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A Hybrid Modeling of Soil - Structure Interaction Problems

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SYNOPSIS: A hybrid method is proposed in this research for dynamic soil-structure interaction analysis of embedded structures within a multilayered elastic half-space. A near field region, containing the structure and a portion of soil surrounding it, is modeled by finite elements while the far field formulation is obtained through the classical wave propagation theory based on the assumption that the actual scattered wave fields can be represented by a set of line sources located at a depth corresponding to the center of mass of the structure under investigation. Traction reciprocity between the two regions is satisfied exactly while displacement continuity across the common interface is enforced in a least-squares sense. The two-dimensional system is excited by seismic body waves (P and SV) propagating with oblique incidence and harmonic time dependence.

INTRODUCTION

Two methods of analysis have been evolved for dynamic soil-structure interaction problems: the continuum and the numerical approach. The continuum approach is generally based on the theory of linear elasticity or viscoelasticity and idealized models must be adopted in order to obtain mathematical solutions.

The most popular methods in the numerical approach are the finite element method (FEM) and the boundary element method (BEM). The main advantage of the finite element method is its versatility in problems involving different materials and complex geometries but FEM can not simulate unbounded domains completely. Several schemes have been proposed to overcome this shortcoming, such as the use of imperfect transmitting boundaries, perfect transmitting boundaries, infinite elements, hybrid techniques, the soil-island technique, etc., but most of them have been developped on the assumption that the soil mass can be represented as a homogeneous body in spite of the fact that stratified soil deposits are a common occurrence in nature.

The boundary element method appears to be well suited to model a semi-infinite domain since it is based on fundamentals solutions that extend to infinity *per se*, automatically taking into account the radiation condition. Applications to dynamic soil-structure interaction problems involving a homogeneous half-space were given by Karabalis and Beskos (1984), Abascal and Dominguez (1985), etc. In the case of a layered half-space the corresponding Green's functions must be determined numerically and some specific algorithms have already been reported in the literature (e.g. Luco and Apsel(1983), Kundu (1985)). Based on such Green's functions, an indirect boundary integral equation method has been used by Apsel and Luco (1987), Luco and Wong (1986, 1987) to obtain the dynamic response of rigid cylindrical/hemispherical foundations embedded in a layered viscoelastic half-space.

This paper presents a hybrid method that combines the continuum and the numerical approaches. The far field problem (i.e. a layered elastic half-space excited by seismic body waves) is solved analytically while the near field (i.e. the structure and surrounding soil) is modeled by finite elements. Special consideration of the scattered field permits the discretized region to be kept reasonably small. This paper is also an extension of a previous research (Romanel and Kundu, 1990) but now providing a better modeling of embedded structures (including tunnels) through a set of lines sources vibrating within the half-space. Formerly, these sources have been considered to be applied on the free surface, thus restricting the analysis to soil-structure interaction problems involving shallow foundations. To avoid unnecessary duplication the reader is referred to Romanel and Kundu (1990) for details about the matrix expressions mentioned herein.

BASIC SUPERPOSITIONS

The multilayered half-space is divided into two regions, a near field and a far field (Fig. 1). Stress and displacement fields at the common interface must be computed first since they will be used as forcing functions in the near field finite element analysis. In this hybrid approach the overall effects of the incident, reflected and scattered wave fields are modeled through the combined responses of the four simple problems shown in Fig. 2. The soil mass in each case is represented as a multilayered half-space without any structure or irregularity in it.

Problem 1 considers the incident wave field. Since elastic wave propagation in multilayered media is a well known phenomenon (Ewing et alli, 1957; Kennett, 1983) the ground motion at any point can be obtained without difficulty.

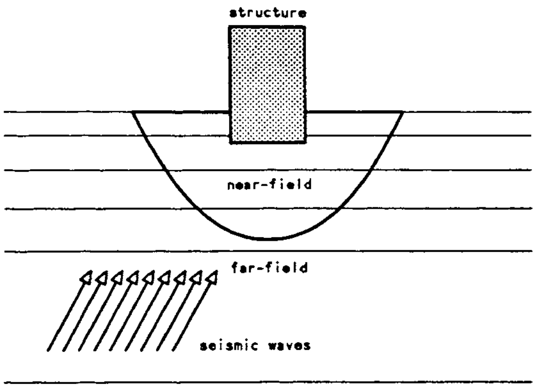

Fig. 1 - Problem geometry.

