3.4. Solutions to Integer LP-based Mine Production Sequence Optimization Problems. LP problems are solved in polynomial time with respect to the size of the binary coding of the input data. Binary integer linear programs (0/1 decision variables) are known to be non-deterministic polynomial (NP) time hard[9] problems (Megiddo 1986, Schrijver 1998). The relationship between computational times for these problems and number of decision variables, in the best case, is polynomial. Mine production systems consists of millions of jobs scheduled over long periods of time. Modeling mine production sequencing problems as binary integer linear programming (BILP) results in large precedence constraints and decision variables with very high computational

[9] A problem is NP-hard if an algorithm for solving it can be converted into one for solving any NP-problem (non-deterministic polynomial time) problem (Weisstein 2009).

complexity. Various methods and algorithms have being developed that are capable of solving NP-hard problems. Such methods include the Lagrangian relaxation method, branch and bound method, branch and cut method, cutting plane, clustering approach and dynamic programming (Osanloo et al. 2008).

The Lagrangian relaxation method is a pre-processing approach used to minimize complete enumeration of the production sequencing problem. The first application was by Dagdelen and Johnson (1986), to solve long-term production scheduling for an open pit mine. The approach entailed decomposing the problem into single period sub-problems based on the Lagrangian. In doing so, the problem can be solved more easily by most algorithms. The problem is also relaxed by incorporating mining and milling constraints into the objective function with Lagrangian multipliers.

The framework of dynamic programming that allows the division of the optimization problem into sub-problems, which makes it easier to find optimal solutions quicker. Compared to other operation research techniques, this approach does not use the traditional mathematical formulation framework.

Many authors have used meta-heuristic approaches such as the genetic algorithm, simulated annealing and tabu search to solve production sequencing problems because of the inability of mathematical optimization approaches to solve complex large scale problems (Eglese 1990, Knosala and Wal 2001, Samanta et al. 2005).

The branch and cut method is the most common approach used to solve integer optimization problems. The method is an exact algorithm that combines the cutting plane with the branch and bound method. The method works by solving the LP relaxation of the sub-problems until a feasible integer solution is found. In order to obtain a quick solution of the ILP problem with minimum computational effort, the cutting plane algorithm improves the relaxation (by making it more restrictive) so as to more closely approximate the IP problem. Typically, branch and bound algorithms are then used to solve the problems using a divide and conquer approach (Mitchell 2009).

The CPLEX optimization software based on the branch and cut method is currently the most powerful tool on the market for solving integer programming problems. There are very few examples in literature where the authors develop a novel branch and cut algorithm to specifically solve a production sequencing problem. In the only example this author could find, Caccetta and Hill (2003) used a branch-and-cut method to solve the production sequencing problem for an open pit mine. To develop such algorithms, the researcher will have to formulate valid cutting planes that are applicable to all instances of the problem. This process is quite challenging (this is further discussed in Section 2.4).

On the contrary, most research on mine production sequencing has combined heuristic and meta-heuristic solution techniques with the branch and cut method to minimize the solution time. The most common approach is block aggregation which reduces the number of binary or integer decision variables (Topal 2008, Boland et al. 2009, Askari-Nasab et al. 2010). In this research, block aggregation is used to minimize the number of binary variables in the production sequencing problem. The research also introduces custom cutting planes with the branch and cut method to minimize computational complexity. Further details on these exact algorithms are discussed in the next section.

2.4. THE BRANCH AND CUT METHOD FOR SOLVING COMBINATORIAL PROBLEMS

Combinatorial optimization problems such as the production sequencing problem is a class of problems in which an optimal solution has to be selected from a finite number of possibilities. Combinatorial optimization problems are classified as NP-hard problems. Branch and cut is an efficient non-deterministic approximating algorithm that can solve production sequencing problems in which the decision variables take on binary values.

2.4.1. The Branch and Cut Algorithm. Developed in 1991, the branch and cut method is an exact algorithm that combines the cutting plane method with the branch and bound method. The branch and cut is used to obtain near optimal solutions to pure integer programming problems with finite bounds on the integer variable (Chen et al. 2010). For integer LP problems, the method works by solving successive LP relaxations of the integer programming problem and implementing cutting planes that improves the LP relaxations of the sub-problems, thus closely approximating the integer problem. The aim at each node is to generate tighter bounds before branching and pruning. Branch and cut has being used to solve many combinatorial optimization problems (Brunetta et al. 1997, Bixby and Lee 1998).

The next sections explain in detail the components of the branch and cut method.

2.4.1.1. Branch and bound algorithm. The goal of the branch and bound method is to solve integer and discrete optimization problems without complete enumeration. Due to the exponential increase in possible solutions to the problem with the number of decision variables, the algorithm aims to solve a small number of optimistic solutions while ignoring the large number of inferior solutions. The method uses bounds and the current value of the best solution to search parts of the solution space (Clausen 1999).

The scheme employed is known as the “divide and conquer.” The algorithm divides (branching) the integer problem into sub-problems. Each of the sub-problems is solved exactly or approximately to obtain an upper bound[10] (bounding) on the objective function value. The upper bound obtained is compared to the objective function value of an existing integer solution obtained by solving other sub-problems. If the upper bound is less that the objective function value, the solution to the original integer problem cannot be found within the feasible space associated with the sub-problem. The upper bound is used as a guide to obtain optimality with minimal enumeration. The concept behind obtaining an upper bound of the solution of the sub-problem is the relaxation of the problem. The most common relaxation is solving the LP equivalent of the problem (Mitchell 2008). The method is developed based on the realization that solutions to integer and discrete problems have an upside down tree structure. For example, consider the complete enumeration of an integer programming problem with binary (0, 1) variables x_1, x_2 and x_3 (Figure 2-1).

The top (all solutions) is the r*oot* (root node) of the tree with the *leaves* (leaf nodes) below it. The leaves represent the actual enumeration (branching and bounding) of the integer problems.

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[10] This discussion assumes a minimization problem. There is no loss of generality with this assumption as a maximization problem can be converted to a minimization one.

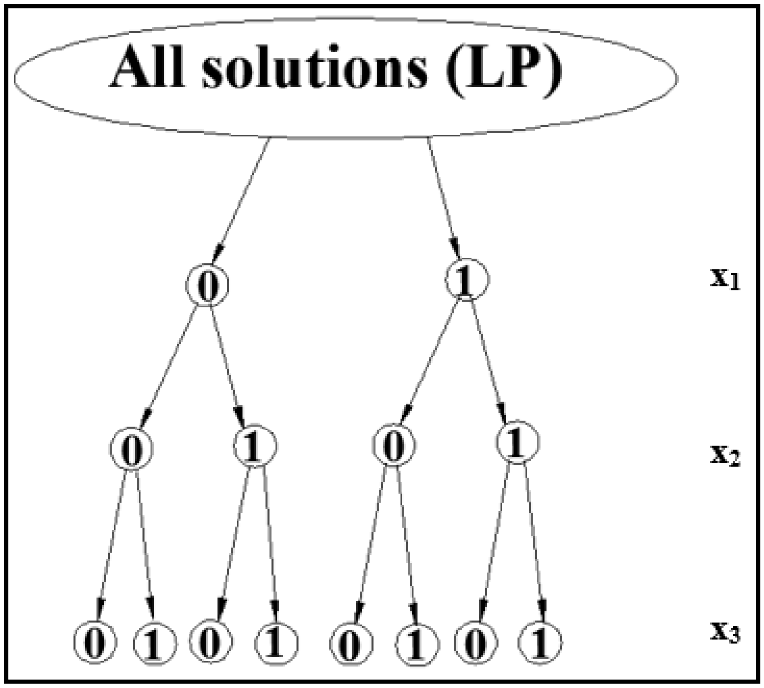

Figure 2-1 A complete enumeration tree

The nodes represent possible solutions that can be obtained by growing the tree. For two nodes connected in the branch and bound tree, the one closer to the leaves is known as the *child* node, and the one closer to the root is the *parent* node. The concept of branch and bound is to avoid growing the entire tree (complete enumeration) by growing only the most optimistic nodes at any instance. An important aspect of the algorithm, is pruning the tree. This represents cutting off or permanently removing nodes if it can be proven that none of its children will ever be feasible or optimal.

Some of the terms used to describe the algorithm are defined below (Chinneck 2006):

- Node: a partial or complete solution (i.e. only some of the variables have values)
- Bud (bud node): a feasible or infeasible partial solution. Similar to a tree, it is a node that is yet to be grown further.

- Leaf: a complete solution in which the values to all the variables are known.
- Bounding function: is the optimistic estimator of the objective function value at a bud node. The function overestimates or underestimates, if the aim is to maximize or minimize the objective function, respectively.
- Branching: is the process of growing a child node from a bud node.
- Incumbent: the current best complete feasible solution found.

In order to avoid complete enumeration, the algorithm must define policies with regards to selecting the next node to be grown, the next variables, how to prune, and termination criteria for the algorithm. These policies must be defined before solving integer problems with branch and bound algorithms.

There are three common policies for the next node to be grown: best first, depth first, and breadth first. The best first approach selects the bud node with best bounding function value on the tree. The depth first approach selects only from new sets of bud nodes just created. This policy is usually used when the tree is very deep and integer solutions are rare. The breadth first grows bud nodes in the same order in which they are created. This policy is usually used if the analyst is aware that the solution is not far from the root node. Other methods include the sum of integer infeasibilities, best estimate using pseudo-costs and best projection.

The variable selection policy determines the next variable to be chosen in order to create the child node of a bud node. A simple approach is to select the variables in their natural order (x_1, x_2, x_3), although an efficient policy can be tailored to the optimization problem. Some of the policies that have been implemented included the most/least infeasible integer variable, Pseudocost estimate and Driebeck-Tomlin penalties (Mitchell 2008). A general approach to implement a pruning policy is by comparing the bounding function at the bud node to that of the incumbent solution. If the value is worse than that of the objective function value at the incumbent solution, the bud node can be pruned. In certain instances where best possible objective function value obtainable by expansion can be seen directly, the bud node expansion can be halted. This is known as *fathoming* a node (Chinneck 2006).

To solve an integer programming problem using branch and bound, the general algorithm is as follows:

1. Initialization: define the set of active sub-problems and the initial upper bound (bounding function value) and the incumbent objective function value.
2. Termination: if no active sub-problems exist, then the integer solution that yielded the incumbent solution is optimal (if it is within acceptable distance to optimality). If no such integer solution exists, the integer problem is infeasible.
3. Problem selection and relaxation: select and solve the relaxation for a sub-problem.
4. Fathoming and pruning: if the optimal objective function value for the relaxation (in step 3) is worse that the incumbent value, go to Step 2. If the objective function value is better, make it the new incumbent value. Delete all sub-problems with objective function values worse than the new incumbent value. Go to step 2

5. Partitioning: partition a constraint set of the problem into multiple sub-problems. Each new sub-problem is bounded by the new incumbent solution. Go to Step 2

Although the branch and bound algorithm is better than full enumeration, it is not very efficient in solving problems with a large number of possible solutions. The algorithm still evaluates too many solutions. One of the challenges faced by the branch and bound algorithm is the availability of integer feasible solutions during execution. Since pruning is only possible after obtaining a fathomed solution (Step 4 requires an incumbent solution), when integer solutions are not readily available pruning a node becomes impossible. Thus, branch and bound can fail to find an optimal solution due to inadequate memory as a result of excessive accumulation of active nodes (Lee and Mitchell 2000).

These limitations of the branch and bound algorithm are overcome when used in conjunction with cutting planes in the branch and cut algorithm.

2.4.1.2. Branch and cut algorithm. Developed in the 1950s, the cutting plane method is a convex mathematical technique used to solve integer and mixed integer linear programming problems. A cutting plane is a linear inequality, in that is generated (when needed) in the course of solving an integer linear program problem as a sequence of linear programs (Lee 2004). The computational complexity of the branch and bound method can be improved immensely by implementing cutting planes at the root node or at every leaf node. For the past decades, general inequalities such as the Gomory cutting planes, Knapsack problems based cutting plane, Fenchel cutting planes and the lift and project cutting planes have been developed that prove useful for most problems (Mitchell

2009). The general framework of the cutting plane method for solving an integer linear program is as follows:

1. Define the linear programming (LP) relaxation of the integer problem
2. Solve for the optimal extreme-point solution (**x***) of LP
3. If the solution in step 2 (**x***) is all integer subject to existing constraint, terminate the algorithm because it is optimal.
4. If the solution is not optimal, find an inequality that is satisfied by all feasible solutions of the ILP, but violated by the extreme-point solution (**x***). Add on the inequality to LP, and go back to Step 1.

The optimal solutions to the LP problem become a sequence of upper bounds on the optimal value of the integer problem as cutting planes are successively added on. The most challenging aspect of this method is finding valid cutting planes. By incorporating the cutting plane method into the branch and bound method, the branch and cut algorithm becomes:

1. Initialization: define the set of active sub-problems and the initial upper bound (bounding function value) and the incumbent objective function value.
2. Termination: if no active sub-problems exist, then the integer solution that yielded the incumbent solution is optimal. If no such integer solution exists, the integer problem is infeasible.
3. Problem selection: select and delete sub-problem from active set.
4. Relaxation: solve the relaxation for a sub-problem. If the problem is infeasible go to Step 6, otherwise if it is feasible, add the solution of the sub-problem to the set of feasible solutions.

5. Add cutting planes: if needed, search for cutting planes (inequality) that are violated by the feasible solution to the sub-problem. If found add it to the LP relaxation of the sub-problem and return to Step 4 (solve the problem again).

6. Fathoming and pruning: if the optimal objective function value for the relaxation is worse that the incumbent value, go to Step 2. If the objective function value is better, make it the new incumbent value. Delete all sub-problems with objective function values worse than the new incumbent value. Go to step 2

7. Partitioning: partition a constraint set of problem (select node for expansion) into multiple sub-problems. Each new sub-problem is bounded by the incumbent solution. Go to Step 2

A simple two variable example is used to illustrate the branch and cut algorithm (Mitchell 2002). Consider the integer problem in Equation (2-16) .

$$\begin{aligned} & Min\ z = -6x_1 - 5x_2 \\ & subject\ to \quad 3x_1 + x_2 \leq 11 \\ & \qquad -x_1 + 2x_2 \leq 5 \\ & \qquad x_1, x_2 \geq 0,\ integer \end{aligned} \tag{2-16}$$

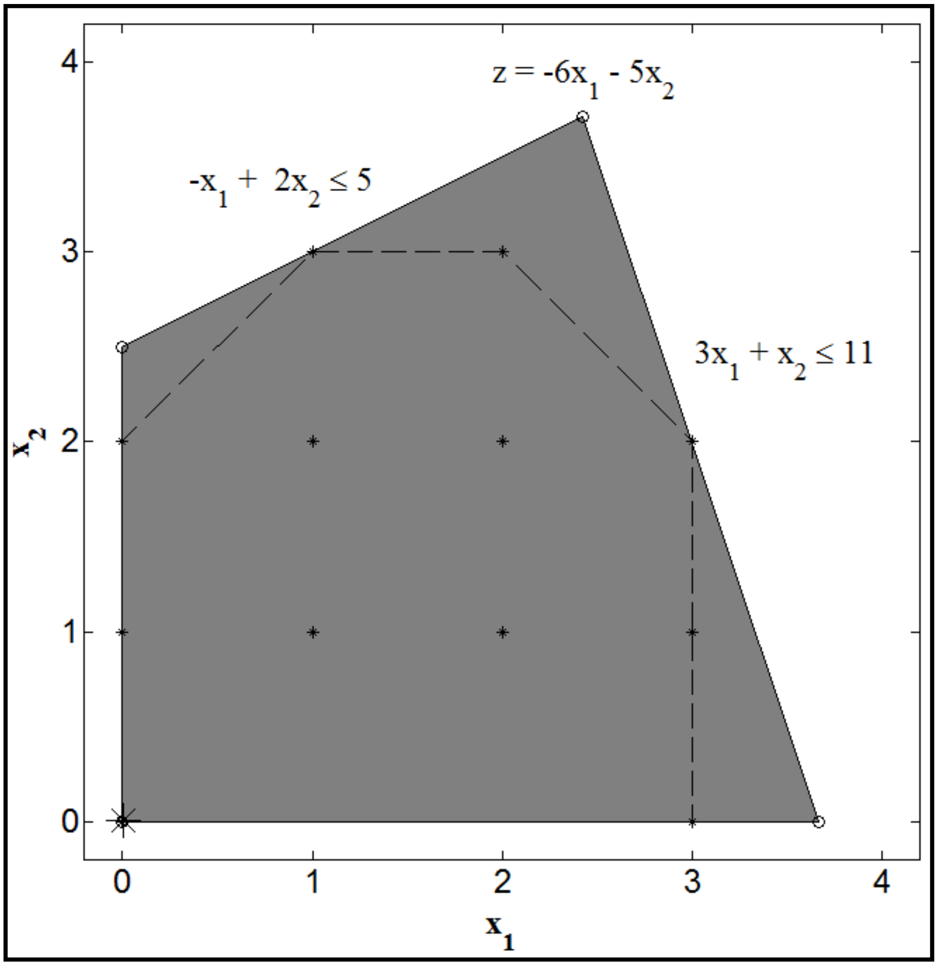

Figure 2-2 Graphical solution to Equation (2-16)

The region (Figure 2-2) marked by the polygon contained in (gray area) solid lines contains continuous and integer solutions to the LP relaxation of the problem. From Figure 2-2, it can be shown that the minimum objective function value is obtained by the integer solution x_1 = 3, x_2 = 2.

Using the branch and cut algorithm, the first step is to solve the LP relaxation of the problem by ignoring the integer restriction. The solution to the problem is x_1 = 2.43, x_2 = 3.71 with an objective function value (upper bound) of -33.14. The next step is to decide whether to divide the problem into sub-problems (branch) or improve the LP solution using a cutting plane (cut).

Dividing the problem on x_1 result in the following sub-problems: Equation (2-17) and Equation (2-18) with solutions of x_1 = 3, x_2 = 2 and x_1 = 2, x_2 = 3.5, respectively. The objective function values are -28 and -29.5, respectively.

Sub-problem 1:

$$\begin{aligned} \min\ z &= -6x_1 - 5x_2 \\ \textit{subject to} \quad 3x_1 + x_2 &\le 11 \\ -x_1 + 2x_2 &\le 5 \\ x_1 &\ge 3 \\ x_1, x_2 &\ge 0,\ \text{integer} \end{aligned} \tag{2-17}$$

Sub-problem 2:

$$\begin{aligned} \min\ z &= -6x_1 - 5x_2 \\ \textit{subject to} \quad 3x_1 + x_2 &\le 11 \\ -x_1 + 2x_2 &\le 5 \\ x_1 &\le 2 \\ x_1, x_2 &\ge 0,\ \text{integer} \end{aligned} \tag{2-18}$$

The first integer solution is obtained by solving sub-problem 1, which now becomes the incumbent solution. Sub-problem 2 needs to be solved further to determine if a better integral solution can be obtained since its bounding function value is lower than the incumbent solution. A cutting plane is implemented to improve the LP solution for sub-problem 2. A valid cutting plane will be violated by the solution $x_1 = 2$, $x_2 = 3.5$ but satisfied by all integer feasible solutions in sub-problem 2. A new sub-problem (Sub-problem 3) is obtained by adding such a cutting plane.

Sub-problem 3:

$$
\begin{aligned}
& \min\ z = -6x_1 - 5x_2 \\
& \textit{subject to} \quad 3x_1 + x_2 \leq 11 \\
& -x_1 + 2x_2 \leq 5 \\
& x_1 \leq 2 \\
& 2x_1 + x_2 \leq 7 \ \text{(Cutting plane)} \\
& x_1, x_2 \geq 0,\ \text{integer}
\end{aligned}
\tag{2-19}
$$

The solution to the LP relaxation of sub-problem 3 is $x_1 = 1.8$, $x_2 = 3.4$ with an objective function value of -27.8. The optimal value obtained from the LP relaxation of sub-problem 3 is greater than the current incumbent solution. That is, any integer solution obtained from sub-problem 3 will be worse than the current incumbent solution. The incumbent solution, is therefore, the solution to the original integer problem.

2.4.2. Generating Valid Cutting Planes. As explained earlier, the most challenging aspect of the branch and cut algorithm is how to generate valid cutting planes for the particular problem. For a general integer LP solver, valid cutting planes have to be generated for all possible integer LP problems. There are some methods available for doing that. However, these cutting planes may not be the best cutting planes to ensure efficient solution for any particular problem (e.g. integer LP problem of mining production sequencing) since they do not take into account the peculiar characteristics of the problem. Nonetheless these cutting planes are useful because they are implemented in commercial integer optimization solvers (e.g. CPLEX), which are used to solve most optimization problems. The common cutting plane methods (including Chvátal-Gomory cutting planes, cutting planes based on polyhedral theory and lift and project cutting planes) are discussed below.

Chvátal-Gomory cutting planes were initially developed by Gomory (1958) using the simplex tableau. The convergence to an optimal solution was very slow and made the approach numerical instable. Chvátal (1973) introduced and implicitly described the concept of integer rounding. Integer rounding entails combining the linear inequalities of the current linear programming relaxation subject to the integer variables. The positive inequality constraints of the problem are summed up and the coefficient of the resulting constraints rounded down to the nearest integer. To obtain a convex hull[11] of feasible

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[11]Convex hull of a set is the smallest convex (non-intersecting) polygon that contains all the points in the set (Goutsias 2006).

integer points, the number of iterations needed is known as the Chvátal rank. The Chvátal-Gomory cutting plane is the most common cutting plane implemented in branch and cut algorithms.

One of the main contributions towards solving integer problems using the cutting plane method was the introduction of polyhedral theory in the 1980s, which allows the generation of strong cutting planes. The polyhedral theory is based on the implementation of cutting planes that use the facets of the convex hull of integer feasible points as cuts. Facets are faces of a polytope[12] with one less dimension than that of the polytope. In the example given in Figure 2-2, all the dashed lines are facets. If the convex hull of all integer feasible points is known, the integer problem can be solved as an LP problem that minimizes the objective function over the convex hull.

Other cutting planes have been proposed based on various ideas relating to integer optimization problems. Some of the common ones include knapsack problem based, Fenchel and lift and project cutting planes. The knapsack problem is a single constraint optimization problem. Most integer programming problems can be formulated as combinations of multiple Knapsack problems. The method entails generating facets and strong cutting planes for the Knapsack problems and adding them as cutting planes to the LP relaxation of the integer problem (Mitchell 2009). The Fenchel cutting plane method solves the separation problem rather than the traditional method of using explicit knowledge of the polyhedral structure of the problem. The cutting plane excludes the

[12] Polytopes are geometrical figures with flat sides bounded by portions of planes, lines, or hyperplanes. It exists in any number of dimensions. A polygon is a 2-dimensional polytope (Coxeter 1973).

feasible solution of the current sub-problem without excluding the convex hull (Boyd 1994). Lift and project cutting planes involve generating of higher dimensional representations of the convex hull (lifting) that is projected back to generate multiple cutting planes. The aim is to reduce the size of the LP needed to generate the cutting plane (Balas et al. 1993).

2.4.3. Role of Pre-Processing in Efficiency of Branch-and-Cut Algorithm. In most cases finding integer solutions to the LP relaxation can be cumbersome. Research to date has demonstrated that a large scale real-world integer problems cannot be solved without using some form of heuristics or pre-processing approaches. Pre-processing approaches are often used to convert fractional solutions to integer solutions that result in the pruning of other sub-problems (Guignard 2010). Many pre-processing techniques in the literature are implemented before initiating the branch and cut algorithm. This includes simply eliminating implicit slack variables (removing empty columns and rows), removing redundant constraints, strengthening bounds on each constraint (right-hand-side vector), coefficient reduction, aggregation and the use of specialized cuts (Mitchell 2009). The most common pre-processing techniques currently used in mining are the aggregation (Topal 2008, Boland et al. 2009, Askari-Nasab et al. 2010) and specialized cutting plane technique (Bley et al. 2010). In this research, the effect of block aggregation and specialized cutting planes on computational complexity were investigated for the room and pillar mine sequencing problem.

The aggregation approach is such that multiple variables can be combined as one, which minimizes the size of the constraints matrix, as well as the number of integer (binary) variables. This is possible if satisfying the bounds on some variables implies satisfying the bounds of other variables. However, not all variables have this property and can satisfy this condition. Consequently, the solution to the integer problem may vary significantly from the optimal solution depending on the size of the aggregates and aggregation method used. Ramazan (2007) reduces the number of binary variables from 37, 800 to 4,920 using the aggregation approach. He solved the problem, which could not otherwise be solved in 36 minutes. Ramazan et al. (2005) aggregated ore and waste blocks to decrease the number of binary variables in the integer programming model. The block aggregation was performed using a mathematical programming approach to minimize loss of information pertaining to each block. Boland et al. (2009) introduced a disaggregation approach in conjunction with block aggregation to solve the production scheduling problem. The authors realized that block aggregation, which is often referred to as “binning,” minimizes the size and computational time of the problem. However, in all these cases, the aggregation strategy affects the optimal solution and many authors have called for further research to determine optimal aggregation strategy (Askari-Nasab et al. 2010)

Specialized cutting planes can be added to the problem to eliminate sub-problems that the analyst knows are sub-optimal. This approach requires in-depth knowledge of the problem and the nature of optimal solutions to the problem. For example, mine engineers are aware of sections of the mine (such as the development area) that has to be mined in specific periods over the planning horizon. A specialized cutting plane can be included to eliminate solutions that do not mine such blocks in an expected period. Chvátal-Gomory cutting planes can be formulated from existing active constrains (such as the precedence and production constraints) to eliminate infeasible sub-problems. For example, Bley et al. (2010) solved the open pit production scheduling problem presented by Caccetta and Hill (2003) using cutting planes generated by combining the production and precedence constraints into precedence constrained knapsack problems. The authors noted the decrease in solution time when cutting planes are implemented. This approach shows real potential to reduce the computational times associated with mine production sequence optimization.

This approach is not possible for generalized integer optimization solvers because it requires intimate knowledge of the particular optimization problem. However, for a specific problem (e.g. room and pillar mine production sequencing problem) it is a viable strategy to reduce the computational time required to solve the problem. The most challenging aspect of this idea is the time it takes to generate the cutting plane. Specialized cutting planes are efficient if the computational time required to construct and apply the cutting planes are more than compensated for by the savings in solution times of the problem (Bley et al. 2010). This research explores the effect of different specialized cutting planes, specific to underground mining operations on the solution time.

3. APPLICATION OF DISCRETE EVENT SIMULATION IN OPTIMIZATION COAL MINE ROOM AND PILLAR PANEL

3.1. INTRODUCTION

This chapter focuses on the use of discrete event simulation to determine the optimal panel width that increases productivity and minimizes cost. An approach is presented based on simulation experiments to accomplish this goal. An existing room and pillar coal mine in Illinois was used as a case study to demonstrate the practical application of the approach.

One of the most important aspects of selecting an optimal panel design is the rate at which coal is extracted. A smaller panel width may result in a faster extraction (higher productivity[13]) if all other parameter are optimal for the selected width. For an existing mine, selecting a smaller or larger panel width may reduce the productivity, if the existing fleet size is not optimal for the system. Too large a fleet in the system (due to a small panel width) results in longer queues and higher waiting times leading to under-utilization of the haulage fleet. However, a less than optimal number of cars will also under-utilize the CM. The width of the coal panel should, therefore, be selected to optimize the productivity given specific mining conditions.

The panel width also affects the unit cost of operation (which includes all fixed costs and variable costs incurred during production). Some of the costs associate with R&P production systems include, the operating cost of the shuttle car, continuous miner (CM), belt feeder, as well as cost associated with roof support. As the width of the panel changes so does the cost incurred to extract a unit of coal. For example, larger panels

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[13] Productivity is the rate of output (material production) per unit input in a period.

result in longer tramming distances and increased ground control requirements resulting in higher fuel, maintenance and labor costs.

Current methods (*e.g.* pressure arch concept) used to determine panel width are primarily based on geotechnical properties (such as over burden thickness, pillar shape and size), especially, when pillar recovery is a consideration (Standridge and Nicholas 2012, Luo 2015). Most engineers use experience and practical experimentation to select the *optimal* panel width or optimize other aspects of the production process such as the haulage system to maximize productivity. While these factors are essential in panel design, it is important to optimize the productivity and unit cost as a function of panel width at the initial stage of mine design. This should be done rigorously to ensure the selected panel width maximizes productivity or minimizes unit costs, given all other constraints.

3.2. FRAMEWORK FOR PANEL WIDTH OPTIMIZATION USING DES

Ultimately, optimizing a design parameter is an optimization problem as described by Equation (3-1). The decision variable, vector $\mathbf{x}$, represents variables that affect the objective function $f(\mathbf{x})$. Possible values that these variables can take make up the set of feasible solutions (alternative designs).

$$\text{minimize } f(\mathbf{x}) \qquad (3\text{-}1)$$

$$\text{subject to } \mathbf{x} \in \Omega$$

In the case of panel width design, the objective function could reflect the desire to maximize mining recovery and productivity as well as minimize unit operating costs.

This work focuses on the dual objective of maximizing productivity and minimizing unit costs. Decision variables can be panel width, cut sequences, the number of continuous miners (CMs), and the number of haulage units assigned to each CM.

If the objective function can be written mathematically (explicit) in terms of the decision variables and all constraints can be described similarly, there are many techniques to solve such optimization problems. Often, however, the objective function is highly nonlinear and implicit.[14] In such cases, simulation is one of the very few techniques that can solve the problem (Kleijnen 1998). In the case of panel width optimization, productivity and unit costs associated with cutting, loading, and hauling as a function of the panel width, equipment fleet, and cut sequence are nonlinear and implicit. DES offers a means to estimate the unit cost and productivity for a given panel width, equipment fleet, and cut sequence.

The approach taken in this work is to:

1. Build a valid DES model of the coal loading and hauling operations;
2. Determine a feasible set of decision variable values (panel widths, fleet, and cut sequences);
3. Estimate objective function values (productivity and unit costs) for each possible solution from the feasible set; and
4. Select the optimal solution based on the objective function to reflect relative importance of productivity and unit costs.

[14] An implicit objective function, as used here, refers to an objective function that cannot be expressed as a function of variables in a particular time step alone. Such functions require knowledge of the variables at multiple time steps. See, for instance, Zou (2012).

Generally, simulation optimization methods are used to find the best input variable values from all possible values (the feasible set) without explicitly analyzing each possibility. Common simulation optimization methods currently used for system optimization include the gradient search method, stochastic approximation, response surfaces methodology, heuristic methods, and statistical methods (Carson and Maria 1997). Response surface and gradient based methods are designed to solve optimization problems with continuous variables. The decision variables of the panel width optimization problem (number of entries/panel width, number of cars, and cut sequence) are discrete and cannot be solved using continuous-based methods.

Statistical methods (e.g. ranking and selection) are computationally exhaustive. These methods evaluate all system alternatives instead of a finite set. The methods are also limited to only small problems since they examine the entire feasible set.

Heuristic and metaheuristic (such as simulated annealing and genetic algorithm) methods can be used to optimize discrete stochastic problems. The development of computer simulation software has significantly minimized the complexity and time needed to solve large optimization problems. DES software generally adopts deterministic metaheuristic optimization algorithms such as tabu search and genetic algorithm to solve discrete optimization problems with minimal analytical effort. In contrast, metaheuristic algorithms (*e.g*. genetic algorithm and simulated annealing) converge too slowly for practical application.

The panel width optimization problem is complex with each estimate of the objective function requiring significant time (each estimate requires running the simulation model for the required number of iterations). The use of algorithms such as genetic algorithm and simulate annealing will require extensive effort to develop computer codes and application program interface (API) to ensure the optimization algorithm can interact with Arena®, which is the estimator of the objective function values.

