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Sun-Rong Huo
State Seismological Bureau, Harbin, China

Yuxian Hu
State Seismological Bureau, Beijing, China

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Attenuation Laws Considering M and R Uncertainties

un-Rong Huo
octor at Institute of Engineering Mechanics, State eismological Bureau, Harbin 150080, China

Yuxian Hu
Professor at Institute of Geophysics, State Seismological Bureau, Beijing 100081, China

SYNOPSIS: Accounting for the measuring errors or uncertainties of magnitude and distance in addition to ground motion, the attenuation laws for peak horizontal ground acceleration(PGA), velocity(PGV), and displacement(PGD) are studied with a weighted consistent least-square regressional method (WCLSRM) proposed by the authors to obtain the functional relation among several random variables(R.V.). The saturation of ground motions near epicenter with respect to both magnitude and distance is also emphasized in the models. The result is that the scaling factors for distance and the linear term of magnitude in the attenuation relations regressed by the suggested method are about 16 to 28 percent in average larger than those from the routine method. The ground motion estimates are somewhat higher than the routine values for large earthquakes or at near field for small earthquake and saturate near the source. The prediction from the new model is very well consistent with the observed data from the three large earthquakes in the South America recently occurred and not considered in the regression.

INTRODUCTION

In the past decades, many empirical formulas were obtained for ground motion attenuation, all of which gave similar and acceptable results within the regions of sufficient data but diverged outside. One important factor, which has strong influence on the behavior of the formula outside the data range, is the form of the attenuation formula, and another important factor is the principle of regression, i.e. which variables are considered as random. The former factor is well known but the later factor is still not realized by many investigators. This paper will emphasize it at the beginning and followed by some typical results.

The selection of the function form of ground motion attenuation depends on the physical relationship among the parameters. If the selected data base can cover relatively uniformly the necessary range, or the predictions are restricted within the range of observed data (such as defined in Fig.1), then a simple empirical function would be appropriate. However, if the attenuation relation is to be extrapolated to or out of the edge of the data base, the selection of the functional form becomes then very important.

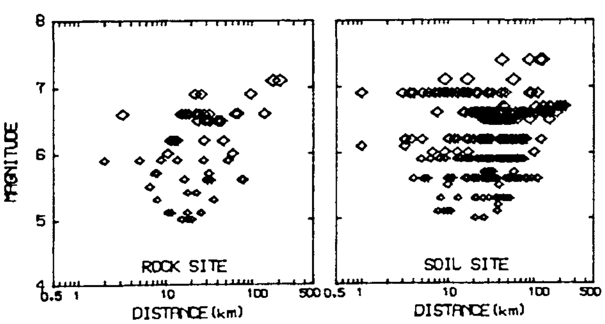


Fig. 1 Distribution in M and R of the data.

The ordinarily used form of attenuation may be expressed in the following general expression:

$$Y=b_0 f_1(R) f_2(M,R) f_3(M) f_4(P_i)\,\varepsilon \qquad (1)$$

where Y is the studied ground motion parameter, such as PGA, PGV, PGD or response spectra, etc.; ε is a random variable, representing the uncertainty in predicting Y; M and R are earthquake magnitude and distance respectively. Functions f_1 and f_2 represent the attenuations due to the material damping and scattering and the geometrical spreading away of seismic waves from the source, respectively. The most common form of f_1 is:

$$f_1(R)=\exp(b_7 R) \qquad (2)$$

To consider the finite dimension of the earthquake source for near field ground motion, it is sometimes to combine this effect together with geometric spreading as in the following form

$$f_2(M,R)=[R+b_5\exp(b_6 M)] \qquad (3)$$

where b_5 and b_6 are positive constants. The geometrical attenuation represented by Eq.(3) considers the facts of the distance saturation and, part of the magnitude saturation of ground motion near epicenter.

The magnitude scaling function of ground motion, f_3, follows commonly the magnitude definition and takes the following form:

$$f_3(M)=\exp(b_2 M) \qquad (4)$$

However, on the basis of actual records and theoretical reasoning, ground motion at or very close to the causative fault of a very strong earthquake does not always proportionally increase with magnitude, and it is not the whole fault, but only the local part of the fault very close the site, that effects the peak acceleration (Hanks and McGuice, 1981). This phenomena is the so called magnitude saturation near the source. Because Eq.(3) can only reflect magnitude saturation to a certain extent, the scaling function for magnitude of the following form has been proposed (Campbell and Duke, 1974; Trifunac,1976; Trifunac and Lee, 1985)

$$f_3(M)=\exp(b_2M-b_3M^2) \qquad (5)$$

Function $f_4(p_i)$ may be used to consider some other factors, such as those related to the earthquake background, site condition, etc. The empirical form is sometimes taken as:

$$f_4(p_i)=\exp(\sum C_i p_i) \qquad (6)$$

Usually p_i is assigned some discrete integer. Its disadvantage is forcing the attenuation curves parallel for various M and R values, which is not necessarily true.

