12 Mar 1991, 2:30 pm - 3:30 pm

A Model for Predicting Liquefaction Induced Displacement

Peter M. Byrne
University of British Columbia, Vancouver, B.C., Canada

Proceedings: Second International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, March 11-15, 1991, St. Louis, Missouri, Paper No. 7.14

A Model for Predicting Liquefaction Induced Displacement

Peter M. Byrne
Professor of Civil Engineering, University of British Columbia, Vancouver, B.C.

SYNOPSIS: A simple model for predicting liquefaction induced displacement is presented. The method is based on a single-degree-of-freedom system that incorporates the post-liquefaction stress-strain response of sand. The key parameters are the residual strength and the limiting shear strain, and considerable data presently exists on these two parameters from correlation with SPT $(N_1)_{60}$ values. Based on this data, liquefaction induced displacements from the model are compared with both field and laboratory measurements. The model predictions are found to be in excellent agreement with the measurements and indicate that liquefaction induced displacements are very sensitive to the density or $(N_1)_{60}$ value. The large observed displacements appear to be associated with $(N_1)_{60}$ values less than 8. Much smaller displacement are predicted for denser sands with $(N_1)_{60}$ values in excess of 12.

INTRODUCTION

Displacements induced by liquefaction of soil can be very large and result in severe damage to earth and earth supported structures including embankment dams and general life-line facilities. It is important, therefore, to be able to predict such displacements, and much research effort has recently been applied in that direction.

A simple empirical equation for predicting the liquefaction induced displacements of one-dimensional slopes has been proposed by Hamada et al. (1987). This equation is based upon field measurements during past earthquakes. However, it has a severe shortcoming in that the density of the sand is not considered, so that slopes comprised of a medium dense sand or a loose sand that are triggered to liquefy would be predicted to have the same displacements. Although the medium dense sand would require a higher level of shaking to induce liquefaction, it is unlikely that the displacements would be as large as for the loose sand. A displacement model based on soil mechanics principles and calibrated with field experience would be very useful in practice.

A rigorous 2-D dynamic analysis of earthquake induced displacements of saturated sandy soils requires a complex stress-strain relation which takes into account cyclic shear-volume coupling effects. This coupling involves shear induced volumetric strains that arise from slip at grain contacts and results in the generation of excess porewater pressure which in turn reduces the shear modulus and can lead to large displacements. A rigorous coupled analysis of this type has been developed by Prevost (1981).

A simpler loose-coupled approach following the concepts of Martin et al. (1975) has been developed by Finn et al. (1986). In this approach a simple incremental elastic-shear stress-strain law is used, and the plastic strains are introduced through a separate shear-volume coupling equation. The coupling equation can only predict pore pressure change at the completion of a strain cycle or at best at the 1/2 cycle, so the effect of generated pore pressure can only be accounted for at these discrete intervals rather than at every time step and leads to a "loose-coupled analysis" procedure. Finn, et al., however, have shown this procedure to be adequate in predicting both centrifuge model behaviour and field experience. While this analysis is very efficient compared to a rigorous coupled one, it is still a very complex procedure.

Simple analysis procedures exist for predicting the zones of initial liquefaction. Once liquefaction is triggered, either in the laboratory or in the field, the strains and displacements become very large as compared to the cyclic strains that occurred prior to liquefaction. For example, the strains to trigger liquefaction are generally less than 0.5%, whereas the strains upon liquefaction could be well in excess of 5%. The post liquefaction strains and displacements depend on the post-cyclic stress-strain characteristic behaviour of the soil, and the geometry of the slope, and are not significantly influenced by the strains that occurred prior to liquefaction.

It would therefore seem that liquefaction induced displacement could be computed from a simple analysis that takes into account the post-liquefaction behaviour of the soil. Newmark (1965) presented a simple method for predicting earthquake induced displacements of slopes. His method is based upon a single-degree-of-freedom model and a rigid plastic soil. While his method can be adapted to account for the reduced strength of the soil upon liquefaction, it cannot in its present form account for the large strains and displacements that occur within the zone of liquefaction. Herein a procedure is developed for the infinite slope that considers both the strength loss and the modulus reduction within the liquefied layer, and the concept is extended to 2-D slopes in a manner similar to that outlined by Newmark (1965).