In this work, the author fully enumerates all possible solutions in the feasible set because this approach is not very time consuming, in this case. The problem is well constrained using engineering judgment. For example, it is impractical to mine a 15 entry panel using one shuttle car, therefore, this option will be excluded from the feasible set. Once that is done, the feasible set contains tens of combinations rather than hundreds or thousands. For instance, in the particular case of the case study, the feasible set contains 36 combinations. For situations where the feasible set is large, an optimization algorithm may be useful. This should be explored in future work.

In this study, the objective function is a dual objective function made up of productivity and unit cost. To find *the* optimal solution, one would have to determine the relative significance of productivity and unit cost to the decision. Since this varies from one situation to another, the researcher chose not to attempt finding a single optimal solution, but to present a discussion of results relative to productivity and unit costs. In situations where this can be done, an objective function (Equation (3-2)) can be formulated for such situations. The coefficients (η_1 and η_2) should be selected to scale the units as well as describe the relative importance of the two objectives.

$$F(x) = \eta 1 * (productivity) + \eta 2 * (unit\ costs) \tag{3-2}$$

3.3. CASE STUDY

A case study of an actual coal mine is presented in this section to illustrate the approach discussed in Section 3.2. The discussion here follows the general steps of the approach as discussed in Section 3.2. The discussion of the simulation modeling (Step 1) is presented following the major steps of a typical simulation study (Kelton and Sturrock 2003).

3.3.1. Step 1: Build Valid DES Model.

3.3.1.1. Problem formulation. The objective of the panel width optimization study is to evaluate the impact of panel width on the unit cost and productivity of an underground R&P operation. A DES model with variables that characterize the coal cutting, loading and hauling system was built using Arena®. The model predicts unit mining cost and productivity at different panel widths using user specified cut sequences and fleet. The DES model was validated with shift production data obtained from a R&P coal mine in Illinois. The defined performance metric was that the relevant simulated output should be within 15% of actual values from the mine.

3.3.1.2. System and simulation specification. The mine used for this study is located in southern Illinois. The mine produces approximately 7 million tons of coal per year from the Herrin No. 6 seam using R&P mining methods with a panel recovery rate of 54%. Eight Joy Model 14CM27 CMs (two for each panel) cut and load coal at up to 40 tons per minute with a maximum cutting height of 11.2 ft. Coal is hauled from CMs to feeder-breakers by 20-ton Joy Model BH20 battery-powered haulage units. A feeder-breaker is located at the center of each production panel to transfer mined coal from haulage units to conveyor belts. As the panel advances, the feeder-breaker is moved forward in three-crosscut increments. The full width of the panel is mined in six-crosscut increments. The panel is mined by first advancing (mining) the center of the panel ahead of its flanks. The mine has experimented with different panel widths and mining sequences. Currently, the strategy of advancing the central 11 or 13 entries before mining rooms on the flanks is the most common. Minimum and maximum panel widths are 11 and 23 entries, respectively. During normal operations, each CM mines up to seven entries on one side of a panel.

The objective of the simulation is to develop a valid DES model that predicts unit mining costs and productivity. Also, the model should provide basic animation for verification. Input data used in the model were obtained from time studies done at the mine (Figure 3-1 to Figure 3-6). Raw data were analyzed to fit statistical distributions using the Chi-squared goodness-of-fit test as shown in Table 3-1. Input data include loading and dumping times, payloads, and battery change data, which are sampled from the distributions. Model output includes production per shift, tons per hour, total operating costs including equipment costs, and the calculated cost per ton for a given panel width.

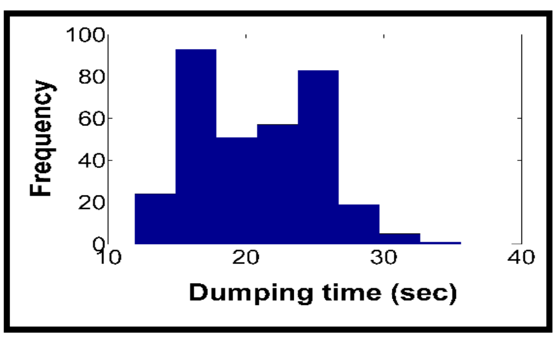

Figure 3-1 Haulage unit dumping time

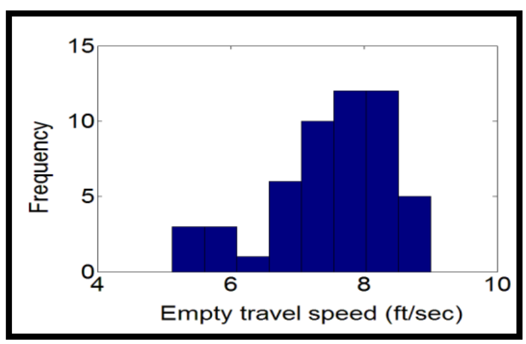

Figure 3-2 Empty haulage unit travel speed

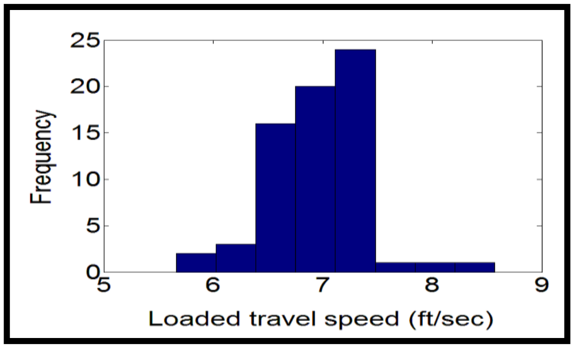

Figure 3-3 Loaded haulage unit travel speed

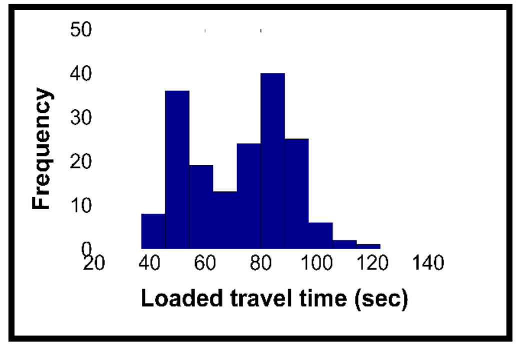

Figure 3-4 Loaded haulage unit travel time

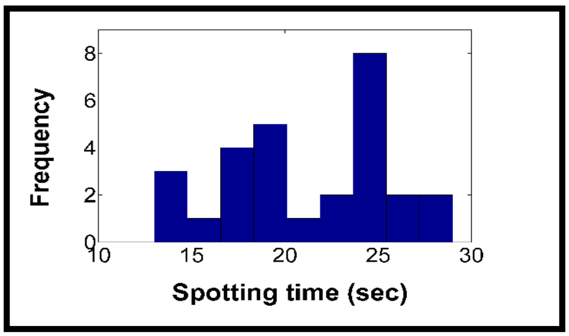

Figure 3-5 Haulage unit spotting time

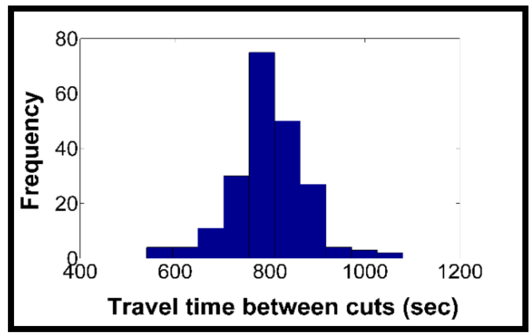

Figure 3-6 CM travel time between cuts

Table 3-1 Input data

Data (s)	**Distribution(s)**	**P- value**
Payload (ton)	12	-
Empty speed (ft s^{-1})	6.11 + GAMM (0.327, 4.97)	< 0.005
Loading time (s)	28 + ERLA (3.63, 3)	< 0.005
Dumping time (s)	6 + GAMM (2.79, 5.36)	< 0.005
Battery change (s)	TRIA (5,7,10)	< 0.005
Loaded speed (ft s^{-1})	6 + GAMM (0.261, 4.24)	< 0.005
Time between cuts (s)	NORM (797, 87.7)	< 0.005
Spotting time (s)	12.5 + GAMM (4.22, 2.11)	< 0.005

3.3.1.3. Model formulation: CM and haulage logic. The DES modeling framework requires the entities, resources, and processes of the system to be specified by the analyst. To initiate the model, entities go through defined processes in a logical manner waiting for needed resources to become available at each process (*i.e.* resources are "busy" if they are being used by other entities) before they go through the process. The CM is modeled as a resource used for the loading process and can only load one haulage unit at a time. Loads of coal are modeled as entities with specific attributes (entity number, payload, and cut sequence – the cut sequence was assigned to each entity to ensure the information is available to "route" loads to the active cut). Battery-powered haulage units are modeled as guided transporters used for hauling loads (entities). A guided transporter is an Arena®-specific modeling construct for material haulage (Rockwell Automation Inc. 2012). Transporters use entries and crosscuts as haulage routes, which are modeled to restrict traffic flow such that any point on a haulage route can only accommodate one haulage unit at a time since the mine openings are not wide enough for them to pass each other. The feeder-breaker is also modeled as a stationary resource used for dumping loads (entities). The feeder-breaker and each cutting face are modeled as stations, which are points in the model where transporters transfer entities. Haulage routes between stations are modeled as network links to capture varying haulage distances. Distances for each network link are an input to the model. Figure 3-7 shows the logic used to model the system.

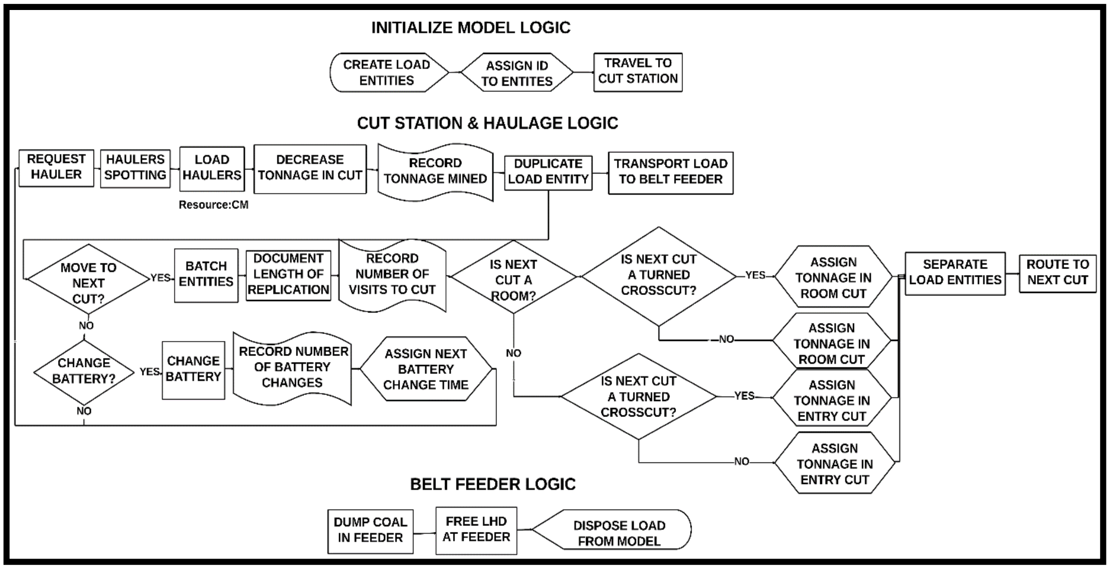

Figure 3-7 DES model logic

3.3.1.4. Verification and validation. An animation of the system was designed and used to verify that the model performs as intended. The resource, transporters, stations, and network links are modeled as part of the animation for loading and transporting coal (entities). Shift production data from the mine was used to validate the model. For validation, the simulation model predicted coal production (load count/shift) and shift duration, which was compared with data from a time-and-motion study conducted in one of the sections of the mine in question where the panel was being advanced with 13 entries. The time-and-motion study collected data for 11 CM cuts completed during the course of a shift. During the 8-hour shift, 6.33 hours were spent making 11 cuts with the remaining time spent on conveyor belt and CM repairs. The coal was hauled by four haulage units with an average payload of 12 tons. According to the CM's onboard monitoring system, the mine produced 2,448 tons of coal from 204 loads in the shift.

In the validation experiment, 150 replications were conducted to obtain estimates of load count and total coal production, mining duration, and other output. The number of replications was selected such that the half-width[15] of the mining duration (the most uncertain output) is less than 1% of the estimated duration. The cut sequence used in the validation experiment duplicated that used during the time-and-motion study. Each replication stops when all specified cuts have been mined in the simulation.

Table 3-2 shows the results of the validation experiments for the production shift. The model takes a bit longer (30 minutes more) to mine the 11 cuts and also loads 24 more haulage units than the observed system. The key performance measures are the number of loads mined from the 11 cuts and the duration of mining, which are within 11% and 8%, respectively, of the actual values. Both values are within the 15% specified earlier. The model was thus deemed valid and used for all the experiments.

Table 3-2 Results of validation experiment

Parameter	**Actual**	**Simulated**	**Difference**
Duration of mining (hours)	6.33	6.83	8%
Production (tons)	2,448	2,748	12%
Number of haulage unit loads	204	226	11%
Half-width of duration (hours)	--	0.012	--

[15] Half-width = $t_{n-1,1-\alpha/2}\frac{s}{\sqrt{n}}$, $t_{n-1,1-\alpha/2}$ = critical values from t tables, n = number of replications, s = sample standard deviation.

3.3.2. Step 2: Determine Feasible Set. This step is similar to the experimental design steps in classical simulation studies. The main distinguishing feature is that the experimental design should cover all possible combinations of the decision variables so that the entire feasible set is described in the experiments. The decision variables that affect the objective function (mining cost and productivity) are panel width (number of entries), number of haulage units assigned to each CM, and the cut sequence. At the mine, the staff has experimented with cut sequences that advance 11 or 13 entries first before expanding into rooms, if necessary. Hence, these experiments were to evaluate whether to advance with 11 or 13 entries before mining rooms leading to two possible sequences. Each sequence is based on work done by Hirschi (2012) and with specific input from Dr. J. Hirschi.[16] Once the initial advance is mined, the mine has mined anywhere from zero to five additional rooms on each side depending on the designed width of the panel. The mine has also experimented with three shuttle cars and is currently using four shuttle cars in each section. The engineers plan to increase the number of shuttle cars to five in large panels. To avoid further field experimentation and account for all previous and planned scenarios, the number of shuttle cars was varied between three and five in the feasible set.

Hence the experiment includes three factors:

- Number of initial entries (11 or 13) for advance;
- Number of rooms (0, 1, …, 5 rooms on each side of the panel); and

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[16] Dr J Hirschi optimized the cut sequences for the panel widths analyzed. He has years of experience in identifying the optimal sequence for R&P coal mines including the collaborating mine.

- Number of haulage units assigned to each CM (3, 4, or 5).

This leads to a total of $2 \times 6 \times 3 = 36$ combinations of experiments that describe the entire feasible set. For each experiment, 150 replications were run to estimate the productivity and unit cost. Each replication was run until all cuts in the sequence have been mined.

The cut sequence was provided as an input based on mining practices at the mine. Mining faces in the 11- or 13-entry initial advance are mined using the cut sequence shown in Figure 3-8(a) and (b), respectively. Rooms are mined using optimal cut sequences based on Hirschi (2012), such as the one shown in Figure 3-9(a) and (b) for a 15-entry panel width. The experiment evaluates a mining system with two CMs (one on each side of the section). The conveyor belt is located in the center entry of the panel.

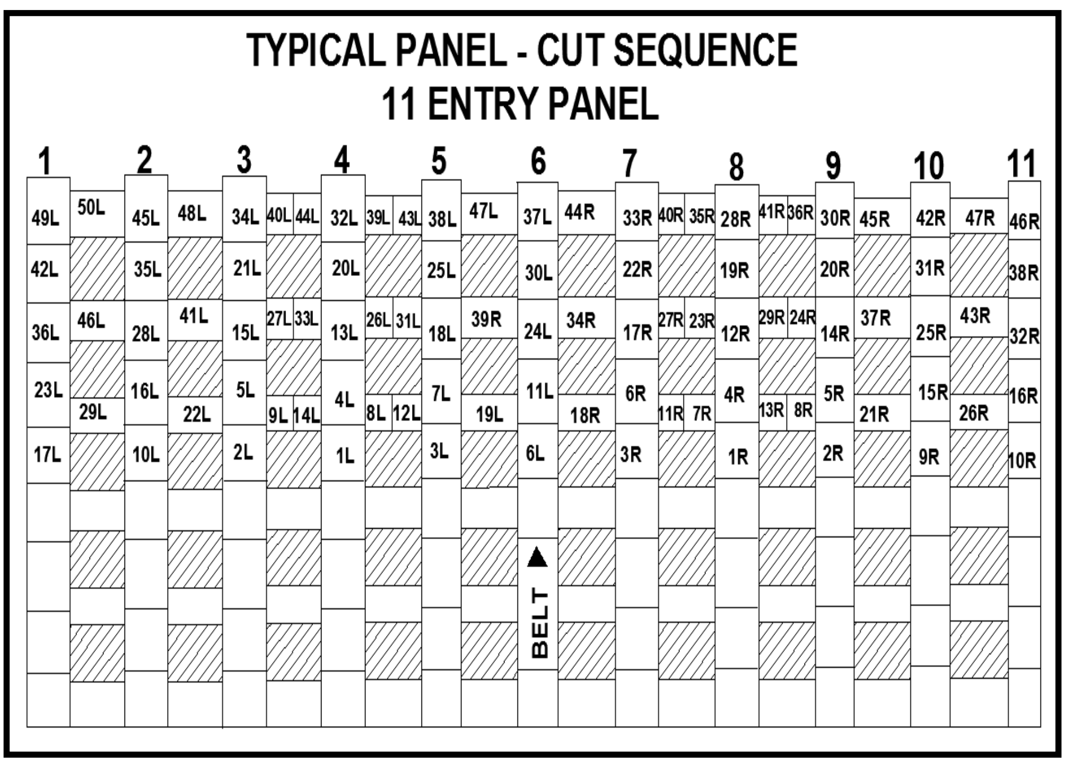

Figure 3-8(a) Cut sequence for 11-entry initial advance

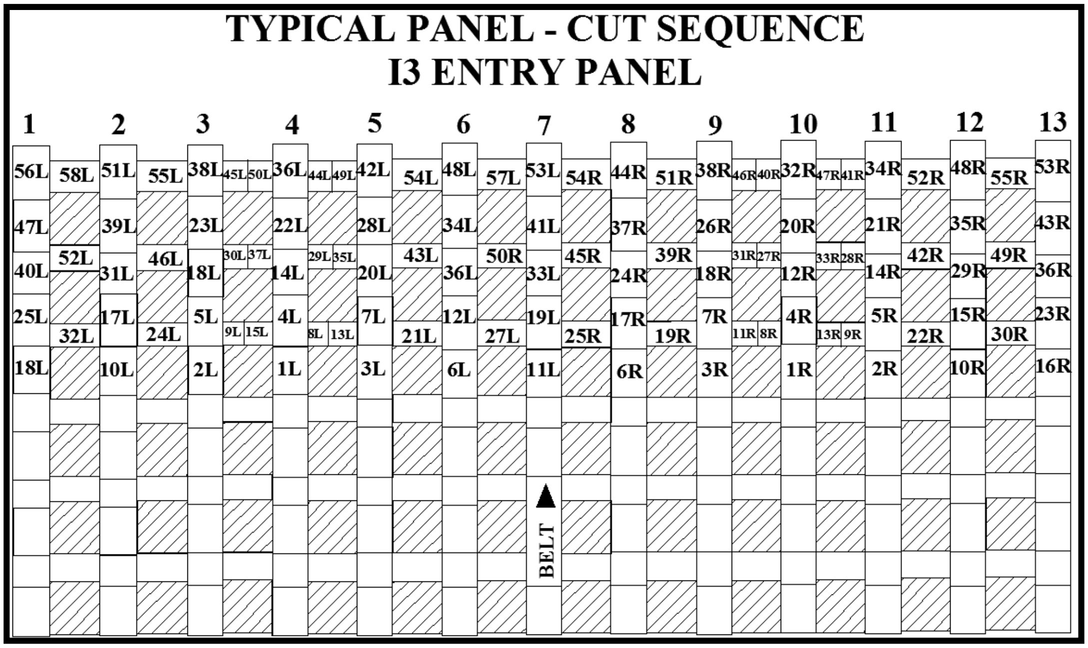

Figure 3-8(b) Cut sequence for 13-entry initial advance

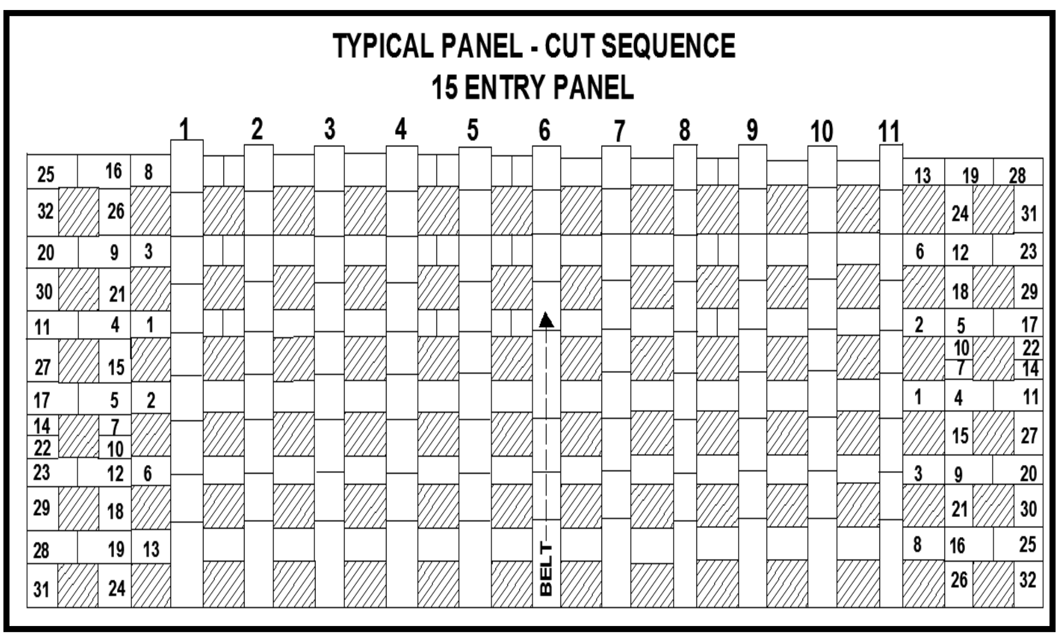

Figure 3-9(a) Room cut sequence for 11-entry initial advance with two additional rooms on each side

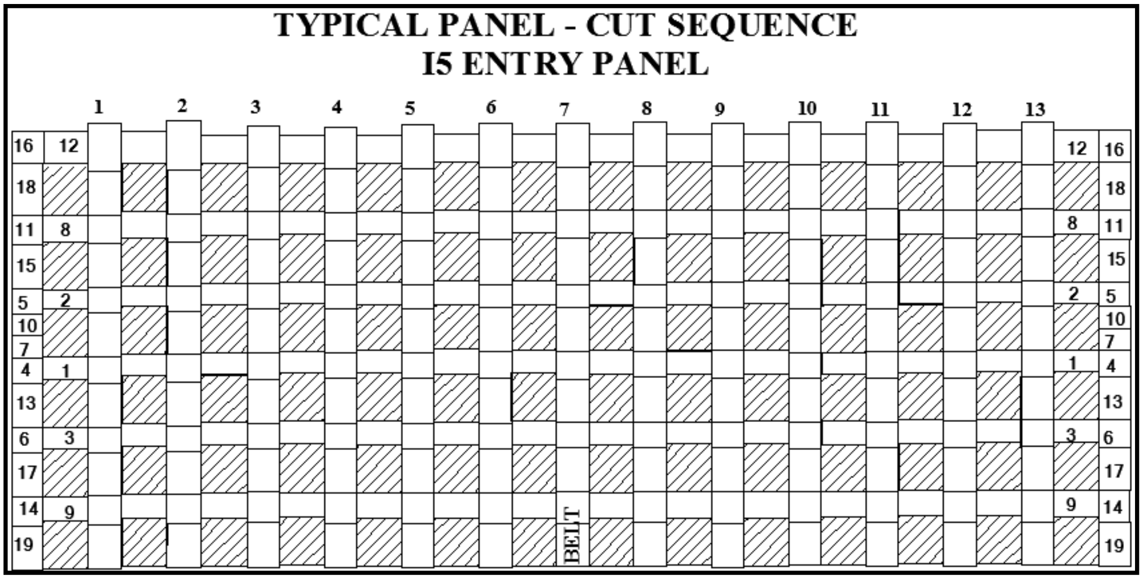

Figure 3-9(b) Room cut sequence for 13-entry initial advance with one additional room on each side

The simulation output includes production data (*e.g.* load count and total production tonnage), duration of mining, and percentage of time the CM spends loading haulers. Other outputs include total cost of mining and estimated unit costs (Equation (3-3)).

$$\text{Unit costs (\$/ton)} = \frac{(n_{CM} C_{CM} + n_H C_H) t_r + C_F}{\text{Total production}} \qquad (3\text{-}3)$$

where n_{CM} and n_H are the number of CMs and haulage units, respectively; t_r is the duration of the simulation run; C_{CM} and C_H are hourly costs for CMs and haulage units, respectively; and C_F is fixed costs, which include labor and equipment for advancing belt and power systems.

3.3.3. Step 3: Estimate Objective Function Values. Table 3-3 shows the results for all 36 experiments. The table includes the unit cost and productivity for all combinations of the three factors determined in Section 3.3.2.

Table 3-3 Productivity and unit cost of all 36 simulations

#	No. of initial advance entries	No. of additional rooms	No. of haulage units	Productivity (tons/hour)	Unit cost ($/ton)
1	11	0	3	535	3.33
2	11	1	3	541	3.02
3	11	2	3	540	2.87
4	11	3	3	540	2.70
5	11	4	3	537	2.62
6	11	5	3	534	2.52
7	13	0	3	532	3.08
8	13	1	3	536	2.85
9	13	2	3	534	2.74
10	13	3	3	532	2.61
11	13	4	3	529	2.55
12	13	5	3	525	2.47
13	11	0	4	550	3.29
14	11	1	4	555	2.98
15	11	2	4	556	2.83

Table 3-3 Productivity and unit cost of all 36 simulations. Cont.

#	No. of initial advance entries	No. of additional rooms	No. of haulage units	Productivity (tons/hour)	Unit cost ($/ton)
16	11	3	4	558	2.65
17	11	4	4	557	2.56
18	11	5	4	557	2.45
19	13	0	4	549	3.03
20	13	1	4	553	2.80
21	13	2	4	554	2.68
22	13	3	4	555	2.55
23	13	4	4	554	2.48
24	13	5	4	553	2.39
25	11	0	5	550	3.28
26	11	1	5	556	2.97
27	11	2	5	557	2.82
28	11	3	5	559	2.64
29	11	4	5	558	2.55
30	11	5	5	558	2.44
31	13	0	5	549	3.02
32	13	1	5	553	2.79
33	13	2	5	554	2.67
34	13	3	5	555	2.54

Table 3-3 Productivity and unit cost of all 36 simulations. Cont.

#	No. of initial advance entries	No. of additional rooms	No. of haulage units	Productivity (tons/hour)	Unit cost ($/ton)
35	13	4	5	554	2.47
36	13	5	5	554	2.38

3.3.3.1. Effect of panel width. Figure 3-10 to Figure 3-17 show simulation results for experiments with the default number of haulage units (four per CM). These results indicate the effect of panel width (number of entries) on productivity and unit cost. Figure 3-10 and Figure 3-11 show that total production and duration of mining increase with increasing number of entries. This is what one would expect, if the model is performing well. Figure 3-12 and Figure 3-13 show that the percentage of production time the CM spends loading haulage units initially increases with increasing panel width until an *optimal* panel width is reached. This indicates that there is excess haulage unit capacity in the system with less than optimal number of entries. CM operations are inefficient due to the excessive spotting time resulting in long wait times and bunching; however, expanding panel width beyond the optimal results in inadequate haulage unit capacity and under-utilization of the CM. This is confirmed by Figure 3-14 and Figure 3-15 showing that the optimal panel width. Initial expansion of the panel reduces the haulage unit cycle time (minimizes waiting time). However, further expansion of the panel increases haulage unit cycle times because haul distances become longer, leading to a haulage unit constrained operation. Adding more haulage units will increase productivity and CM utilization as discussed further in Section 3.3.3.2.

These trends (cycle time and CM loading times) directly result in the observed trend in productivity (Figure 3-16). Panel widths of 17 and 19 entries result in maximum productivity when advancing with a base width of 11 and 13 entries, respectively. However, this trend is not mirrored in the unit cost results (Figure 3-17) due to the effect of fixed costs that make larger panels more cost effective even with sub-optimal productivity. In Figure 3-17, unit costs are estimated using Equation (3-3). Hourly costs of haulage units and CM are estimated at $104.13 and $122.40 (InfoMine 2013)[17]. Fixed costs for moving the belt are estimated at $81,050.

The following observations can be made from these results:

- Systems which advance initially with 11 entries outperform those that advance with 13 entries under similar conditions (cut sequences and equipment);
- Haulage unit cycle times correlate very well with productivity and CM loading times;
- There appears to be an optimal panel width for a given number of haulage units based on productivity analysis; and
- Unit costs decrease with increasing number of entries due to the effect of fixed costs.

[17]InfoMine cost data is used in this study to protect the mine's confidential data.

