# Deformation of Soil to Seismic Waves Reflected on Foundation

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Annotation: Interacting with constractions foundations during earthquake elastic seismic waves form reflected waves field which interference with coming waves lead to an increase of stresses near by foundation. These stresses can exceed the earth limit of elasticity and become the reason of limited plastic deformations region formation and residual foundations seismosettlements. In order to describe this phenomenon solution of the problem of non-standard elastic wave interaction with shifting base of foundation is obtained for conditions of caused by interference reflected plastic wave generation behind the front of which medium load relief takes place which can be described using bilinear elastic-plastic model with reinforcement.

Foundation reflected waves combine with seismic waves field resulting in increase of stresses near by foundation which can exceed the limit of elasticity. Limited residual soil deformations region is formed in such conditions leading to residual foundation shift. In order to illustrate this phenomenon let us analyse using simple scheme stressed soil state below the base of foundation in conditions of seismic action, difraction on boundaries of the base of foundation is not taken into consideration.

We study interaction of seismic wave with foundation in the plane of variables: y - depth, t - time, using method of wave equation solution characteristics (Fig.1). In this original Euler's coordinates system (y,t) for axial stresses $\sigma$ and mass velocities $v$ of medium moving with density $\rho$ continuity equation

$$\frac{\partial}{\partial y}(\rho v) + \frac{\partial \rho}{\partial t} = 0 \qquad (1)$$

and momentum equation

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial \sigma}{\partial y} = 0 \qquad (2)$$

have to be satisfied.

Let us introduce Lagrangian coordinates h,t, where variable "h" is defined by mass of single area soil column situated between the foundation base and considered horizontal section at a depth y. This moving coordinate system appreciably simplifies wave equation solution and is bound with unmovable Euler's coordinate system (y) according to relationship:

$$h = \int_{y(0,t)}^{y(h,t)} \rho(h,t)\,dy \qquad (3)$$

In new coordinate system h,t, equation (1) and (2) have more simplified form:

$$\frac{\partial v}{\partial t} + \frac{\partial \sigma}{\partial h} = 0\ ; \quad A_o^2\frac{\partial v}{\partial h} + \frac{\partial \sigma}{\partial t} = 0 \qquad (4)$$

where $A_o = \rho\sqrt{d\sigma/d\rho} = \sqrt{E_o \rho_o}$ - impedance which in h,t-coordinates is a disturbance propagation speed.

Let us use elastic-plastic Prendl scheme with linear reinforcement. This model for uniaxial stress state can be represented by three equations:

$$\sigma = E_o\varepsilon \quad \text{for} \quad \sigma \le \sigma_s\ ; \qquad (5)$$

$$\sigma = \sigma_s + E_1(\varepsilon - \varepsilon_s) \quad \text{for} \quad \sigma > \sigma_s\ ; \qquad (6)$$

$$\sigma = \sigma_m + E_o(\varepsilon - \varepsilon_m) \quad \text{for load relief} \quad \sigma_m > \sigma_s \qquad (7)$$

where $E_1$ - linear soil reinforcement modulus above the limit of elasticity; index "s" for stresses $\sigma$ and deformations $\varepsilon$ corresponds to the limit of elasticity, and index "m" - to maximum variables values in loading conditions. In region 2 behind the front of reflected elastic waves $h = A_o t$ plastic waves front $h=A_1 t$ propagates, where $A_1 = \sqrt{E_1 \rho}$ - is reflected

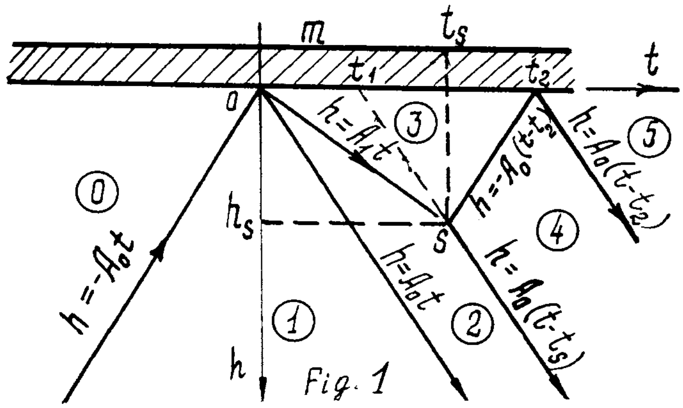

Fig. 1

plastic wave propagation speed in h,t-coordinates. In region 0 ($t < 0$) dynamic stress and speed is equal to zero. In region 1 the values of stresses $\sigma_1$ are equal to those of the incident wave front and soil mass speed is: $v_1 = -\sigma_1/A_0$. In region 2, i.e. behind the front of reflected elastic wave $h = A_0 t$ the stresses are equal to $\sigma_s$. The combined conditions of mass continuity and momentum conservation for characteristic $h = A_0 t$ have the following form:

$$\sigma_s - A_0 v_2 = \sigma_1 - A_0 v_1 \equiv 2\sigma_1 , \qquad (8)$$

whence we obtain speed value which is constant for the whole region 2:

$$v_2 = - \frac{2\sigma_1 - \sigma_s}{A_0} . \qquad (9)$$

The condition of soil plastic deformations formation during interaction of elastic waves with foundation is expressed by inequality:

$$\sigma_1 < \sigma_s < 2\sigma_1 , \qquad (10)$$

whence follows that numerator in formula (9) is positive and $v_2 < 0$, i.e. in reflected waves region particles speed is also directed in the direction of incident wave propagation and $|v_2| < |v_1|$. Plastic deformations region 3 is formed behind the front of reflected plastic wave $h = A_1 t$. Stresses $\sigma_{32}$ and speed $v_{32}$ in region 3 on the boundary with region 2 should satisfy the condition of mass and momentum conservation:

$$\sigma_{32} - A_1 v_{32} = \sigma_s - A_1 v_2 ,$$

which allowing for relationship (9) has a form:

$$\sigma_{32} - A_1 v_{32} = \frac{A_0 - A_1}{A_0}\sigma_s + \frac{2A_1}{A_0}\sigma_1 . \qquad (11)$$

If $h=0$ and $t=0$ allowing for initial condition $v(0,0)=0$, which expresses unmovable state of construction in the moment of seismic wave income formula for maximum initial stress $\sigma_{max}$ determination below the base of foundation follows from equation (11)

$$\sigma_{max} = \sigma(0,0) = \frac{A_0 - A_1}{A_0}\sigma_s + \frac{2A_1}{A_0}\sigma_1 \qquad (12)$$

Let us express stresses and speeds in region 3 with the help of function $\varphi_1$ and $\varphi_2$, which are to be determined and characterize incident and reflected waves, in order to study soil plastic deformation when $\sigma_1^* < \sigma_1 < \sigma_s$:

$$\left.\begin{aligned} \sigma_3(h,t) &= \varphi_1\left(t - \frac{h}{A_0}\right) + \varphi_2\left(t + \frac{h}{A_0}\right); \\ v_3(h,t) &= \frac{1}{A_0}\left[\varphi_1\left(t - \frac{h}{A_0}\right) - \varphi_2\left(t + \frac{h}{A_0}\right)\right]. \end{aligned}\right\} \qquad (13)$$

Plastic wave $h = A_1 t$ is a load relief-wave, therefore its wave front stresses for each soil layer are maximum. Let us denote these stresses $\sigma_m = \sigma_{32}$, and speeds $v_m = v_{32}$, then using expression (13) we have:

$$\left.\begin{aligned} \sigma_m(t) &= \varphi_1(\alpha t) + \varphi_2(\beta t); \\ A_0 v_m(t) &= \varphi_1(\alpha t) - \varphi_2(\beta t), \end{aligned}\right\} \qquad (14)$$

where

$$\alpha = \frac{A_0 - A_1}{A_0} \quad ; \quad \beta = \frac{A_0 + A_1}{A_0} .$$

Functional equations system (14) solution has a form:

$$\left.\begin{aligned} \varphi_1(t) &= \frac{A_0}{2A_1}\left[\beta\sigma_m\left(\frac{t}{\alpha}\right) - \sigma_{max}\right]; \\ \varphi_2(t) &= \frac{A_0}{2A_1}\left[\sigma_{max} - \alpha\sigma_m\left(\frac{t}{\beta}\right)\right], \end{aligned}\right\} \qquad (15)$$

where $\sigma_{max}$ value is determined by formula (12). Expressions (13) for $h=0$ and allowing for equation (15) have a form:

$$\left.\begin{aligned} \sigma(0,t) &= \frac{A_0}{2A_1}\left[\beta\sigma_m\left(\frac{t}{\alpha}\right) - \alpha\sigma_m\left(\frac{t}{\beta}\right)\right]; \\ v(0,t) &= \frac{1}{2A_1}\left[\beta\sigma_m\left(\frac{t}{\alpha}\right) + \alpha\sigma_m\left(\frac{t}{\beta}\right) - 2\sigma_{max}\right], \end{aligned}\right\} \qquad (16)$$

which gives a possibility allowing for boundary condition

$$\sigma_3(0,t) + m\,\frac{d v_3(0,t)}{dt} = 0 \qquad (17)$$

to obtain differential-functional equation in $\sigma_m$:

$$\frac{d}{dt}\left[\beta\sigma_m\left(\frac{t}{\alpha}\right) + \alpha\sigma_m\left(\frac{t}{\beta}\right)\right] = \frac{A_0}{m}\left[\alpha\sigma_m\left(\frac{t}{\beta}\right) - \beta\sigma_m\left(\frac{t}{\alpha}\right)\right]. \qquad (18)$$