OBSERVED LIQUEFACTION INDUCED DISPLACEMENTS

Liquefaction induced displacements have been the subject of great research effort in the past 5 years. In particular, the study reported by Hamada et al. (1987) and the numerous papers in both the first Japan-US Workshop, 1988, and the second US-Japan Workshop, 1989, contain very useful information on observed displacements.

The most comprehensive compilation of field data has been presented by Hamada et al., who found that liquefaction induced displacements were strongly related to slope - either the surface slope, or the slope of the base of the liquefied layer. Some of their data is shown in Fig. 1. and indicates that

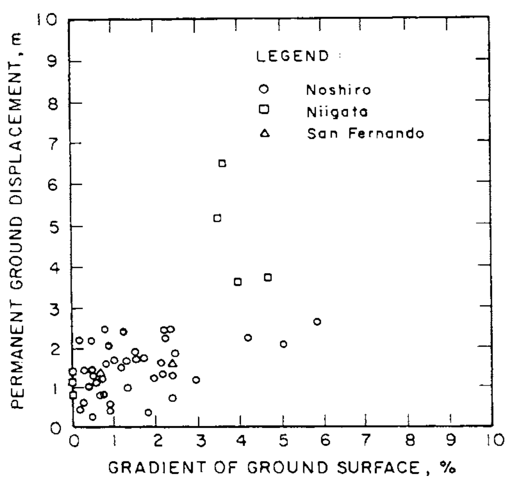

Fig. 1. Observed Liquefaction Inuced Displacements versus Gradient of the Ground Slope, Hamada et al., 1987.

displacements of up to 2-1/2 m can occur on slopes of less than 2%. The Hamada data is mainly from sites that liquefied during the 1964 Niigata and 1983 Nihonkai-Chuba earthquakes which involved events about M7.5, and clean sands some of which were very loose. Hamada et al. proposed the following equations for permanent earthquake induced displacements,

$$D = 0.75\ H^{1/2}\ \theta^{1/3} \qquad (1)$$

in which D = displacement in metres, H = the thickness of the liquefied layer, in metres, and θ = the ground slope in %. The actual data varied from about 1/2 to twice the values predicted by the equation so that the equation represents a mean rather than an upper bound as shown in Fig. 2. Their data indicates that the magnitude of the observed displacement is approximately proportional to H rather than $H^{1/2}$, as shown in Fig. 3.

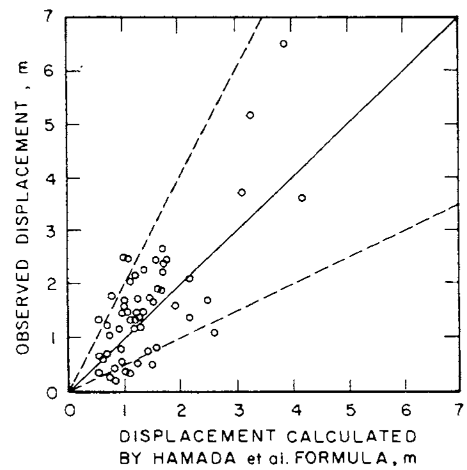

Fig. 2. Comparison of Predicted and Observed Displacements, Hamada et al., 1987.

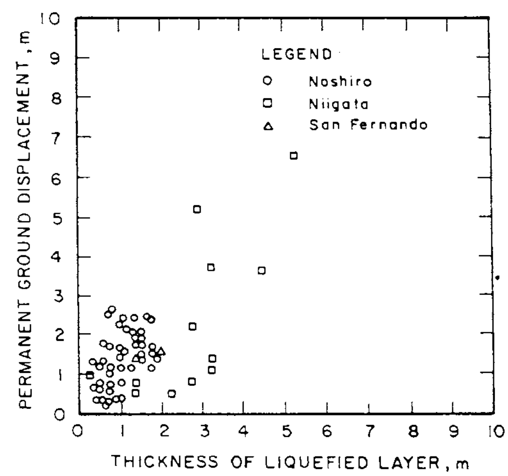

Fig. 3. Observed Liquefaction Induced Displacement versus Layer Thickness, Hamada et al., 1987.