The appropriate form, simple or complicated, of the attenuation model mentioned above should be consistent with the number and the distribution of the data base, and some coefficients may be assigned through physical consideration.

UNCERTAINTY OF MAGNITUDE AND DISTANCE

The uncertainty in ground motion is well known. But M and R are always taken as determinatively given parameters with no consideration of their uncertainties. But in fact, there are many uncertainties in the procedure of dertermining M and R in seismology, which have important effects on the reliability of the results of SHA and earthquake-resistant design of structures.

Magnitude Uncertainty

Magnitude is generally defined as follows:

$$M=\log[F(A,T)]+f(R,h)+C_S+C_r \qquad (7)$$

where A and T are ground peak displacement and the corresponding period respectively, R the epicentral distance, h the focal depth, C_S a station correction factor, and C_r a regional correction factor. Function F(A,T) has different form for different kinds of magnitude, such as A or A/T. Magnitude calibrating function f(R,h) is determined by combining empirical with physical reasons, used to compensate the effects of distance and focal depth, and is different for different type or different components of waves.

There exist uncertainties in the procedure of measurement and discretization of seismic wave and computation, which will result in certain errors. The famous seismologist M. Båth (1981) had pointed out that magnitude errors of 0.2-0.3 can be reached for the best case. In fact, difference between seismic station networks in different regions can often reach 0.5.

It is then clear that magnitude not only has random error, but also can not reflect the frequency content of source energy distribution. It bounds to introduce uncertainties in earthquake magnitude if a single constant is used in place of spectra of varying shapes.

Uncertainty in Propagating Path

The distance R is defined as the length of a straight line from the source or the epicenter of the earthquake to the site. But the seismic waves are radiated from many different parts of a big volume of the source and travel along quite different, curvilinear or zigzag paths to the site. On the other hand, the error in locating hypicenter or epicenter may reach several or several tens kilometers, and the actual source is not a point but a plane or a volume where energy is propagating outward. No matter which one of the original rupture point, energy releasing center or any other kind of point or a fault line is considered as the starting point, a straight line to the sites can never represent the real propagating path of seismic wave without error. For sites within a distance of a few or a few tens kilometers of great engineering interest, the distance is greatly uncertain. The distance data used by different investigators may be different according to the definitions of distance used, such as distance to epicenter, energy center and hypicenter, and shortest distance to fault, etc.

REGRESSIONAL MODELS FOR MULTIPLE RANDOM VARIABLES

From the viewpoint of statistics, the routine least-square regressional method (RLSRM) can be used only for the case that all the independent variables (X) in the relationship are controlled or exactly measured, and only the functional variable (Y) is observed or measured with random error. This regressional principle assumes that the best relationship makes the sum square of the random error of the function variable, $\sum(\Delta Y)^2$, minimum on the sense of least square. While in the study of ground motion attenuation law Y(M,R), ground motion parameter Y and M,R are all random or with uncertainties and the results from RLSRM can not give the correct relationship of random variables Y, M and R. Nevertheless, in SHA, the inversed function R(M,Y) derived directly from Y(M,R) is also required. Strictly speaking, R(M,Y) by RLSRM should be from another set of original data of R, M and Y with M and Y controlled and R measured and makes $\sum(\Delta R)^2$ minimum instead of $\sum(\Delta Y)^2$. The inversed function R(M,Y) is different from the result directly obtained with RLSRM letting R be the function variable, even in case that same set of data is used. Of course, these two ways are both unreasonable because the uncertainty from only one of the three random variables is accounted for in regression.