In problems 2, 3 and 4, an oscillating normal line force, a shear line force and a line moment are acting within the stratified half-space at a depth corresponding to the center of mass of the buried part of the structure. These three types of excitation are being considered here because when a structure is subjected to seismic waves of angular frequency ω the three possible reactions that it can exert on the soil mass are an oscillating normal line force of amplitude P, a shear line force of amplitude S and a line moment of amplitude M, all vibrating at the same angular frequency ω. From our knowledge about the response of a multilayered half-space to point or line sources (Kennett, 1983; Kundu, 1983) we can compute displacements and stresses at any point by a systematic matrix method. Thus problems 2, 3 and 4 are solvable too.

The total stress and displacement fields at the near field boundary can be given by

$$\sigma = \sigma_1 + P\sigma_2 + S\sigma_3 + M\sigma_4 \qquad (1.a)$$

$$u = u_1 + Pu_2 + Su_3 + Mu_4 \qquad (1.b)$$

$$w = w_1 + Pw_2 + Sw_3 + Mw_4 \qquad (1.c)$$

where σ stands for the stress component (σ_{xx}, σ_{zz}, σ_{xz}), u stands for the horizontal displacement component and w represents the vertical displacement component. The subscripts 1,2,3,4 indicate solutions to problems 1,2,3,4, respectively.

Consistent load vectors acting at the near field boundary could now be obtained and used as forcing functions for the near field finite element problem if the amplitudes P, S and M were known. How to determine them through a hybrid technique will be discussed in the next section.

HYBRID TECHNIQUE

The hybrid technique follows an adaptation of the global-local finite element method (GLFEM) suggested by Goetschel et alli (1982).

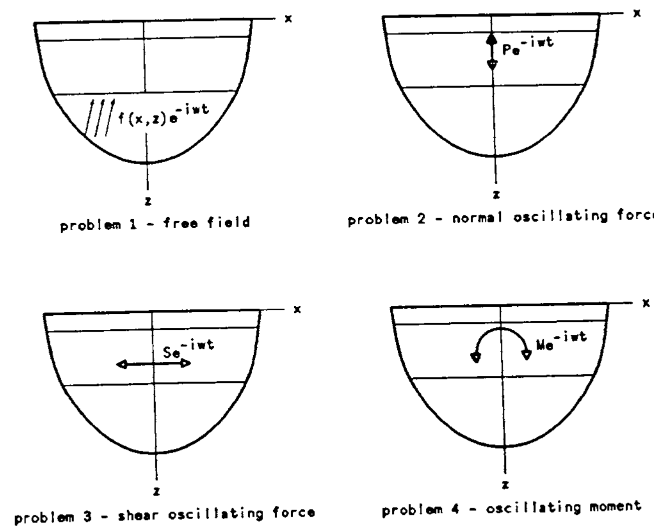

Fig. 2 - The response on an embedded structure as the superposition of four basic problems.

Let m be the number of nodal points on the near field boundary. Let U_b^i and U_b^s denote two (2m x 1) arrays of the horizontal and vertical boundary nodal displacements due to the incident (superscript i) and scattered (superscript s) fields, respectively. The method is based on two possible ways of evaluating U_b^i and U_b^s. One way is to calculate them analytically in terms of the unknowns amplitudes P, S, M.

$$U_{ba}^s = GX \quad \text{where} \quad X^T = (P\ S\ M) \qquad (2.a)$$

and G is a (2m x 3) matrix whose columns are the boundary nodal displacements found by considering line sources similar to those of problems 2, 3, 4 but of unit amplitudes (Green's functions).

$$G = \begin{pmatrix} u_2^1 & u_3^1 & u_4^1 \\ w_2^1 & w_3^1 & w_4^1 \\ u_2^2 & u_3^2 & u_4^2 \\ w_2^2 & w_3^2 & w_4^2 \\ \vdots & \vdots & \vdots \\ u_2^m & u_3^m & u_4^m \\ w_2^m & w_3^m & w_4^m \end{pmatrix} \qquad (2.b)$$