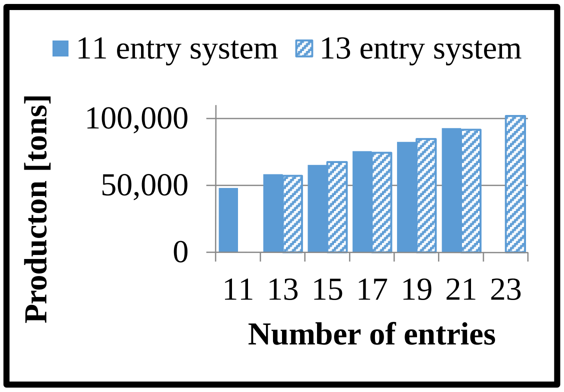

Figure 3-10 Total production

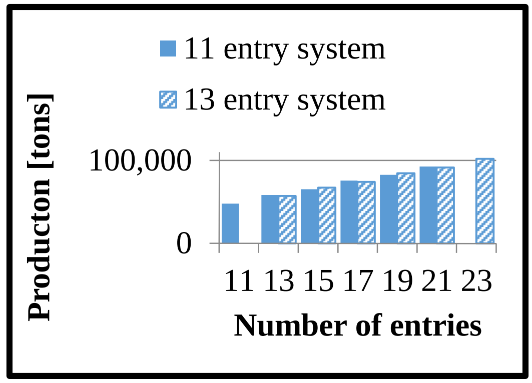

Figure 3-11 Duration of mining

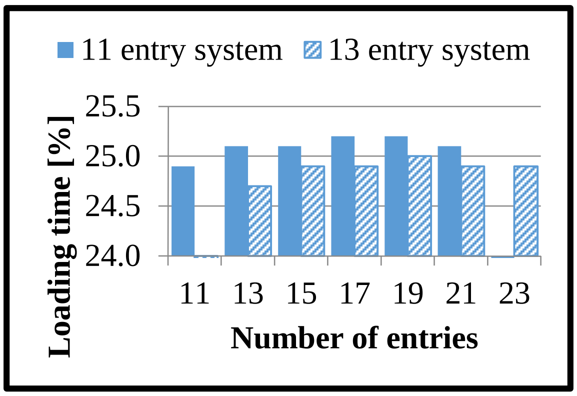

Figure 3-12 CM time spent loading (LHS)

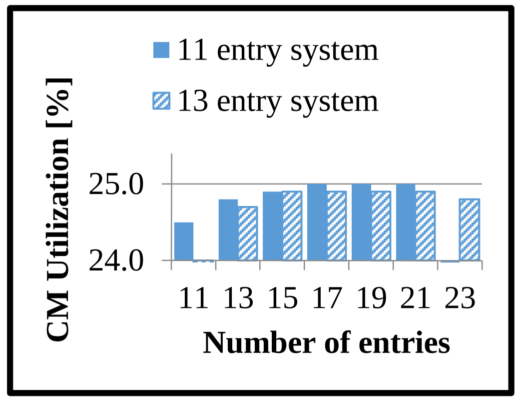

Figure 3-13 CM time spent loading (RHS)

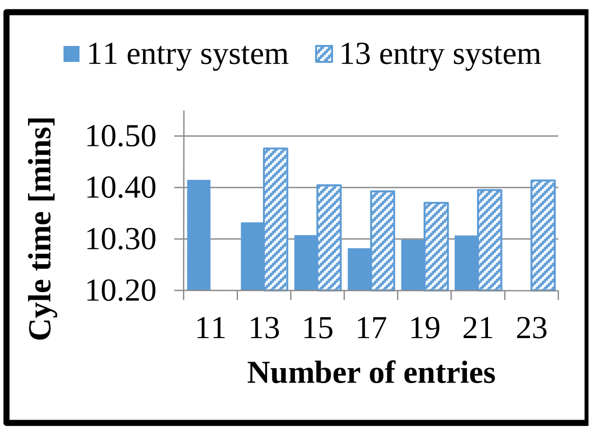

Figure 3-14 Average cycle times (LHS)

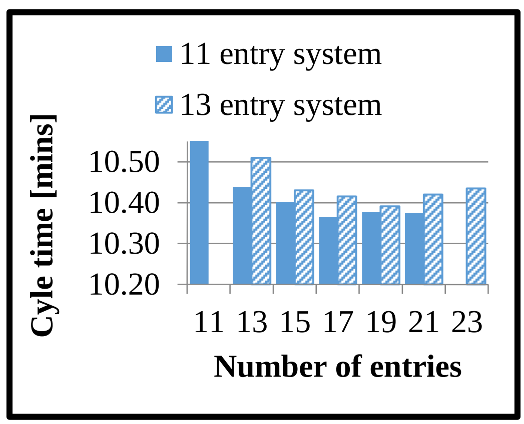

Figure 3-15 Average cycle times (RHS)

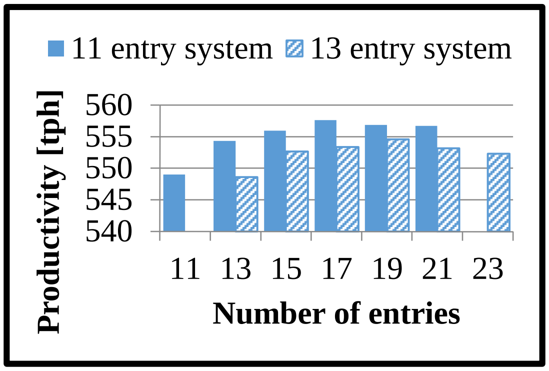

Figure 3-16 Productivity

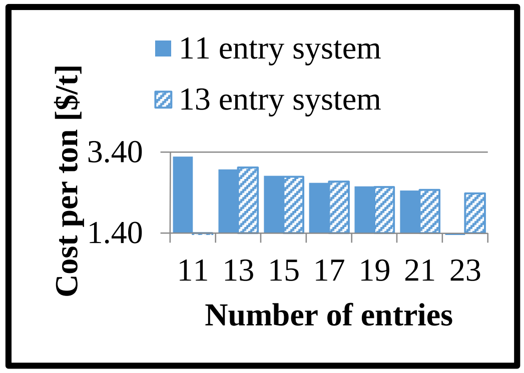

Figure 3-17 Unit costs

3.3.3.2. Effect of number of haulage units. Figure 3-18 and Figure 3-19 show the sensitivity of productivity to the number of haulage units. It can be observed that with the addition of each haulage unit, productivity increases; however, the increase when the number of haulage units increases from three to four is much more significant than the increase when the number of haulage units increases from four to five. Also, the number of haulage units can affect optimal panel width. For example, Figure 3-18 shows that optimal panel width with three haulage units assigned to each CM is 13 entries, whereas with four haulage units, optimal panel width is 17 entries. This is because the number of assigned haulage units affects the width at which the system becomes limited by haulage unit capacity.

Figure 3-20 and Figure 3-21 show the sensitivity of unit cost results to number of haulage units. With each additional haulage unit, unit costs increase for both systems 11 and 13 entries in the initial advance.

Based on these results we can observe the results are sensitive to the number of haulage units as follows:

- Productivity increases with additional haulage units, and

- Optimal number of entries changes with varying number of haulage units.

Cost increases outpace productivity increases with each additional haulage unit leading to higher unit costs.

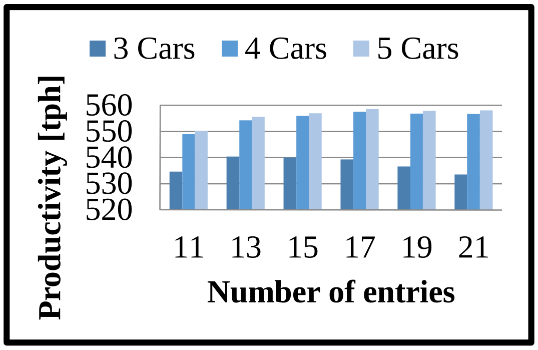

Figure 3-18 Effect of number of haulage units on productivity for 11-entry system

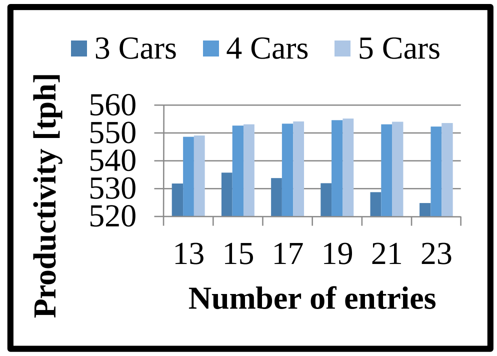

Figure 3-19 Effect of number of haulage units on productivity for 13-entry system

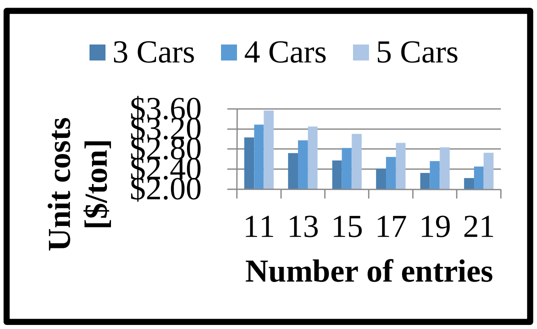

Figure 3-20 Effect of number of haulage units on unit costs for 11-entry system

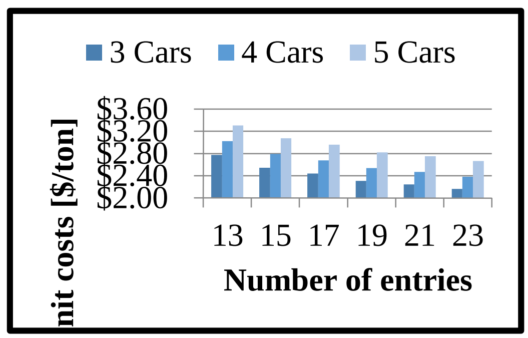

Figure 3-21 Effect of number of haulage units on unit costs for 13-entry system

3.3.3.3. Effect of fixed costs. From Equation (3-3), if fixed costs are negligible, the unit cost curve should be the inverse of the productivity relationship. However, Figure 3-16 and Figure 3-17 do not show this relationship indicating that fixed costs significantly affect the unit cost relationship. Figure 3-22 shows the sensitivity of the unit cost relationship to fixed costs using results for the sequences where 11 entries are advanced initially with four haulage units (same as Figure 3-17). Figure 3-22 shows that the unit cost relationship will indeed show an optimal at 17 entries if fixed costs are less than or equal to $1,000. Fixed costs as low as $2,000 more than compensate for any decline in productivity due to under-resourced CMs. That is, with high fixed costs (≥$2,000), unit costs for mining larger panels will be lower, even though productivity will be sub-optimal after the panel width exceeds the optimal panel width for productivity. From a cost perspective, larger panels are advantageous because of fixed costs included in moving the conveyor belt and power.

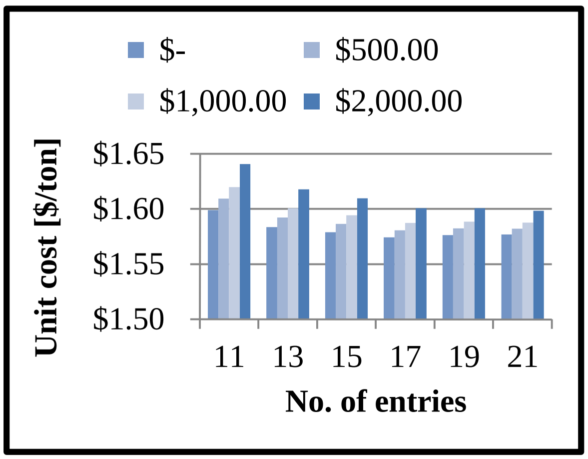

Figure 3-22 Effect of fixed costs on unit cost relationships

3.3.4. Step 4: Select Optimal Value. In this step, the optimal solution is selected based on the objective function values estimated in Step 3. The objective was to maximize the productivity whiles minimizing unit cost (Equation (3-2)) subject to existing mining conditions. Prior to the analysis, the mine's engineers and the author expected that the optimal panel width will have the highest productivity and the lowest unit cost. At the initial stage where the scope was defined for this problem, the engineers at the collaborating mine decided that productivity and unit cost were equally significant in choosing the optimal panel width. However, once the author incorporated fixed cost into the analysis of unit cost (Section 3.3.3.3), it became clear that the choice of an optimal panel width has to be made either by prioritizing productivity or unit cost. Based on the results, the productivity was deemed more important than unit cost by the mine's engineers. Based solely on productivity (i.e. η_2 in Equation (3-2) is set to zero), the revised objective function value increases significantly as the panel width increases until an optimal value is reached, beyond which the value decreases. For the existing mining condition, a panel width of 17 entries and an 11-entry initial advance is deemed optimal to maximize productivity (Table 3-3).

3.4. SUMMARY

This research effort presents an approach, based on discrete event simulation, to optimize productivity and unit costs as a function of panel width. The 4-step approach has been successfully illustrated with a case study of a real underground coal mine in Illinois. The research has successfully built a discrete event simulator that can be used to facilitate panel width design. The simulator is capable of evaluating the effect of panel width (number of entries) on R&P mine productivity and unit costs. The simulator has successfully been validated for the case study mine. The validated model has been used to evaluate the effect of panel width on productivity and unit costs of the mine.

Based on results of the case study, the following general conclusions can be made:

- For particular operating conditions (equipment, cut sequence, *etc.*), there exist an optimal panel width that maximizes productivity.

- For particular operating conditions, an optimal panel width exists that minimizes unit costs, only if the fixed costs are negligible. For any significant fixed cost, larger panels will always result in lower unit costs.

For the cooperating mine, in particular, the following recommendations can be made:

- Mining with sequences that initially advance 11 entries is better than mining with sequences that advance 13 entries initially.
- The optimal panel width under simulated conditions is 17 entries (3 rooms on each side of the 11-entry base width).
- Four (4) haulage units should be assigned to each CM in the panel.
- The practice of moving the belt after mining three crosscuts to ensure haul distances to rooms is reasonable.

4. INCORPORATING CHANGING DUTY CYCLES IN CM-SHUTTLE CAR MATCHING USING DISCRETE EVENT SIMULATION

4.1. INTRODUCTION

The cut sequence used to mine a coal panel in room and pillar mines differ, depending on production, ground control and ventilation requirements. In most mines, the direction of mining and cut sequence change as mining progresses, resulting in varying shuttle car and CM cycle times. Multiple cuts are mined in a single shift with varying distances from the conveyor belt feeder, as well as from one cut to the other. This results in frequent tramming by the CM from cut to cut. As the cuts change, so do the cycle times of the shuttle cars as they travel to and from the belt feeder. The duty cycles may vary significantly as mining progresses, depending on the cut sequence, which determines distance from the loading point to the conveyor belt. The CM and shuttle cars may be underutilized, if changing duty cycles is not accounted for in matching a CM to an optimal number of shuttle cars.

As shown in Chapter 3, the size of the haulage fleet affects the productivity in a panel; although a higher number of shuttle cars does not always lead to higher productivity. Thus, mine managers desire an optimal haulage fleet size that maximizes its objectives (e.g. productivity) while meeting all constraints. To optimize the haulage fleet size, it is important to consider the operating cycle of the haulage equipment (including loading, traveling loaded, dumping, traveling empty and waiting, as necessary).

In coal R&P mining, the duty cycles of production equipment are a function of the panel width and cut sequence. Modeling and simulation could be used to account for the effect of varying duty cycles in determining the optimal fleet size.[18] To capture varying cycle times in the modeling process, the panels have to be divided into different segments to discretize the process (otherwise, the analysis has to be done for each infinitesimal instance in time). The real challenge is how to define the panel segments. If the segments are too small, they lose practical relevance for mine management. Alternatively, if the segments are too large, the duty cycles within the segment itself will vary significantly. Once the segments are defined, the next challenge is how to model and run simulation experiments for the different segments, as well as optimize the fleet size for each segment without excess computational cost.

DES can be used to model an operation to predict the resources needed, fleet type and availability, as well as the current performance of the system. The defined resources usually include loading and off-loading equipment, maintenance personnel, and other equipment. The typical input data needed for fleet size optimization models include cycle times and equipment speed, loading, dumping/delivery and production rates, and travel distances. The output parameters are dependent on the objective of the optimization. Typical output variables include queue length, waiting time, resource utilization, duration of mining, unit cost, and productivity. Longer queue lengths indicate excess equipment fleet capacity in the system, and vice versa. Similarly, under-utilization of loading and dumping resources indicate the fleet size is less than optimal. Experiments are conducted, which typically include varying the size of the fleet and evaluating the impact on the

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[18] See Section 3.2 for reasons why simulation is a good approach for optimizing productivity of coal R&P mining in panels.

output variables. Determining the optimal fleet size is dependent on a balance between these variables.

Very limited work can be found in literature that incorporates changing duty cycles in determining the optimal fleet size for a mining operation. None of these applications can be found in room and pillar mining, where changes in duty cycle can be observed frequently. There is no comprehensive approach in the literature that has been used to evaluate the effect of changing duty cycles on system efficiency in underground mines.

This chapter focuses on incorporating changing duty cycles in determining the optimal number of shuttle cars. A modeling approach is presented capable of defining mining segments that capture changing equipment cycle times in an operation. The approach is demonstrated and validated using real-life R&P mining data.

4.2. PROPOSED APPROACH

The proposed approach consists of three major components: (1) building a valid DES model; (2) defining operating segments; and (3) conducting simulation experiments. These components are described in general here, while a case study is used to illustrate how to apply the approach in Section 4.3.

4.2.1. Building DES Model. Kelton and Sturrock (2003) describe steps for a successful simulation analysis. These steps include problem formulation; solution methodology; system and simulation specification; model formulation and construction; verification and validation; experimentation and analysis; and documenting, reporting and dissemination. These steps have been successfully used to optimize mine production systems (Awuah-Offei et al. 2003, Miwa and Takakuwa 2011, Awuah-Offei et al. 2012, and Michalakopoulos et al. 2015). This section will take a closer look at the specific steps that are unique when the process is applied to study the effect of duty cycles on optimal CM-shuttle car matching.

4.2.1.1. Problem formulation. The first step in optimizing any system is defining and formulating the problem. This includes the system to be simulated, stakeholder's expectations, system constraints, and performance metrics used to measure the quality of the simulated system under study. In this application, the objective is to determine the optimal number of shuttle cars allocated to a continuous miner in a room and pillar coal mining system while accounting for varying duty cycles (continuous miner and shuttle car). For a model to accomplish these objectives, it should be capable of predicting the output(s) that are used in the optimization decision (i.e. the model should be able to serve as the objective function). These could be the same objectives used for other fleet optimization but these outputs need to be sensitive to the duty cycle, otherwise they will not achieve the objective. For example, productivity is suitable for evaluating the performance of the material handling system in a particular section of an underground mine, whereas production may not be. This is because the production (total amount of ore that can be mined from that section) is fixed but the rate at which the mining is done is affected by different fleets. Other suitable outputs can be mining duration (time to complete mining of a particular section), queue length, utilization of loading equipment, and unit costs.

4.2.1.2. Model formulation and construction. Understanding and defining the system specification makes it easier for an analyst to visualize and design a simulation model that meets the set objectives. Generally, the model constructed for fleet optimization in mining includes the loading, hauling and dumping logic. The loading and dumping equipment are defined as resources and the hauling equipment as entities or transporters (an Arena® modeling construct for material handling applications). The loading resource is under-utilized if there are fewer than optimal cars in the system and vice versa. In this application, the modeling approach should be such that, the model is capable of evaluating the sensitivity of the output variables to changing duty cycle times. For example, the time it takes to mine different segments in the coal panel using the same fleet size will be longer in sections with higher cycle times. To account for changing duty cycle in optimizing the number of shuttle cars, it may be necessary to develop different models for each segment of the panel. It is also important to take into account the cut sequence, haul routes and distances unique to each segment. The analyst should be able to conduct necessary experiments efficiently using the constructed model.

4.2.2. Defining Operating Segments. The duty cycles of the CM and shuttle cars change mainly because of the cut sequence (i.e. the CM has to keep changing where loading occurs), although one could argue they change continuously. Without defining segments, the optimal fleet size has to be defined for each infinitesimal instance in time where there is a significant difference in duty cycle. Hence, defining segments is necessary to discretize the problem for meaningful analysis. It is ideal that the duty cycles within a segment remain near constant (otherwise, you have the same problem as the panel). However, the different segments should also capture the changing duty cycles as mining progresses in the panel. In other words, intra-segment variation in duty cycles is undesirable whereas inter-segment variation is acceptable.

Defining too many segments (e.g. each cut is a segment) is computationally expensive and practically meaningless for mine management (i.e. engineers and foremen cannot allocate a different number of shuttle cars for each cut). Defining too few segments (e.g. two segments) would result in segments where the duty cycles vary significantly within the segment. This defeats the purpose of this sort of analysis. The optimal number and size of segments, is somewhere between these two extremes. Also, for an existing mine, the segments have to be defined such that using variable fleet sizes will not significantly change the allocation or dispatch practices at the mine. For example, if the segments are defined so that it is optimal to change the number of cars in the middle of a shift, this will result in operational delays, equipment underutilization or personal scheduling challenges. Therefore, a good rule of thumb would be to select segments that start at the time when fleet assignments are made. This is usually at the beginning of a shift. This is the approach taken in this research.

After establishing a guiding principle for determining segment sizes (e.g. segments that coincide with shifts), it is still not trivial to determine the number and size of segments. Especially in cases where the mine or panel is being planned and no data exists on how long it takes to mine a cut or certain number of cuts. Two approaches are suggested for existing and planned operations. For existing operations, an analyst can examine the number of cuts the crew typically mines in a shift and use that to define the size of a segment. For planned operations, the same simulation model that will be used for the segment-by-segment fleet analysis should be used to simulate the entire segment first. The simulation results can then be used to determine the average number of segments mined in the period it will take to complete a shift (e.g. eight hours). The first approach is illustrated by the case study in Section 4.3.

4.2.3. Simulation Experiments. Experimental analysis is an essential step that allows the analyst to evaluate and identify optimal scenarios that maximizes the system efficiency. In fleet sizing, the primary experimental factor is the size of the fleet. The number of experiments conducted is a function of the current fleet size being used, as well as stakeholder's expectations. Usually the analyst, evaluates the sensitivity of the defined simulation output to decreasing and/or increasing fleet size in the system. In most experiments, if there is no significant change to the output when the number of cars is increased (or decreased), the analyst does not introduce additional cars in the system. To account for changing duty cycles of production equipment, a generalized model is defined that incorporates all cuts and distances in the system for a particular panel width. By doing so, the only input that changes in the model is the cut sequence. The validated model can be replicated without developing new models for each segment. The process analyzer tool in Arena® can be used to vary the number of cars in the system without manually changing it in the model. In order to take advantage of the process analyzer tool, the number of cars in the model is defined as a variable in Arena® along with an initial value. The output for each experimental scenario can simply be obtained by adding the range of fleet sizes to be evaluated.

4.3. CASE STUDY

A case study of a coal mine is used to illustrate the approach presented in Section 4.2. The case study is presented using the same outline as the proposed approach.

4.3.1. Building DES Model. A detailed description of the simulation model of the system in this case study can be found in Chapter 3. This section provides a summary description with emphasis on the unique aspects relating to accounting for the changing duty cycle as mining progresses through the panel, which is not addressed in Chapter 3.

The studied mine is a room and pillar coal mine in Illinois. The mine mines coal from the Herrin Number 6 seam with eight Joy Global 14CM27 continuous miners for cutting and loading (with up to two CMs for each panel). Each CM loads up to 40 tons per minute of coal with a maximum cutting height of 11.2 ft. The mine produces approximately 7 million tons of coal per year with a panel recovery rate of 54%. The mine uses 20-ton Joy Global BH20 battery operated shuttle cars to haul the cut coal from the mining cut to a conveyor belt feeder. The conveyor belt feeder is located at the centre of each production panel and moved forward every three crosscuts. The optimal panel width recommended for operation, based on the study presented in Chapter 3, is 17 entries with a fleet of two CMs, each assigned four shuttle cars. The recommended fleet is optimal for mining the entire panel and does not account for variations in duty cycle as mining progresses.

The input data used in the model was obtained from time and motion studies done at the mine, as well as data from equipment monitoring systems. The raw data was analyzed to fit statistical distributions using the Chi-squared goodness-of-fit test. The model input data includes spotting, loading and dumping times, payload and battery change data. (The data is presented in Section 3.3.1.2.)

As explained in Chapter 3, the CM is modeled as a resource used for the loading process and can only load one shuttle car at a time. The truck loads of coal are modeled as entities with specific attributes (entity number, payload, and cut sequence). The shuttle cars are modeled as guided transporters used for hauling the loads (entities). The conveyor belt feeder is also modeled as a resource needed to dump the load entities. The belt feeder and the cuts are modeled as stations. To capture the varying haulage distances, the haulage routes between stations are modeled as network links. The distance for each network link is an input to the model. Figure 3-7 (on page 81) shows the logic used to model the cutting, loading and haulage practices of the mining system.

The cut sequence (Figure 4-1 and Figure 4-2) was provided as an input based on mining practices at the mine. Since the analysis in this chapter only focuses on the optimal panel width from Chapter 3 (17-entries wide), only the optimal cut sequence for the optimal panel width (found to be the cut sequence based on Hirschi (2012) where 11 entries are initially advanced before mining the three additional entries on each side - Figure 4-1 and Figure 4-2) is considered in this analysis. Mining cuts in the 11 entries at the center of the panel are mined using the cut sequence shown in Figure 4-1. Rooms in the remaining 6 entries are mined using the cut sequence shown in Figure 4-2.

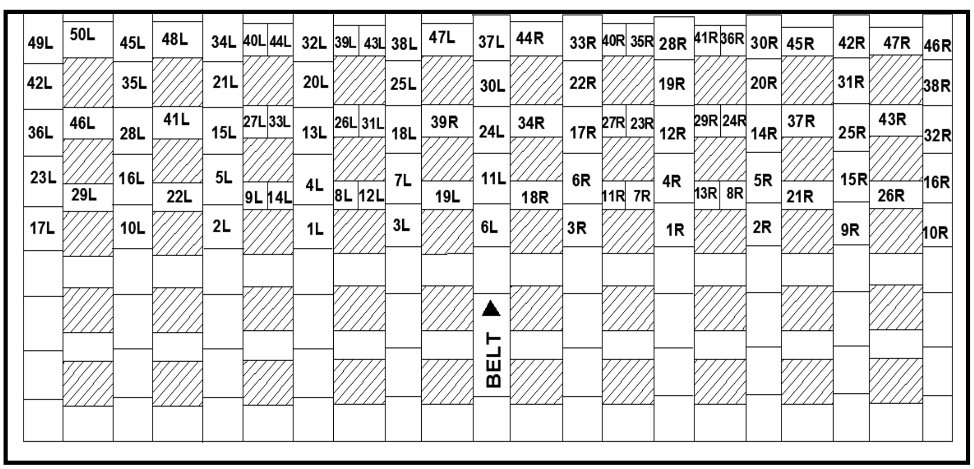

Figure 4-1 Cut sequence for the 11 entries at the center of the panel

The simulation output includes duration of mining, cycle times, average waiting time in loading queue, productivity (tons per hour) produced and percentage of time the CM spends loading cars.

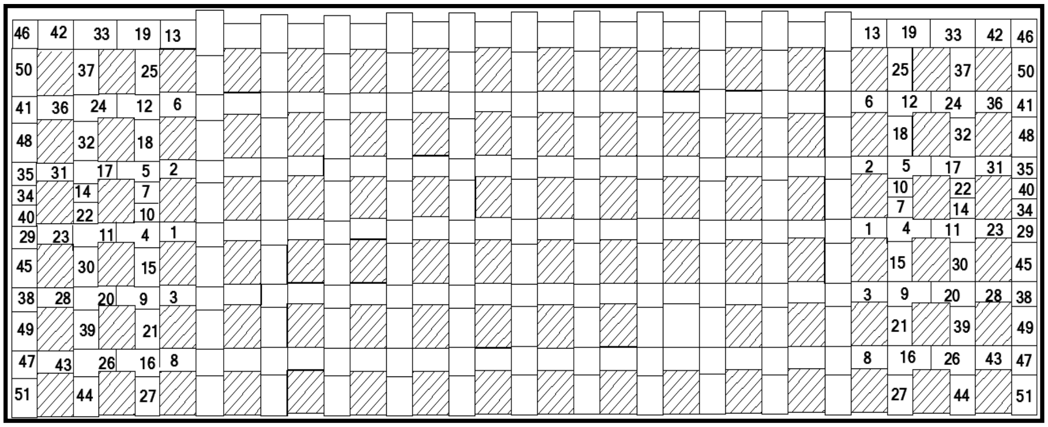

Figure 4-2 Cut sequence for the three additional entries on each side

As presented in Chapter 3, the model was validated using coal production data collected from a representative eight-hour shift at the mine. During the shift, 6.33 hours were spent to complete 11 cuts with the remaining time spent on conveyor belt and CM repairs. The total coal production and mining duration (time it takes to mine the 11 cuts) predicted by the DES model from the studied shift is compared to the actual data from the mine. A performance measure of ±15% deviation based on stakeholder's expectations was set for this research. The model took a bit longer (30 minutes more) to mine the 11 cuts and also loaded 24 more cars than the observed system. The number of loads mined from the 11 cuts and the duration were within 11% and 8%, respectively, of the actual values. Both values were within the 15% specified earlier. The model was thus deemed valid and used for all the experiments.

4.3.2. Selecting Number of Operating Segments. As discussed in Section 4.2.2, this approach depends on how segments are defined in the analysis. In this case study, the 17-entry panel is divided into segments of up to 11 cuts each based on the typical shift used for validation. The goal was to define segments that can be mined in a shift, since car assignments are made at the beginning of a shift at this particular mine.

Only the first six crosscuts of the 17-entry panel, with a total of 146 and 151 cuts on the left- and right-hand sides of the panel, respectively, were analysed in this work. Six crosscuts completely mine out the width of the panel using the mines cut sequences (Figure 4-1 and Figure 4-2). This resulted in a total of 14 mining segments on each side of the panel with exactly 11 cuts in each, except for the last segment on each side. The last segment contained three and eight cuts on the left- and right-hand sides, respectively. Segments 1 to 8 are found in the entries mined with the cut sequence in Figure 4-1. Segment 9 is mined using both cut sequences and segments 10 to 14 are mined with the cut sequence in Figure 4-2.

4.3.3. Simulation Experiments and Analysis. Experimental analysis was conducted using the validated model to determine the optimal number of shuttle cars required in each panel segment. As at the mine, one CM is assigned to work on each side of the belt feeder in the panel. In the experiments, the number of shuttle cars assigned to each CM was varied from one to six. Preliminary analysis indicated that a fleet of more than six cars assigned to each CM has no further significant impact on the model outputs. This leads to a total of 84 (14 segments × 6) experiments. For each experiment, 150 replications were run for the analysis.[19] Each replication was run until all cuts in the segment had been mined.

Results of the simulation experiments are discussed in the next section.

4.3.4. Results and Discussions. The simulation results are shown in Figure 4-3 to Figure 4-7. Figure 4-3 shows the duration of mining in each segment using variable

[19] As in Chapter 3, the number of replications was selected to ensure that the half-width of the mining duration is less than 1% of the estimated duration.

number of shuttle cars. As expected, the tonnage in each segment is mined out at a slower pace with fewer than optimal cars. Duration of mining starts to decrease significantly, initially as the fleet size is increased, until it reaches an optimal value, beyond which further increases in the number cars results in no significant reduction in the time it takes to mine out the segment. This correlates well with the utilization of the CM for loading shuttle cars (Figure 4-6. In segments 1, 2, and 7, the duration of mining approaches a constant after a fleet size of three. There is no significant change in the duration of mining when the number of cars increases from three to four. For the remaining segments, a constant value is reached after four shuttle cars are used in the system. The duration of mining increases by 41 minutes on average when the number of cars increases from three to four. This is because the average distances from the cut to the belt feeder in segments 1, 2 and 7 are relatively short compared to other segments.

Figure 4-4 shows the effect of varying fleet size on the average cycle time in each segment. The overall average cycle time increases as the number of shuttle cars increases. This correlates well with the waiting time in the loading queue (Figure 4-5). As the number of cars increase, the cars wait longer in queue to be loaded and therefore, the overall cycle time is increased. Figure 4-6 shows that the average percentage of production time the CM spends loading the shuttle cars, as opposed to tramming or waiting on cars, in each segment. The time spent loading increases with the increasing number of shuttle cars, until an optimal fleet size is obtained. This indicates that there is limited haulage unit capacity in the system at the beginning and, therefore, the CM is under-utilized. Once an optimal number of cars is reached, any additional car results in inefficient CM operations (over-matched). Excess cars in the system results in longer waiting times and no productivity gains. There is no significant change in the CM utilization after an optimal fleet size is reached. In Segments 1, 2, and 7 the percentage of production time spent loading the shuttle cars approaches a constant after a fleet size of three. For the remaining segments a constant value is reached after four shuttle cars are used in the system. The same trend can be seen in the productivity (Figure 4-7) as the number of cars changes from one to six. There is no significant change in productivity as the number of shuttle cars is increased from three to four in Segments 1, 2, and 7. For the remaining segments, increasing the fleet size from three to four increases the productivity by 5% on average.

By accounting for changing duty cycle in selecting the optimum fleet size for the CM-shuttle car mine system, the optimal number of cars needed to mine the 17-entry panel reduces as compared to the estimate determined in Chapter 3. The analysis done in Chapter 3 shows that it is optimal to mine the 17-entry panel with 8 shuttle cars (four cars for each CM) at all times. Incorporating changing duty cycle times in the fleet size optimization analysis suggests that the mine can meet its productivity target using six shuttle cars in three out of the 14 segments of the panel. Based on the operating cost ($104.13 per hour) of the battery operated shuttle cars, the overall cost of mining the coal in segments 1, 2 and 7 decreases by $5,862[20] per panel. In addition to significantly

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[20] The cost to mine each segment was estimated based on the number of cars assigned to the segment, the unit cost of operating a shuttle car and the duration of mining (Figure 4 3). The duration of mining was 9.04, 9.58 and 9.53 hours for segments 1, 2, and 7, respectively. The cost of mining segment 1 is estimated to be $5,646 and $7,529 using 3 and 4 cars per CM, respectively. The cost of mining segment 2 is estimated to be $5,984 and $7,979 using 3 and 4 cars per CM, respectively. The cost of mining segment 7 is estimated to be $5,955 and $7,940 using 3 and 4 cars per CM, respectively.

reducing the operating cost, this analysis allows equipment which will otherwise be underutilized in segments 1, 2 and 7 to be freed up for other activities. Accounting for changing duty cycles in determining the optimal fleet size needed for mining, especially at the early stages of investment minimizes capital cost and avoids unnecessary expenditure. The results of this study demonstrates that the fleet size needed for an operation can be overestimated if changing duty cycles is not accounted for.