Based on the weighted consistent least-square regressional method (WCLSRM) proposed by the authors (Huo et al,1987; Hu,1988; Hu and Huo,1988, Huo, 1989) for a relationship involving m random variables y_i (i=1,2,...,m) and L controlled variables x_i (i=1,2,...,L), the regressional function may be written as :

$$A+\sum B_i y_i+\sum C_i x_i=0 \qquad (8)$$

the regressional principle is to make the sum of the residual errors of all random variables minimum, i.e.

$$J = \sum_i [W_i \sum_k (\Delta Y_{ik})^2] = \min \qquad (9)$$

where ΔY_{ik} is the regressional residual of the normalized random variable y_i for the kth sample; W_i is a weighting factor considering the randomness of y_i, and will automatically reduce to zero if y_i is deterministic. The normal equations for the regressional coefficients determined by Eq.(9) are a group of nonlinear simultaneous equations. For the case of attenuation relationship Y(M,R), the normalized regressional function is:

$$Z=aX+bY$$

and the normal equations determined by $\partial J/\partial a=0$ and $\partial J/\partial b=0$ are as follows:

$$\left.\begin{aligned} &W_x\left(\frac{\rho_{xz}-b\rho_{xy}}{a^2}-\frac{1+b^2-2b\rho_{yz}}{a^3}\right)+W_y\left(\frac{\rho_{xy}}{b}-\frac{\rho_{xz}-a}{b^2}\right) \\ &\qquad -W_z(\rho_{xz}-a-b\rho_{xy})=0 \\ &W_x\left(\frac{\rho_{xy}}{a}-\frac{\rho_{yz}-b}{a^2}\right)+W_y\left(\frac{\rho_{yz}-a\rho_{xy}}{b^2}-\frac{1+a^2-2a\rho_{xz}}{b^3}\right) \\ &\qquad -W_z(\rho_{yz}-b-a\rho_{xy})=0 \end{aligned}\right\} \quad (10)$$

where ρ_{ij}(i,j=x,y,z) is the correlation factor between variables i and j. For the case of many random variables(R.V.), Eq.(10) is very complicated and not easy to solve, and a direct iterative method proposed by the authors (Huo, 1989) can be used efficiently.

The attenuation relation Y(M,R) determined by principle of Eq.(9) will be a unique correlation of Y, M and R which is not affected by the different selections of functional variable and inversed function can be used directly.

ATTENUATIONS CONSIDERING UNCERTAINTIES IN M AND R

Data used here for regressional analysis of attenuation relation are those listed in Table 1 and the M-R coverage is shown in Fig.1. For each record, two horizontal components are taken and treated as two data in order to maintain their actual randomness. From Fig.1, it can be seen that data concentrate in a region of moderate distances and moderate magnitudes, rather unevenly distributed, statistically speaking, which makes the regressed results not reliable when used on the border or outside the data concentrated region, such as prediction of ground motion near epicenter of a very large earthquake. For this reason, weights will be given according to the number of data used for each M and R subdivision. The intervals used here are as follow: for magnitude, M<5.5, 5.5-5.9, 6.0-6.4, 6.5-6.9, 7.0-7.5, >7.5; for distance, R<3, 3-9.9, 10-29.9, 30-59.9, 60-99.9, 100-300, >300km. Equal weights are assigned to each subdivision, and any data point in a subdivision i of n_i data will have a weight $1/n_i$.

On the basis of the past studies, attenuation model in this study is taken as:

TABLE 1. Earthquake Information Used

EVENT	DATE	M_L	Ms
IMPERIAL VALLEY	MAY 18, 1940	6.7	7.1
NORTHWEST CAL.	OCT 07, 1951	5.8	6.0
KERN COUNTY, CAL.	JUL 21, 1952	7.2	7.7
SAN FRANCISCO	MAR 22, 1957	5.3	
HOLLISTER	APR 08, 1961	5.7	5.6
BORREGO MOUNTAIN	APR 08, 1968	6.4	6.7
LONG BEACH	MAR 10, 1933	6.3	6.5
HELENA, MONTANA	OCT 31, 1935	6.0	
WESTERN WASHINGTON	APR 13, 1949	7.1	
NORTHERN CAL.	SEP 22, 1952	5.5	
PUGET SOUND, WASH.	APR 29, 1965	6.5	
PARKFIELD, CAL.	JUN 27, 1966	5.6	6.0
SECOND NORTH CAL.	DEC 10, 1967	5.8	
SAN FERNANDO	FEB 09, 1971	6.4	6.6
BORREGO VALLEY	OCT 21, 1942	6.5	
HELENA MOUNTAIN	NOV 28, 1935	5.0	
SANTA BARBARA	JUN 30, 1941	5.9	
NORTHERN CAL.	JUN 05, 1960	6.0	5.7
SOUTHERN CAL.	NOV 21, 1952	5.5	6.2
NORTHERN CAL.	SEP 12, 1966	6.3	6.4
LYTLE CREEK	SEP 12, 1970	5.4	
COYOTE LAKE	AUG 06, 1979	5.6	5.9
IMPERIAL VALLEY	OCT 15, 1979	6.6	6.9
COALINGA	MAY 02, 1983	6.2	6.5
MORGAN HILL	APR 24, 1984	6.1	6.2