Youd and Bartlett (1988) reported experience at sites in the United States. Their results are shown in Fig. 4 and exhibit more scatter than does the Japanese data, with a significant proportion of the data plotting well below the Hamada line. Some of the data from the M8+ event, San Francisco, 1906, and Alaska, 1964, plot above the Hamada line and gravelly soils plotted significantly below the line.

The data presented by Hamada et al. and by Youd and Bartlett is for slopes that retained

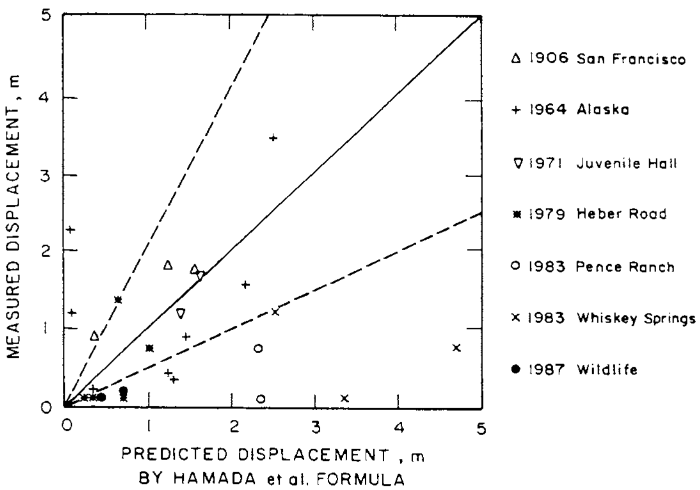

Fig. 4. Comparison of Predicted and Observed Liquefaction Induced Displacements, Youd and Bartlett, 1988.

sufficient residual strength to prevent a large change in geometry or flow slide from occurring. Their data indicates that for slope less than 6% (3.5°) flow slides did not occur.

Shaking table model studies on liquefaction induced displacements have been carried out by a number of researchers including Byrne et al. (1982) and Towhata et al. (1988). The pattern of displacement for a model slope from Byrne et al. is shown in Fig. 5 and

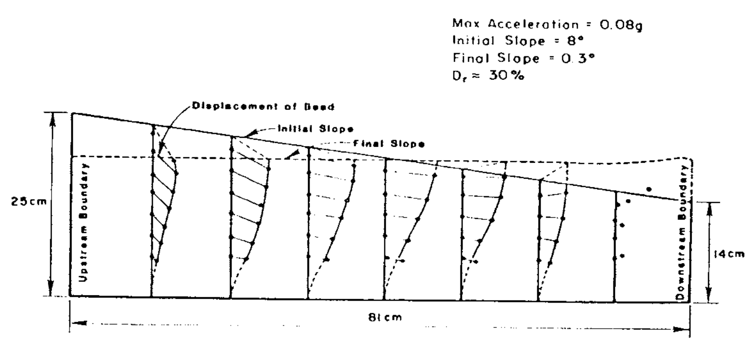

Fig. 5. Liquefaction Induced Displacements from Model Tests, Byrne et al., 1982.

indicates that when triggered to liquefy, an initial slope of 8° flattened to a final slope of 0.3°. The material was loose Ottawa sand with a relative density, $D_r \approx 30\%$. The downslope displacement pattern is slightly curved with depth but can be approximated by a linear distribution with zero at the base and the maximum value at the top of the liquefied layer. The observed displacements correspond with a shear shear strain of about 40%. This pattern of displacement is consistent with field data reported by Hamada et al.