Following developed by H.A. Rahmatulin (1) method of deformation determination on load relief wave let us represent stresses distribution along plastic wave $h = A_1 t$ by infinite power series:

$$\sigma_m(t) = \sum_{n=0}^{\infty} b_n t^n . \qquad (19)$$

In such a case equation (18) has a form:

$$\frac{\beta}{\alpha}\sum_{n=1}^{\infty} n b_n \left(\frac{t}{\alpha}\right)^{n-1} + \frac{\alpha}{\beta}\sum_{n=1}^{\infty} n b_n \left(\frac{t}{\beta}\right)^{n-1} + \frac{A_0}{m}\left[\beta\sum_{n=0}^{\infty} b_n\left(\frac{t}{\alpha}\right)^n - \alpha\sum_{n=0}^{\infty} b_n\left(\frac{t}{\beta}\right)^n\right] = 0, \qquad (20)$$

whence for $t=0$ we have:

$$b_1 = b_0\,\frac{A_0\alpha\beta(\alpha - \beta)}{m(\alpha^2 + \beta^2)} , \qquad (21)$$

where $b_0 = \sigma_m(0) = \sigma(0,0) = \frac{A_0 - A_1}{A_0}\sigma_s + \frac{2A_1}{A_0}\sigma_1$.

Putting equal coefficients of identical powers of $t$ in expression (20), we get recurrent formula

$$b_n = b_{n-1}\,\frac{A_0\alpha\beta(\alpha^n - \beta^n)}{mn(\alpha^{n+1} + \beta^{n+1})} \qquad n \geqslant 1 , \qquad (22)$$

and then an expression for depth distibution of maximum stresses:

$$\sigma_m(h) = \sigma_{max}\,G(rh), \qquad (23)$$

where

$$G(S) = 1 + \sum_{n=1}^{\infty} (-1)^n Q(n) \frac{S^n}{n!} \;; \tag{24}$$

$$r = \frac{A_0 - A_1}{A_1 m} \;; \tag{25}$$

$$Q(n) = \prod_{k=1}^{n} \frac{1 - \xi^k}{1 + \xi^{k+1}} \;; \tag{26}$$

$$\xi = \frac{\alpha}{\beta} = \frac{A_0 - A_1}{A_0 + A_1} \tag{27}$$

For convenience of calculations let us approximate natural argument function Q(n) by power relationship

$$Y(n) = q_1 + q_2 q_3^n , \tag{28}$$

its parameters should satisfy according to least-squares technique to a system of equations:

$$\sum_{n=1}^{\infty} [Q(n) - Y(n)] \frac{\partial Y}{\partial q_j} = 0 \quad (j = 1,2,3) \tag{29}$$

Limiting by first three members of series (29) we get trivial conditions:

$q_1 + q_2 q_3 = Q(1)$; $q_1 + q_2 q_3^2 = Q(2)$; $q_1 + q_2 q_3^3 = Q(3)$,

whence allowing formula (26) follows:

$$q_1 = \frac{(1-\xi^2)(1-\xi)^2}{(1+\xi^2)(1+\xi^3)(1-\xi+\xi^2+\xi^4)} ;$$
$$q_2 = \frac{\xi(1+\xi^4)^2}{(1+\xi^2)(1+\xi^3)(1-\xi+\xi^3+\xi^4)} ; \tag{30}$$
$$q_3 = \frac{\xi(1-\xi^2)}{1+\xi^4} .$$

Putting relationship (28) instead of Q(n) into (24) let us represent expression (23) in a form:

$$\sigma_m(h) = \sigma_{max} \left[1 + q_1 \sum_{n=1}^{\infty} (-1)^n \frac{(rh)^n}{n!} + q_2 \sum_{n=1}^{\infty} (-1)^n \frac{(q_3 rh)^n}{n!}\right] . \tag{31}$$

Allowing for formula of expansion of exponential function into a power series let us carry out series convolution to exponential function in (31), which gives an approximate solution of equations (18) in a form covenient for practical calculations:

$$\sigma_m(h) = \sigma_{max}\left[1 - q_1(1 - e^{-rh}) - q_2(1 - e^{-q_3 rh})\right] . \tag{32}$$

