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SYNOPSIS: A hybrid method, in which soil media is modeled by 3—D boundary elements and the pile by 1—D finite elements, for an analysis of dynamic pile-soil interaction is investigated. A new series of equations and an effecient method to deal with singularity of integrals are presented. Several effects including pile length, stiffness of the pile and surrounding soil on dynamic response are studied. Finally, the characteristics of impaired and intact piles are investigated and compared; and some case studies are presented.

I. Introduction

Pile foundation is one of the most widely used foundation types for structures. Engineers feel concern about qualification and bearing capacity of the piles, since they are directly related to safty and reliability of the structure. Because of time consuming and cost of load test on pile and drilling out, they are selected as occasion demands. On the other hard, engineers are looking for a facile and inexpensive method, that is the reason why studies of dynamic technique are so active in recent years.

Vibration of pile foundation is one of typical dynamic soil-structure interaction problems, which can not be solved analytically for its complicatedness. On the other hand, there are some difficulties on dealing with an infinite space or a half space domain by monotonic finite element method. Boundary element method, by which the controlled equations are saticfied within the domain and only the boundaries of the domain are needed to be discribed, can be employed for solving problems with infinite domain efficiently (Brebbia, 1978).

Authors establish motion equations for soil medium by boundary element method and for pile by finite element method. By consisdering the consistent condition at the interface between pile and soil, calculation of dynamic stiffness of the pile can be obtained, as a result, dynamic response curves of different types of piles and the influence of characteristic parameters of soil and pile on dynamic stiffness of the pile can be investigated.

II. Mechanical Modelling

As shown in Fig. 1 a, following assumptions can be valid approximately for pile-soil system:

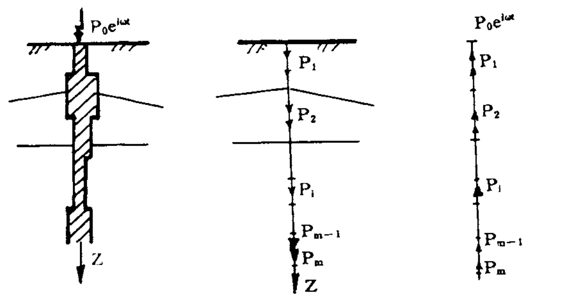
(1)Pile is a linear elastic rod with a changeable cross-section for modelling neck-narrowed or neck-widdened.

(2)Soil is a layered system, parallel each other, filled with a homogeneous and isotropic material in each layer.

(3)There exists continuity between pile and its surrounding soil during vibration.

(4)Pile is of unique function for transmitting vertical load to the surrounding soil, neglecting the effects of lateral dimension on soil.

With the assumptions mentioned above, the pile-soil system shown in Fig. 1 can be devided into two sub-systems, provided the existance of continuity along the boundary of pile and its surrounding soil. In 3—D dynamic system with infinite in Z direction as shown in Fig. 1 b, there are forces acting in the domain transmitted from pile and no forces acting on the boundary, which is suitable for BEM. A system of 1—D vibration of variational cross-section compression rod shown in Fig. 1c can be modelled by FEM. The pile is devided into m elements and soil resistance to pile distributes uniformly along the axis in each element. The soil resistance to pile and the force transmitted to the soil from pile are pair of action and reaction forces, therefore, the latter distributes evenly in each segment too.



(a)Soil-Pile System (b)Soil (c)Pile

Fig. 1 Modelling of Soil-Pile System

1. Motion Equations of Soil System

For simplicity, at first a circumstance of homogeneous soil medium is studied, then the motion equations of layered soil system can be obtained by combining boundary element method and transfer matrix.