The observed displacements from the Towhata et al. tests are shown in Fig. 6. Their

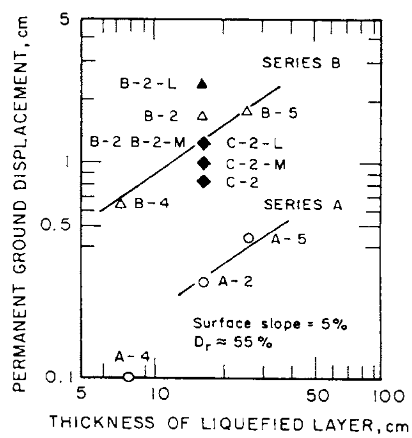

Fig. 6. Liquefaction Induced Displacements from Model Tests, Towhata et al., 1988.

values are much smaller than predicted by Hamada's equation. For a layer thickness of 20 cm the test results indicate a maximum displacement of about 2 cm as compared to 57 cm from Hamada's equation.

There are a number of possible reasons for this:

1) The material is medium dense (Dr ≈ 55%), and would behave as an even denser material because of the low confining stresses of the test.
2) There are end effects in the testing box which would restrict displacement, and
3) The $H^{1/2}$ Term in the Hamada equation is likely incorrect. Both the field and the laboratory data indicate displacement are proportional to thickness of the liquefied layer H. This would significantly reduce the Hamada prediction for small values of H.

The field and laboratory data suggest that the post liquefaction stress-strain and strength properties of sand will be important factors influencing displacements.

Considerable data now exists on the post-liquefaction behaviour of sand in terms of both residual strength and strains and this will be examined in the section to follow.

POST-LIQUEFACTION RESPONSE OF SAND

Observed stress-strain response prior to, and after liquefaction is shown in Fig. 7. Prior

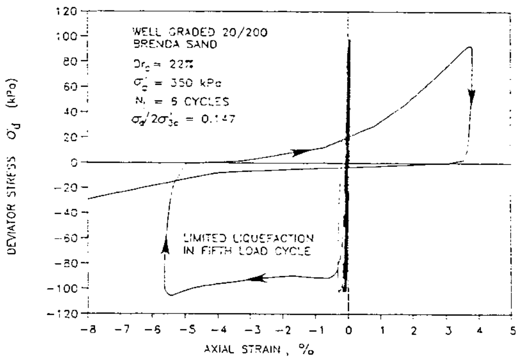

Fig. 7. Stress-Strain Response of Sand, Pre- and Post-Liquefaction, Kuerbis, 1989.

to liquefaction the strains are small, but upon liquefaction large strains occur. When loaded after liquefaction the sand initially deforms with an essentially zero stiffness which then increases with the level of strain, suggesting a limit strain, γ_{Lim}. This is depicted in Fig. 8a together with a limit or residual strength, s_r. This stress-strain response in which the soil stiffens with increasing strain is opposite to the usual response of soil.

The unusual stress-strain response for liquefied soil results from the fact that, upon shearing the soil dilates causing the effective stress to increase as shown in Fig. 8b. The amount of dilation controls the maximum increase in effective stress and hence the residual strength that the material can develop as shown in Fig. 8. The residual strength will be strongly dependent on the relative density of the soil.

Residual strengths based upon back analysis of field experience together with laboratory testing are shown as a function of the normalized standard penetration test, $(N_1)_{60}$, value in Fig. 9. This data is from Seed and Harder (1990) and currently represents the state of the practice on residual strength estimated from penetration tests.