Using expressions (6), (7) and (32) we can obtain formulars of maximum plastic $\varepsilon_m$ and residual $\varepsilon_0$ deformations distribution in base depth:

$$\varepsilon_m(h) = \frac{\sigma_m(h)}{E_1} - \left(\frac{1}{E_1} - \frac{1}{E_0}\right)\sigma_s =$$
$$= \frac{\sigma_{max}}{E_1}\left[1 - q_1(1 - e^{-rh}) - q_2(1 - e^{-q_3 rh})\right] - \left(\frac{1}{E_1} - \frac{1}{E_0}\right)\sigma_s ; \tag{33}$$

$$\varepsilon_0(h) = \varepsilon_m(h) - \frac{\sigma_m(h)}{E_0} =$$
$$= \left(\frac{1}{E_1} - \frac{1}{E_0}\right)\left\{\sigma_{max}\left[1 - q_1(1 - e^{-rh}) - q_2(1 - e^{-q_3 rh})\right] - \sigma_s\right\} . \tag{34}$$

The lower limit of plastic deformations region $h_s$ we obtain from condition $\sigma_m(h_s) = \sigma_s$ which allowing for relation (32) leads to a formula

$$h_s = -\frac{1}{r} \ln z = -\frac{A_1}{A_0 - A_1} m \ln z , \tag{35}$$

where $z$ is determined by means numerical solution of power equation its parameters being dependent only on soil deformational properties The value of foundation residual settlement we determine integrating residual deformations distribution function (34) with respect to depth over an interval of plastic deformations region, necessarily passing from coordinate h to coordinate of displacement:

$$S_0 = \frac{A_1 m}{(A_0 - A_1)} \int_0^{h_s} \varepsilon_0(rh)\, dh = m \frac{A_0 + A_1}{A_0^2 A_1} \left\{\sigma_{max}\left[rh_s(1 - q_1 - q_2) + q_1(1 - e^{-rh_s}) + \frac{q_2}{q_3}(1 - e^{-q_3 rh_s})\right] - \sigma_s rh_s\right\} . \tag{37}$$

Analysis of the obtained approximal solutions shows that during seismic wave and foundation elastic interaction the maximum stress value below the base of foundation is equal to double stress value on the front of incident wave, and in case of soil plastic deformations forming for the same seismic wave intensity dynamic action on construction turns out to be lesser. Let us determine the ratio of maximum stress during earth plastic deformations forming $K_\sigma$ to maximum stress during wave-foundation elastic interaction:

$$K_\sigma = \frac{\sigma_{max}}{2\sigma_1} = \frac{1}{2\sigma_1}\left[\sigma_s + \frac{A_1}{A_0}(2\sigma_1 - \sigma_s)\right] . \tag{38}$$

Taking into account the main inequality (10) in formula (38) we can have $K_\sigma < 1$ , i.e. we come to a conclusion that soil plastic deformations formation below the base of foundation reduces construction dynamic load caused by seismic waves action.

The time of soil plastic deformations and foundation settlement formation $t_s = h_s / A_1$ is too small in comparison with seismic waves oscillation period since their wavelength is many times as large as the depth of plastic deformations region below the base of foundation $y_s = h_s / \rho_0$ , and though reflected plastic wave speed is less than elastic waves speed, it can be measured by the same order values. So the incident seismic wave parameters changing during the time of plastic deformations forming $t_s$ is too negligible and hence do not considerably effect the value of residual foundation settlement.

CONCLUSIONS:

In the process of elastic seismic waves interaction with constructions foundations dynamic stresses near by foundation increase in comparison with stress values on the front of propagating waves. Foundation reflection of elastic seismic waves can generate reflected plastic wave resulting in soil residual deformation region formation below foundation and additional residual foundation settlement.

Residual foundation settlements do not take place during action of seismic waves if wave stress amplitude do not exceed half of the soil limit of elasticity.

Soil plastic deformations formation near by foundation appreciably reduces construction dynamic load in comparison with the case of pure elastic interaction between the wave and foundation, other conditions being identical.

The time during which residual foundation settlement takes place is small in comparison with seismic oscillations period. During this period of time reflected plastic wave penetrates on the depth which is relatively small in comparison with seismic wavelength and then degenerate into elastic wave. The maximum stresses and soil deformations takes place directly below the base of the foundation which intensively decrease approximatelly according to exponential law as far as moving off foundation base.

Foundation residual settlement and the depth of soil plastic deformations region are proportional to foundation static pressure on soil owing to the weght of construction. That's why to avoid irregular additional settlements it is recommended to provide approximately equal specific soil pressure near by the bases of all fondations of construction or its separate sections isolated by antisettlement joints during design of constructions in seismic regions.

Literature:

1. Rahmatulin H.A., Demyanov U.A., "Strength for intense short-term stresses", Fismatgis, Moscow, 1961, 400 p.