According to principle of virtual work, one can establish boundary integral equations for the system shown in Fig. 1 b (Dominguez, 1981; Chen, 1989)

$$\frac{1}{2}u_i(P) = \int_\Gamma u_{ij}^*(P,Q)p_j(Q)d\Gamma - \int_\Gamma p_{ij}^*(P,Q)u_j(Q)d\Gamma + \sum_{s=1}^{m}\int_s^{s+1} u_{is}^*(P,Z)p_s dZ \qquad (2.1)$$

$i=1, 2, 3;\ s=1, 2, \cdots m$

in which Γ is the boundary of the system in concern, u_i and p_j, displacement in i direction and boundary force in j direction, respectively, $u_{ij}^*(P,Q)$ and $p_{ij}^*(P,Q)$ are fundamental solutions which represent the amplitudes of displacement and force at point Q in j direction as an unit harmonic force acting at point P in i direction in an infinite elastic space, respectively.

$$u_{ij}^*(P,Q) = \frac{1}{4\pi G}(\psi\delta_{ij} + \chi_{r,i}\cdot r_{,j}) \qquad (2.2)$$

$$p_{ij}^*(P,Q) = \frac{1}{4\pi}\left\{\frac{\partial r}{\partial n}\left(\frac{d\psi}{dr} - \frac{x}{r}\right)\delta_{ij} - 2\left(\frac{dx}{dr} - \frac{2x}{r}\right) + \left[\left(\frac{c_1^2}{c_2} - 2\right)\left(\frac{d\psi}{dr} - \frac{dx}{dr} - \frac{2x}{r}\right) - \frac{2x}{r}\right]r_{,j}n_j + \left(\frac{d\psi}{dr} - \frac{x}{r}\right)r_{,j}n_i\right\} \qquad (2.3)$$

Where

$$\psi = \left(1 - \frac{c_2^2}{\omega^2 r^2} + \frac{c_2}{i\omega r}\right)\frac{1}{r}e^{-i\omega r/c_2} - \left(\frac{c_2}{c_1}\right)^2\left(-\frac{c_1^2}{\omega^2 r^2} + \frac{c_1}{i\omega r}\right)\frac{1}{r}e^{-i\omega r/c_1} \qquad (2.4)$$

$$\chi = \left(1 - \frac{3c_2^2}{\omega^2 r^2} + \frac{3c_2}{i\omega r}\right)\frac{1}{r}e^{-i\omega r/c_2} - \left(\frac{c_2}{c_1}\right)^2\left(-\frac{3c_1^2}{\omega^2 r^2} + \frac{3c_1}{i\omega r} + 1\right)\frac{1}{r}e^{-i\omega r/c_1} \qquad (2.5)$$

As boundary force $p_j(Q)=0$, Eq(3. 1) can be rewritten

$$\frac{1}{2}u_i(P) + \int_\Gamma p_{ij}^*(P,Q)u_j(Q)d\Gamma = \sum_{s=1}^{m}\int_s^{s+1} p_s u_{is}^* dZ \qquad (2.6)$$

$i = 1,2,3;\quad s = 1,2,3,\cdots\cdots m$

For facilitation, Eq(2. 6) can be expressed in form of matrix, let displacement vector, at point P

$$U(P) = \begin{Bmatrix} u_1(P) \\ u_2(P) \\ u_3(P) \end{Bmatrix} \qquad (2.7)$$

bounday force vector at point P,

$$P(P) = \begin{Bmatrix} p_1(P) \\ p_2(P) \\ p_3(P) \end{Bmatrix} \qquad (2.8)$$

plane force matrix of fundamental solution

$$P^*(P,Q) = \begin{bmatrix} p_{11}^* & p_{12}^* & p_{13}^* \\ p_{21}^* & p_{22}^* & p_{23}^* \\ p_{31}^* & p_{32}^* & p_{33}^* \end{bmatrix} \qquad (2.9)$$

displacement vector of fundamental solution

$$U_s^* = \begin{Bmatrix} u_{13}^* \\ u_{23}^* \\ u_{33}^* \end{Bmatrix} \qquad (2.10)$$

Eq(2. 6) can be rewritten again as

$$\frac{1}{2}U(P) + \int_\Gamma P^*(P,Q)\ U(Q)\ d\Gamma = \sum_{s=1}^{m}\int_s^{s+1} P_s U_s(P,Z)dZ \qquad (2.11)$$

Based on discreteness of Eq. (2. 11) and N elements considered, Eq. (2. 11) can be rewitten:

$$C_r U_I + \sum_{I=1}^{N} H_{IJ}U_J = \sum_{s=1}^{m}\int_S^{S+1} P_S U_I(P,Z)dZ \qquad (2.12)$$