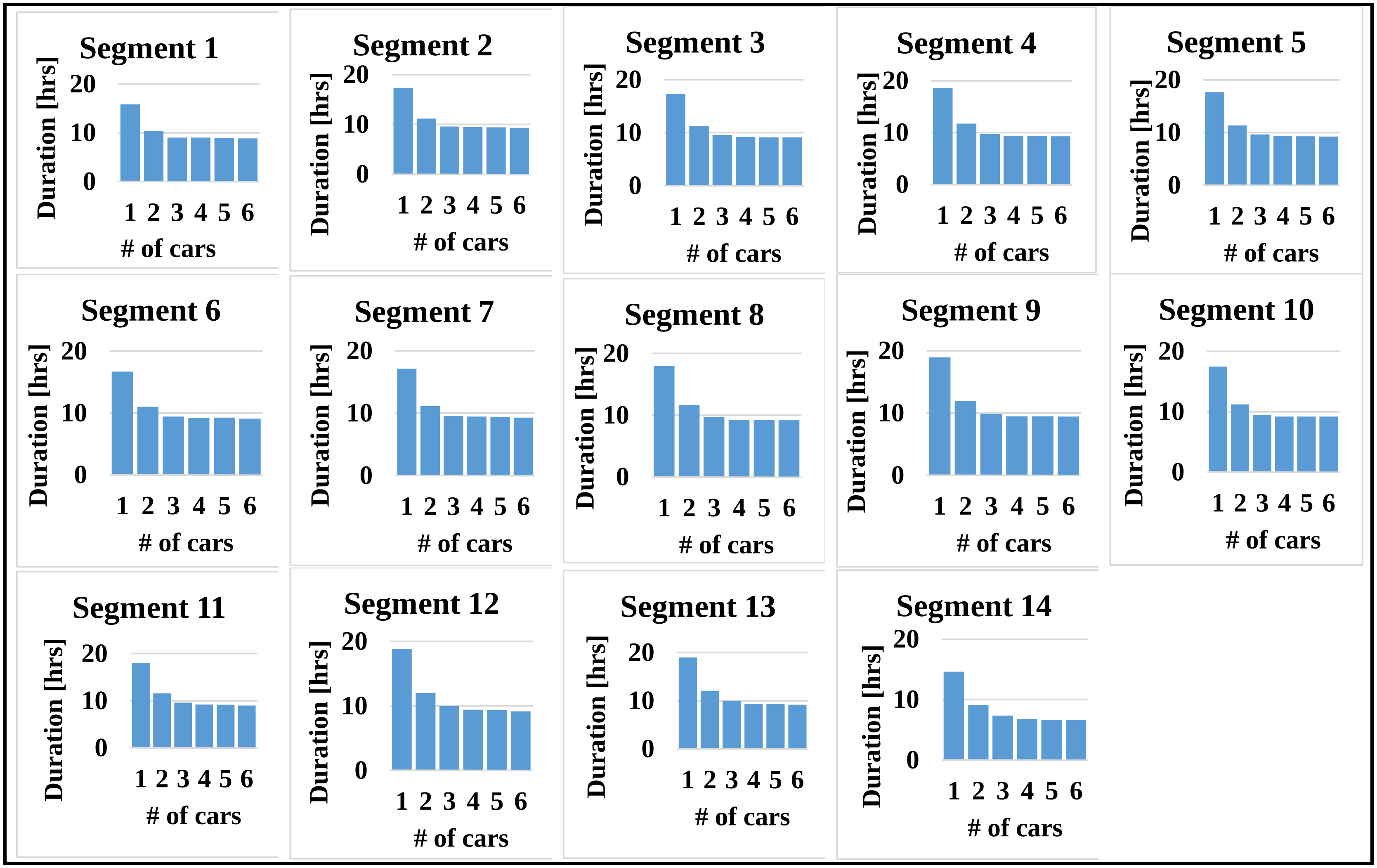

Figure 4-3 Duration of mining for all segments for varying number of cars

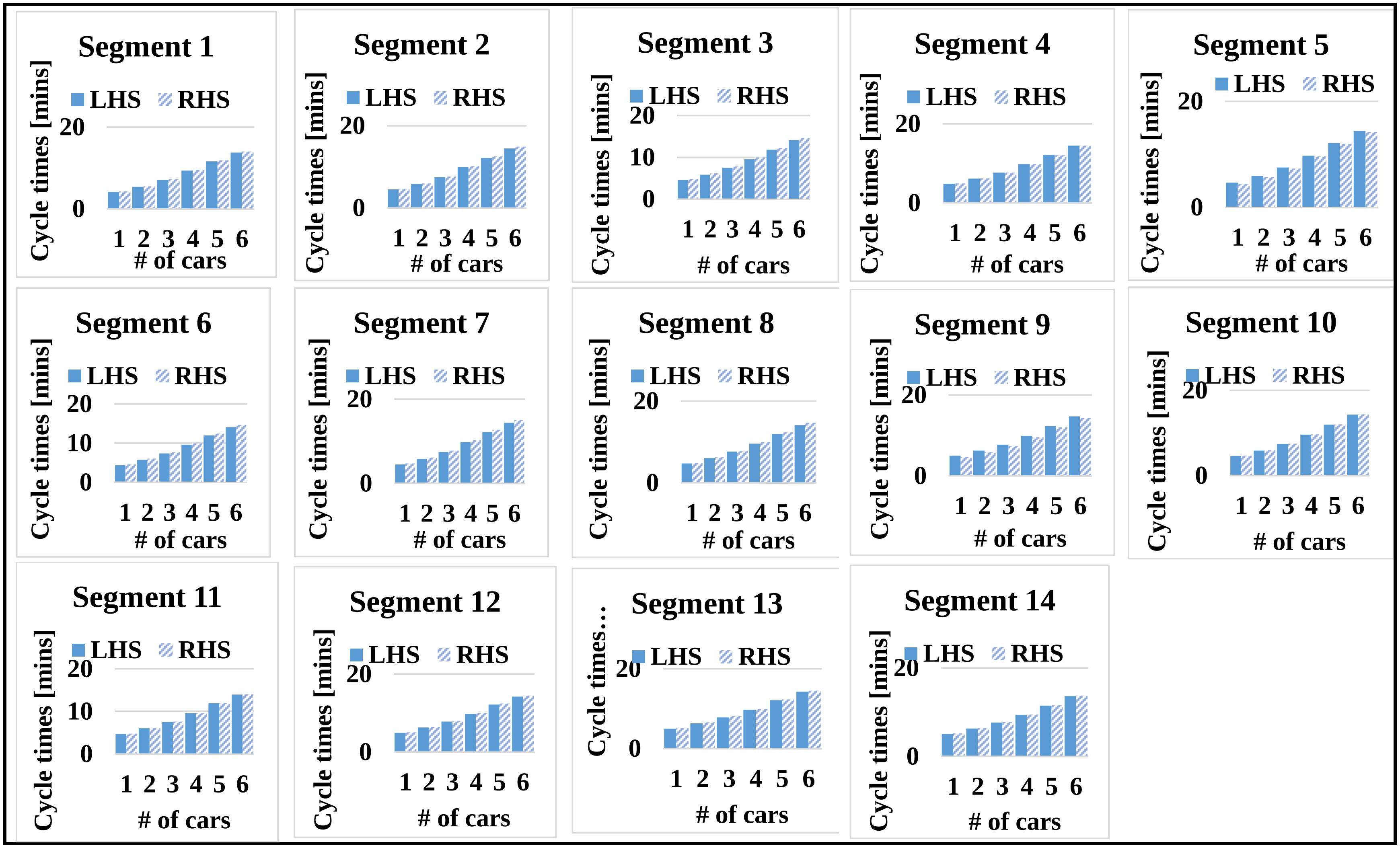

Figure 4-4 Average cycle time for all segments for varying number of cars

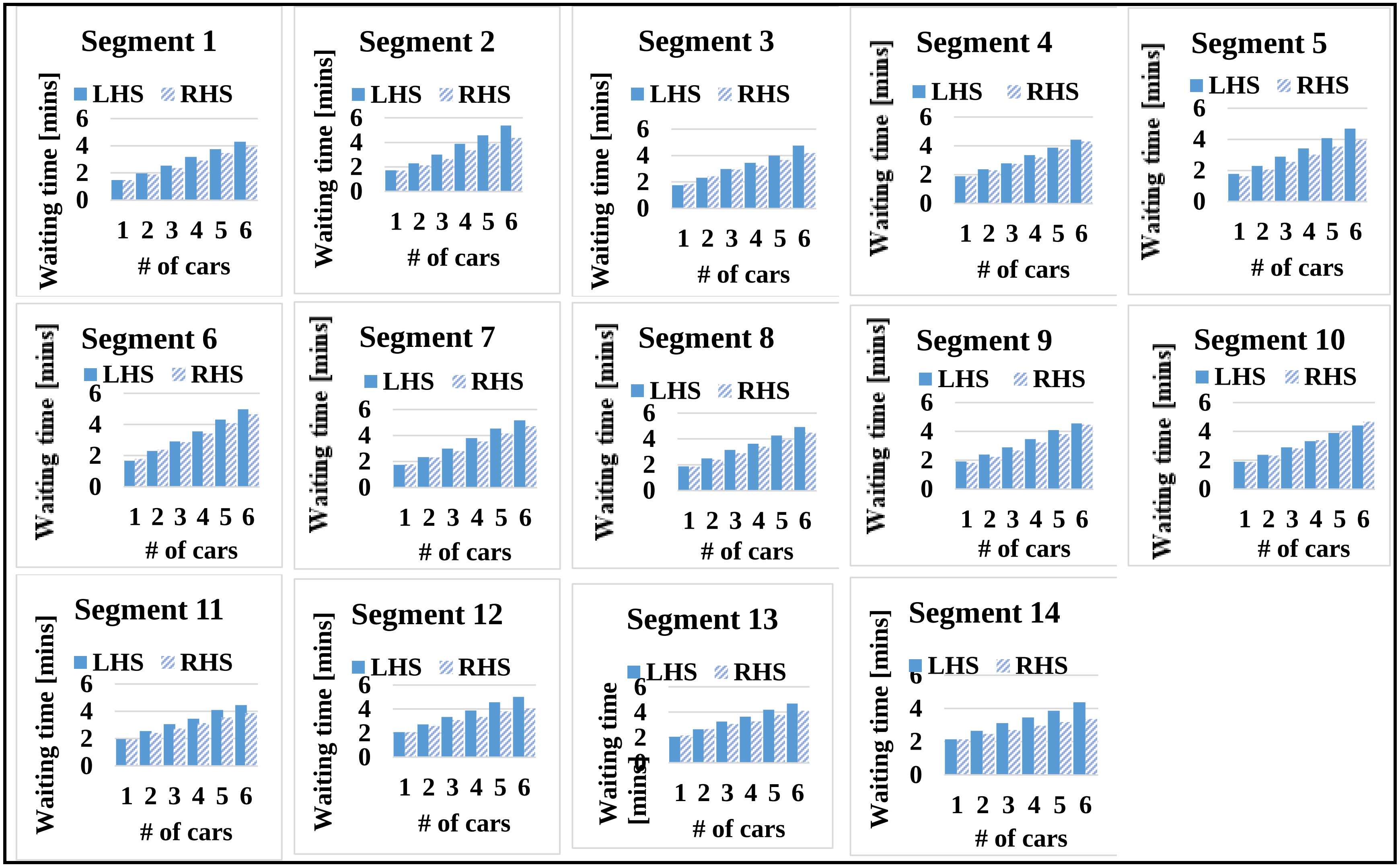

Figure 4-5 Average car waiting time in queue at CM for all segments for varying number of cars

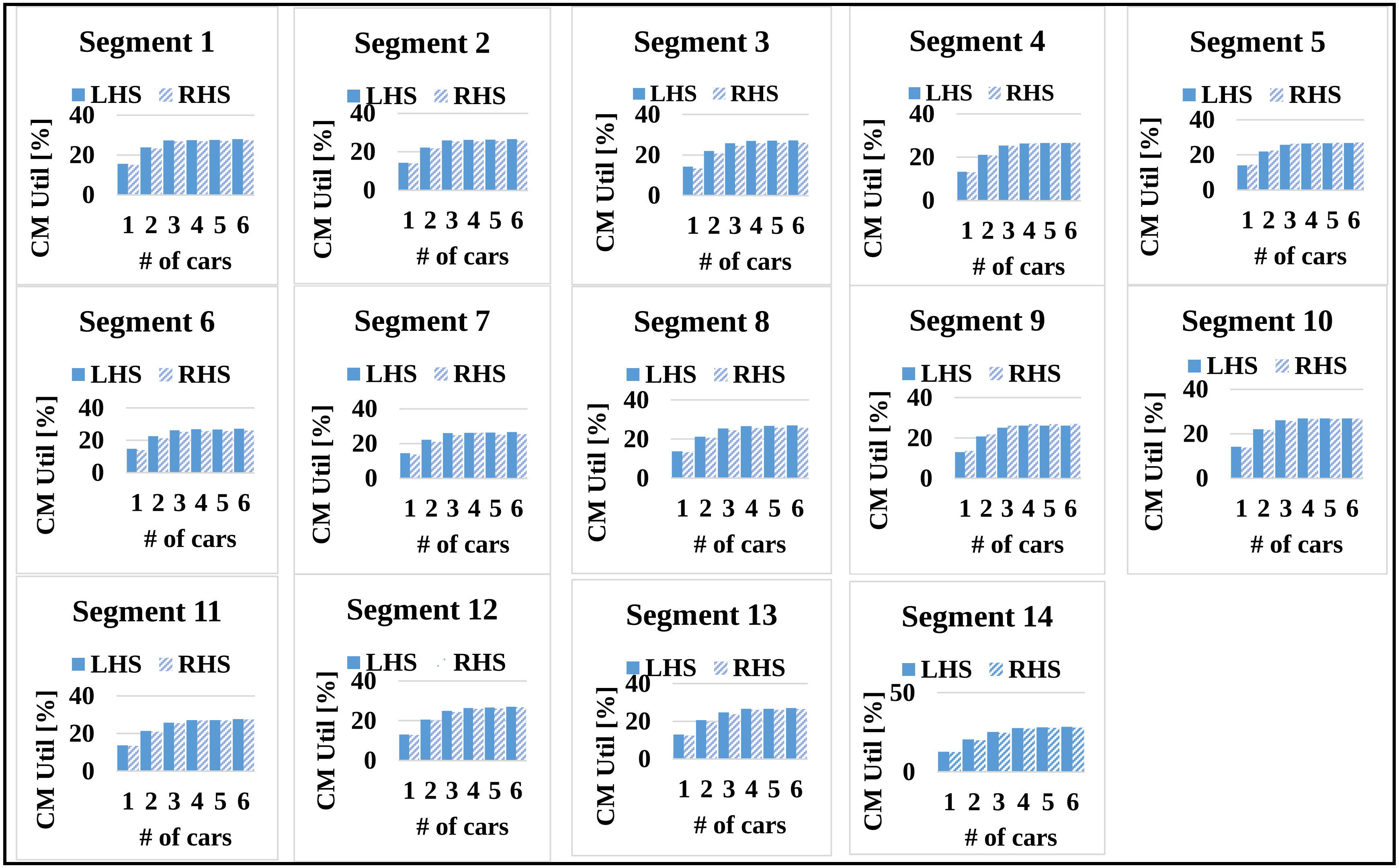

Figure 4-6 Percentage of time CM spent loading shuttle cars for all segments for varying number of cars

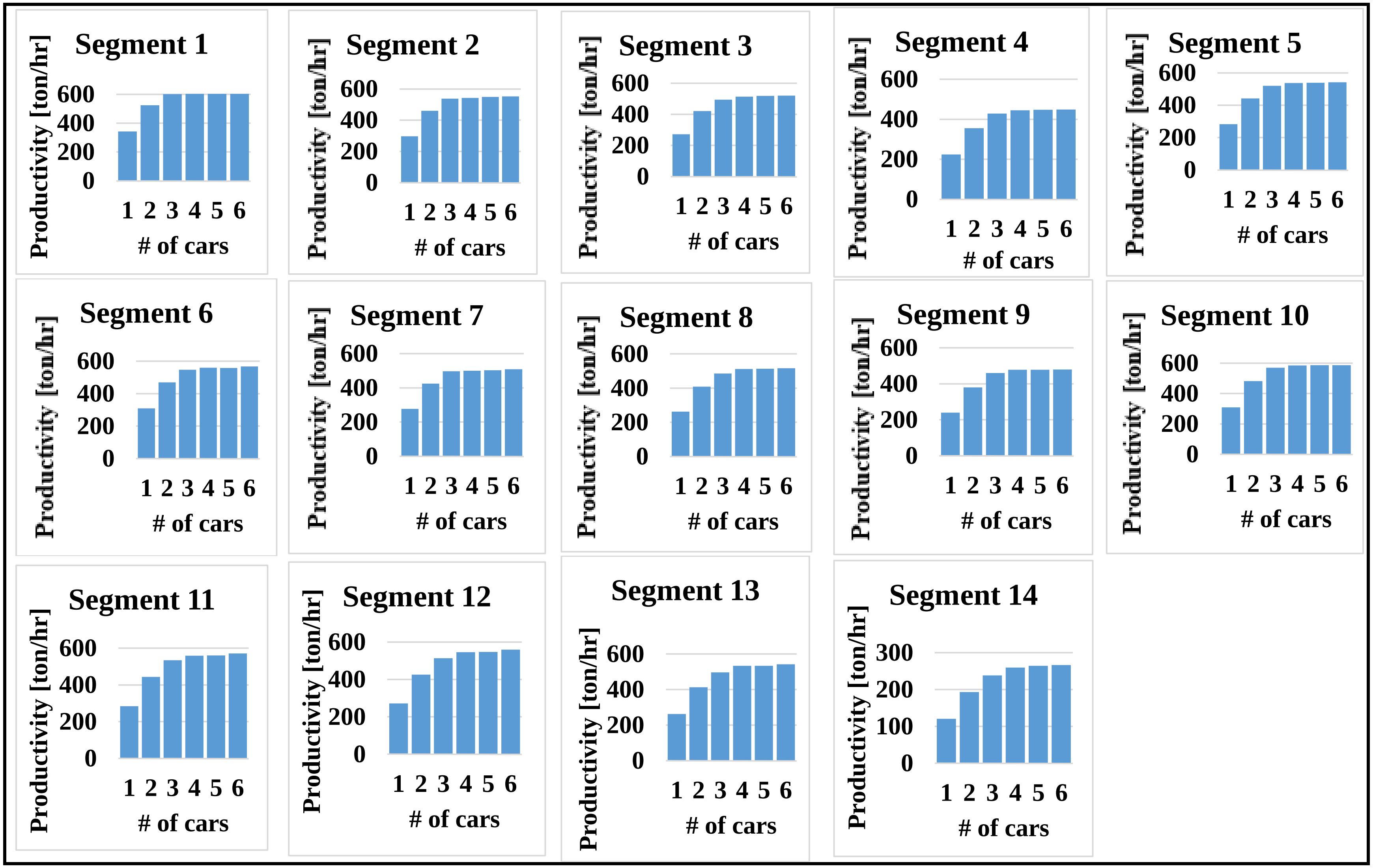

Figure 4-7 Productivity for all segments for varying number of cars

4.4. SUMMARY

A simulation approach is proposed to investigate whether the shuttle car fleet size used to mine a particular panel width is optimal in different segments of the panel. The proposed approach includes criteria used to define segments that reflect changing equipment cycle times. The research also includes experimental analysis that minimizes computational cost and evaluate the effect of changing duty cycles on the productivity, cycle times and the duration of mining. The results indicate that, for particular operating conditions (equipment, cut sequence, *etc.*), the optimal fleet size is different for different segments of the panel. Changes in haul distance and cut sequence affect the optimal number of cars required in each segment. The total distance travelled by the shuttle cars in segments 1, 2, and 7 is shorter than the remaining segments. Therefore, fewer number of cars are needed to mine these segments**.** For the mining system evaluated and the defined segments, a fleet size of four shuttle cars is optimal for 80% of the segments (Table 4-1). An optimal fleet size of three is observed for the remaining segments. The mine can dispatch the excess shuttle cars to other areas of the operation, once these segments are scheduled to be mined. Otherwise, the mine can continue to use four cars for all segments, if the change in the actual unit cost by adding a shuttle car is minimal compared to the gain in productivity.

Table 4-1 Optimal number of shuttle cars in each segment

Segment	Coal tonnage in Segment	Optimal fleet size
1	5,437	3
2	5,158	3

Table 4-1 Optimal number of shuttle cars in each segment. Cont.

Segment	Coal tonnage in Segment	Optimal fleet size
3	4,740	4
4	4,182	4
5	5,018	4
6	5,158	4
7	4,740	3
8	4,740	4
9	4,529	4
10	5,403	4
11	5,117	4
12	5,117	4
13	4,971	4
14	1,754	4

5. A DETERMININSTIC FRAMEWORK FOR INCORPORATING RISK IN ROOM-AND-PILLAR MINE PRODUCTION SEQUENCING USING BILP

5.1. INTRODUCTION

Linear programming (LP) is one of the main optimization tools used for mine production sequencing. The ability to model complex systems with a variety of constraints makes LP versatile compared to other mathematical models. In mine production sequencing, researchers most often define the decision variables as the proportion of a block mined in a period. In most cases, a block is either mined completely in a particular period or not at all. If this constraint is imposed on the decision variables of the LP problem, the resulting problem is a binary integer linear programming (BILP) problem. A BILP model is an integer LP model in which each decision variable can only take on a value of zero or one. Modeling the many activities associated with mine production sequencing as binary integer variables subject to strict sequencing requirements, results in large and complex problems which are NP-hard.[21] As a result, past modeling attempts only solve large-scale sequencing problems in a limited time frame (Newman and Kuchta 2007). Other researchers relax this constraint (binary integer variables) for some of the decision variables leading to mixed integer linear programming (MILP) problems (Gershon 1982, Barbaro and Ramani 1986, Askari-Nasab et al. 2010, Eivazy, and Askari-Nasab 2012). However, such models can lead to infeasible solutions (solutions in which a block is mined over several time periods) that require sub-optimal post-processing to be meaningful for mining.

[21] A problem is NP-hard if an algorithm for solving it can be converted into one for solving any NP-problem (nondeterministic polynomial time) problem (Weisstein 2009).

In this chapter, a BILP approach instead of an MILP one is used in order to assure practically feasible solutions. (The BILP model is compared to an MILP one to evaluate the effect of block precedence constraints on solution complexity.) The objective of this study is to develop a deterministic BILP model that is capable of incorporating multiple mining risks while maximizing the net present value (NPV) of the operation. The model is referred to as a “long-term” production sequencing model because NPV is often not as important in short-term sequencing. It is important to note that the model can solve any number of time periods so long as NPV and/or risk are the desired objective functions. The disadvantage of this approach is that the resulting problem is more difficult to solve. The optimization problem is solved using the CPLEX (IBM, Armonk, NY) solver through the CPLEX API for Matlab®. A simulated lead room and pillar (R&P) mine data set is used to verify the model and demonstrate the ability to model multiple mining risks as BILP. The modeling of mining risk is based on the stochastic modeling approach by Dimitrakopoulos and Ramazan (2004) used to characterize uncertainty in open pit mine production sequencing. Block aggregation techniques are used to minimize the computational complexity[22] associated with solving these sequencing problems.

5.2. MODELING R&P PRODUCTION SEQUENCING AS BILP

The objective of the BILP model is to maximize the overall net present value of the R&P operation and minimize the discounted value of risk while meeting all constraints. In the context of this study, risk is defined as the probability of a block property deviating (in an undesirable way) from the desired property. There are two

[22] In this work, computational complexity is used to refer to how difficult a problem is to solve and it is measured with computational time and the number of iterations required to solve the problem.

important aspects to modeling the discounted value of risk: risk cost penalty and discount rate of risk. Conceptually, the risk cost penalty is the extra cost needed to take recourse. For example, for geotechnical risks, the risk cost penalty will be the cost for additional support (on top of what is planned), if the actual geotechnical property is 'worse' than predicted. For ore grade, it will be the cost to ensure the period's metal production target is met. For the case study solved in this work, grade and geotechnical risks are used to verify that BILP can be used to model multiple mining risks. The discount rate of risk is applied to discount the risk cost penalty. It has the practical effect of deferring mining of high risk blocks till later periods and reduces risk since more data (knowledge) may become available, which will reduce the risk associated with mining such blocks.

The notations used in defining decision variables, parameters, and constraints are defined in the nomenclature.

5.2.1. Objective Function. Equation (5-1) is a dual objective function model that maximizes the NPV and minimizes the discounted cost of risk in each mining period. The model defines separate variables for mining rooms as well as pillars. The discounted profit depends on the market price p_i of the ore mined, the recovery r_i, tonnage t_i, average grade of each block q_i^u, and the unit cost of mining c_t. The model incorporates pillar extraction as an integral part of determining the optimal NPV. This is important in metal room-and-pillar production sequencing where high grade ore is left in support pillars.

$$Max\ \underbrace{w_e\left[\sum_i\sum_t\left\{\frac{p_t\cdot r_i\cdot t_i\cdot q_i^{\mu}-c_t\cdot t_i}{(1+d)^t}\right\}\cdot x_{it}+\sum_k\sum_t\left\{\frac{p_t\cdot r_k\cdot t_k\cdot q_k^{\mu}-c_t\cdot t_k}{(1+d)^t}\right\}\cdot y_{kt}\right]}_{NPV} \tag{5-1}$$

$$\underbrace{-\sum_{\eta} w_r^{\eta}\left[\frac{\sum_i\sum_t R_i^{\eta}\cdot x_{it}\cdot c_i^{\eta}\cdot t_i+\sum_k\sum_t R_k^{\eta}\cdot y_{kt}\cdot c_k^{\eta}\cdot t_k}{\left(1+d_r^{\eta}\right)^t}\right]}_{Discounted\ risk}$$

5.2.2. Constraints. Equations (5-2) to (5-7) are basic constraints needed to obtain a feasible production sequence. Combined, these result in $(I + K + J) \cdot T$ decision variables and $(T+1) \cdot (I + K + J) + T \cdot (A + U)$ constraints.

$$R_t^{\alpha,l} \le \sum_{i,k}(r_i^{\alpha}\cdot x_{it}+r_k^{\alpha}\cdot y_{kt}) \le R_t^{\alpha,u} \quad \forall\alpha,\forall t \tag{5-2}$$

$$N_i\sum_{t=1}^{T}x_{it}-\sum_{i\in O_i}\sum_{t=1}^{T}x_{i't}\le 0 \quad \forall i,\forall t \tag{5-3a}$$

$$N_k\sum_{t=1}^{T'}y_{kt}-\sum_{i,k\in O_{ik}}\sum_{t=1}^{T'}\left(x_{i't}+y_{k't}\right)\le 0 \quad \forall k,\forall t \tag{5-3b}$$

$$N_j\sum_{t=1}^{T'}z_{jt}-\sum_{j\in O_j}\sum_{t=1}^{T'}z_{j't}\le 0 \quad \forall j,\forall t \tag{5-3c}$$

$$\sum_{t}^{T}x_{it}\le 1, \qquad \forall i \tag{5-4a}$$

$$\sum_{t}^{T} y_{kt} \leq 1, \qquad \forall k \tag{5-4b}$$

$$\sum_{t}^{T} z_{jt} \leq 1, \qquad \forall j \tag{5-4c}$$

$$MR_t^l \leq \sum_{i,k} (t_i \cdot x_{it} + t_k \cdot y_{kt}) \leq MR_t^u \qquad \forall t \tag{5-5}$$

$$q_t^{\mu,l} \leq \frac{\sum_{i,k} (q_i^{\mu} \cdot x_{it} + q_k^{\mu} \cdot y_{kt})}{\sum_{i,k} (x_{it} + y_{kt})} \leq q_t^{\mu,u}, \qquad \forall \mu, \forall t \tag{5-6}$$

$$x_{it} = z_{j't} \qquad \forall i \in O_{j'}^{s}, \forall t \tag{5-7a}$$

$$y_{kt} = z_{j't} \qquad \forall k \in O_{j'}^{s}, \forall t \tag{5-7b}$$

5.2.2.1. Resource constraint. Equation (5-2) constrains the model from exceeding available resource capacity ($R_t^{\alpha,u}$) or under-utilizing available resources ($R_t^{\alpha,l}$) in a particular period. This results in T (number of scheduling periods) constraints for each modeled resource leading to a total of $A \cdot T$ constraints. Mining resources may include production and development equipment, labor, and other auxiliary equipment required to extract the material being mined. Mine haulage truck fleet is the only resource used in the case study in Section 5.4 to validate this constraint.

5.2.2.2. Precedence constraint. Equation (5-3) constrains block and pillar mining precedence, which is the single most significant contributor to the problem's complexity (Bienstock and Zuckerberg 2010). The equation results in $(I+K+J)\cdot T$ constraints ensuring that a block, pillar, or section cannot be mined until the set of blocks, blocks and pillars, or sections that restrict access to it are all mined first. This constraint allows for practical mining of blocks, pillars, and sections. For each block, pillar, and section, a set $\{O_i, O_{ik}, O_j\}$ of blocks, blocks and pillars, and sections, respectively, are defined to be mined prior to its extraction. To minimize the complexity and number of constraints, the mine is divided into sections, which could be a section of the mine, or other aggregate of blocks and pillars as appropriate for mine planning purposes.

It is entirely possible to use only block-pillar constraints (Equation (5-3a) & (5-3b)) alone to model this problem. However, this will make each block constraint complex with all preceding blocks included in the constraint. By including the section precedence constraint in Equation (5-3c), along with block-pillar precedence in Equations (5-3a) and (5-3b), the complexity of each constraint is reduced (reduce the number of decision variables in each constraint). That is, with the exception of blocks in the same section as the block under consideration, all other blocks preceding the block, which are in other sections, can be represented by just the section decision variables. In instances of the problem where all three constraints are used, the section decision variables were modeled as continuous (as opposed to binary) variables, which allows for partial mining of sections in a period and selectivity of mining blocks. By defining section decision variables as continuous, the model becomes a mixed integer linear program.

This study explores ways to solve the R&P sequencing problem without the block-pillar constraints (Equations (5-3a) & (5-3b)) in order to save computational time. To accomplish this, the section precedence constraints (Equation (5-3c)) can be used without the block-pillar precedence constraints by adding a block-in-section constraint (Equation (5-7)). This constraint is written such that when a section is mined, all the blocks in that section are mined as well. In this instance of the problem, all block, pillar and section decision variables are defined as binary variables. Then the problem can be formulated without the block-pillar precedence constraints. In that case, the block precedence constraints constitute only $J \cdot T$ constraints instead of $(I+K+J) \cdot T$ constraints. However, this adds on an additional $(I + K) \cdot T$ block-in-section constraints (see Section 5.2.2.6 for details).

5.2.2.3. Reserve constraint. Equation (5-4a) to (5-4c) are reserve constraints, which ensure that the ore reserve mined at the end of a time period is less or equal to the available reserve. When a block or pillar is mined in a particular period, it cannot be mined in other periods. This constraint results in $(I+K+J)$ constraints.

5.2.2.4. Mining rate constraint. Equation (5-5) ensures that the total tonnage of material mined in each period is within production targets (MR_t^u and MR_t^l). The constraint is such that upper and lower limits can be set on production targets for each period, if necessary. This constraint controls the mining rate and results in T constraints.