In the matrix above, the superscript identifies the boundary node and the subscript is associated with the problem number indicated in Fig. 2. In equation (2.a) subscript a is also used to identify the method of constructing the displacement matrix U_b^s analytically. The incident/reflected field is simply given by

$$\left(U_{ba}^{i}\right)^{T} = \left(u_1^1 \quad w_1^1 \quad u_1^2 \quad w_1^2 \quad \cdots \quad u_1^m \quad w_1^m \right) \qquad (3)$$

The total boundary nodal displacements can be obtained by combining equations (2.a) and (3)

$$U_{ba} = U_{ba}^{i} + U_{ba}^{s} = U_{ba}^{i} + GX \qquad (4)$$

The other way of evaluating the nodal displacements is to solve the near field problem numerically (FEM). The stresses computed at the common interface in problems 1,2,3,4 can be converted into consistent load vectors $\mathbf{F}_b$ which for the scattered field have the following form

$$F_b^s = HX \qquad (5.a)$$

where

$$H = \begin{pmatrix} F_{x2}^1 & F_{x3}^1 & F_{x4}^1 \\ F_{z2}^1 & F_{z3}^1 & F_{z4}^1 \\ F_{x2}^2 & F_{x3}^2 & F_{x4}^2 \\ F_{z2}^2 & F_{z3}^2 & F_{z4}^2 \\ \vdots & \vdots & \vdots \\ F_{x2}^m & F_{x3}^m & F_{x4}^m \\ F_{z2}^m & F_{z3}^m & F_{z4}^m \end{pmatrix} \qquad (5.b)$$

and for the incident/reflected field

$$\left(F_b^i\right)^T = \left(F_{x1}^1 \quad F_{z1}^1 \quad F_{x1}^2 \quad F_{z1}^2 \quad \cdots \quad F_{x1}^m \quad F_{z1}^m \right) \qquad (5.c)$$

In expressions (5.b) and (5.c) F_{xk}^j and F_{zk}^j represent consistent loads acting at the j^{th} boundary node along the x and z directions, respectively. The subscript k stands again for the problem number from which the consistent load vectors have been derived.

The discretized governing equation for the near field problem can now be written in the frequency domain as

$$\left(K - \omega^2 M\right)U = F \qquad (6)$$

where the stiffness $\mathbf{K}$, the consistent mass matrix $\mathbf{M}$ and the consistent load vector $\mathbf{F}$ are obtained using the Ritz finite element method.

Solution to equation (6) considering forcing functions as given by equations (5.a) and (5.c) yields all nodal displacements. Boundary nodal displacements, thus obtained, may be written as U_{bf}^s and U_{bf}^i where the subscript f indicates that the solution has been computed through a finite element analysis.

$$U_{bf}^s = WX \qquad (7)$$

where the columns of the (2m x 3) matrix $\mathbf{W}$ are the boundary nodal displacements for the force distributions given by the corresponding columns of $\mathbf{H}$ (equation 5.b). In the same way, U_{bf}^i is the numerical solution for the nodal force distribution given by equation (5.c).

The total boundary displacements can now be expressed as

$$U_{bf} = U_{bf}^i + U_{bf}^s = U_{bf}^i + WX \qquad (8)$$

At this point, displacements at the near field boundary are available from the finite element analysis (equation 8) and the far field analytic approach (equation 4). In both, the vector $\mathbf{X}$ of generalized coordinates is the unknown. Traction reciprocity at the mesh boundary is fully satisfied in virtue of the equivalence between the far field problem and the consistent load vectors used in the near field finite element analysis. Only displacement continuity across the interface remains to be satisfied.

To enforce displacement continuity, equation (4) is set equal to equation (8)

$$U_{ba}^i + GX = U_{bf}^i + WX \qquad (9.a)$$

$$(G - W)X = U_{bf}^i - U_{ba}^i \qquad (9.b)$$

$$AX = B \qquad (9.c)$$

Solution to equation (9.c) may be obtained by least-squares error minimization without difficulty. Specific details about the required algorithm can be found elsewhere (e.g. Dahlquist and Björck, 1974).

FAR FIELD ANALYSIS

Problem 1

Fig. 3 shows a multilayered soil profile that does not contain any soft layer and the layers are all assumed to be linear elastic, isotropic, homogeneous and perfectly bonded at the interfaces.