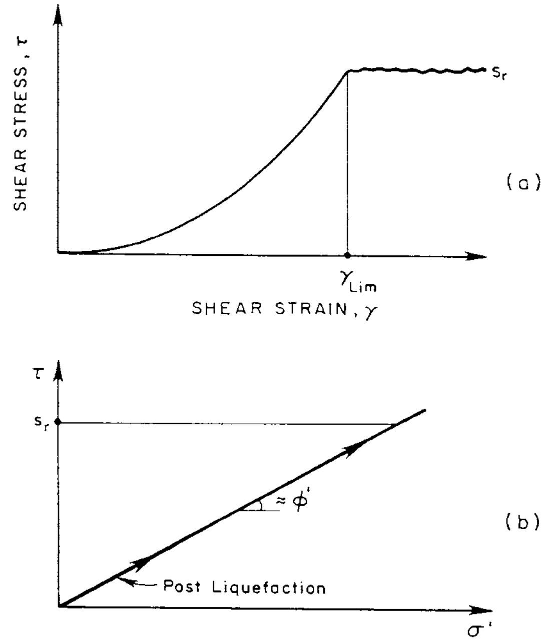

Fig. 8. Idealized Stress-Strain Response.

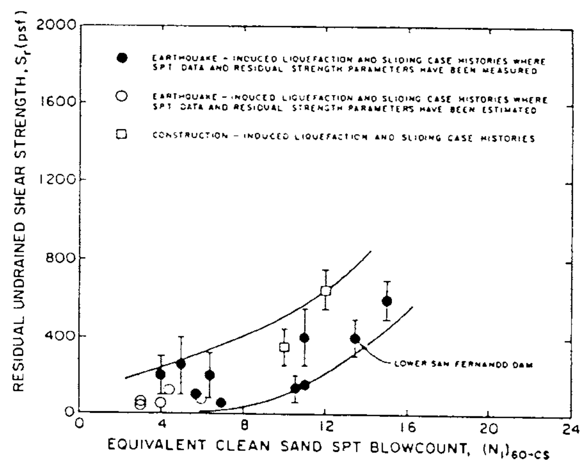

Fig. 9. Relationship Between $(N_1)_{60}$ Blowcount and Undrained Residual Strength, Seed and Harder, 1990.

The limiting strains, γ_{Lim}, that sand will undergo upon liquefaction, are shown in Fig. 10. This data is from Seed et al. (1984) and is based on laboratory tests including tests on undisturbed samples of frozen cored samples. The range and average values of the residual strength, s_r, and the limit strain, γ_{Lim}, from Figs. 9 and 10 are given in Table 1.

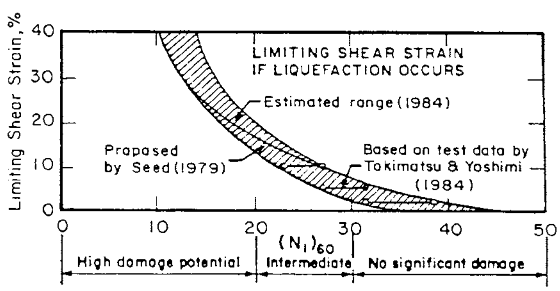

Fig. 10. Relationship Between Limiting Shear Strain and $(N_1)_{60}$ Blowcount.

TABLE 1

$(N_1)_{60}$	s_r Range Psf	s_r Avg. Psf	γ_{Lim}% Range	γ_{Lim} Avg.
4	0-240	120	>40	100
6	0-320	160	>40	80
8	30-430	230	>40	63
10	120-500	310	40-Large	50
12	200-680	440	32-Large	40
16	550-1100	825	20-30	25
20	>2000	>2000	13-20	16
30	>2000	>2000	3-7	5
40	>2000	>2000	0-3	1.5
50	>2000	>2000	0	0

The average s_r and γ_{Lim} values can be approximated by:

$$s_r = 3\ (N_1)^2_{60}\ \frac{2000}{Pa} \tag{2}$$

in which Pa = the atmospheric pressure.

$$\gamma_{Lim} = 10^{(2.2\ -\ 0.5\ (N_1)_{60})} \tag{3}$$

Test data from Vaid (1990) suggests that the residual strength of very loose rounded sand under simple shear conditions is unlikely to be less than:

$$s_r = 0.087\ \sigma'_{vo} \tag{4}$$

in which σ'_{vo} is the effective vertical consolidated pressure, and this lower bound will be used when considering "average" values.

The residual strength and limiting strain are the basis of the displacement model presented in the next section.