5.2.2.5. Quality constraint. Equation (5-6) ensures that the solution meets quality requirements. For each quality property of interest, there is a separate equation. Quality properties of interest include grades and deleterious elements or minerals content. Each set will result in T constraints to ensure that average constituents (grade and contaminants) mined in a period are within the desired range ($q_t^{\mu,l}$ and $q_t^{\mu,u}$) for that period. Thus, this results in a total of $U \cdot T$ constraints. This constraint forms a basis for blending and quality control. The case study problem solved in this work does not account for multiple metals which sometimes occur, in commercial quantities, with lead mineralization. Only lead grades are considered in the case study.

5.2.2.6. Block-in-section constraint. Equation (5-7) ensures that once a section is scheduled for mining, all blocks in the section are mined in the same period. This constraint is used only when block and pillar precedence constraints (Equations (5-3a) and (5-3b)) are not used in the model. For it to work properly, all decision variables have to be binary. This results in $(I+K) \cdot T$ constraints, compared to $(I+K+J) \cdot T$ block-pillar precedence constraints.

5.3. SOLUTION FORMULATION

Equations (5-2) to (5-7) are solved using the CPLEX solver through the CPLEX API for Matlab®. The CPLEX software uses branch and cut search to solve discrete optimization problems. A generalized binary or mixed integer linear program is formulated mathematically in the form of Equation (5-8). Inputs required by CPLEX are the cost (or benefit) coefficient vector ($\mathbf{c}$), generated from the objective function; the equality constraint matrix ($\mathbf{A}_{eq}$); the inequality constraint matrix ($\mathbf{A}$); and limits (right hand-side of constraint equations) of inequality and equality constraints ($\mathbf{b}$ and $\mathbf{b}_{eq}$), respectively. The CPLEX solver also requires other constraints on decision variables (integer, binary, etc.).

$$
\begin{aligned}
&\min z = cx \\
&\textit{subject to}: \\
&\mathbf{A}_{eq}\mathbf{x} = b_{eq} \\
&\mathbf{Ax} \le \mathbf{b}
\end{aligned}
\tag{5-8}
$$

$$
\mathbf{x} = \begin{bmatrix} x_{11} \\ \vdots \\ x_{1T} \\ \vdots \\ x_{IT} \\ y_{11} \\ \vdots \\ y_{1T} \\ \vdots \\ y_{KT} \\ z_{11} \\ \vdots \\ z_{1T} \\ \vdots \\ z_{JT} \end{bmatrix}
\qquad
\mathbf{c} = \begin{bmatrix} c^x_{11} \\ \vdots \\ c^x_{1T} \\ \vdots \\ c^x_{IT} \\ c^y_{11} \\ \vdots \\ c^y_{1T} \\ \vdots \\ c^y_{KT} \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

Vector of decision variables Vector of cost coefficient

$$
c^x_{it} = w_e \sum_i \sum_t \left\{ \frac{p_t.r_i.t_i.q^\mu_i - c_t.t_i}{(1+d)^t} \right\} - \sum_\eta w^\eta_r \left[\sum_i \sum_t \frac{R^\eta_i.c^\eta_{it}}{(1+d^\eta_r)^t} \right]
$$

$$
c^y_{kt} = w_e \sum_k \sum_t \left\{ \frac{p_t.r_k.t_k.q^\mu_k - c_t.t_i}{(1+d)^t} \right\} - \sum_\eta w^\eta_r \left[\sum_k \sum_t \frac{R^\eta_k.c^\eta_{kt}}{(1+d^\eta_r)^t} \right]
$$

From Equation (5-1), elements of **c** are present values of blocks less the discounted risk penalty associated with mining these blocks in a particular period. Hence, the length of **c** is the number of decision variables. The solution algorithm developed by the author in this research allows the user to provide a set of blocks and time periods, block properties (grades, tonnages, etc.), discount rates, risks, and other input, which the algorithm uses to generate the vector **c**, as shown above. The solution algorithm also creates constraint matrices using information provided by the user. Equation (5-7) results in an equality constraint used to formulate $\mathbf{A}_{eq}$ and $\mathbf{b}_{eq}$ in Equation (5-8). All other constraints are inequality constraints. For each constraint, the solution algorithm needs to formulate a matrix which becomes part of $\mathbf{A}$ in Equation (5-8) and a right-hand side (RHS) vector, which becomes part of $\mathbf{b}$ in Equation (5-8).

The solution algorithm is implemented in Matlab® version R2014a and consists of a master function and various other sub-functions, which are used to formulate a specific cost coefficient vector or constraints. The Matlab® program is set up so that the user can provide the amount of each resource required to mine each block and the total resource available in each period. Thus, resources can be controlled for the life of the mine to maintain a feasible mining schedule and efficient use of resources. To formulate the mining rate matrix, the algorithm requires the tonnage of material in each block and the production demand for each period. The program requires the user to provide information on the blocks and pillars in each section. The program also requires the user to provide the average grade or deleterious material content of each block and limits on them for each period. Thus, the user can set upper and/or lower limits on grades and others for each period. The precedence constraint requires users to provide indices for each block and pillar, or section. For each block, the set of blocks that precede it are in the same sections (there is no need to add blocks from other sections because sections are added to the precedence sets). A set of indices are used to describe each section such that sections that precedes another section can be mined first. All inequality constraint matrices and RHS vectors are concatenated into a single matrix and a single vector. Along with the cost coefficient vector, these serve as input for the CPLEX solver.

The algorithm then calls the CPLEX solver via the CPLEX Matlab API. Once a solution is obtained from CPLEX, the solution is post-processed to obtain a meaningful sequence and a visual plot of the solution.

5.4. CASE STUDY

5.4.1. Case Study Problems. A simulated data set was used in this paper to verify and illustrate the model and solution algorithm. A geologic block model of lead mineralization was created with geostatistical methods using the Geovia Surpac® software. The mineable reserve in the model was determined using a regular (spacing of pillars) room-and-pillar lead mine design. The mining system is simulated as a single-level lead room and pillar operation. Figure 5-1 shows the mine layout and the simulated lead grades.

Each mining block was assigned geologic attributes including the grade, grade risk and geotechnical risk. The data was exported to text files as input along with block indices for the BILP model. The verification problem included 2,361 blocks, each containing approximately 250 tons of ore. The production sequencing problem did not account for primary developments such as drift development. The problem did not include pillar recovery, although the model is capable of solving problems with pillar recovery. The project and risk discount rates may vary depending on the situation and management's tolerance for risk. For this case study, the cash-flows and risks costs were discounted at 8% and 5%, respectively.

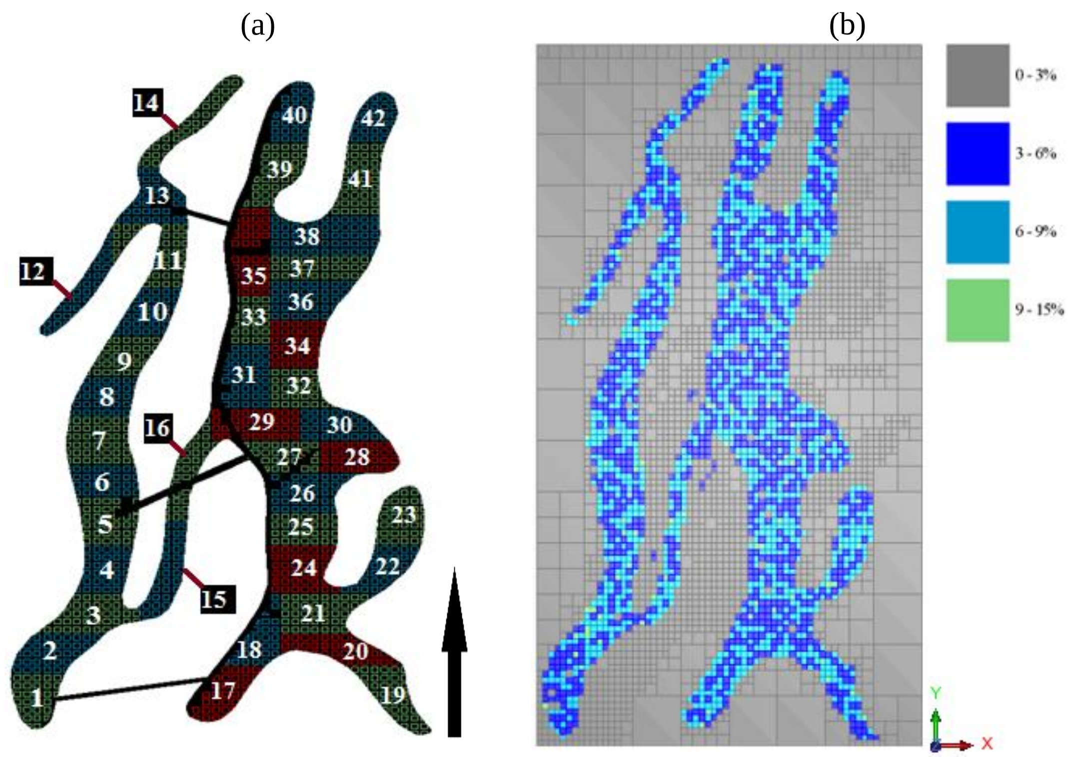

Figure 5-1 Case study: (a) mine layout, colored to illustrate sections; (b) grade distribution

Other input data needed to verify the model is shown in Table 5-1. The problem evaluated risk associated with lead grades and rock strength (geotechnical risk).

Table 5-1 Model input data

Parameter	**Value**	**Parameter**	**Value**
cost of mining ($/ton)	19.96	Resource capacity per period (loads)	2,192
lead price ($/kg)	1.70	Minimum production (ton/period)	32,857
Unit cost of risk (grade, $/ton)	15.40	Maximum production (ton/period)	53,571
Unit cost of risk (geotechnical, $/ton)	7.60	Number of sections	42
Discount rate (economic)	8%	Recovery (overall)	90%
Discount rate (risk)	5%	lead target (cutoff)	3.2%
Block tonnage (ton)	250	lead grade (mean)	6.2%
Resource capacity per block (loads)	13	Block grade std. dev.	1.59%

The verification problem included 42 sections. Table 5-2 shows the section precedence used in the examples. The section precedence is defined such that parallel mining of sections is feasible and respects the development plan (Figure 5-1). The block precedence data is too large to show in this section.

Table 5-2 Section precedence constraint

Section #	**Precedence Set**	**Section #**	**Precedence Set**	**Section #**	**Precedence Set**
1	{ }	15	{3}	29	{17,18,24,25,26,27}
2	{1}	16	{}	30	{17,18,24,25,26,28}
3	{1,2}	17	{}	31	{17,18,24,25,26,27}
4	{1,2,3}	18	{17}	32	{17,18,24,25,26,28,30}
5	{4,27}	19	{20,21}	33	{17,18, 24,25,26,27, 29,31}
6	{5}	20	{19,21}	34	{17,18,24,25,26, 28,30,32,}
7	{5,6}	21	{17,18}	35	{17,18, 24,25,26,27, 29,31,33}
8	{5,6,7}	22	{19,20,21}	36	{17,18,24,25,26, 28,30,32,34}
9	{5,6,7,8}	23	{19,20,21,2 2}	37	{17,18,24,25,26,28,30,3 2,34,36}
10	{5,6,7,8,9 }	24	{17,18}	38	{17,18,24,25,26,28,30,3 2,34,36,37}
11	{5,…,10}	25	{24}	39	{17,18,24,..,27,29,31,33 ,35}
12	{11,13}	26	{17,18, 25}	40	{17,18,24,..,27,29,31,33 ,35,39}
13	{5,…,11}	27	{17,18,24,2 5,26}	41	{19,..,23,25,26,28,30,32 ,36,37,38}
14	{5,..,11,13}	28	{17,18,24,2 5,26}	42	{19,..,23,25,26,28,30,32 ,36,37,38,41}

The optimization problem was solved using CPLEX version 12.2, with the Matlab® API, which uses the branch and cut method to solve integer linear programs. Computational experiments are performed to evaluate the effect of solving the production sequencing problem with and without the block precedence constraints (note that there are no pillar precedence constraints because there is no pillar extraction in this case). In this experiment, the problem is solved for only two periods to illustrate the differences between the problems.[23] In the first scenario, the MILP problem is solved with the block precedence constraints, which results in 4,722 block binary decision variables, 84 continuous section decision variables and 7,219 inequality constraints. This scenario allows for the partial mining of sections. Further computational experiments were done to examine the effect of block precedence on the complexity of the problem. A smaller precedence set is defined for each block by reducing the number of decision variables in each constraint. This was done by eliminating some of the blocks in each precedence set. The effect of reducing the precedence set by up to 32% and 48% on the solution time and number of iterations was evaluated.

In the second scenario, the problem is solved without the block precedence constraints. In order to use the block-in-section constraints, all the decision variables are defined as binary. It does not allow for partial mining of sections; thus each section can only be mined once in a single period. The problem is thus a BILP problem, which

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[23] Two periods are used because the point was to illustrate the effect of the constraints. A simple problem allows one to solve the problem many times in a short period for the analysis.

includes 4,806 binary decision variables, 4,722 equality constraints (block-in-section constraints) and 2,497 inequality constraints.

In a second set of experiments, the BILP problem (with no block precedence constraints) is solved for 14 periods. The problem includes 33,642 binary variables, 33,054 equality constraints (block-in-section constraints) and 3,061 inequality constraints. This problem is more realistic and used to illustrate the ability of the model to solve realistic mining problems. Two instances of this problem are solved: one where the ratio of ‘NPV’: ‘grade risk’: ‘geotechnical risk’ is 1:1:1 and another where the ratio is 1:2:2. These instances are used to illustrate the effect these ratios can have on the solution.

The output data obtained includes, the solution found by the optimization function (period each block is to be mined in), the optimal objective function value, execution time of the algorithm, and number of iterations. A gap tolerance of 0.001 is set for the problem. Both scenarios are run on a 64-bit Dell Precision T5610 computer with twin quadcore Intel Xeon E5-2609 (2.5 GHz, 10 MB) processors and 32 GB RAM

5.4.2. Results and Discussion. Each solution was verified, after post-processing, to ensure that the optimal production sequence respects all the constraints. Figure 5-2 shows optimal solutions of the two-period problem used to investigate the influence of the block-precedence constraints on computational complexity.

Table 5-3 shows the effect of the block precedence constraints on the number of iterations and CPLEX algorithm’s execution time. By eliminating the 4,722 block precedence constraints, 97% fewer iterations are required to obtain a solution even though 4,722 equality constraints are added to the problem and all variables are binary.

The computational time required to solve the problem reduced from 2.34 to 0.58 minutes. This is more than a four-fold reduction in computational time. This trend is probably going to be the same or worse (i.e. introduction of block precedence will require more than four times the computational time) for larger problems. Hence, by aggregating geologic blocks into sections and completely eliminating block precedence constraints, larger problems can be solved using the BILP in more reasonable time. This is very important for these kinds of problems because mine engineers tend to run many variants of the sequencing problem (where input parameters are varied in each instance) in order to determine the 'optimal' sequence. For large problems, with hundreds of thousands of blocks, the time savings to the mine can be significant.

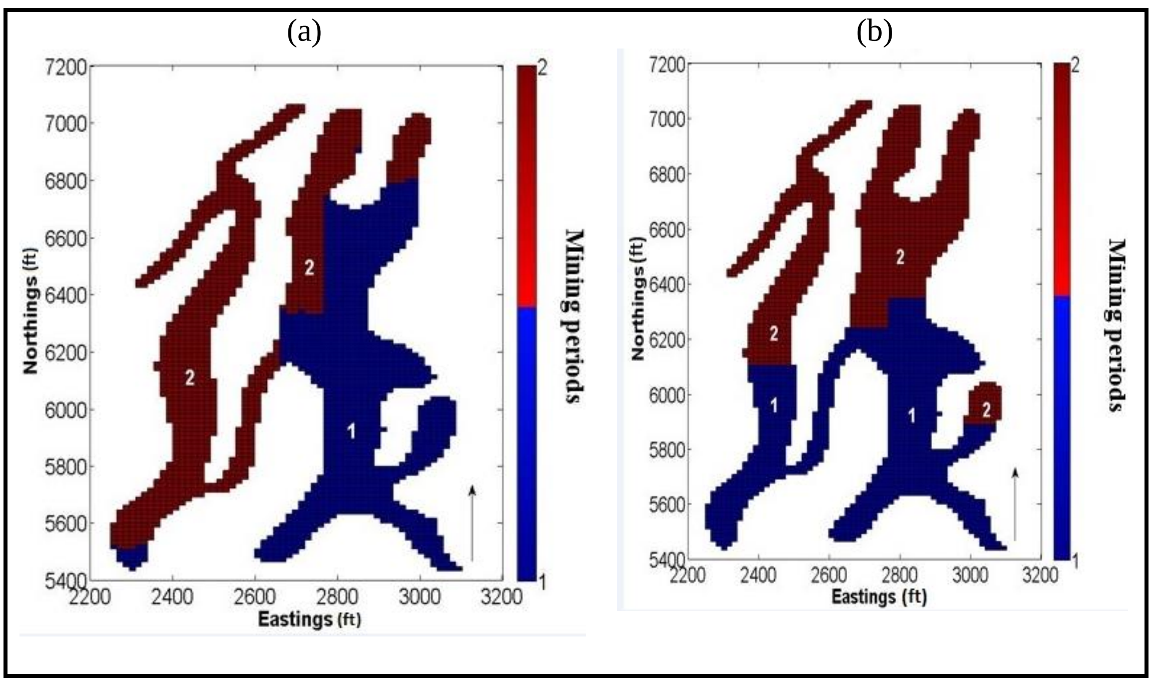

Figure 5-2 Two-period optimal production sequence: (a) with block precedence constraints; (b) without block precedence constraints.

Table 5-3 Effect of block precedence on computational complexity

Percent of original variables in block precedence constraints	Precedence constraints	Computational time (mins)	No. of iterations	Objective function value ($)
100%	With block precedence constraints	2.34	36,381	4516.2M
100%	Without block precedence constraints	0.58	966	4515.9M
68%	With block precedence constraints	1.80	6,711	4522.0M
68%	Without block precedence constraints	0.51	683	4526.0M
52%	With block precedence constraints	1.39	6,260	4526.3M
52%	Without block precedence constraints	0.51	983	4526.0M

The effect of the nature of the block precedence constraints on the observation, that adding block precedences significantly increases computation complexity, was examined with further experiments. That is, the work examined whether the time savings is more or less pronounced for less complicated block precedence constraints. This analysis was done reducing the size of precedence set for the same problem (same number of decision variables and constraints).

Table 5-3 shows the result of the analysis which includes reducing the number of decision variables in each precedence constraint by 32% and 48%.[24] The result shows

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[24] Note that these problems do not result in practically feasible solutions. They are just used to examine whether the four-fold time savings is dependent on the complexity of the block precedence constraints.

that as the block precedence constraints become less complicated (few decision variables – fewer blocks in the precedence set), relative savings in solution time and number of iterations reduces. For instance, with 32% fewer decision variables in the constraints, the computational time is 3.5 times (compared to 4 times) smaller and the number of iterations decreases by 90% (compared to 97%) compared to when block precedence constraints are excluded. Also, with 48% fewer decision variables in the constraints, the computational time is 2.7 times smaller and the number of iterations decreases by 84% compared to when block precedence constraints are excluded. It is important, however, to still note that, even for the simplest block precedence constraints in these examples, the computational time is still almost 300% higher, when the block precedence constraints are included in the model.

As one would expect, the optimal production sequence obtained for the two problems (with and without block precedence) were different (Figure 5-2 and

Table 5-3). The flexibility in including the block precedence constraint may result in a significant difference in the optimal sequence and objective function values. The differences in the optimal sequence and objective function values will depend on the way the blocks are aggregated into sections. In some instances, sections can be meaningful for managing production (i.e. mine engineers often divide the mine up into sections for ventilation and other requirements). In other instances, smaller sections may be defined to primarily serve as a means to aggregate blocks for sequencing. The effect of block aggregation on the solution is beyond the scope of the current work and should be explored as part of future work.

Figure 5-3 to Figure 5-6 and Table 5-4 show the results of the 14-period problem, which was also used to evaluate the effect of the effective ratios on the solution. The figures show the production, resource (truck loads) scheduled and average grade per period, respectively. The optimal solutions respect all constraints (for precedence constraints compare Table 5-2 and Table 5-4). The BILP problem was tested for multiple optima by implementing Balas and Jeroslow (1972) binary cut (Equation (5-9)) for excluding the existing optimal solution. The results indicate that multiple optima with the same objective function value exist for the problem.

$$\sum_{i \in B} x_{it} - \sum_{i \in C} x_{it} \le |B| - 1, \;\; B = \{i | x_{it} = 1\}, \; C = \{i | x_{it} = 0\} \qquad (5\text{-}9)$$

Where:

x_{it} Binary decision variable

B Set of decision variables with solution $x_{it} = 1$

C Set of decision variables with solution $x_{it} = 0$

Given that the production, resource and grade constraints are never active, the solution is driven by precedence and reserve constraints. (The reserve constraints are always active since all the material is mined to maximize NPV.) The precedence constraints are particularly important for underground mines since the nature of development activities means strict precedence have to be observed so that development can proceed ahead of mining activities.

Table 5-4 Optimal production sequence

Period	1:1:1 solution (sections)	1:2:2 solution (sections)
1	{17,18,24}	{17,18,24}
2	{25,26,27}	{25,26,27}
3	{28,29,31}	{28,29,31}
4	{33,35,39}	{33,35,39}
5	{30,32,40}	{30,32,40}
6	{34,36,37}	{34,36,37}
7	{1,2,3}	{1,2,3}
8	{4,5,15}	{4,5,6}
9	{6,7,21}	{7,8,9}
10	{8,9,10}	{10,11,13}
11	{11,13,14}	{19,20,21}
12	{16,19,20}	{12,22,38}
13	{12,38,22}	{14,15,23}
14	{23,41,42}	{16,41,42}

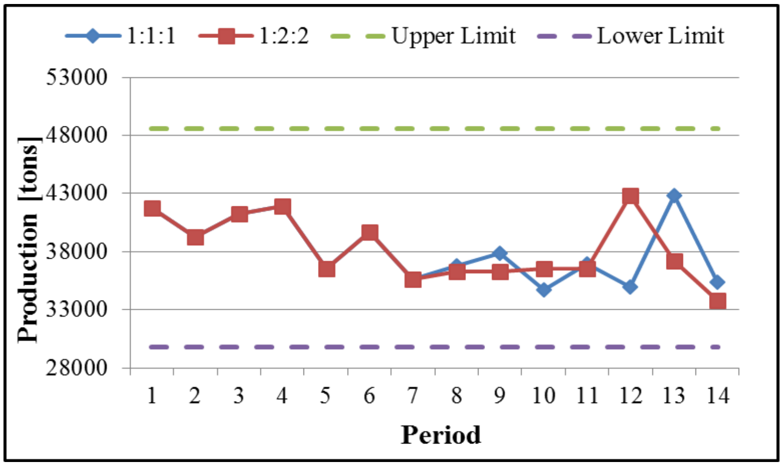


Figure 5-3 Production per period

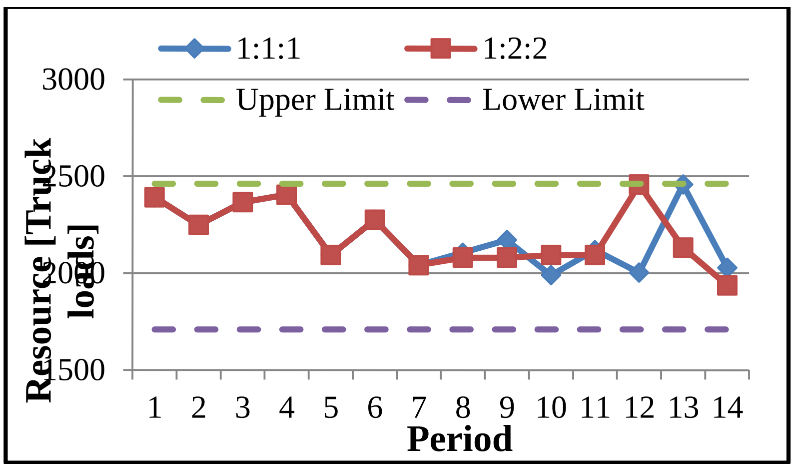


Figure 5-4 Amount of resources used in each period

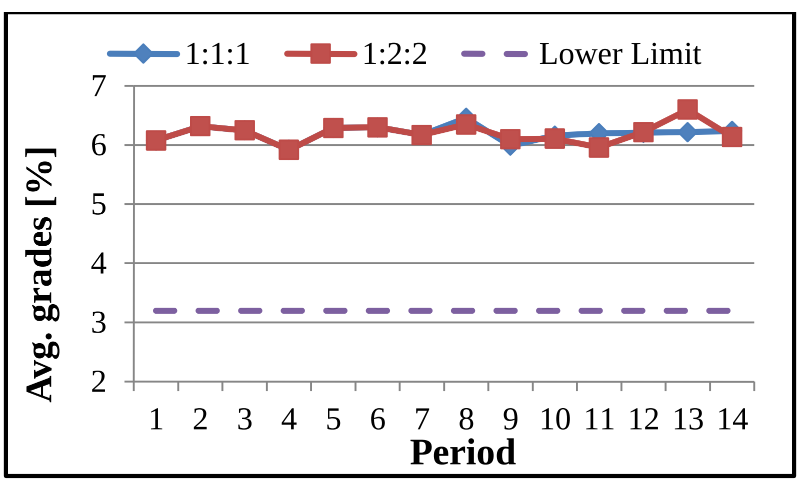


Figure 5-5 Average lead grade mined in each period

One would expect a model that maximizes NPV to schedule higher grade sections first. However, Figure 5-5 shows that grades do not drive the optimal solution. Instead, the solution seems to be driven by precedence constraints, as discussed earlier. Sections in the mine development areas are mined in earlier periods regardless of the average grade of blocks in them or risks associated with them.

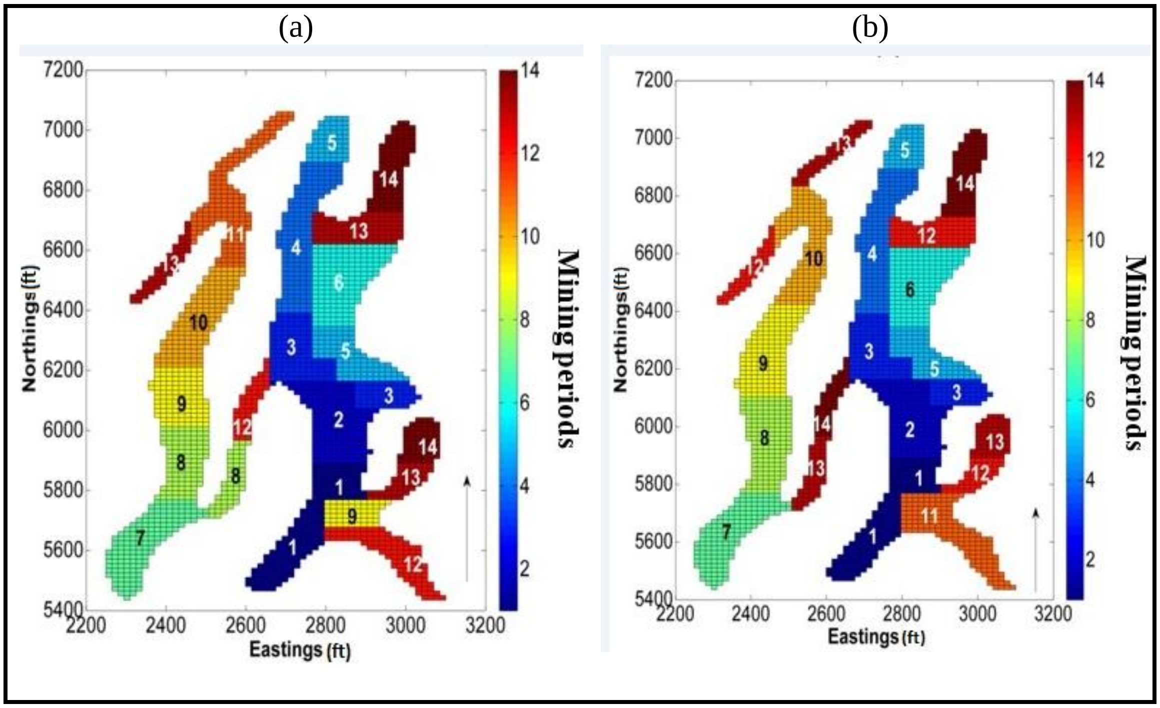


Figure 5-6 14-period optimal production sequence: (a) 1:1:1 ratios; (b) 1:2:2 ratios

Figure 5-6 and Table 5-4 show the effect of different effective ratios on the production sequence. In Figure 5-6(a) the significance of NPV and risk on the production sequence are the same with an effective ratio of 1:1:1. Further investigation is conducted to evaluate the effect of increasing the significance of risk by using a ratio of 1:2:2. Figure 5-6(b) shows the impact of NPV to risk (grade and geotechnical) ratio of 1:2:2 on the optimal sequence. In both scenarios, three sections are mined in each production period. The combinations of sections mined in Periods 8 to 14 are different when the relative significance of risk is increased. For instance, section 14 is scheduled for mining in Period 11 and 13, respectively, as the effective ratios are changed from 1:1:1 to 1:2:2. The average grade of blocks in section 14 is 6.4% which is well above the cutoff grade of 3.2% and the overall average grade of the deposit of 6.19%. The average grade and geotechnical risks of the blocks are also 46% and 50%, respectively with both risk for the entire deposit less than 25%. Consequently, with an emphasis on risk, it is preferred to delay mining of this block till period 13. In a practical application, this will allow engineers and geologist time to improve the uncertainty surrounding the grade and geotechnical model estimates.

This presents the BILP model as an effective risk management tool that aims to maximize the overall profit. The economic penalty of risk is a function of the type of risk modeled, the associated cost penalty and discount rate. The effect of a particular risk on the production sequence is therefore a function of the mining system and the optimization problem solved.

5.5. SUMMARY

The ability to model mining risk in room-and-pillar underground sequencing using a deterministic binary integer linear programming framework is demonstrated in this Chapter. This was achieved by incorporating risk as a discounted cost penalty in the objective function. Although recent stochastic integer programming approaches demonstrate the significant impact of uncertainty in production sequencing, the deterministic modeling framework developed in this study provides a substantial advantage over traditional approaches without increasing the computational time.

Multiple risks can be accounted for without increasing the computational time with each additional risk factor. The significance of risk on the optimal mine sequence is controlled by assigning effective ratios to the risk model in the objective function. The mine sequencing model has been verified with a sample lead mine problem that includes 2,361 cuts in 42 sections. The study demonstrates that the complexity and number of block precedence constraints affects the computational complexity (number of iterations and execution time). Therefore, approaches that completely eliminate the need for block-pillar precedence constraints will significantly reduce the computational complexity. In the simple two-period example, presented in this chapter, there was a four-fold reduction in computational time and 15% reduction in number of iterations. The same sample data is used to examine the effect of the relative importance of risk on the solution. The results show that altering the importance of risk can significantly change the production sequence. In the verification problem, different sections are sequenced to be mined in Periods 8 to 14 when the relative importance of risk is increased by a factor of 2. This study shows that it is possible to incorporate risks into room-and-pillar production sequencing using BILP and carefully examine factors that affect computational complexity. This provides insight that will be useful for researchers and industry alike.

Further work is required to evaluate other factors that affect the computational time of this BILP problem used to solve room-and-pillar mine sequencing problems. For example, the effect of the number of decision variables (number of blocks and sections) on the computational time needs to be evaluated with carefully defined test problems. As part of this, the effect of sections (or block aggregation) should be examined and optimal aggregation strategies recommended. It will also be helpful to use this model to solve a large scale real-life (non-proprietary) production sequencing problem.