TABLE 2. Results of Regressed Attenuation Relations

$$\log(Y)=C_1+C_2M+C_3M^2+C_4\log(R+C_5\exp(C_6M))+\varepsilon$$

PARAMETER	MODEL	R. V.	C1	C2	C3	C4	C5	C6	σ_ε
PGA (gal)	I	1	2.1630	0.4389	0.0000	-1.8430	14.0000	0.0000	0.1812
		3	1.4640	0.5890	0.0000	-1.9990	14.0000	0.0000	0.1931
	II	1	0.6430	0.7000	0.0000	-1.9050	0.3268	0.6135	0.1801
		3	0.0650	0.8290	0.0000	-2.0490	0.1818	0.7072	0.1893
	III	1	-0.9350	1.2410	-0.0460	-1.9040	0.3268	0.6135	0.1802
		3	-1.8220	1.4480	-0.0520	-2.0180	0.1818	0.7072	0.1868
PGV (cm/s)	I	1	-0.0457	0.5818	0.0000	-1.7290	14.0000	0.0000	0.2571
		3	-0.6924	0.7352	0.0000	-1.9300	14.0000	0.0000	0.2747
	II	1	-1.4480	0.8241	0.0000	-1.7940	0.3268	0.6135	0.2582
		3	-2.1550	0.9841	0.0000	-1.9810	0.1818	0.7072	0.2697
	III	1	-4.4720	1.8460	-0.0855	-1.7970	0.3268	0.6135	0.2552
		3	-5.0450	1.9820	-0.0865	-1.9460	0.1818	0.7072	0.2629
PGD (cm)	I	1	-0.4464	0.4834	0.0000	-1.4190	14.0000	0.0000	0.3140
		3	-1.4310	0.7280	0.0000	-1.7690	14.0000	0.0000	0.3488
	II	1	-1.5790	0.6728	0.0000	-1.4470	0.3268	0.6135	0.3178
		3	-2.6030	0.9199	0.0000	-1.7790	0.1818	0.7072	0.3450

$$\log[Y(M,R)]=C_1+C_2M+C_3M^2+C_4\log[R+Ro(M)] \qquad (11)$$

$$Ro(M)=C_5\exp(C_6M) \qquad (12)$$

where coefficients C_i (i=1,2,...,6) are determined through regression. And C_5 and C_6 of near field saturation factor Ro(M) are derived before regressional analysis of the attenuation relationship. Based on the acceleration data of those earthquakes of enough data and with a rather wide variation of magnitude (from 5.2 to 7.7) in Table 1, the values of Ro obtained from regression and a second step of regression gives the following result:

$$Ro(M)=\begin{cases}0.3268\exp(0.6135M), & \text{Ro random}\\ 0.1818\exp(0.7072M), & \text{Ro \& M random}\end{cases} \qquad (13)$$

Knowing the function Ro(M), Eq.(11) is used to obtain the attenuation of ground motion on rock site according to regressional principle Eq.(9) to consider Y, M and R all random variables(R.V.), where weights W_i (i=Y,M,R) are assigned equal, i.e. $W_i=1$.

Three cases for Eq.(11) given in the following are compared:

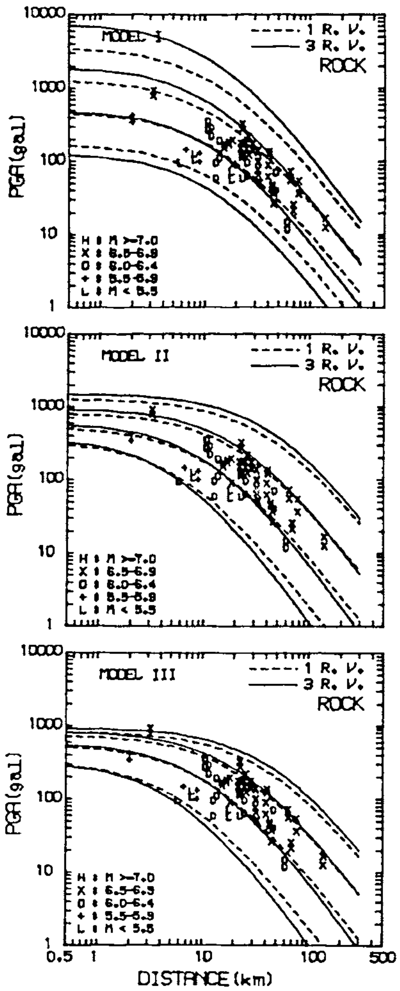


Fig. 2 Regressions and observed data for PGA, (M=5,6,7,8)