$I = 1,2,3,\cdots\cdots N$

in which C_r is a diagonal matrix with konwn constant elements, U_I and P_s are displacement vector of Ith element and transmitting force to the soil from Sth segment in the pile, respectively, the elements in matrix H_{IJ} can be expressed

$$(h_{ij})_{IJ} = \left(\int_\Gamma P_{ij}^*(P,Q)d\Gamma\right)_{IJ} \qquad (2.13)$$

Eq(2. 12) which is boundary integal equation by means of numerical method can be simplified

$$H_{N,N}U_N = G_{N,M}P_M \qquad (2.14)$$

where $H_{N,N}$ can be determined by Eq(2 .13) and matrix, C_r;

$G_{N,M}$ can be obtained by integrating $u_i^*(P,Q)$ in each segment; u_N represents displacement vector of a node at boundary; while P_M, load vector, consists of m unifomly distributed loads. Eq(2. 14) is the motion equation for soil medium, which infers the relationship beetween displacemts at soil surface, U_N, and loads on soil transmitting from pile.

The relationships of the displacement within soil medium and boundary force, boundary displacement and concentrited load in domain can be derived from principle of work equivalency

$$u_i(P) = -\int_\Gamma p_{ij}(P,Q)\, u_j(Q)d\Gamma + \sum_{s=1}^{m}\int_s^{s+1} p_s u_{is}^*(P,Z)dZ \tag{2.15}$$

where $u_i(P)$ represents the displacement at point P in the domain. similarly, it can be simplified in forms of matrix and calculated numerically

$$W_M = -H'_{M,N}U_N + G_{M,M}P_M \tag{2.16}$$

where

$$W_M = (w_2\ w_3 \cdots w_{m+1})^T \tag{2.17}$$

U_N and P_M are the same as showm in Eq(2. 14), and $G_{M,M}$ can be integrated by $u_{is}^*(P,Q)$ at each segment.

2. Motion Equations of Pile

Motion equations for pile elements

$$M\frac{\partial^2 w(z,t)}{\partial t^2} + c\frac{\partial w(z,t)}{\partial t} - E_PA\frac{\partial^2 w(z,t)}{\partial z^2} = 0 \tag{2.18}$$

where M represents mass of unit length of pile, $M=\rho A$; c, internal damping; E_P and A are Young's modulus and cross-section area of each pile segment, respectively; H, here the length of each segment.

Let $w(z,t)=w(z)e^{i\omega t}$, substituting into Eq(4.1), it yields

$$\frac{d^2w(z)}{dz^2} + \left(\frac{A}{H}\right)^2 w(z) = 0 \tag{2.19}$$

with a solution

$$w(z) = B\cos A\frac{z}{H} + C\sin A\frac{z}{H} \tag{2.20}$$

where

$$A = H\sqrt{(M\omega^2 - i\omega c)/E_PA} \tag{2.21}$$

One can get the relationship between terminal force and displacement at each element.

$$\begin{Bmatrix} p_i \\ p_{i+1}\end{Bmatrix} = \frac{E_PA}{H}\begin{bmatrix} ctgA & -\dfrac{1}{sinA} \\ -\dfrac{1}{sinA} & ctgA\end{bmatrix}\begin{Bmatrix} w_i \\ w_{i+1}\end{Bmatrix} \tag{2.22}$$

Stiffness matrix of each element

$$K_w = \frac{E_PA}{H}\begin{bmatrix} ctgA & -\dfrac{1}{sinA} \\ -\dfrac{1}{sinA} & ctgA\end{bmatrix} \tag{2.23}$$

Lumping all stiffness matrixes together, the relationship between external forces and displacements at every node on the pile can be obtained

$$\begin{Bmatrix} P_0 - N_1 \\ -N_1 - N_2 \\ \cdot \\ \cdot \\ \cdot \\ -N_{m-1} - N_m \\ -N_m \end{Bmatrix} = K_{(m+1),(m+1)}\begin{Bmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ \cdot \\ w_{m+1}\end{Bmatrix} \tag{2.24}$$