Consider the m^{th} layer bounded by the $(m-1)^{th}$ and m^{th} interfaces. As a result of multiple reflections a system of upgoing and downgoing waves will exist. Let us represent the unknown wave amplitudes as the elements of $D_m^T = \left(a_m \quad b_m \quad c_m \quad d_m \right)$ and let us define a displacement-stress vector as $S_m^T = \left(u_m \quad w_m \quad \sigma_m \quad \tau_m \right)$ where σ_m and τ_m denote σ_{zz} and τ_{xz}, respectively.

Thomson-Haskell (1950, 1953) matrix formulation is a forward marching algorithm that relates the response at the $(n-1)^{th}$ interface to the response at the free surface by a product of layer matrices A_m ($1 \le m \le n-1$).

$$S_n^{n-1} = E_n^{n-1} D_n = A_{n-1} A_{n-2} \cdots A_1 S_1^0 \qquad (10.a)$$

or

$$D_n = \left(E_n^{n-1}\right)^{-1} A_{n-1} \cdots A_1 S_1^0 = J S_1^0 \qquad (10.b)$$

where the superscripts refer to the interfaces and the subscripts to the layers.

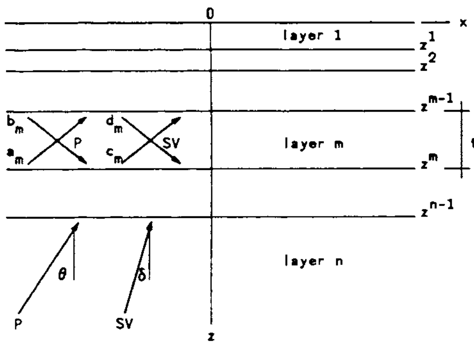

Fig. 3 - A multilayered elastic half-space excited by P and SV seismic waves.

Considering the regularity conditions (incident wave amplitudes a_n and c_n are known) and the boundary conditions, it is possible to determine $S_1^0 = \left(u_1^0 \quad w_1^0 \quad 0 \quad 0\right)^T$ in equation (10.b). Consequently, the displacement-stress vector S_m can be evaluated at any depth in the generic m^{th} layer by

$$S_m^{z^*} = E_m^{z^*} D_m = E_m^{z^*}\left(E_m^{m-1}\right)^{-1} A_{m-1} \cdots A_1 S_1^0 \qquad (11)$$

where $z^* = z - z^{m-1}$ is the depth measured from the top of the layer.

Problem 2

For problems 2, 3, 4 we have to consider the elastodynamics equations in a more complex form as wave diverging from a line source within the multilayered half-space. This can be done by integration of plane waves through a Fourier transform with respect to the spatial coordinate x, thus bringing our problem from the ω-plane to the k-plane.

After solving the problem in the k-plane by the Thomson-Haskell method just described, the response in the frequency domain may be evaluated by means of a Fourier synthesis given by

$$u(x,z,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{u}(k,z,\omega)\, e^{ikx}\, dk \qquad (12)$$

where the tilde symbol indicates response in the k-plane.

A major and long standing problem associated with the Thomson-Haskell matrix method in its original form is the presence of growing terms in the layer matrix A_m. This creates numerical difficulties involving overflow, underflow and loss of precision as the wave number k varies over the entire real axis in equation (12). Several schemes have been proposed to overcome this problem and in this research we have adopted the delta matrix method (Thrower, 1965; Dunkin, 1965) which is based on a (6x6) matrix, referred as the delta matrix, constructed from the subdeterminants of the (4x4) layer matrix.

The application of the Thomson-Haskell matrix method in association with the delta matrix technique makes possible to obtain, after some algebraic manipulation, the corresponding expressions for D_m and S_m which, through a relationship similar to (11), will permit the evaluation of the stress and displacement fields at any selected point of the multilayered half-space.

Problem 3

The Green's functions for the horizontal excitation are derived following the same procedure as for problem 2.

Problem 4

The Green's functions for the rocking excitation can be derived as a combination of line forces with line moments (Fig. 4), the moments being about the same axis and of the same sign and the forces acting perpendicularly to each other. The line forces are separated by a small distance h and have amplitude 0.5/h.