I: $y=C_1+C_2M+C_4\log(R+C_5)$ (14a)

II: $y=C_1+C_2M+C_4\log(R+C_5\exp(C_6M))$ (14b)

III: $y=C_1+C_2M+C_3M^2+C_4\log(R+C_5\exp(C_6M))$ (14c)

where y represents log(Y) and Y may be any ground motion parameter. The results are all listed in Table 2 and some curves in Fig. 2.

ANALYSIS OF RESULTS

Characteristics of Attenuation Relations Considering All Random Parameters

Results of all three models, I, II and III, of attenuation given in Table 2 reveal that the scaling factors C_2 and C_4 of magnitude and distance respectively for all random parameters Y,M,and R are in general 28 percent and 16 percent greater than the corresponding values for the routine method of only one random parameter Y. The routine attenuation model underestimates the ground motion Y (PGA, PGV and PGD) for large earthquakes and near field for small ones, as shown in Fig. 2. As for variance, the suggested method gives the minimum of the sum $\sum\sigma_i^2$, while the others only minimum of one variance, as shown in Table 3.

Table 3. Comparison of Standard Deviation

Random Variable	Y	M	logR	Y,M,logR
σ_Y	0.2274	0.2934	0.2527	0.2452
σ_M	0.5035	0.3723	0.4562	0.3947
$\sigma_{\log R}$	0.1523	0.1678	0.1370	0.1445

Differences of Results By Different Models

Because of lack of near field data for very large earthquakes, ground motion for this case is usually estimated by extrapolation and thus controlled by moderate distance data from moderate earthquakes where data concentrated. In order to avoid this unreasonable situation, attenuation model should be selected based upon theoretical study in addition to weighting of the data to correct the uneven data distribution. Models II and III allow the near field distance saturation by the term Ro(M) and the magnitude saturation of high frequency ground motion by both C_3M^2 and Ro(M), which agrees also with observed data, as shown by models II and III in Fig. 3, avoids the unreasonably high acceleration for very large earthquake at epicentral region by the routine model I, and remains almost the same for moderate distance range to match the observed data. This selection of model satisfies both observed data and theoretical requirements and is therefore reasonable and reliable. Fig. 4 shows a comparison of tendency of ground motion saturation of the results from three models and observed data at epicenter.

Comparison With Other Results:

Only some recently suggested attenuation laws are compared here as examples. Because some studies

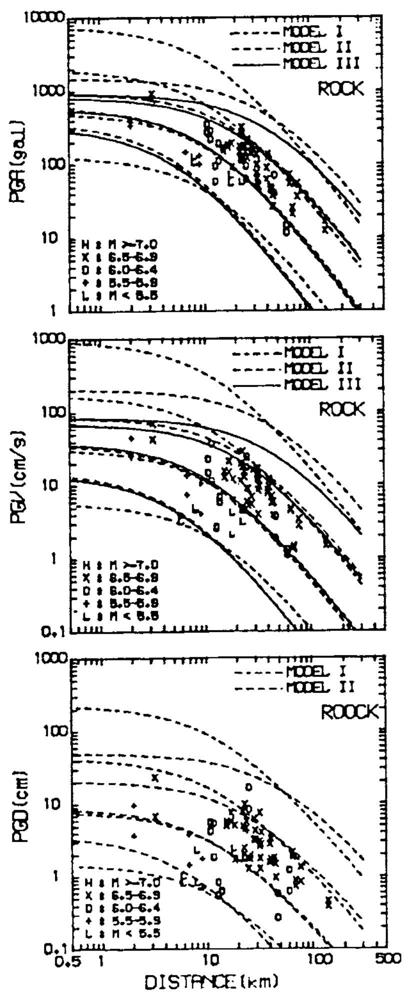


Fig. 3 Comparison of results from different models, (M=5,6,7,8)