6. MINIMIZING THE COMPUTATIONAL COMPLEXITY OF PRODUCTION SEQUENCING PROBLEMS USING THE CUTTING PLANE METHOD

6.1. INTRODUCTION

To obtain the highest possible value from a mine operation, it is essential to optimize the sequence of ore and waste extraction. Mine operations consist of a variety of activities, most of which are performed sequentially. Common commercial software such as Carlson, Geovia MineSched, Maptek and XPAC are often used to schedule these activities. These software use heuristic methods that only find approximate solutions to production sequencing problems. Even so, the mining industry still relies heavily on their use. Current research uses mathematical optimization approaches, which can solve production sequencing problems and provide optimal solutions. The limitation to this approach is the complexity of the problem which requires vast computational power (speed and memory) and time to solve the problems. This diminishes incentive in industry to apply mathematical optimization methods.

To minimize the computational complexity of the problem, pre-processing techniques (such as block aggregation, coefficient reduction, and Lagrangian relaxation methods) can be used to reduce the number of enumerations required to obtain an optimal solution. Apart from generalized cutting planes developed as part of the branch and cut algorithm, problem-specific cutting planes can be used to pre-process the problem in other to minimize the computational time and the number enumerations required to solve the problem. Generating such cutting planes require extensive knowledge of the problem to be solved.

In this PhD research, the author has applied block aggregation pre-processing techniques in solving the R&P production sequencing problem in Chapter 5. This chapter continues the research by testing the hypothesis that heuristic pre-processing can be used to minimize the complexity of the production sequencing problems before solving it with the generalized branch and cut method. The hypothesis was validated using the BILP R&P production sequencing problem in Chapter 5. Three cutting plane pre-processing techniques are introduced and tested to investigate their effect on the number of iterations and computational time of the problem. The first cutting plane implemented ensures that solutions to sub-problems that mine the highest valued (blocks that maximize the objective function) blocks in later periods subject to the precedence constraint are eliminated from the feasible space. The second technique defines cutting planes such that geologic sections with no preceding sections will be mined in earlier periods. Therefore, the technique eliminates solutions that mine such sections in later periods from the search space. Lastly, a third technique introduced a cutting plane that eliminates feasible solutions that mine sections in the primary development area in later periods. The performance of these techniques are evaluated with the case study.

6.2. SOLVING PRODUCTION SEQUENCING PROBLEMS WITH PRE-PROCESSING CUTTING PLANES

As explained in Chapter 2, the branch and cut algorithm is one of the most advanced exact methods for solving combinatorial optimization problems such as production sequencing problems. In this work, the BILP room and pillar production sequencing problem is solved using the branch and cut algorithm implemented in ILOG CPLEX®.

CPLEX® uses the traditional branch and cut algorithm to solve integer problems. The CPLEX® software is capable generating different types of cutting planes including the Gomory fractional, cliques, flow path and disjunctive cutting planes. These cutting planes are generalized formulations that apply to a wide range of integer problems. Similar to the conventional branch and cut algorithm, the problem is either divided into sub-problems or cutting planes are added in order to generate tighter bounds on subsequent sub-problems. Once an integer solution is found, it is made the incumbent solution as well as the new bound on the sub-problems. All sub-problems with objective function values worse than the incumbent[25] solution are pruned. CPLEX® also gives the user an option to define a gap tolerance between the best integer solutions so far and the true optimal (estimated using the most optimistic bounding function). The gap tolerance is defined based on the level of accuracy desired by the user for a particular problem. The tolerance parameters can either be defined as absolute objective function difference or relative objective function difference (ILOG CPLEX® 2011).

Pre-processing a problem with specifically defined cutting planes can improve the performance of branch and cut algorithms significantly (Darby-Dowman 1998, Bley et al. 2010). This improvement in performance is gained by reducing the search space, within the feasible set, that the branch and cut algorithm searches for the optimal solution. However, there are two key challenges to overcome: (1) how to define these cutting planes without excluding the optimal solution; and (2) to ensure the computational time required to generate the cutting plane does not exceed the savings in time. The first issue can result in situations where the branch and cut algorithm converges to a solution other than the optimal, regardless of how quickly it finds that solution. This challenge can be overcome by studying the problem in question to understand the nature of the optimal

[25] Refer to Chapter 2 for detailed definitions of the branch and cut terminologies.

solutions. The second issue can result in instances where the combined solution time (the sum of the time it takes to generate the cutting plane and the time it takes to solve the subsequent problem) is greater than the time it takes branch and cut without any pre-processing to solve the same problem. This can be overcome by developing efficient algorithms to generate the cutting planes so that the computational time it takes to generate the cutting planes are minimal.

In the remainder of this chapter, the author presents three algorithms, motivated by an intimate understanding of the LP-based mine production sequencing problems, which have the potential to lead to efficient pre-processing. The goal is to evaluate whether any of these strategies can increase the computational efficiency of the CPLEX® branch and cut algorithm when solving the case study problem. Computational efficiency is measured by the computational time and number of iterations needed to solve the problem. If any of the methods is able to increase computational efficiency, then the hypothesis would be proven and the result can be motivation for developing efficient pre-processing algorithms.

6.3. SPECIALIZED CUTTING PLANES FOR BILP R&P PRODUCTION SEQUENCING PROBLEMS

The use of specialized (problem-specific) cutting planes as a pre-processing technique depends primarily on the characteristics of the problem. In this section three pre-processing cutting plane techniques specific to most mining operations are discussed. These are cutting planes based on: (1) a greedy (bin packing) algorithm; (2) mine sections with no precedence constraints first; and (3) mine sections in the development area first.

6.3.1. Based On a Greedy (Bin Packing) Algorithm. The greedy algorithm is a heuristic algorithm which only considers the current best solution at any instance without considering the overall solution. The assumption is that a global solution can be obtained by choosing local optimal solutions. The greedy bin packing problem is a well-studied optimization problem. Assume you are required to place a set of items into a fixed number of bins to fill the bins while minimizing the weight of items in the bins. Each of the items in the set has a defined weight and volume and the bins have limited volume. The best solution is the solution that fills each bin to capacity with the least amount of weight. For each bin, a greedy algorithm selects the lightest item in the set. A different bin cannot be opened until the current bin is full, thus the last bin will have the heaviest items. The optimal solution is obtained using a simple iterative approach (Yap 2005).

In production sequencing, the bin and items are synonymous to each planning period and geologic blocks (or groups of blocks if aggregation is used), respectively. The weight of each item is similar to the value (contribution of the block or section to the objective function) of each block. For production sequence optimization, the aim is to fill each period to capacity such that each period has the highest objective function value possible. The capacity of each period is limited to the production targets (such as mining rate and quality target) in that period. Mine production sequencing problems are known to be similar to the greedy packing problem (Martinez and Newman 2011, Chicoisne et al. 2012).

In order to reach optimality faster, the author hypothesizes that a valid cutting plane based on the greedy algorithm can be used to pre-process the production sequencing problem. The aim is to minimize the feasible search space by eliminating solutions that do not prioritize “high valued” blocks subject to the production constraints.

In so doing, the highest valued integer solution should be obtained in the shortest possible time.

To implement this cutting plane, an algorithm is required to identify the high valued blocks. In LP based production sequencing, the value of each geologic block is assigned as an attribute prior to sequencing. Therefore, it is possible to select the highest valued blocks for implementing the cutting plane. There are two main things to consider:

a. Is the block of significantly high value compared to other blocks (ranking based on contribution to the objective function)?; and

b. If the block is of high value, is it feasible to mine it in the initial planning periods.

To address the first issue, a ranking of the blocks in order of decreasing value is necessary. This is a simple sorting algorithm so long as one can define what the "value" of a block is. The value is the contribution of the block to the objective function independent of when it is scheduled to be mined. For example, if the objective function is the NPV, then the undiscounted profit is the block value. In most cases, the coefficients of the block decision variables for the first period (or any period for that matter) can be used as block values for the sorting algorithm.

To address the second issue, some heuristic strategies are necessary. It is important to select blocks that result in a feasible solution. For instance, if the goal is to restrict the mining of the top five blocks to the first two periods, it must be feasible to mine those blocks in the first two periods. If mining the top five blocks in the first two periods violates the production, quality, stockpile or processing plant capacity constraints then the cutting plane would result in an infeasible solution. The approach proposed in this work is to first select the number of periods and then select the number of blocks that is only a small proportion (α) of the production target. The strategy depends on the fact that a small proportion of blocks is very unlikely to violate the other constraints (quality, blending etc.).

If O_v is the set of high valued blocks that are feasible to mined in the first t periods, then the cutting plane (added in the form of an inequality constraint) must exclude (cut) all sub-problems where the solution mines these blocks after period t. Let $\{t+1,\ t+2,\ ....,\ T\}$ be the set of periods after time t.

$$\sum_{t'} x_{it} \leq 0 \quad i \in O_V \quad t' \in \{t+1,\ t+2,\ ....,\ T\} \tag{6-1}$$

In summary, the algorithm can be presented as follows:

1. Solve LP relaxation of the given production sequencing problem.
2. If LP relaxation is infeasible, STOP. Problem is infeasible.
3. Use sorting algorithm to rank blocks by value.
4. If $\alpha < \alpha_{\min}$, STOP. Algorithm fails.
5. Select α ($0 < \alpha < 1$) of blocks required to meet production in the first t periods.
6. Create cutting plane constraint using Equation (6-1).
7. Solve LP relaxation with new cutting plane.
8. If solution is infeasible, increase α by μ. Go to Step 4.

The user is required to provide the initial α, μ, the step size of α, and the value of t. If α is selected carefully, the algorithm should be able to find a feasible cutting plane in the very first step. Admittedly, there are some challenges in optimizing the gain in computational time as a function of α. Theoretically, α should be chosen such that it minimizes the computational time of the ensuing branch and cut algorithm without excluding the optimal solution. Without any guidance on how to choose α, analysts are likely to over-estimate it out of caution and the resulting cutting plane may not restrict the feasible set enough, even when this is possible. On the other hand, if α is under-estimated, the resulting cutting plane may be too restrictive leading to an infeasible problem or a problem that does not contain the true optimal solution.

6.3.2. Based On Blocks with No Precedence. Mine production sequencing problems modeled as LP include some form of precedence constraints. The precedence constraint ensures that, if access to block b is restricted by block a laterally or vertically, b cannot be scheduled for mining until a has been scheduled for mining. Practically, blocks with no restrictions (there is immediate access to them) in the development area tend to be mined first. The direction of mining also gives the analyst an idea of blocks that are likely to be mined in the early stages of the planning horizon as part of an optimal solution.

The author hypothesizes that a specialized cutting plane can be developed to ensure solutions that mine blocks with no precedence constraint (i.e. blocks that can be accessed immediately) in later periods are eliminated from the feasible search space. This should minimize the computational time needed to reach optimality. The number of solutions cut from the search space will depend on the number of blocks with no precedence constraints, other production constraints, and relative value of the blocks. For instance, Figure 6-1 shows the direction of mining for seven geologic blocks. There are three blocks (1, 2, and 3) in the figure with no precedence constraints. Assume the mining capacity is such that in each mining period only two blocks can be mined and we desire to sequence the blocks over four periods. Assume also that each block cannot be mined until the block before (in the direction of mining) has been mined (i.e. block 2 has to be mined before 5 and blocks 2 and 5 have to be mined before 7). Given these facts, it is likely that the optimal solution includes mining blocks 1, 2, and 3 in the first two periods. Of course, to maximize the net present value of the operation, a different optimal solution may be obtained that does not mine blocks 1, 2, and 3 in the first two periods (e.g. the optimal solution may be Period 1: {2, 5}, Period 2: {1, 7}, Period 3: {3, 4} and Period 4: {6} if block 7 is a really high value block). Therefore, the pre-processing cutting plane should be implemented such that the optimal solution is not excluded from the feasible space.

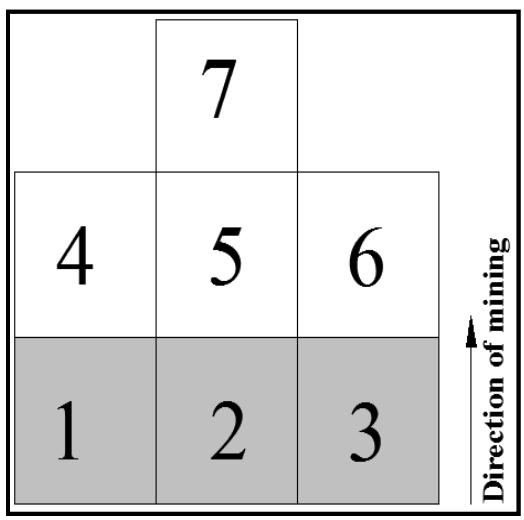


Figure 6-1 A simple example of production sequencing problem

The steps for implementing a pre-processing technique approach based on blocks with no precedence constraint is as follows:

1. Solve the LP relaxation of the given production sequencing problem.

2. If LP relaxation is infeasible, STOP. Problem is infeasible.

3. Determine the number of blocks in each precedence set (N_i) for all blocks. In most LP formulations of the production sequence problem, the number in the set is already determined as it is required to specify the precedence constraint (Equation 5-3).

4. If $t \geq T$, STOP. Algorithm fails.

5. Determine blocks i such that $i \in O_i$, where O_i is the set of blocks with empty precedence sets ($N_i = 0$).

6. Create cutting plane (inequality constraint) constraint that is violated by solutions that require blocks i ($i \in O_i$) to be mined in a period later than t .

7. Solve LP relaxation with new cutting plane.

8. If solution is infeasible, increase *t* by μ. Go to Step 4.

Let $\{t+1,\ t+2,\ ....,\ T\}$ be the set of periods greater than t . The cutting plane is given as:

$$\sum_{t'} x_{it} \leq 0 \quad i \in O_i,\ \ t' \in \{t+1,\ t+2,\ ....,\ T\} \tag{6-2}$$

Much like the greedy algorithm, the user has to provide the value *t*, which determines the limits placed on the solution. Too small a *t* and the problem may become infeasible or the true optimal might be excluded. Too large a *t* and the solution set may not be restricted enough to provide significant gains in computational time. The choice of step size is also an important choice that determines how efficient the algorithm is at generating feasible sub-problems.

6.3.3. Based On Blocks in the Development Area. Mine development includes the extraction of ore or waste material to create an opening allowing access to ore. There are different types of development in production sequencing. These include primary, secondary and tertiary development. The type of development depends on the stage in the mining process in which development blocks are sequenced to be mined. Primary development is done at the initial stages of mining to gain access to the production blocks. These include drifts, entries, crosscuts, and shaft raises. Secondary and tertiary development can be postponed to periods when they are needed.

As part of the sequencing problem, precedence constraints are defined that ensure developments blocks are mined prior to production blocks. In addition to the precedence constraints, production sequence problems can be pre-processed with specialized cutting planes to ensure that primary development blocks are mined in earlier periods, therefore eliminating sub-problems with integer solutions that mine primary development blocks in later periods.

The following algorithms can be used to generate pre-processing cutting planes based on a strategy to force blocks in the development area to be mined first:

1. Solve the LP relaxation of the given production sequencing problem.
2. If LP relaxation is infeasible, STOP. Problem is infeasible.
3. Identify blocks in the development area (O_d).
4. If $t \geq T$, STOP. Algorithm fails.

5. Create a cutting plane (inequality) that restricts the mining of development blocks to the first t periods.
6. Solve LP relaxation with new cutting plane.
7. If solution is infeasible, increase t by μ. Go to Step 4.

The cutting plane is such that feasible solutions that mine development blocks after t periods are eliminated (Equation (6-3)). Let $\{t+1,\ t+2,\ ....,\ T\}$ be the set of periods greater than t. The cutting plane is given as:

$$\sum_{t'} x_{it} \leq 0 \quad i \in O_d,\ \ t' \in \{t+1,\ t+2,\ ....,\ T\} \tag{6-3}$$

Just as in the previous two techniques, the user has to provide the value *t*, which determines the limits placed on the solution. The same considerations apply to the choice of *t*.

The development of pre-processing cutting planes depends on the characteristics of the production sequencing problem. Unlike the other two techniques, in the case of development blocks, other cutting planes can be developed for secondary and tertiary development areas if the analyst is aware of the optimal time frame in which they are to be mined.

In the next section, the effect of the cutting plane pre-processing on the computational efficiency of the branch and cut procedure is investigated using a case study based on the same problem in Chapter 5.

6.4. CASE STUDY

6.4.1. Data and Problem. A lead room and pillar mining data is simulated in the Geovia Surpac® software (Figure 5-1). The room and pillar production sequencing problem was modeled as a binary integer linear program. The objective of the problem was to maximize the net present value and minimize the grade and geotechnical risk subject to mining constraints. The production constraints included (refer to Chapter 5 for detailed BILP model):

- Resources constraints which ensured that the amount of equipment resources needed to mine scheduled blocks in a particular period does not exceed the available resources in that period.
- Reserve constraints that ensured the amount of material scheduled to be extracted does not exceed the material available in each period.
- Quality constraint ensured that the ore quality target in each period is met.
- Mining rate constraint ensured that the production target in each period is met.
- Precedence constraint ensured that blocks are mined in a way that respects the required or desired precedences.

The attributes of each block include the tonnage, value, amount of resource needed, risk factor and quality of the blocks. The problem consisted of 2,631 blocks aggregated into 42 sections (Figure 5-1) with 33,642 binary variables, 33,054 equality constraints and 3,061 inequality constraints to be solved over 14 periods. In order to minimize the number of constraints, the block precedence constraints were replaced with block-in-section constraints. The constraint is such that once a section is mined, all the blocks in the section are mined as well. The problem is solved such that the effective ratios of the NPV to risk (grade and geotechnical risk) in the objective function is 1:2:2, respectively.

The goal of this case study is to test the research hypothesis using this problem. The author pre-processes the problem using the three pre-processing techniques discussed in this section. In each case, the computational efficiency and solution is compared to the solution obtained for the problem without any pre-processing.

6.4.2. Based On a Greedy Packing Approach. For this problem, the geologic blocks are aggregated into sections using the block aggregation approach implemented in Chapter 5. The sequence is, therefore, optimized based on the defined mining sections. Consequently, greedy packing approach is applied to the sections rather than the blocks. The sections were ranked with respect to their value in order to implement the greedy (bin packing) approach. The value of each section is calculated as the sum of the block values in that section. Figure 6-2 shows the sections and their values. For this case study, the highest valued section is Section 38 (Figure 6-2). The production capacity (mining rate constraint) is such that, it is feasible to mine a maximum of three sections per period. The precedence constraints described in Table 5-2 are such that Section 38 cannot be mined until at least 11 sections have been mined. In the optimal solution (Figure 5-6 (b)), Section 38 is scheduled to be mined in period 12. Thus feasible solutions that mine Section 38 in periods 13 and 14 can be safely deleted from the search region.

Two scenarios of the greedy (bin packing) algorithm are analyzed in this case study. The 12 highest valued sections are selected and restricted to be mined: (1) in the first eight (the problem is infeasible when the sections are restricted to fewer periods than eight) periods; and (2) in the first 12 periods. The results are discussed in the Results and Discussion section.

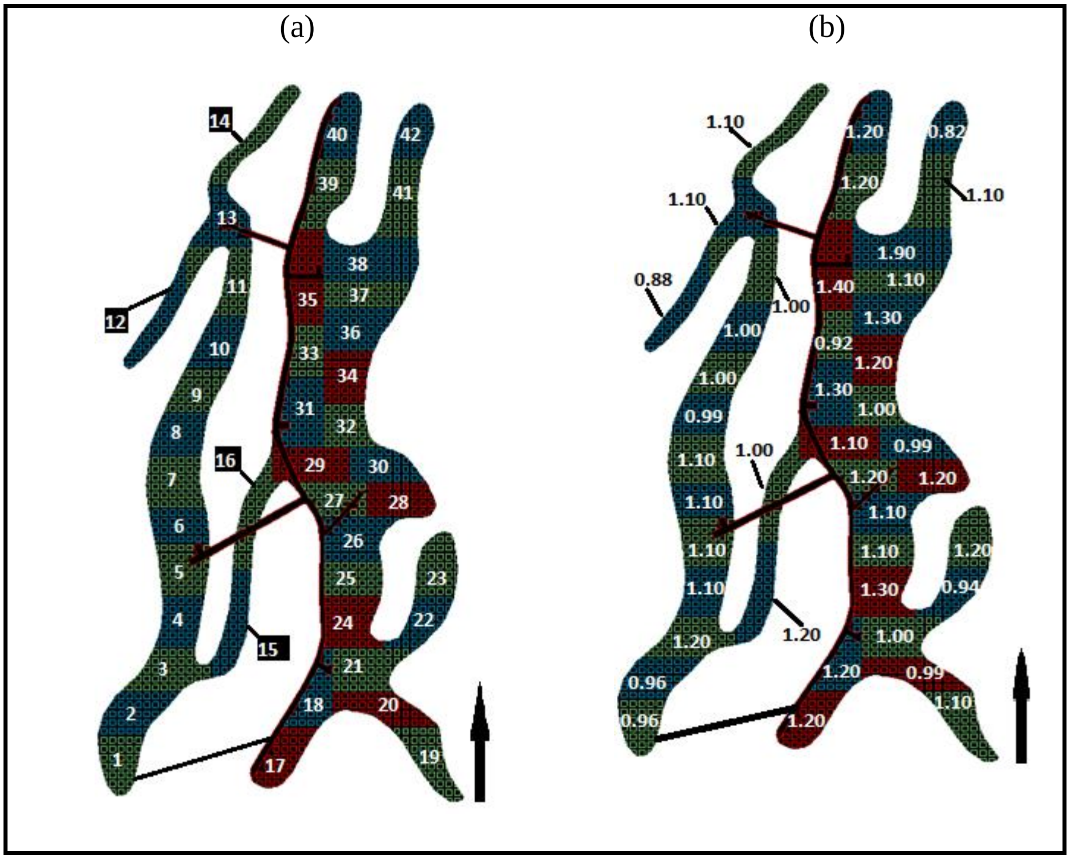


Figure 6-2 R&P mine design and layout showing the: (a) 42 sections; (b) value of each section (\$ x10^8)

Table 6-1 Top 12 highest valued sections

Ranking	**Section**
1	38
2	35
3	31
4	24

Table 6-1 Top 12 highest valued sections. Cont.

Ranking	**Section**
5	36
6	27
7	39
8	40
9	23
10	28
11	34
12	2

6.4.3. Based On Sections with No Precedence Constraints. In this case study, block precedence constraints are replaced with a block-in-section constraint and section precedence constraint. The block-in-section constraint is defined such that is if a section is sequenced to be mined, the blocks in that section are mined as well. The section precedence, therefore, controls the period in which these blocks are mined. For the case study, only three sections (1, 16, and 17) are without precedence constraints (Table 5-2). Thus, if it is feasible to mine these sections in earlier periods, they can be restricted to the first period. Using such a strict bound on the subsequent sub-problems may result in a sub-optimal solution. In Table 5-4, the solution indicates that it is optimal to mine Section 1 in period seven, Section 16 in period 14 and Section 17 in period one. Two scenarios are evaluated with the no precedence based pre-processing technique applied to the problem. The three sections with no precedence are restricted to: (1) the first period and (2) the first 12 periods.

6.4.4. Based On Sections in the Development Area. Figure 6-3 shows the sections in the primary development area. The precedence constraints ensure that the sections in the development area are mined prior to mining production sections. There are 12 sections in the primary development area. In conjunction with the precedence constraints, the development sections can be restricted to be mined in earlier periods. A cutting plane was implemented that restricts the developments sections to be mined in the first five periods. A second analysis was done that restricts the mining of the development blocks to the first 12 periods. The results of the analysis are discussed in the Results and Discussion section.

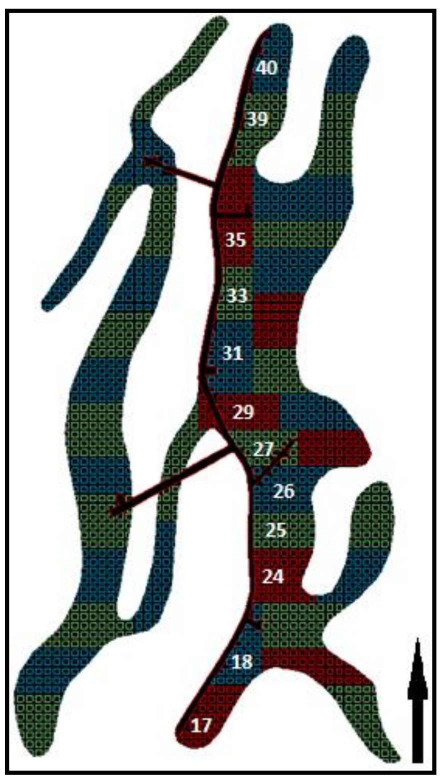


Figure 6-3 Primary development area

6.4.5. Results and Discussion.

6.4.5.1. Based on a greedy packing approach. Table 6-2 shows the change in objective function value, computational time, and number of iterations as well as the pre-processing execution time. Figure 6-4 shows the optimal production sequence obtained after implementing the greedy (bin packing) algorithm as a pre-processing technique for the BILP problem.

The first scenario involved creating a cutting plane (inequality constraint), which eliminates feasible solutions that mine the first 12 highest valued blocks (Table 6-1) after period 8. The production sequence obtained differs from the optimal solution. The objective function value decreased by 1.85 %. The global optimal solution (Table 5-4) was not obtained because it is not optimal to mine Sections 38 and 23 prior to period 8. Thus, the implemented cutting plane eliminated the optimal solution from the feasible space. However, there was a significant improvement in the computational time and the number of iterations required to solve the BILP problem. The computational time and number of iterations decreased by 37.27 % and 39.17 %, respectively. Although there was a significant difference in the computational time, the time required to execute the pre-processing algorithm was greater than the time it took to solve the problem without pre-processing (~ 24.83 seconds). Hence, overall the pre-processing strategy resulted in longer solution times.

The second scenario includes relaxing the cutting plane so that the optimal solution is included in the feasible search space. The high value blocks were restricted to the first 12 periods. The sequence obtained in this scenario was optimal (Figure 6-4 (b)). The computational time and the number of iteration decreased by 17.81 % and 48.18 %, respectively. Compared to the first scenario, although the improvement in computational time was not as significant, the number of iteration required to reach optimality was significantly less.

The application of the greedy algorithm as a pre-processing approach to the R&P mine production sequencing reduces the computation complexity (time and number of iterations) significantly although the reduction of the feasible search space was minimal. The main limitation of this application is the execution of the pre-processing technique. A more efficient algorithm, if possible is needed to minimize the execution time of pre-processing.

Table 6-2 Effect of greedy algorithm based pre-processing on the objective function value, computational time and number of iteration

Greedy packing approach restricted to the first:	**Pre-processing execution time (%)**	**Change in optimal objective function value (%)**	**Change in computational time (%)**	**Change in no. of iterations (%)**
8	3.72	-1.85	-37.27	-39.17
12	3.72	0.00	-17.81	-48.18

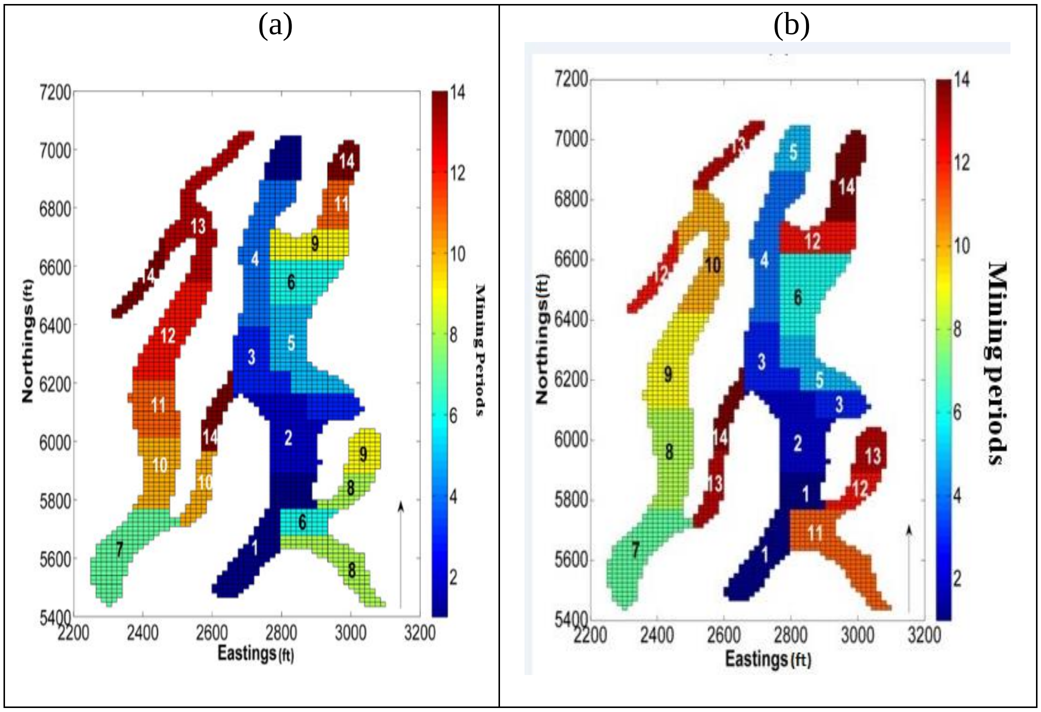


Figure 6-4 Production sequence with highest valued blocks restricted to be mined prior to: (a) the first 8 periods; and (b) to the first 12 periods

6.4.5.2. Based on sections with no precedence. Table 6-3 shows the change in objective function value, computational time, and number of iterations as well as the pre-processing execution time for implementing the cutting plane based on sections with no precedence constraint. Figure 6-5 shows the production sequence obtained by pre-processing the problem with cutting planes based on sections with no precedence. Two scenarios where evaluated by highly restricting and then relaxing the cutting plane. For the case study only three sections had empty precedence sets (refer to Table 5-2).

In the first scenario, the sections with no precedence (Sections 1, 16, and 17) where restricted to be mined in the first period. Figure 6-5 (a) shows the sequence obtained for that scenario. The sequence obtained differs from the optimal solution (Figure 5-6 (b)) significantly. Although these sections have no precedence constraint, only section 17 is part of the primary development area. The precedence constraint is such that, the blocks in the development area were mined first. Therefore, by eliminating solutions that mine Sections 1 and 16 in other periods, the optimal solution is deleted from the feasible space. In this scenario, the objective function value decreased by 0.33 %. By reducing the search space, the computational time and number of iterations decreased by 28.52 % and 44. 76%, respectively. Thus minimal enumerations were needed to find the solution in the feasible space with the highest objective function value.

In the second scenario the author further relaxes the cutting plane by restricting Sections 1, 16 and 17 to be mined before period 12. Although the cutting plane was significantly relaxed, the optimal solution is still excluded from the feasible space. From the solution obtained in Chapter 5 (Figure 5-6 (b)), it is optimal to mine Section 16 in period 14. Thus implementing a cutting plane that included the optimal solution based on all sections with no precedence for this case study will be redundant. Although the solution (Figure 6-5 (b)) is still not optimal in the second scenario, it is significantly improved. The optimal solution suggests that it is optimal to mine Section 1 in period 7, therefore relaxing the cutting plane suggest the same solution. It can also be noted that, it is optimal to mine Section 16 in a much later period and therefore the algorithm mines the section in the latest period possible (period 12). The computational time and number of iterations increases by 18.21 % and 15.79 %, respectively.

Although applying a cutting plane based on sections with no precedence constraints as a pre-processing approach does not result in an optimal solution, the pre-processing execution time is minimal. The pre-processing algorithm takes advantage of the precedence constraints already created to identify the sections with no precedence constraint. Hence, it takes far less time to generate the constraint than for the other two constraints. This approach has potential to save significant computational time in the instances where it is beneficial because of the low computational time needed to generate the constraint.