Because of uniform distribution of resistance force to the pile at each element, it is proper to assume that the uniformly distributed forces acting along the pile at each element can be substituted by two concentrited loads, with a value equal to half of the sum at each element acting on the terminal points of each element, thus $N_1=\frac{1}{2}p_1H_1$, $N_2=\frac{1}{2}p_2H_2$, $N_3=\frac{1}{2}p_3H_3$, $\cdots$, $N_{m-1}=\frac{1}{2}p_{m-1}H_{m-1}$, $N_m=\frac{1}{2}p_mH_m$.

3. Global Motion Equations of Pile-Soil System

Combining Eq(2.14), (2.16) and (2. 24), it yields

$$H_{N,N}U_N = G_{N,M}P_M \tag{3.1}$$

$$W_M = -H'_{M,N}U_N + G'_{M,M}P_M \tag{3.2}$$

$$\begin{Bmatrix} P_0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0\end{Bmatrix} - \begin{Bmatrix} N_1 \\ N_1 + N_2 \\ \cdot \\ \cdot \\ \cdot \\ N_m\end{Bmatrix} = K_{(M+1),(M+1)}W_{M+1} \tag{3.3}$$

Eqs(3.1) to (3.3) are global motion equations of pile-soil system. Solve U_N from Eq(3.1) and substitute into Eq(3.2)

$$U_N = H'_{N,N}G_{N,M}P_M \tag{3.4}$$

$$W_M = -H'_{M,N}H^{-1}_{N,N}G_{N,M}P_M + G'_{M,M}P_M \tag{3.5}$$

$$= (-H'_{M,N}H^{-1}_{N,N}G_{N,M} + G'_{M,M})P_M \tag{3.6}$$

Let $A_{M,M} = -H'_{M,N}H^{-1}_{N,N}G_{N,M} + G'_{M,M}$

Eq(3. 5) can be rewritten as

$$W_M = A_{M,M}P_M \tag{3.7}$$

or

$$P_M = A^{-1}_{M,M}W_M \tag{3.8}$$

Substituting Eq(3. 8) into Eq(3. 3), it yields

$$\begin{bmatrix} P_0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} - \begin{bmatrix} B_{M,M}W_N \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ B_{M,M}W_M \end{bmatrix} = K_{(M+1),(M+1)}W_{(M+1)} \tag{3·9}$$

where

$$W_M = \begin{bmatrix} w_2 \\ w_3 \\ \cdot \\ \cdot \\ \cdot \\ w_{m+1} \end{bmatrix} \tag{3·10}$$

and

$$W_{M+1} = \begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ \cdot \\ w_{m+1} \end{bmatrix} \tag{3·11}$$

The dynamic Stiffness of pile is defined as the amplitude of harmonic force acting at the top of the pile which causes unit displacement of the same point in vertical direction.

Let the first element in $W_{M+1}, w_1 = 1$, thus the value of P_0 is equal to that of dynamic stiffness, K_0. Substituting $w_1 = 1$ and $P_0 = K_0$ into Eq(3. 9), K_0 can be Solved .

In dynamic testing ,admittance curves are always applied for describing dynamic response of the pile (Davis ,1976), which is defined as the ratio of velocity amplitude to force amplitude of the top of the pile ,that is

$$\left|\frac{V_0}{P_0}\right| = \left|\frac{\omega A_0}{P_0}\right| = \left|\frac{\omega}{K_0}\right| = 2\pi\left|\frac{f}{K_0}\right| \tag{3. 12}$$

As a result ,the curves relating $\left|\frac{V_0}{P_0}\right|$ to f can be found from Eq(3. 12).

Ⅲ. Influences of Nature of Impairment of the Pile on Admittance Curves

A programme combining BEM and FEM in FORTRAN for analyzing dynamic response of single pile is prepared by the authors , which can be employed to evaluate response curves of a single intact or impaired pile in homogeneous and isotropic layered soil system . It can be seen from Fig. 2,3, and 4 that there are some visualized changes in the curves for neck-narrowed and -widdened piles compared for those for intact one . The frequency difference between larger peaks in the curves for impaired piles generally indicates wave reflection from impaired spots and the length calculated from this frequency difference infers the distance from top of the pile to the impaired spot . The frequency difference between two smaller peaks within the first larger peak usually indicates wave reflection from the bottom of the pile , from which whole length of the pile can be calculated .