1-D LIQUEFACTION DISPLACEMENT MODEL

An idealized slope is shown in Fig. 11a and is modelled as the single-degree-of-freedom system shown in Fig. 11b. The model comprises a mass, M, and a nonlinear spring, K, representing the stiffness and strength of the liquefied layer. The model parameters are based on a unit column of soil normal to the slope and are as follows.

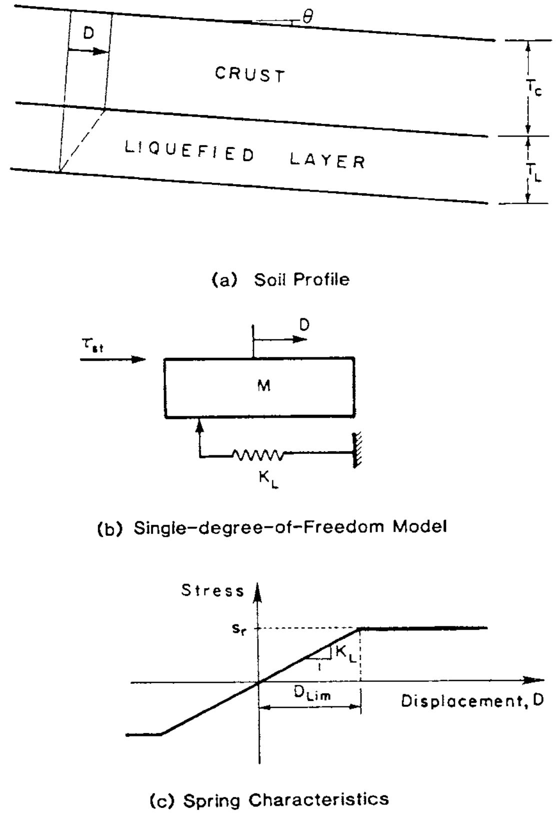

Fig. 11. Single-Degree-of-Freedom Model for Liquefaction Induced Displacements.

The soil mass, M, is given by:

$$M = (T_C\gamma_C + \frac{1}{2}\ T_LG_L)/g \tag{5}$$

T_C and T_L are the thickness of the crust and liquefaction layers respectively, γ_C and γ_L are their respective unit weights, and g is the acceleration of gravity.

The driving stress, τ_{st}, is given by:

$$\tau_{st} = (T_C\gamma_C + \frac{1}{2}\ T_L\gamma_L)\ \sin\theta \tag{6}$$

in which θ is the surface slope.

The spring stiffness, K_L, depends on the shear modulus of the liquefied soil, G_L, which in turn depends on the residual strength, s_r, and the limit displacement, γ_{Lim}, as follows:

$$G_L = s_r/\gamma_{Lim} \tag{7}$$

and the spring stiffness, K_L, is given by:

$$K_L = G_L/T_L \tag{8}$$

The displacement of the crust, D_{st}, due to the static driving stress applied to the softened liquefied layer is given by:

$$D_{st} = \tau_{st}/K_L \tag{9}$$

If we consider that the onset of liquefaction is a very sudden event and that just prior to liquefaction the mass has a velocity, V_0, and a displacement $\approx$ o, then upon liquefaction the post cyclic stress strain curve is appropriate and will arrest the motion when the work done by the external forces acting on the mass equal the change in kinetic energy. The static driving stress, τ_{st}, produces positive work, whereas the resisting spring force does negative work as depicted in Fig. 12. Equating the net work done to

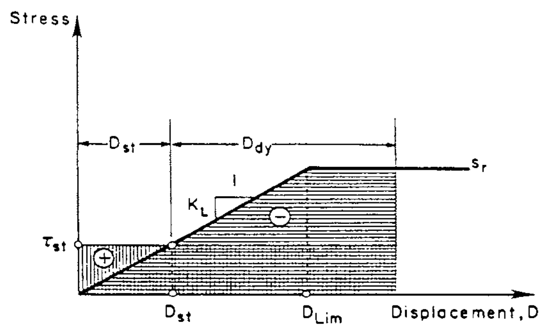

Fig. 12. Work Done by External Forces.