Table 6-3 Effect of sections with no precedence based pre-processing on the objective function value, computational time and number of iteration.

Sections with no precedence restricted to the first:	**Pre-processing execution time (%)**	**Change in optimal objective function value (%)**	**Change in computational time (%)**	**Change in no. of iterations (%)**
1	-99%	-0.24	-28.52	-44.76
12	-99%	0.00	18.21	15.79

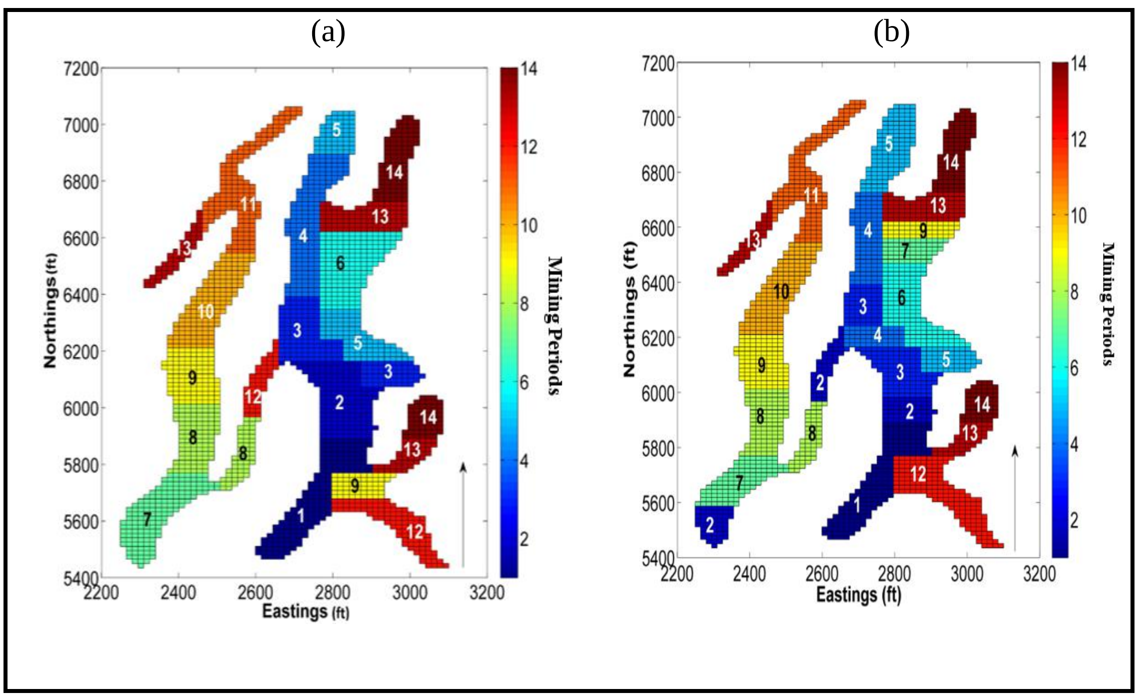


Figure 6-5 Production sequence with sections with no precedence restricted to: (a) the first period; and (b) to the first 12 periods

6.4.5.3. Based on sections in the development area. Table 6-4 shows the change in objective function value, computational time, and number of iterations as well as the pre-processing execution time for implementing the cutting plane based on sections in the development area. Figure 6-6 shows the production sequence obtained when the cutting planes based on sections in the development area was implemented as a pre-processing technique to the BILP problem. The application of this techniques involved evaluating two different scenarios. Similar to the previous techniques, a strict cutting plane is implemented to reduce the feasible search space. The cutting plane was further relaxed to evaluate the effect on the computational time, number of iteration and the objective function value. In this case study, 12 sections (Figure 6-3) are found in the development area. From the solution (Figure 5-6 (b)) in Chapter 5, it is optimal to mine all 12 blocks before period 6.

In the first scenario, the development sections are restricted to the first 5 periods. The sequence (Figure 6-6) obtained was optimal with significant decrease in the computational time and the number of iterations. The computational time and the number of iterations decreased by 22.95 % and 17.24 %, respectively.

In the second scenario, the cutting plane was relaxed to evaluate the effect on the complexity of the problem. The sections in the development area were restricted to be mined in the first 12 periods. Although the solution obtained (Figure 6-6) was optimal, pre-processing the problem with the cutting plane did not improve the computational time or the number of iterations. In fact, the number of iterations increased by as much as 76.48 % with a 37.55 % increase in the computational time.

The pre-processing execution time of the algorithm alone was 3.72% higher (25.79 seconds), given that CPLEX can solve the problem in 24.83 seconds without any pre-processing.

Table 6-4 Effect of sections in development area based pre-processing on the computational complexity of the problem.

Sections in the development area restricted to the first:	**Pre-processing execution time (%)**	**Change in optimal objective function value (%)**	**Change in computational time (%)**	**Change in no. of iterations (%)**
5	3.72	0.00	-22.95	-17.24
12	3.72	0.00	37.55	76.48

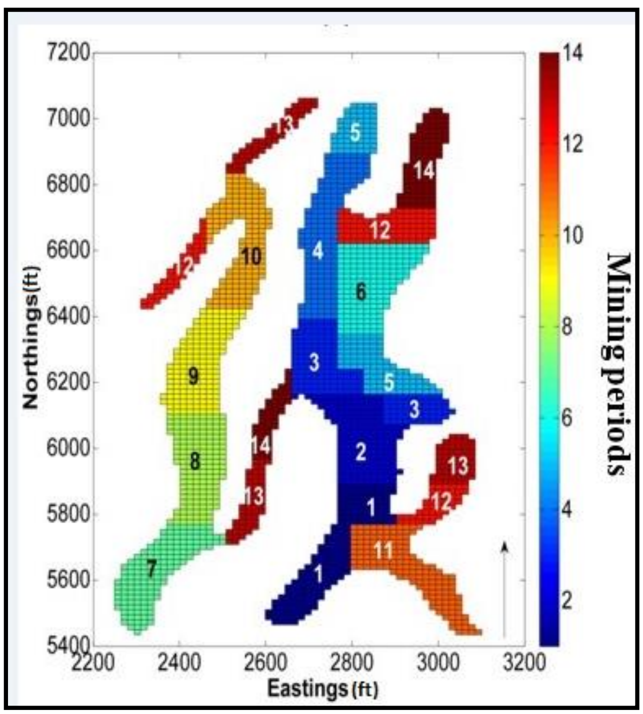


Figure 6-6 Production sequence based on pre-processing with development sections

6.4.5.4. General discussions. Of the three pre-processing techniques evaluated, the cutting plane generated based on sections in the development area resulted in optimal solutions in both scenarios. An optimal solution was obtained by implementing the greedy algorithm with very little restrictions on the search space. To minimize computational time needed to solve the BILP problem, the best pre-processing approach is the implementation of the cutting plane based on development sections. The pre-processing algorithm must be significantly improved to fully benefit from the development sections based approach.

From these results, it appears the effectiveness of cutting planes used to pre-process production sequencing problems depends on the characteristics of the problem. For the case study evaluated, the precedence constraints imposed by the primary development drive the solution (see Section 5.4.2). From Table 5-2 it can be seen that the problem is heavily constrained. The effect of the precedence constraint can be seen in the optimal solution as well. Therefore, a pre-processing technique based on the development sections is the best approach for the problem. Further experiments are necessary to determine whether development based pre-processing cutting planes will be superior for all types of R&P mine sequencing problems.

The objective of this research was to test the hypothesis that heuristic pre-processing can be used to improve the computational efficiency of the branch and cut solution to the R&P BILP problem. The sections in development area and greedy algorithm-based cutting planes resulted in significantly less computational time and number of iterations. The execution time, however, exceeded the computational time prior to pre-processing, resulting in a higher computational time overall. Based on the

BILP R&P problem characteristics, the sections in development area-based cutting plane is likely to improve the computational efficiency of the branch and cut solution. The execution time of the sections in development based-cutting plane, however, exceeds the computational time saved after pre-processing. A more efficient algorithm is needed to generate valid cutting planes in order to appreciate the full benefit of the pre-processing approach. There is, therefore, no substantial evidence that heuristic pre-processing can improve the computational efficiency of branch and cut solutions. Further work is necessary to determine whether the results obtained in this analysis can be extended to other instances of this problem.

Although the overall pre-processing execution time for the sections with no precedence-based cutting planes was significantly small, it did not improve the computation efficiency for the R&P BILP problem. If the problem was such that, it is optimal to mine sections with no precedence in the defined periods, the application of this pre-processing approach could potentially improve the branch and cut solution. Further work is needed to investigate the optimal conditions under which the sections with no precedence pre-processing approach will be beneficial.

6.5. SUMMARY

In this chapter, the effect of pre-processing techniques on the computational efficiency (computational time and number of iteration) of a production sequencing problem is evaluated. Three techniques were used to generate problem specific cutting planes to reduce the feasible search space before solving with the branch and cut algorithm. Three cutting planes were developed and analyzed using the simulated R&P lead mine sequencing problem developed in Chapter 5. The cutting planes were developed based on: (1) on the greedy algorithm; (2) sections with no precedence constraints; and (3) sections in the development area. The objective was to test the hypothesis that heuristic pre-processing can be used to increase the computational efficiency of branch and cut solutions to the BILP problem of R&P mine scheduling.

Based on the results obtained in the experiments here, this author concludes there is not enough evidence to determine whether heuristic pre-processing increases the computational efficiency of branch and cut solutions or not. Although some of the cutting planes evaluated resulted in optimal solutions with less computational time and fewer number of iterations, the computational time required to generate these cutting planes was very high. This resulted in higher computational time overall. The one cutting plane algorithm with very little computational time did not perform well (with respect to number of iterations or computational time for the branch and cut algorithm). At this time, it is not clear whether these observations can be extended to all instances of the R&P BILP problem.

The solution indicates that implementing a cutting plane based on the sections in the development area significantly reduces the number of iterations and solution time needed to solve the problem. This is most likely because of the nature of the problem, which is highly constrained with solutions controlled significantly by precedence constraints of the primary development. The optimal solution was driven by the precedence constraints instead of the section values. Therefore, pre-processing with the greedy algorithm based cutting planes did not reduce the complexity of the problem significantly. By relaxing the greedy algorithm based constraints, the search space included the solutions that mine the development blocks in earlier periods as constrained by precedence. This resulted in an optimal solution. Implementing the cutting plane based on sections with no precedence constraint, however, did not result in an optimal solution.

The changes in computational time and number of iterations depends on the nature and size of the problem, and the restrictions imposed by the cutting plane. Future work must include developing a more efficient algorithm for generating the sections in development area-based cutting planes. It is also necessary to determine whether the results obtained can be extended to other instances of the R&P BILP problem. Further investigation is needed to determine the optimum conditions under which the section with no precedence pre-processing approach will be beneficial. As part of the cutting plane pre-processing method, an algorithm should be developed that evaluates the validity of each cutting plane generated prior to pre-processing. A valid cutting plane reduces the feasible search space without eliminating the optimal solution. The effect of pre-processing a real-life problem with cutting planes on computational time and the number of iterations should be evaluated as well.

7. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

7.1. SUMMARY

A key part of mine planning is the need to optimize the production system. Historically, engineers have used various tools to optimize mine plans and designs. The pioneering approaches were trial and error methods. However, the complex nature of mining systems makes trial and error methods sub-optimal in maximizing the value of the mine. The continuous improvement of computational power and technology has made more techniques available that enable engineers to model complex systems and analyze different alternatives. One such technique is operations research and management science (ORMS), which is the single most prominent method used to optimize systems today.

The choice of mine design parameters affects the way mines are run through all stages of a mine. The sequence in which blocks are extracted significantly affects the value of the mine. Based on current technology and economic conditions, a mining sequence can be optimized to meet production and quality targets in each period.

Due to the ever challenging operating and market conditions, mining methods such room and pillar used to mine coal resources must be optimized to minimize the unit cost of production. Design parameters such as panel width affect the recovery, productivity, unit cost, and haulage efficiency of the operation. The dimensions of the panel affects the production (cut) sequence with larger panels resulting in more complex cut sequencing and tramming by the CM and smaller panels resulting in equipment congestion. It is therefore essential that dimensions of the coal panel are optimized.

Although production sequencing in R&P mines is essential, the uniformity of most coal deposit makes the extraction sequence less complex. On the other hand, the erratic nature of metal deposits makes sequencing in R&P metal mines a challenge.

Production sequencing in R&P pillar mining must account for risk (such as geological, production and market risk) associated with material extraction.

The goal of this research was to use operations research techniques to model, evaluate and select optimal alternatives to some critical R&P mine design parameters as well as mine production sequences. In accordance with the overall goal of this research the specific objectives are:

1. Apply discrete event simulation (DES) to determine the optimal width of coal R&P panels under specific mining conditions;
2. Investigate whether the shuttle car fleet size used to mine a particular panel width is optimal in different segments of the panel;
3. Test the hypothesis that binary integer linear programming (BILP) can be used to account for mining risk in R&P long range mine production sequencing; and
4. Test the hypothesis that heuristic pre-processing can be used to increase the computational efficiency of branch and cut solutions to the BILP problem of R&P mine sequencing.

The first two objectives apply to *coal* R&P mines. A discrete event simulation of an existing coal R&P mine, capable of evaluating the effect of varying optimal panel width on productivity and cost, was built using Arena ® simulation software. The developed simulator was also used to study whether the shuttle car fleet size used to mine a particular panel width is optimal in different segments of the panel.

The last two objectives apply to *metal* R&P mines. To achieve the third objective, a binary integer linear program was developed that maximizes the overall net present value of the operation while minimizing multiple risks. The model was subject to resource, mining rate, quality, reserve, and precedence constraints. The BILP model was coded in Matlab® and solved using ILOG CPLEX® solver through the CPLEX API for Matlab®. The fourth objective examines the effect of pre-processing techniques based on the problem's characteristics on the computational time and number of iterations needed to solve the problem. The pre-processing techniques involved developing specialized cutting planes that minimizes the feasible search space before solving the problem with CPLEX®. The pre-processing techniques are demonstrated using the simulated BILP problem.

7.2. CONCLUSIONS

Based on the work done in this dissertation several conclusions can be drawn:

1. With respect to the first objective (coal R&P panel width optimization):
    a. The research shows it is possible to use a DES approach to determine the optimal panel width. A valid DES model has been built and successfully used to determine the optimal width for an operating mine.
    b. For the mining system evaluated, increasing the number of entries (panel width) increases the total production and the duration of mining. It can also be concluded that a smaller panel width for initial advance outperforms a larger panel width. Specifically, it is optimal for the mine to use an 11-entry initial advance rather than a 13-entry one. For a given number of shuttle cars, there appear to be an optimal panel width that optimizes productivity.

c. The unit cost of mining decreased as the panel width was increased. This was due to the fact that the fixed cost significantly outweighed the variable cost (Equation (3-3)). Based on productivity alone, a 17-entry panel was observed to be the optimal panel width for the existing system which operates with four cars per CM.

d. Sensitivity analysis of unit cost and productivity to changing shuttle car fleet size showed that, the productivity and unit cost increases as the number of cars were increased. However, the change in unit cost due to an additional shuttle car outweighs the changes in productivity. The optimal number of entries that maximizes productivity changes as the number of shuttle cars changes. Based on the results, a fleet size of four was deemed to be optimal for mining the entire panel.

2. With respect to the second objective (the effect of changing duty cycles on CM-Shuttle car matching):

   a. This research successfully promulgated a DES-based approach for accounting for changing duty cycles in optimal CM-shuttle car matching. The proposed approach includes criteria used to define segments that reflect changing equipment cycle times.

   b. From the case study, it can be concluded that the fleet size used to mine a particular panel width was not always optimal for all segments of the panel. Any analysis that ignores the varying duty cycles in different segments of a panel (such as the analysis in Chapter 3) is likely to overestimate the optimal number of shuttle cars needed to meet production demands.

c. For the panel width evaluated, it was found that three out of the 14 panel segments can attain a maximum productivity level using a three shuttle cars per CM instead of four. Using three shuttle cars per CM to mine these segments saves the mine $5,862 per panel.

3. With respect to the third objective (accounting for risk in R&P production sequencing models):

    a. The research demonstrates that a deterministic binary integer linear programming approach can be used to model and incorporate multiple mining risks in R&P production sequence optimization.

    b. The significance of risk in production sequencing can be controlled by introducing effective ratios. Verification with a simulated lead room and pillar production sequencing problem indicates that changing the importance of mining risk in production sequencing can significantly change the optimal sequence.

    c. The use of a block aggregation technique minimized the number of constraints in the BILP problem. For the problem evaluated the computational time reduced four fold when the blocks were aggregated. Further experiments showed that this gain is true (if not as pronounced) even when the optimization problem has less complicated precedence constraints.

   d. The aggregation of geologic blocks into sections and the elimination of block precedence constraint for the problem minimizes mining selectivity and, thus, affects the optimal value.

4. With respect to the fourth objective (investigating the effect of pre-processing on the computational efficiency of production sequencing problems):
   a. Implementing a cutting plane based on the sections in the development area significantly reduces the number of iterations and solution time needed to solve the problem. This is most likely because of the nature of the problem, which is highly constrained with solutions controlled significantly by precedence constraints of the primary development. Pre-processing with the greedy algorithm based cutting planes did not reduce the complexity of the problem significantly. Implementing the cutting plane based on sections with no precedence constraint, however, did not result in an optimal solution.
   b. There is not enough evidence (based on the analysis in this work) to determine whether heuristic pre-processing increases the computational efficiency of branch and cut solutions or not. Some of the cutting planes developed in this research resulted in optimal solutions with less computational time and less number of iterations. However, the computational time required to generate these cutting planes were very high resulting in higher computational time overall.
   c. The most promising cutting plane algorithm is the cutting plane based on blocks with no precedence constraint. This algorithm required very

little computational time and could therefore result in lower solution times, overall. However, the cutting plane did not perform well (with respect to number of iterations or computational time for the branch and cut algorithm) in the experiments in this work.

7.3. CONTRIBUTION OF THE PHD RESEARCH

1. This research was the first attempt to optimize productivity and unit mining cost as a function of coal panel width. The implicit nature of the objective function makes it difficult to use analytical tools to solve the problem. This challenge was overcome by modeling the mining system as a stochastic discrete event-based simulation capable of evaluating alternative panel width.
2. This research extends the work on accounting for changing duty cycles in fleet size optimization to a R&P mining system. A novel frame work on how to discretize the system into reasonable periods of operation (segments) to facilitate realistic solutions was introduced. The approach balances the need to optimize for changing duty cycles with realistic and reasonable operating periods. The case study shows that this approach is viable and when used properly can lead to savings in production costs.
3. An experimental approach was introduced that investigates the sensitivity of productivity, cycle times, utilization, and duration of mining to changing duty cycle with minimum computational effort. This is a contribution to simulation methods as a novel simulation experiment design approach.
4. This work makes a contribution to the research on how to account for risk in mine production sequencing models. A novel deterministic framework was

presented that models multiple mining risks using a binary integer linear programming framework.

5. This work confirms that aggregation methods, used properly, can reduce the computational time and number of iterations required to solve mine production sequencing problem. This work, more than any that the author is aware of, presents a systematic evaluation of this result with respect to the complexity of the precedence constraints.
6. This work is the first attempt to investigate the effect of pre-processing techniques, based on multiple specialized cutting planes, on the number of iterations and computational time of a R&P production sequencing problem.
7. The research has proposed algorithms for three specialized cutting planes which can be used as pre-processing techniques prior to solving with branch and cut algorithms.

7.4. RECOMMENDATIONS FOR FUTURE WORK

The following recommendations for future work could improve and add to the body of knowledge from this research:

1. Optimization methods for other important design parameters that affect unit mining cost and productivity in coal panels:

   In order to further improve the efficiency of mining operations in coal R&P mines, researchers have to develop optimization methods for other important design parameters such as those associated with conveyor belt, roof support, panel, roof span, panel pillars, haul routes, and barrier pillars. The parameters often optimized in room and pillar mines are those related to geotechnical properties of the ore and hanging wall. Other factors that affect the productivity and unit cost are rarely optimized. R&P mines have lower production levels compared to longwall and surface coal mines. To improve the efficiency of this mining method, all mine design parameters should be optimized to increase productivity and minimize unit mining cost.

2. Optimizing coal recovery as a function of panel dimensions:

In this research, the panel width was optimized with regards to the unit cost and productivity. The dimension of the panel significantly affects the coal recovery as it affects the number and size of barrier pillars. Also, the orientation and the size of a panel will affect the overall amount of coal recovered from the mine. It is therefore essential to optimize coal recovery as a function of panel dimensions as well.

3. Accounting for input correlation in fleet size optimization:

In this study, the input data was assumed to be independently and identically distributed. In most operations correlation exists between mine parameters (Que et al. 2015). Disregarding correlation when it exists may result in under/over estimating the number of haulage equipment needed to maximize productivity. Future work should evaluate whether correlation exists between the various input variables and determine how to account for any correlation in the model.

4. Accounting for pillar extraction in R&P production sequencing:

Although the model in this research included variables that represent the extraction of pillars, the effect of pillar extraction on the mining sequence was not evaluated. The model can be used to analyze the impact of pillar extraction as an integral part of long range planning on the overall net present value of the mine.

5. Comprehensive study of the computational complexity of the BILP R&P mine sequencing problem:

Further work is required to evaluate other factors that affect the computational complexity (solution time and number of iterations) of the BILP R&P mine sequencing problem. For example, the effect of the number of decision variables (number of blocks and sections) on the computational time needs to be evaluated with carefully defined test problems. The effect of sections (or block aggregation) should be examined and optimal aggregation strategies recommended as well. Also, research is necessary to explore what specific instances of the problem are more difficult to solve than others.

6. Evaluating problem specific cutting plane-based pre-processing techniques:

The research evaluated three cutting plane-based pre-processing techniques. Other pre-processing techniques can be developed and tested based on the characteristics of the problem. Also, further work is required to test the effectiveness of the three techniques developed and any additional techniques using well designed test problems. The goal will be to demonstrate the conditions under which a particular cutting plane will be effective in reducing the computational time and number of iterations.

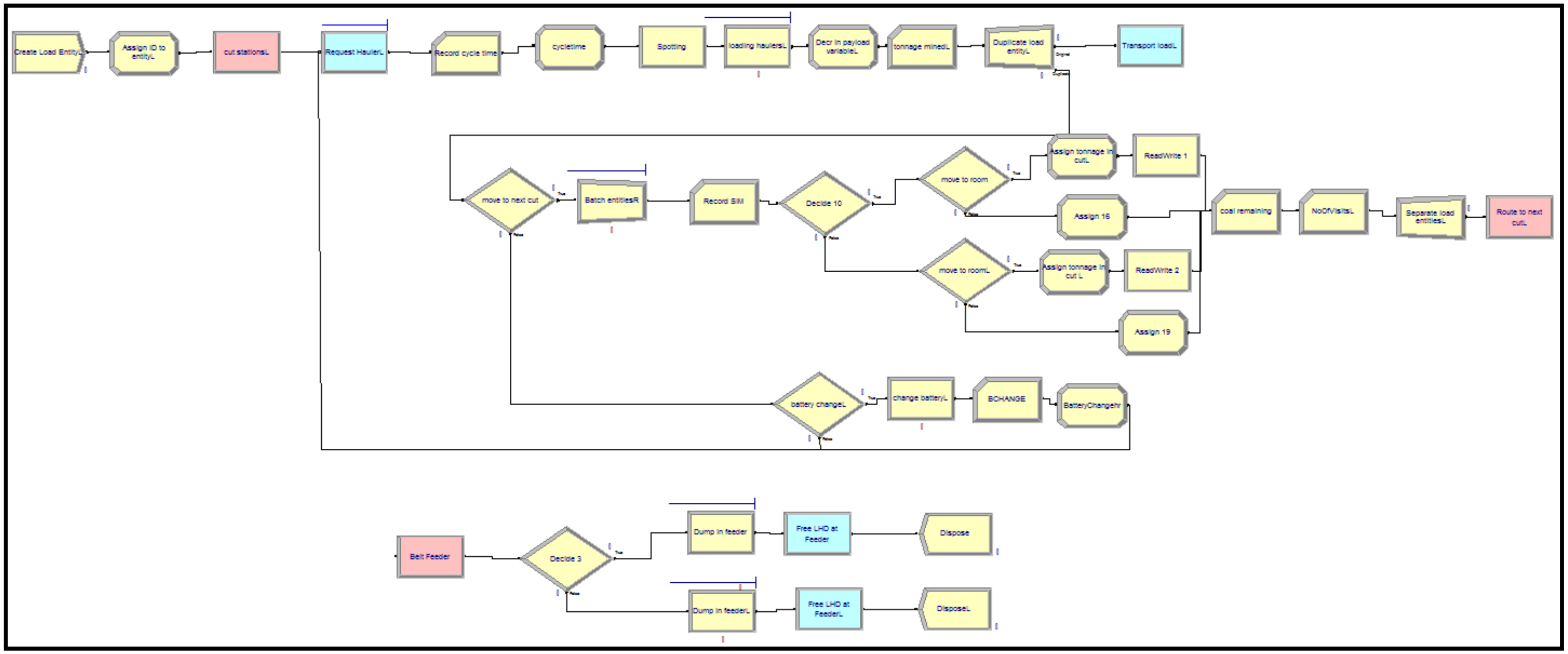
7. Developing tests to evaluate the validity of pre-processing cutting planes:

The research did not develop an efficient algorithm for testing the validity of a cutting plane. The only test used in this research was solving the LP relaxation of the problem with the cutting plane to see if it is feasible or not. The results from the application of the greedy algorithm and sections with no precedence based cutting planes, showed that the optimal solution was eliminated from the feasible space. This showed that the test was inadequate (i.e. the LP relaxation could be feasible but still eliminate the global optimal). Using a trial and error approach to determine the validity of the cutting plane will be time consuming and may not address the issue. Further work must be done as part of the development of pre-processing algorithms to validate the specialized cutting planes generated.

APPENDIX A.

ARENA DES MODEL AND HAULAGE DISTANCES

APPENDIX A: PANEL WIDTH OPTIMIZATION



Arena DES model

Haul distances for the left-hand-side of the panel based on the 11 entries cut sequence in Figure 3-8(a).

Cut Number	Cut Entry	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
1	4	2 down, 1 right, 1 down, 1 right	338	458	796
2	3	2 down, 2 right, 1 down, 1 right	398	518	916
3	5	3 down, 1 right	278	398	676
4	4	2.5 down, 1 right, 1 down, 1 right	368	488	856
5	3	2.5 down, 2 right, 1 down, 1 right	428	548	976
6	6	2 down, 1 left, 1 down, 1 right	338	338	676
7	5	3.5 down, 1 right	308	428	736
8	between 4 & 5 turn	3 down, 1 right, 1 down, 1 right	398	518	916
9	between 3 & 4 turn	3 down, 2 right, 1 down, 1 right	458	578	1036
10	2	2 down, 3 right, 1 down, 1 right	458	458	916
11	6	2.5 down, 1 left, 1 down, 1 right	368	368	736
12	between 4 & 5 hole	3.5 down, 1 right, 1 down, 1 right	428	548	976
13	4	3.5 down, 1 right, 1 down, 1 right	428	548	976
14	between 3 & 4 hole	3.5 down, 2 right, 1 down, 1 right	488	608	1096
15	3	3.5 down, 2 right, 1 down, 1 right	488	608	1096
16	2	2.5 down, 3 right, 1 down, 1 right	488	488	976

Haul distances for the left-hand-side of the panel based on the 11 entries cut sequence in Figure 3-8(a). Cont.

Cut Number	Cut Entry	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
17	1	2 down, 4 right, 1 down, 1 right	518	518	1036
18	5	4 down, 1 right	338	458	796
19	between 5 & 6	1 left, 3 down, 1 right, 1 down, 1 right	458	458	916
20	4	4 down, 1 right, 1 down, 1 right	458	578	1036
21	3	4 down, 2 right, 1 down, 1 right	518	638	1156
22	between 3 & 2	1 right, 3 down, 1 right, 1 down, 1 right	458	458	916
23	1	2.5 down, 4 right, 1 down, 1 right	548	548	1096
24	6	3 down, 1 left, 1 down, 1 right	398	398	796
25	5	5 down, 1 right	398	518	916
26	between 4 & 5 turn	4 down, 1 right, 1 down, 1 right	458	578	1036
27	between 3 & 4 turn	4 down, 2 right, 1 down, 1 right	518	638	1156
28	2	3.5 down, 3 right, 1 down, 1 right	548	548	1096
29	between 2 & 1	1 right, 3 down, 2 right, 1 down, 1 right	518	518	1036
30	6	4 down, 1 left, 1 down, 1 right	458	458	916
31	between 4 & 5 hole	4.5 down, 1 right, 1 down, 1 right	488	608	1096
32	4	4.5 down, 1 right, 1 down, 1 right	488	608	1096
33	between 3 & 4 hole	4.5 down, 2 right, 1 down, 1 right	548	668	1216

Haul distances for the left-hand-side of the panel based on the 11 entries cut sequence in Figure 3-8(a). Cont.

Cut Number	Cut Entry	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
34	3	4.5 down, 2 right, 1 down, 1 right	548	668	1216
35	2	4 down, 3 right, 1 down, 1 right	578	578	1156
36	1	3.5 down, 4 right, 1 down, 1 right	608	608	1216
37	6	4.5 down, 1 left, 1 down, 1 right	488	488	976
38	5	5.5 down, 1 right	428	548	976
39	between 4 & 5 turn	5 down, 1 right, 1 down, 1 right	518	638	1156
40	between 3 & 4 turn	5 down, 2 right, 1 down, 1 right	578	698	1276
41	between 3 & 2	1 right, 4 down, 1 right, 1 down, 1 right	518	518	1036
42	1	4 down, 4 right, 1 down, 1 right	638	638	1276
43	between 4 & 5 hole	5.5 down, 1 right, 1 down, 1 right	548	668	1216
44	between 3 & 4 hole	5.5 down, 2 right, 1 down, 1 right	608	728	1336
45	2	4.5 down, 3 right, 1 down, 1 right	608	608	1216
46	between 2 & 1	1 right, 4 down, 2 right, 1 down, 1 right	578	578	1156
47	between 5 & 6	1 left, 5 down, 1 right, 1 down, 1 right	578	578	1156
48	between 3 & 2	1 right, 5 down, 1 right, 1 down, 1 right	578	578	1156
49	1	4.5 down, 4 right, 1 down, 1 right	668	668	1336
50	between 2 & 1	1 right, 5 down, 2 right, 1 down, 1 right	638	638	1276

Haul distances for the right-hand-side of the panel based on the 11 entries cut sequence in Figure 3-8 (a). Cont.

Cut Number	Cut Entry	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
1	8	2 down, 1 left, 1 down, 1 left	338	458	796
2	9	2 down, 2 left, 1 down, 1 left	398	518	916
3	7	3 down, 1 left	278	398	676
4	8	2.5 down, 1 left, 1 down, 1 left	368	488	856
5	9	2.5 down, 2 left, 1 down, 1 left	428	548	976
6	7	3.5 down, 1 left	308	428	736
7	between 8 & 7 turn	3 down, 1 left, 1 down, 1 left	398	518	916
8	between 9 & 8 turn	3 down, 2 left, 1 down, 1 left	458	578	1036
9	10	2 down, 3 left, 1 down, 1 left	458	458	916
10	11	2 down, 4 left, 1 down, 1 left	518	518	1036
11	between 8 & 7 hole	3.5 down, 1 left, 1 down, 1 left	428	548	976
12	8	3.5 down, 1 left, 1 down, 1 left	428	548	976
13	between 9 & 8 hole	3.5 down, 2 left, 1 down, 1 left	488	608	1096
14	9	3.5 down, 2 left, 1 down, 1 left	488	608	1096
15	10	2.5 down, 3 left, 1 down, 1 left	488	488	976
16	11	2.5 down, 4 left, 1 down, 1 left	548	548	1096
17	7	4.5 down, 1 left	368	488	856

Haul distances for the right-hand-side of the panel based on the 11 entries cut sequence in Figure 3-8 (a). Cont.