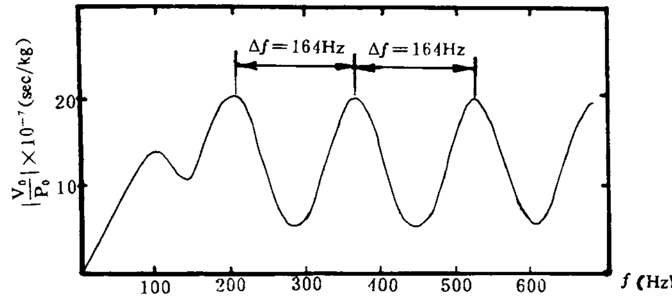


Fig. 2 Admittance Curve for Intact Pile

Ⅳ. Effects of Characteristic Parameters of Pile and Soil on Dynamic Sfiffness of the Pile

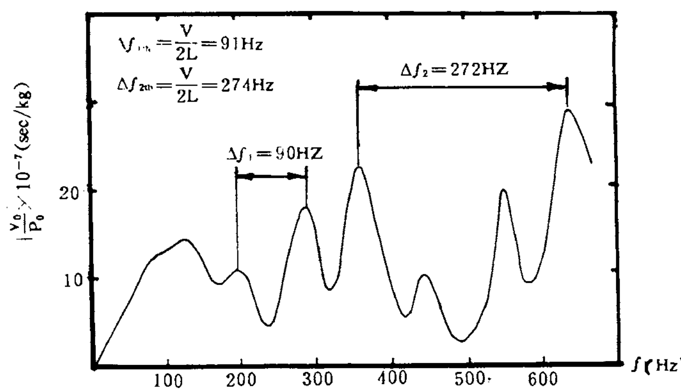


Fig. 3 Admittance Curve for Neck-Narrowed Pile

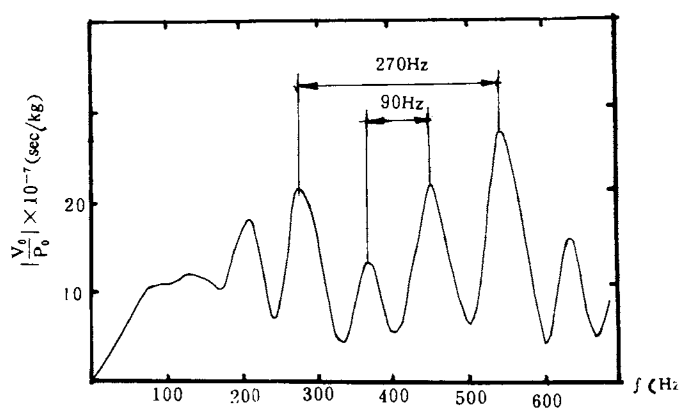


Fig. 4 Admittance Curve for Neck-Widened Pile

1. Effects of Length Ratio

Fig. 5 shows the variation of dynamic stiffness of the pile as length increasing ,if the area of cross-section of the pile keeps Constant. It can be seen from Fig. 5 that at very beginning dynamic stiffness of the pile increases with increasing of length and then the slope decreses as it reaches a certain value.

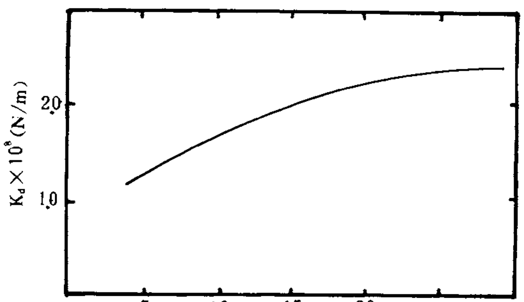


Fig. 5 Effects of Length of Pile on Stiffness

2. Effects of Young's Modulus and Ratio of Pile to Soil

Fig . 6 indicates the effects of dynamic stiffness of pile as Young's modulus of soil increasing ,if Young's modulus of pile keeps constant . At beginning ,dynamic stiffness of the pile increases almost linearly with incresing of Young's modulus of the soil.