As $h \to 0$ the Green's functions for the unit rocking source can be obtained as follows

$$\phi(x,z,\omega) = \lim_{h\to 0}\left\{\frac{1}{2}\left(\frac{f(x-h/2,z,\omega) - f(x+h/2,z,\omega)}{h}\right)\right.$$

$$\left. + \frac{1}{2}\left(\frac{g(x,z+h/2,\omega) - g(x,z-h/2,\omega)}{h}\right)\right\} \qquad (13.a)$$

$$\phi(x,z,\omega) = \frac{1}{2}\left(\frac{\partial g(x,z,\omega)}{\partial z} - \frac{\partial f(x,z,\omega)}{\partial x}\right) \qquad (13.b)$$

where $\phi(x,z,\omega)$ represents the general form in the frequency domain of the Green's functions for the rocking source, $g(x,z,\omega)$ the corresponding Green's functions for the horizontal source and $f(x,z,\omega)$ the Green's functions for the vertical source .

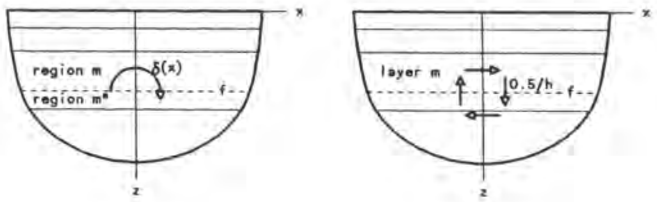

Fig. 4 - Two representations for the unit rocking source in the ω plane.

NUMERICAL INTEGRATION

In problems 2, 3 and 4 numerical evaluation of the stress and displacements fields in the frequency domain requires quadrature of improper integrals, as indicated by equation (12). Let us represent the Fourier synthesis symbolically by

$$I=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{f}(k,z,\omega)\,e^{ikx}\,dk=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\tilde{F}(k,z,\omega)}{\tilde{R}(k,\omega)}e^{ikx}\,dk \quad (14)$$

A direct integration of (14) is not possible due to the presence of poles on the path of integration. Such singularities are the simple, real roots of the Rayleigh denominator $\tilde{R}(k,z,\omega)$ and their number, not known a priori, is a function of layer thickness, soil properties, wave frequency, etc. A pole separation procedure which splits the integral into singular pieces that respond to the classical methods of analysis and nonsingular pieces to which approximate quadrature formulae may be applied has been adopted in this research. Details about the algorithm can be found in Kundu (1983, 1985).

EXAMPLE

The numerical example analyzes the kinematic interaction of a rigid, massless strip footing of width 2b = 24m located on the surface (d=0) as well as embedded (d=12m and d=24m) within a three-layered half-space characterized by:

Layer	Thickness (m)	E (MN/m2)	ν	γ (kN/m3)
1	12	725	0.25	20
2	24	1500	0.25	20
3	∞	3500	0.30	24

E = Young's modulus   ν = Poisson's ratio   γ = unit weight

Table 1 - Geometrical and mechanical properties of a three-layered elastic half-space.

Problem geometry and the finite element mesh (160 bilinear elements) is shown in Fig. 5. In order to avoid the phenomenon of filtering at high frequencies, the finite element size has been kept smaller than 1/10 of the shear wave length corresponding to the highest frequency of interest. A perfect bond has been assumed between the rigid footing and the elastic medium (welded contact).

All numerical results have been normalized with respect to the free-field motion $|w_1^o|$ and $|u_1^o|$ observed at points on the soil surface (z = 0) when the multilayered medium is excited by vertically propagating body (P or SV) waves.

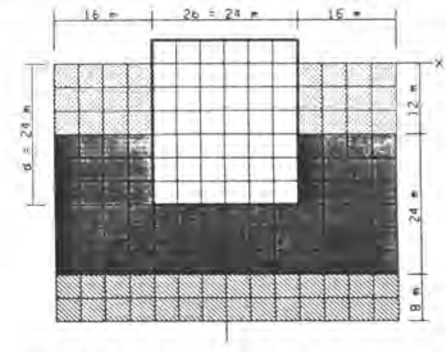

Fig. 5 - Problem geometry and FEM discretization considering normalized depth of embedment d/b=2.