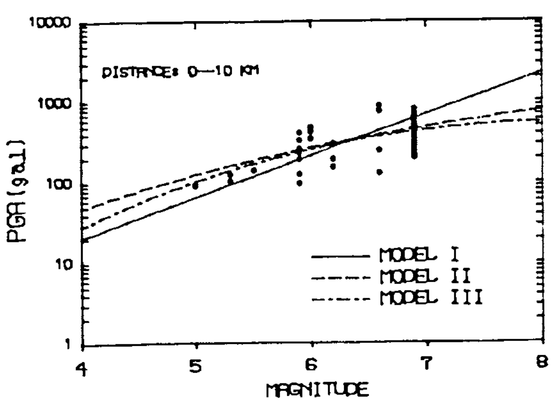
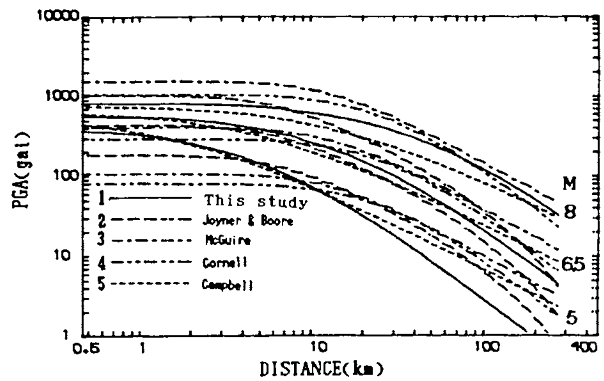


Fig. 4 Comparison of the observed values of PGA near source with the predictions

did not distinguish site conditions, the following relation is given for our case of model II here by combining data on both soil and rock sites.

$$\log(PGA)=0.583+0.651M-1.652\log[R+0.182\exp(0.707M)] \quad (15)$$

Figs. 5 and 6 give the comparison of five sets of results and the predictions for three large South American earthquakes, the September 19, 1985 Mexican Earthquake (M_S=8.1), the March 3, 1985 Chilen Earthquake (M_S=7.8) and the October 18, 1989 Loma Prieta Earthquake (M_S=7.1), near San Francisco, California, by different authors. It can be seen that our predictions are well consistent with the observed data from the large earthquakes for both near region and far field. Well, the other results can only agree with the data in one of two regions.



1、Eq.(15)

2、$y=1.970+0.249M-\log\sqrt{(R^2+7.3^2)}-0.00255\sqrt{(R^2+7.3^2)}$

3、$y=1.477+0.386M-1.170\log(R_H)$

4、$y=2.917+0.373M-1.800\log(R_H+25)$

5、$y=1.200+0.377M-1.090\log[R+0.061\exp(0.7M)]$

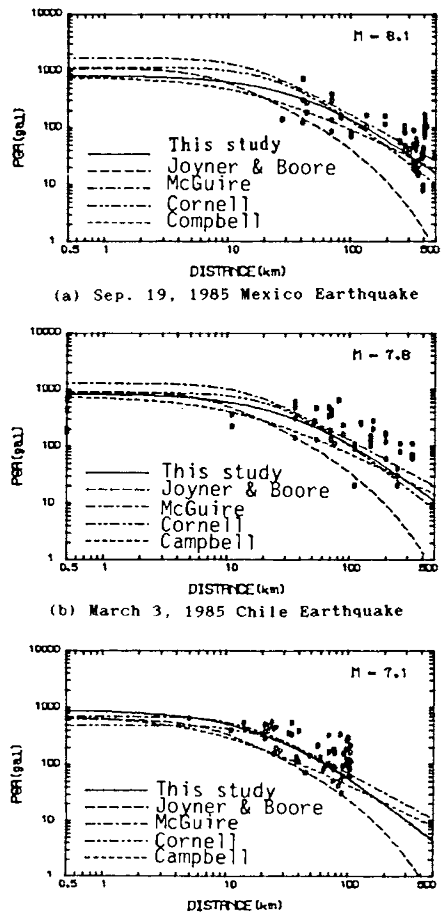
Fig. 5 Comparison of attenuation curves by different researchers

CONCLUSIONS

1. The randomness or uncertainties in the observed data of M and R can not be neglected and has direct influence on SHA. The suggested regressional principle gives a unique attenuation function of Y, M and R.

2. Considering theoretical reasoning of seismology, the model with saturation for large M and at near field is more reasonable.

3. PGA has near field distance and magnitude saturations, but not so significant for PGV and PGD.



(a) Sep. 19, 1985 Mexico Earthquake

(b) March 3, 1985 Chile Earthquake

(c) Oct. 18, 1989 Loma Prieta Earthquake

Fig. 6 Comparison of the observed PGA with predictions

4. Prediction of the model suggested by the authors here are very well consistent with the observed data from three strong earthquakes in the South America.

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