Cut Number	Cut Entry	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
18	between 7 & 6	1 right, 3 down, 1 left, 1 down, 1 left	458	458	916
19	8	4 down, 1 left, 1 down, 1 left	458	578	1036
20	9	4 down, 2 left, 1 down, 1 left	518	638	1156
21	between 9 & 10	1 left, 3 down, 1 left, 1 down, 1 left	458	458	916
22	7	5 down, 1 left,	398	518	916
23	between 8 & 7 turn	4 down, 1 left, 1 down, 1 left	458	578	1036
24	between 9 & 8 turn	4 down, 2 left, 1 down, 1 left	518	638	1156
25	10	3.5 down, 3 left, 1 down, 1 left	548	548	1096
26	between 10 & 11	1 left, 3 down, 2 left, 1 down, 1 left	518	518	1036
27	between 8 & 7 hole	4.5 down, 1 left, 1 down, 1 left	488	608	1096
28	8	4.5 down, 1 left, 1 down, 1 left	488	608	1096
29	between 9 & 8 hole	4.5 down, 2 left, 1 down, 1 left	548	668	1216
30	9	4.5 down, 2 left, 1 down, 1 left	548	668	1216
31	10	4 down, 3 left, 1 down, 1 left	578	578	1156
32	11	3.5 down, 4 left, 1 down, 1 left	608	608	1216
33	7	5.5 down, 1 left	428	548	976
34	between 7 & 6	1 right, 4 down, 1 left, 1 down, 1 left	518	518	1036

Haul distances for the right-hand-side of the panel based on the 11 entries cut sequence in Figure 3-8 (a). Cont.

Cut Number	Cut Entry	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
35	between 8 & 7 turn	5 down, 1 left, 1 down, 1 left	518	638	1156
36	between 9 & 8 turn	5 down, 2 left, 1 down, 1 left	578	698	1276
37	between 9 & 10	1 left, 4 down, 1 left, 1 down, 1 left	518	518	1036
38	11	4 down, 4 left, 1 down, 1 left	638	638	1276
39	between 6 & 5	1 right, 5 down, 1 left	458	458	916
40	between 8 & 7 hole	5.5 down, 1 left, 1 down, 1 left	548	668	1216
41	between 9 & 8 hole	5.5 down, 2 left, 1 down, 1 left	608	728	1336
42	10	4.5 down, 3 left, 1 down, 1 left	608	608	1216
43	between 10 & 11	1 left, 4 down, 2 left, 1 down, 1 left	578	578	1156
44	between 7 & 6	1 right, 5 down, 1 left, 1 down, 1 left	578	578	1156
45	between 9 & 10	1 left, 5 down, 1 left, 1 down, 1 left	578	578	1156
46	11	4.5 down, 4 left, 1 down, 1 left	668	668	1336
47	between 10 & 11	1 left, 5 down, 2 left, 1 down, 1 left	638	638	1276
50	between 2 & 1	1 right, 5 down, 2 right, 1 down, 1 right	638	638	1276

Haul distances for the left-hand-side of the panel based on the 13 entries cut sequence in Figure 3-8(b)

Cut Number	Cut Entry	Outgoing Direction	Segments to Feeder	Segments from Feeder	Distance (ft) l		
					To Feeder	From Feeder	Total
1	4	2 down, 2 right, 1 down, 1 right	6	8	398	518	916
2	3	2 down, 3 right, 1 down, 1 right	7	9	458	578	1036
3	5	2 down, 1 right, 1 down, 1 right	5	7	338	458	796
4	4	2.5 down, 2 right, 1 down, 1 right	6.5	8.5	428	548	976
5	3	2.5 down, 3 right, 1 down, 1 right	7.5	9.5	488	608	1096
6	6	3 down, 1 right	4	6	278	398	676
7	5	2.5 down, 1 right, 1 down, 1 right	5.5	7.5	368	488	856
8	between 4 & 5 turn	3 down, 2 right, 1 down, 1 right	7	9	458	578	1036
9	between 3 & 4 turn	3 down, 3 right, 1 down, 1 right	8	10	518	638	1156
10	2	2 down, 4 right, 1 down, 1 right	8	8	518	518	1036
11	7	2 down, 1 left, 1 down, 1 right	5	5	338	338	676
12	6	4 down, 1 right	5	7	338	458	796
13	between 4 & 5 hole	3.5 down, 2 right, 1 down, 1 right	7.5	9.5	488	608	1096
14	4	3.5 down, 2 right, 1 down, 1 right	7.5	9.5	488	608	1096

Haul distances for the left-hand-side of the panel based on the 13 entries cut sequence in Figure 3-8(b). Cont.

Cut Number	Cut Entry	Outgoing Direction	Segments to Feeder	Segments from Feeder	Distance (ft) l		
					To Feeder	From Feeder	Total
15	between 3 & 4 hole	3.5 down, 3 right, 1 down, 1 right	8.5	10.5	548	668	1216
16	3	3.5 down, 3 right, 1 down, 1 right	8.5	10.5	548	668	1216
17	2	2.5 down, 4 right, 1 down, 1 right	8.5	8.5	548	548	1096
18	1	2 down, 5 right, 1 down, 1 right	9	9	578	578	1156
19	7	3 down, 1 left, 1 down, 1 right	6	6	398	398	796
20	5	3.5 down, 1 right, 1 down, 1 right	6.5	8.5	428	548	976
21	between 5 & 6	1 left, 3 down, 2 right, 1 down, 1 right	8	8	518	518	1036
22	4	4 down, 2 right, 1 down, 1 right	8	10	518	638	1156
23	3	4 down, 3 right, 1 down, 1 right	9	11	578	698	1276
24	between 3 & 2	1 right, 3 down, 2 right, 1 down, 1 right	8	8	518	518	1036
25	1	2.5 down, 5 right, 1 down, 1 right	9.5	9.5	608	608	1216
26	6	4.5 down, 1 right	5.5	7.5	368	488	856
27	between 6 & 7	1 left, 3 down, 1 right, 1 down, 1 right	7	7	458	458	916

Haul distances for the left-hand-side of the panel based on the 13 entries cut sequence in Figure 3-8(b). Cont.

Cut #	Cut Entry	Outgoing Direction	Segments to Feeder	Segments from Feeder	Distance (ft)		
					To Feeder	From Feeder	Total
28	5	4 down, 1 right, 1 down, 1 right	7	9	458	578	1036
29	between 4 & 5 turn	4 down, 2 right, 1 down, 1 right	8	10	518	638	1156
30	between 3 & 4 turn	4 down, 3 right, 1 down, 1 right	9	11	578	698	1276
31	2	3.5 down, 4 right, 1 down, 1 right	9.5	9.5	608	608	1216
32	between 2 & 1	1 right, 3 down, 3 right, 1 down, 1 right	9	9	578	578	1156
33	7	3.5 down, 1 left, 1 down, 1 right	6.5	6.5	428	428	856
34	6	5 down, 1 right	6	8	398	518	916
35	between 4 & 5 hole	4.5 down, 2 right, 1 down, 1 right	8.5	10.5	548	668	1216
36	4	4.5 down, 2 right, 1 down, 1 right	8.5	10.5	548	668	1216
37	between 3 & 4 hole	4.5 down, 3 right, 1 down, 1 right	9.5	11.5	608	728	1336
38	3	4.5 down, 3 right, 1 down, 1 right	9.5	11.5	608	728	1336
39	2	4 down, 4 right, 1 down, 1 right	10	10	638	638	1276
40	1	3.5 down, 5 right, 1 down, 1 right	10.5	10.5	668	668	1336
41	7	4 down, 1 left, 1 down, 1 right	7	7	458	458	916
42	5	4.5 down, 1 right, 1 down, 1 right	7.5	9.5	488	608	1096

Haul distances for the left-hand-side of the panel based on the 13 entries cut sequence in Figure 3-8(b). Cont.

Cut #	Cut Entry	Outgoing Direction	Segments to Feeder	Segments from Feeder	Distance (ft)		
					To Feeder	From Feeder	Total
43	between 5 & 6	1 left, 4 down, 2 right, 1 down, 1 right	9	9	578	578	1156
44	between 4 & 5 turn	5 down, 2 right, 1 down, 1 right	9	11	578	698	1276
45	between 3 & 4 turn	5 down, 3 right, 1 down, 1 right	10	12	638	758	1396
46	between 3 & 2	1 right, 4 down, 2 right, 1 down, 1 right	9	9	578	578	1156
47	1	4 down, 5 right, 1 down, 1 right	11	11	698	698	1396
48	6	5.5 down, 1 right	6.5	8.5	428	548	976
49	between 4 & 5 hole	5.5 down, 2 right, 1 down, 1 right	9.5	11.5	608	728	1336
50	between 3 & 4 hole	5.5 down, 3 right, 1 down, 1 right	10.5	12.5	668	788	1456
51	2	4.5 down, 4 right, 1 down, 1 right	10.5	10.5	668	668	1336
52	between 2 & 1	2 right, 4 down, 2 right, 1 down, 1 right	10	10	638	638	1276
53	7	4.5 down, 1 left, 1 down, 1 right	7.5	7.5	488	488	976
54	between 5 & 6	1 left, 5 down, 2 right, 1 down, 1 right	10	10	638	638	1276
55	between 3 & 2	1 right, 5 down, 2 right, 1 down, 1 right	10	10	638	638	1276
56	1	4.5 down, 5 right, 1 down, 1 right	11.5	11.5	728	728	1456

Haul distances for the left-hand-side of the panel based on the 13 entries cut sequence in Figure 3-8(b). Cont.

57	between 6 & 7	1 left, 5 down, 1 right, 1 down, 1 right	9	9	578	578	1156
58	between 2 & 1	2 right, 5 down, 2 right, 1 down, 1 right	11	11	698	698	1396

Haul distances for the right-hand-side of the panel based on the 13 entries cut sequence in Figure 3-8(b).

Cut Number	Cut Entry	Outgoing Direction	Segments to Feeder	Segments from Feeder	Distance (ft)		
					To Feeder	From Feeder	Total
1	10	2 down, 2 left, 1 down, 1 left	6	8	398	518	916
2	11	2 down, 3 left, 1 down, 1 left	7	9	458	578	1036
3	9	2 down 1 left, 1 down, 1 left	5	7	338	458	796
4	10	2.5 down, 2 left, 1 down, 1 left	6.5	8.5	428	548	976
5	11	2.5 down, 3 left, 1 down, 1 left	7.5	9.5	488	608	1096
6	8	3 down, 1 left	4	6	278	398	676
7	9	2.5 down, 1 left, 1 down, 1 left	5.5	7.5	368	488	856
8	between 10 & 9 turn	3 down, 2 left, 1 down, 1 left	7	9	458	578	1036
9	between 11 & 10 turn	3 down, 3 left, 1 down, 1 left	8	10	518	638	1156
10	12	2 down, 4 left, 1 down, 1 left	8	8	518	518	1036

Haul distances for the right-hand-side of the panel based on the 13 entries cut sequence in Figure 3-8(b). Cont.

Cut Number	Cut Entry	Outgoing Direction	Segments to Feeder	Segments from Feeder	Distance (ft)		
					To Feeder	From Feeder	Total
11	between 10 & 9 hole	3.5 down, 2 left, 1 down, 1 left	7.5	9.5	488	608	1096
12	10	3.5 down, 2 left, 1 down, 1 left	7.5	9.5	488	608	1096
13	between 11 & 10 hole	3.5 down, 3 left, 1 down, 1 left	8.5	10.5	548	668	1216
14	11	3.5 down, 3 left, 1 down, 1 left	8.5	10.5	548	668	1216
15	12	2.5 down, 4 left, 1 down, 1 left	8.5	8.5	548	548	1096
16	13	2 down, 5 left, 1 down, 1 left	9	9	578	578	1156
17	8	4 down, 1 left	5	7	338	458	796
18	9	3.5 down, 1 left, 1 down, 1 left	6.5	8.5	428	548	976
19	between 9 & 8	1 right, 3 down, 2 left, 1 down, 1 left	8	8	518	518	1036
20	10	4 down, 2 left, 1 down, 1 left	8	10	518	638	1156
21	11	4 down, 3 left, 1 down, 1 left	9	11	578	698	1276
22	between 11 & 12	1 left, 3 down, 2 left, 1 down, 1 left	8	8	518	518	1036
23	13	3 down, 5 left, 1 down, 1 left	10	10	638	638	1276

Haul distances for the right-hand-side of the panel based on the 13 entries cut sequence in Figure 3-8(b). Cont.

Cut #	Cut Entry	Outgoing Direction	Segments to Feeder	Segments from Feeder	Distance (ft)		
24	8	4.5 down, 1 left	5.5	7.5	368	488	856
25	between 8 & 7	1 right, 3 down, 1 left, 1 down, 1 left	7	7	458	458	916
26	9	4 down, 1 left, 1 down, 1 left	7	9	458	578	1036
27	between 10 & 9 turn	4 down, 2 left, 1 down, 1 left	8	10	518	638	1156
28	between 11 & 10 turn	4 down, 3 left, 1 down, 1 left	9	11	578	698	1276
29	12	3.5 down, 4 left, 1 down, 1 left	9.5	9.5	608	608	1216
30	between 12 & 13	2 left, 3 down, 2 left, 1 down, 1 left	9	9	578	578	1156
31	between 10 & 9 hole	4.5 down, 2 left, 1 down, 1 left	8.5	10.5	548	668	1216
32	10	4.5 down, 2 left, 1 down, 1 left	8.5	10.5	548	668	1216
33	between 11 & 10 hole	4.5 down, 3 left, 1 down, 1 left	9.5	11.5	608	728	1336
34	11	4.5 down, 3 left, 1 down, 1 left	9.5	11.5	608	728	1336
35	12	4 down, 4 left, 1 down, 1 left	10	10	638	638	1276
36	13	3.5 down, 5 left, 1 down, 1 left	10.5	10.5	668	668	1336
37	8	5 down, 1 left	6	8	398	518	916
38	9	4.5 down, 1 left, 1 down, 1 left	7.5	9.5	488	608	1096

Haul distances for the right-hand-side of the panel based on the 13 entries cut sequence in Figure 3-8(b). Cont.

Cut #	Cut Entry	Outgoing Direction	Segments to Feeder	Segments from Feeder	Distance (ft)		
					To Feeder	From Feeder	Total
39	between 9 & 8	1 right, 4 down, 2 left, 1 down, 1 left	9	9	578	578	1156
40	between 10 & 9 turn	5 down, 2 left, 1 down, 1 left	9	11	578	698	1276
41	between 11 & 10 turn	5 down, 3 left, 1 down, 1 left	10	12	638	758	1396
42	between 11 & 12	1 left, 4 down, 2 left, 1 down, 1 left	9	9	578	578	1156
43	13	4 down, 5 left, 1 down, 1 left	11	11	698	698	1396
44	8	5.5 down, 1 left	6.5	8.5	428	548	976
45	between 8 & 7	1 right, 4 down, 1 left, 1 down, 1 left	8	8	518	518	1036
46	between 10 & 9 hole	5.5 down, 2 left, 1 down, 1 left	9.5	11.5	608	728	1336
47	between 11 & 10 hole	5.5 down, 3 left, 1 down, 1 left	10.5	12.5	668	788	1456
48	12	4.5 down, 4 left, 1 down, 1 left	10.5	10.5	668	668	1336
49	between 12 & 13	2 left, 4 down, 2 left, 1 down, 1 left	10	10	638	638	1276
50	between 7 & 6	1 right, 5 down, 1 left	7	7	458	458	916
51	between 9 & 8	1 right, 5 down, 2 left, 1 down, 1 left	10	10	638	638	1276
52	between 11 & 12	1 left, 5 down, 2 left, 1 down, 1 left	10	10	638	638	1276

Haul distances for the right-hand-side of the panel based on the 13 entries cut sequence in Figure 3-8(b). Cont.

Cut #	Cut Entry	Outgoing Direction	Segments to Feeder	Segments from Feeder	Distance (ft)		
					To Feeder	From Feeder	Total
53	13	4.5 down, 5 left, 1 down, 1 left	11.5	11.5	728	728	1456
54	between 8 & 7	1 right, 5 down, 1 left, 1 down, 1 left	9	9	578	578	1156
55	between 12 & 13	2 left, 5 down, 2 left, 1 down, 1 left	11	11	698	698	1396

Haul distances for the rooms in each section of the panel based on a 17 entries cut sequence (Figure 4-2).

Room Number or Location	Cut no. in Room	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
Rm 3	2nd	2.5 left, 1 up, 3 left, 1 down, 1 left	548	548	1096
Rm 3	3rd	3.5 left, 1 up, 3 left, 1 down, 1 left	608	608	1216
Rm 3	4th	4 left, 1 up, 3 left, 1 down, 1 left	638	638	1276
Rm 3	5th	4.5 left, 1 up, 3 left, 1 down, 1 left	668	668	1336
Rm 3	6th	5.5 left, 1 up, 3 left, 1 down, 1 left	728	728	1456
Rm 4	1st	1 left, 2 up, 4 left, 1 down, 1 left	578	458	1036

Haul distances for the rooms in each section of the panel based on a 17 entries cut sequence (Figure 4-2). Cont.

Room Number or Location	Cut no. in Room	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
Rm 4	2nd	1.5 left, 2 up, 4 left, 1 down, 1 left	608	488	1096
Rm 4	3rd	2.5 left, 2 up, 4 left, 1 down, 1 left	668	548	1216
Rm 4	4th	3 left, 2 up, 4 left, 1 down, 1 left	698	578	1276
Rm 4	5th	3.5 left, 2 up, 4 left, 1 down, 1 left	728	608	1336
Rm 4	6th	4.5 left, 2 up, 4 left, 1 down, 1 left	788	668	1456
Rm 5	1st	1 left, 3 up, 4 left, 1 down, 1 left	638	518	1156
Rm 5	2nd	1.5 left, 3 up, 4 left, 1 down, 1 left	668	548	1216
Rm 5	3rd	2.5 left, 3 up, 4 left, 1 down, 1 left	728	608	1336
Rm 5	4th	3 left, 3 up, 4 left, 1 down, 1 left	758	638	1396
Rm 5	5th	3.5 left, 3 up, 4 left, 1 down, 1 left	788	668	1456
Rm 5	6th	4.5 left, 3 up, 4 left, 1 down, 1 left	848	728	1576
Rm 6	1st	2 left, 4 up, 3 left, 1 down, 1 left	698	578	1276
Rm 6	2nd	2.5 left, 4 up, 3 left, 1 down, 1 left	728	608	1336
Rm 6	3rd	3.5 left, 4 up, 3 left, 1 down, 1 left	788	668	1456
Rm 6	4th	4 left, 4 up, 3 left, 1 down, 1 left	818	698	1516

Haul distances for the rooms in each section of the panel based on a 17 entries cut sequence (Figure 4-2). Cont.

Room Number or Location	Cut no. in Room	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
Rm 6	5th	4.5 left, 4 up, 3 left, 1 down, 1 left	848	728	1576
Rm 6	6th	5.5 left, 4 up, 3 left, 1 down, 1 left	908	788	1696
Btwn 4&3	1st turn	3 left, 2 up, 3 left, 1 down, 1 left	638	518	1156
Btwn 4&3	1st hole	3.5 left, 2 up, 3 left, 1 down, 1 left	668	548	1216
Btwn 4&3	2nd turn	4 left, 2 up, 3 left, 1 down, 1 left	698	578	1276
Btwn 4&3	2nd hole	4.5 left, 2 up, 3 left, 1 down, 1 left	728	608	1336
Btwn 4&3	3rd turn	5 left, 2 up, 3 left, 1 down, 1 left	758	638	1396
Btwn 4&3	3rd hole	5.5 left, 2 up, 3 left, 1 down, 1 left	788	668	1456
Btwn 4&3	4th turn	6 left, 2 up, 3 left, 1 down, 1 left	818	698	1516
Btwn 4&3	4th hole	6.5 left, 2 up, 3 left, 1 down, 1 left	848	728	1576
Btwn 2 & 1	+60	2 down, 3 left, 2 up, 3 left, 1 down, 1 left	758	638	1396
Btwn 2 & 1	1+20	2 down, 4 left, 2 up, 3 left, 1 down, 1 left	818	698	1516
Btwn 2 & 1	1+80	2 down, 5 left, 2 up, 3 left, 1 down, 1 left	878	758	1636

Haul distances for the rooms in each section of the panel based on a 17 entries cut sequence (Figure 4-2). Cont.

Room Number or Location	Cut no. in Room	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
Btwn 2 & 1	2+40	2 down, 6 left, 2 up, 3 left, 1 down, 1 left	938	818	1756
Btwn 3 & 2	+60	1 down, 3 left, 2 up, 3 left, 1 down, 1 left	698	578	1276
Btwn 3 & 2	1+20	1 down, 4 left, 2 up, 3 left, 1 down, 1 left	758	638	1396
Btwn 3 & 2	1+80	1 down, 5 left, 2 up, 3 left, 1 down, 1 left	818	698	1516
Btwn 3 & 2	2+40	1 down, 6 left, 2 up, 3 left, 1 down, 1 left	878	758	1636
Btwn 4 & 5	+60	1 up, 1 left, 1 up, 5 left, 1 down, 1 left	638	518	1156
Btwn 4 & 5	1+20	1 up, 2 left, 1 up, 5 left, 1 down, 1 left	698	578	1276
Btwn 4 & 5	1+80	1 up, 3 left, 1 up, 5 left, 1 down, 1 left	758	638	1396
Btwn 4 & 5	2+40	1 up, 4 left, 1 up, 5 left, 1 down, 1 left	818	698	1516
Btwn 5 & 6	+60	3 up, 6 left, 1 down, 1 left	698	578	1276
Btwn 5 & 6	1+20	3 up, 7 left, 1 down, 1 left	758	638	1396
Btwn 5 & 6	1+80	3 up, 8 left, 1 down, 1 left	818	698	1516
Btwn 5 & 6	2+40	3 up, 9 left, 1 down, 1 left	878	758	1636

Haul distances for the rooms in each section of the panel based on a 17 entries cut sequence (Figure 4-2). Cont.

Room Number or Location	Cut no. in Room	Outgoing Direction	Distance (ft)		
			To Feeder	From Feeder	Total
Btwn 6 & old	+60	4 up, 6 left, 1 down, 1 left	758	638	1396
Btwn 6 & old	1+20	4 up, 7 left, 1 down, 1 left	818	698	1516
Btwn 6 & old	1+80	4 up, 8 left, 1 down, 1 left	878	758	1636
Btwn 6 & old	2+40	4 up, 9 left, 1 down, 1 left	938	818	1756

APPENDIX B.

CM-SHUTTLE CAR MATCHING EXPERIMENTAL OUTPUT

APPENDIX B: CM-SHUTTLE CAR MATCHING EXPERIMENTAL OUTPUT

Duration of mining

		Fleet size					
Cuts	Segment	1	2	3	4	5	6
Entries	1	15.84	10.37	9.04	9.01	8.98	8.87
Entries	2	17.34	11.19	9.58	9.49	9.40	9.34
Entries	3	17.38	11.24	9.59	9.21	9.14	9.10
Entries	4	18.61	11.76	9.75	9.39	9.34	9.31
Entries	5	17.68	11.33	9.64	9.34	9.32	9.25
Entries	6	16.70	10.99	9.44	9.23	9.25	9.09
Entries	7	17.13	11.16	9.53	9.47	9.43	9.30
Entries	8	18.00	11.61	9.75	9.26	9.24	9.17
Entries	9	18.93	11.93	9.84	9.48	9.47	9.45
Rooms	10	17.47	11.22	9.48	9.22	9.21	9.20
Rooms	11	18.02	11.56	9.58	9.17	9.13	8.96
Rooms	12	18.83	12.02	9.97	9.38	9.35	9.15
Rooms	13	18.98	12.05	10.00	9.32	9.32	9.18
Rooms	14	14.60	9.07	7.35	6.76	6.64	6.59

Cycle times LHS

		Fleet size					
Cuts	Segment	1	2	3	4	5	6
Entries	1	4.15	5.42	7.07	9.37	11.65	13.78
Entries	2	4.54	5.85	7.50	9.88	12.18	14.48
Entries	3	4.55	5.87	7.48	9.55	11.83	14.09
Entries	4	4.87	6.15	7.63	9.79	12.15	14.51
Entries	5	4.63	5.92	7.54	9.74	12.13	14.44
Entries	6	4.37	5.73	7.37	9.59	11.97	14.04
Entries	7	4.48	5.82	7.43	9.82	12.19	14.39
Entries	8	4.71	6.06	7.63	9.61	11.95	14.15
Entries	9	4.96	6.22	7.70	9.87	12.32	14.71
Rooms	10	4.57	5.86	7.42	9.61	11.98	14.33
Rooms	11	4.72	6.04	7.50	9.56	11.89	13.95
Rooms	12	4.93	6.28	7.80	9.78	12.16	14.23
Rooms	13	4.97	6.29	7.80	9.71	12.09	14.26
Rooms	14	5.09	6.31	7.65	9.36	11.47	13.62

Cycle time RHS

		Fleet size					
Cuts	Segment	1	2	3	4	5	6
Entries	1	4.27	5.56	7.21	9.57	11.86	14.00
Entries	2	4.62	5.94	7.63	10.16	12.53	14.92
Entries	3	4.82	6.21	7.85	9.97	12.28	14.67
Entries	4	4.93	6.19	7.66	9.78	12.13	14.47
Entries	5	4.47	5.73	7.34	9.62	11.98	14.24
Entries	6	4.62	6.05	7.65	9.96	12.40	14.65
Entries	7	4.73	6.12	7.76	10.25	12.71	15.05
Entries	8	4.83	6.23	7.85	9.99	12.38	14.72
Entries	9	4.71	5.93	7.40	9.57	11.96	14.26
Rooms	10	4.66	5.95	7.52	9.67	12.04	14.38
Rooms	11	4.79	6.14	7.59	9.58	11.95	14.03
Rooms	12	5.06	6.43	7.98	9.95	12.40	14.50
Rooms	13	5.20	6.57	8.13	9.90	12.30	14.53
Rooms	14	5.19	6.44	7.81	9.47	11.59	13.76

CM utilization for loading LHS

		Fleet size					
Cuts	Segment	1	2	3	4	5	6
Entries	1	15.60	23.90	27.40	27.50	27.60	28.00

CM utilization for loading LHS. Cont.

		Fleet size					
Cuts	Segment	1	2	3	4	5	6
Entries	2	14.30	22.10	25.90	26.10	26.30	26.50
Entries	3	14.20	22.00	25.80	26.90	27.10	27.20
Entries	4	13.30	21.10	25.40	26.40	26.60	26.60
Entries	5	14.00	21.90	25.70	26.50	26.60	26.80
Entries	6	14.80	22.50	26.20	26.80	26.70	27.20
Entries	7	14.50	22.20	26.00	26.10	26.30	26.60
Entries	8	13.70	21.30	25.40	26.70	26.80	27.00
Entries	9	13.10	20.80	25.20	26.10	26.10	26.20
Rooms	10	14.20	22.10	26.10	26.90	26.90	26.90
Rooms	11	13.70	21.40	25.80	27.20	27.20	27.70
Rooms	12	13.10	20.60	24.90	26.40	26.60	27.10
Rooms	13	13.00	20.60	24.80	26.60	26.70	27.10
Rooms	14	12.70	20.50	25.30	27.70	28.20	28.40

CM utilization for loading RHS

		Fleet size					
Cuts	Segment	1	2	3	4	5	6
Entries	1	15.10	23.30	26.90	27.00	27.10	27.50
Entries	2	14.00	21.70	25.40	25.40	25.60	25.80

CM utilization for loading RHS. Cont.

		Fleet size					
Cuts	Segment	1	2	3	4	5	6
Entries	3	13.40	20.70	24.60	25.80	26.10	26.10
Entries	4	13.10	20.90	25.30	26.40	26.60	26.70
Entries	5	14.50	22.50	26.30	26.80	26.90	27.10
Entries	6	14.00	21.30	25.20	25.80	25.80	26.10
Entries	7	13.70	21.00	24.80	26.10	25.10	25.40
Entries	8	13.40	20.70	24.60	25.70	25.80	25.90
Entries	9	13.70	21.80	26.10	26.90	26.90	27.00
Rooms	10	13.90	21.70	25.70	26.70	26.70	26.80
Rooms	11	13.50	21.00	25.50	27.00	27.00	27.60
Rooms	12	12.80	20.10	24.20	25.90	26.00	26.60
Rooms	13	12.40	19.60	23.80	26.10	26.10	26.50
Rooms	14	12.50	20.00	24.80	27.30	27.90	28.00

Average waiting time in loading queue LHS

		Fleet size					
Cuts	Segment	1	2	3	4	5	6
Entries	1	1.48	1.97	2.54	3.18	3.75	4.31
Entries	2	1.73	2.29	3.00	3.89	4.57	5.37
Entries	3	1.76	2.33	2.98	3.46	4.01	4.75

Average waiting time in loading queue LHS. Cont.

		Fleet size					
Cuts	Segment	1	2	3	4	5	6
Entries	4	1.89	2.37	2.78	3.36	3.88	4.42
Entries	5	1.78	2.29	2.89	3.42	4.08	4.69
Entries	6	1.67	2.30	2.91	3.56	4.32	4.99
Entries	7	1.73	2.33	2.98	3.80	4.53	5.17
Entries	8	1.87	2.49	3.15	3.62	4.25	4.91
Entries	9	1.91	2.40	2.89	3.47	4.09	4.54
Rooms	10	1.89	2.38	2.89	3.33	3.90	4.40
Rooms	11	1.96	2.55	3.06	3.46	4.09	4.46
Rooms	12	2.09	2.73	3.35	3.88	4.57	5.01
Rooms	13	2.05	2.65	3.26	3.64	4.20	4.69
Rooms	14	2.14	2.65	3.12	3.46	3.87	4.38

Average waiting time in loading queue RHS

	Fleet size					
Segment	1	2	3	4	5	6
1	1.48	1.88	2.37	2.91	3.46	3.92
2	1.71	2.13	2.65	3.36	3.88	4.38
3	1.86	2.44	2.94	3.24	3.67	4.19
4	1.88	2.31	2.75	3.20	3.76	4.30

Average waiting time in loading queue RHS. Cont.

	Fleet size					
Segment	1	2	3	4	5	6
5	1.63	2.06	2.56	3.02	3.54	3.97
6	1.79	2.38	2.87	3.42	4.09	4.67
7	1.78	2.31	2.81	3.53	4.16	4.70
8	1.86	2.38	2.90	3.41	3.93	4.48
9	1.82	2.23	2.68	3.24	3.81	4.46
10	1.88	2.31	2.82	3.40	4.00	4.68
11	1.95	2.42	2.73	3.14	3.54	3.87
12	2.07	2.60	3.09	3.35	3.82	4.06
13	2.15	2.66	3.08	3.29	3.79	4.11
14	2.14	2.47	2.69	2.96	3.18	3.36

Productivity

			Fleet size					
Cuts	Tonnage in Segment	Segment	1	2	3	4	5	6
Entries	5437	1	343	524	602	603	606	613
Entries	5158	2	297	461	538	543	549	552
Entries	4740	3	273	422	494	515	518	521
Entries	4182	4	225	356	429	445	448	449
Entries	5018	5	284	443	521	537	539	542
Entries	5158	6	309	469	546	559	558	567

Productivity. Cont.

			Fleet size					
Cuts	Tonnage in Segment	Segment	1	2	3	4	5	6
Entries	4740	7	277	425	497	501	502	509
Entries	4740	8	263	408	486	512	513	517
Entries	4529	9	239	380	460	478	478	479
Rooms	5403	10	309	482	570	586	587	587
Rooms	5117	11	284	443	534	558	560	571
Rooms	5117	12	272	426	513	546	547	559
Rooms	4971	13	262	412	497	533	534	542
Rooms	1754	14	120	193	239	260	264	266

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