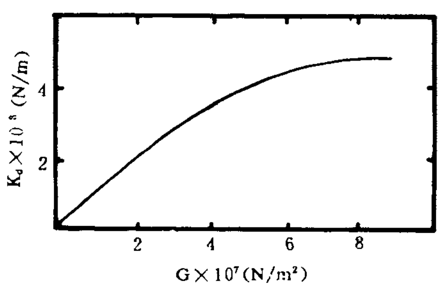


Fig . 6 Effects of E_S on stiffness as E_P keeping Constant

Similarlly ,Fig. 7 shows the effects of Young's modulus of pile material,there are little effects on dynamic stiffness of pile if that of the soil keeps constant.

3. Effects of Location and Characteristis of the Impairment

Table 1 indicates the effects of location of neck-narrowed impairment on dynamic stiffness of the pile . It can be seen that the nearer of the impairment to the bottom , the less effect on dynamic stiffness ,if all other conditionsremain the same . Table 2 Shows the effects of location of neck-widden impairment on dynamic stiffness of the pile. One may find the same tendency as mentioned above .

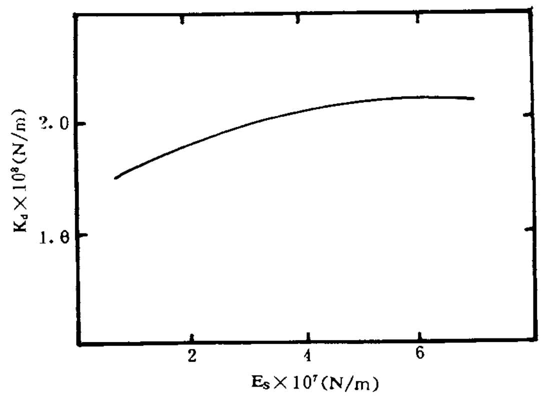


Fig . 7 Effects of E_P on stiffness as E_S keeping Constant

Table 1 The Effects of Locatcon of Neck-Narrowed Impairment

Distance to top of the pile (m)	5. 5	9. 5	13. 5
Dynamic stiffness of the pile (N/m)	0.670×10^9	0.678×10^9	0.680×10^9

Table 2 The Effects of Location of Neck-widden Impairment

Distance to top of the pile (m)	5. 5	9. 5	13. 5
Dynamic stiffness of the pile (N/m)	0.6830×10^9	0.6820×10^9	0.6815×10^9

V. Conclusions

A numerical method of combination of BEM and FEM is employed for analyzing dynamic response of single pile ,avoiding any improper assumptions on interaction between soil and pile in order to make the problem solvable,which enhances the accuracy of solution . Some case studies and comparisons are conducted ,which leads to following points:

(1) The effects of soil medium far away from the pile on the nodes in the interface of soil and pile and its surrounding area is so small that the boundary can be set at the place far enough away from the interface ,neglecting those effects on

the results . Similarly ,there exists a moderate number of pile elements on which one can get a solution accurate enough.

(2) Admittance curves for intact pile are always are even, unlike impaired piles, uneven in shape ,appearing larger peaks followed by smaller peaks. Usually, the frequency difference between two smaller peaks in the first larger peak indicates wave reflection from the bottom of the pile ,from which the whole length of the pile can be calculated. while the frequency difference between two larger peaks indicates wave reflection from the impaired spots, from which the location of the impairment can be estimated.

(3) Dynamic stiffness of the pile increases with increasing length of pile and the slope decreases as the length reaches a specific value .

(4) Young's modulus of pile material has less effects on dynamic stiffness of the pile ,but that of soil does for a same pile .

(5) The value of dynamic stiffness of a intact pile will be greater than that of neck-narrowed one ,but smaller than neck-widdened. Usually ,neck-narrowed piles are of large value of admittance, vice versa, neck- widdened ones, smaller value of admittance.

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