Figs.6 and 7 show the variation of the foundation motion with angle of incidence for both P and SV waves. Amplitudes of the rocking motion have been expressed in terms of the parameter ϕ, indicating the angle of rotation about the y-axis. In this particular example it is clearly seen that the influence of the depth of embedment increases as the angle of incidence approaches 0°. The rocking motion caused by seismic P waves tends to reach a maximum in the vicinity of 45°, and it is practically nonexistent for P waves propagating with either vertical (0°) or grazing (approximately 90°) incidence. In the case of SV waves it is important to point out that vertically propagating waves can produce rocking motion of embedded foundations.

CONCLUSIONS

In this research a hybrid method that combines the finite element method (near field) with an analytical development (far field) has been proposed for the study of deeply embedded structures (including tunnels) in a multilayered half-space. It is based on the hypothesis that the actual scattered field may be represented by wave fields generated by a set of line sources vibrating within the half-space.

The numerical results obtained so far support this assumption with respect to parameters such as frequency and direction of the incident field, depth of embedment, layer thicknesses,etc., although further studies is necessary to check in greater detail the eventual effects of impedance contrasts.

The major disadvantage of the method seems to be the required amount of CPU time. The computational cost of numerical integrations may become prohibitive for those engineering earthquake problems where the response of a system is needed at many discrete values of closely spaced fre quencies and over a fairly large range. Yet, it is important to note that for each frequency the expensive evaluation of the scattered fields is carried out just once for each frequency. Further analyses involving incident wave fields propagating along different directions just add a small fraction (free field motion + finite element analysis) to the total cost.

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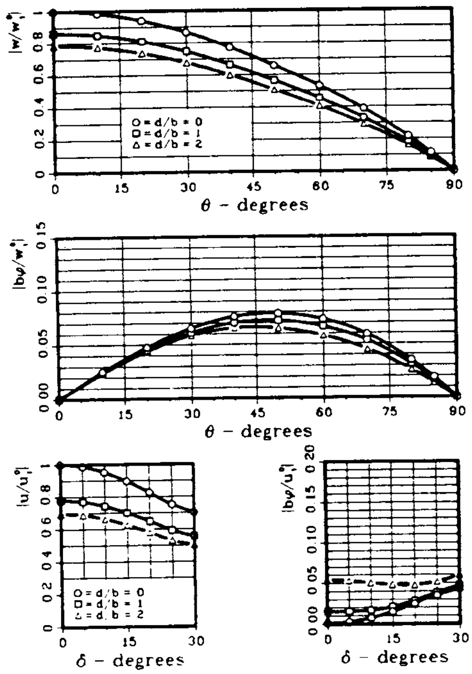

Fig. 6 - Variation of the foundation motion with angle of incidence 0° ≤ θ < 90° (P waves) and 0° ≤ δ ≤ 30° (SV waves) at frequency f = 3 Hz.

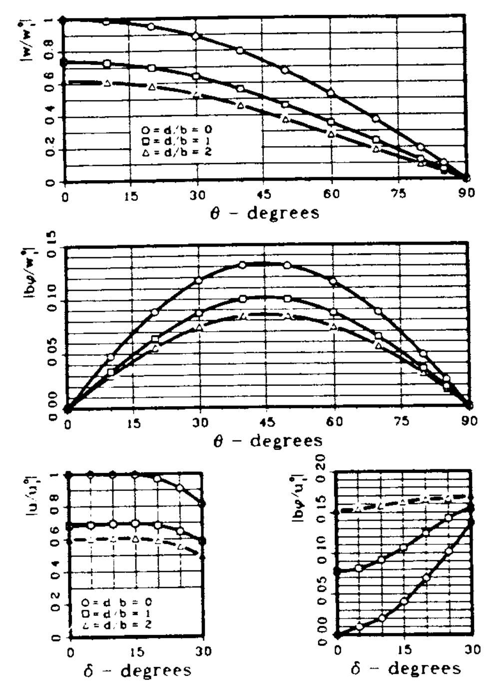

Fig. 7 - Variation of the foundation motion with angle of incidence 0° ≤ θ < 90° (P waves) and 0° ≤ δ ≤ 30° (SV waves) at frequency f = 5 Hz.