the change in kinetic energy as the velocity decreases from V_0 to zero results in the following equation for D_{dy}:

$$D_{dy} = [\frac{M}{K_L} V_0^2 + D_{st}^2]^{1/2} \tag{10}$$

For $D_{dy} < (D_{Lim} - D_{st})$

and

$$D_{dy} = \frac{1}{2} [D_{Lim} - D_{st} + \frac{D_{st}^2 + MV^2/K_L}{D_{LIM} - D_{st}}] \tag{11}$$

For $D_{dy} > (D_{Lim} - D_{st})$, where

$$D_{Lim} = \gamma_{Lim} \cdot T_L \tag{12}$$

If it is assumed that the spring stores no energy, in agreement with the laboratory data of Fig. 7, then D_{dy} is a permanent displacement, i.e., there is no rebound.

The total displacement, D, is given by:

$$D = D_{st} + D_{dy} \tag{13}$$

The above equations are appropriate for the linear stress-strain relations shown as the dashed line in Fig. 13. The laboratory data,

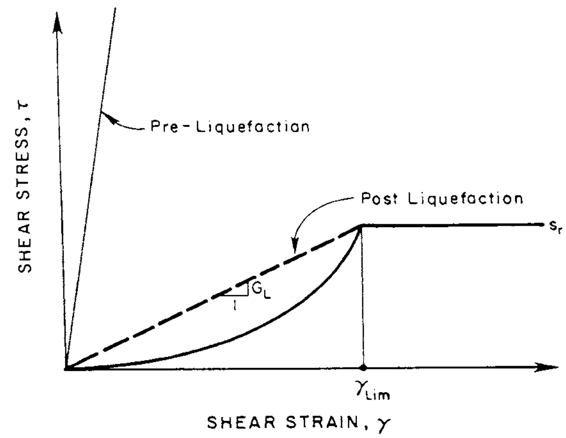

Fig. 13. Linear and Nonlinear Stress-Strain Models.

however, suggests that the response follows the solid line. This nonlinear response can be better approximated at any strain level, γ, by a secant G specified as follows:

$$G = G_L \frac{\gamma}{\gamma_{LIM}} \tag{14}$$

Equation 14 is appropriate for $\gamma \le \gamma_{LIM}$. For $\gamma > \gamma_{LIM}$, $\tau = s_r$.

The appropriate static displacement for the nonlinear conditions is given by:

$$D_{st} = (\frac{\tau_{st}}{s_r})^{1/2} \cdot D_{LIM} \tag{15}$$

The total displacement, D, from energy considerations can now be obtained from:

$$\frac{D^3 \cdot K_L}{3 \cdot D_{LIM}} - D \cdot \tau_{st} - 1/2\ M \cdot V_0^2 = 0 \tag{16}$$

For $D < D_{LIM}$, and

$$D = (1/2\ M \cdot V_0^2 - 1/3 \cdot K_{LIM} \cdot D_{LIM}^2 + s_r \cdot D_{LIM})/(s_r - \tau_{st}) \tag{17}$$

For $D > D_{LIM}$.

For $D < D_{LIM}$, Eq. (16) involves the solution of a cubic equation. This can be solved using Newton's method. The use of a trial D equal to twice the value of D obtained from the linear analysis will converge to the correct root.

MODEL PREDICTIONS

The model predictions are first compared with the Hamada et al. (1987) equation. Since most of the data on which this equation was based involved an average layer thickness of 1.5 m, this thickness was used in the comparison. The Hamada et al. prediction as a function of ground slope is shown as a solid line in Fig. 14. The predicted displacements for the various $(N_1)_{60}$ conditions are also shown. The s_r and γ_{Lim} implied by these $(N_1)_{60}$ values correspond with the average values shown in Table 1. The following additional parameters were used: T_C = 1.5 m, T_L = 1.5 m, V_0 = 0.2 m/sec.

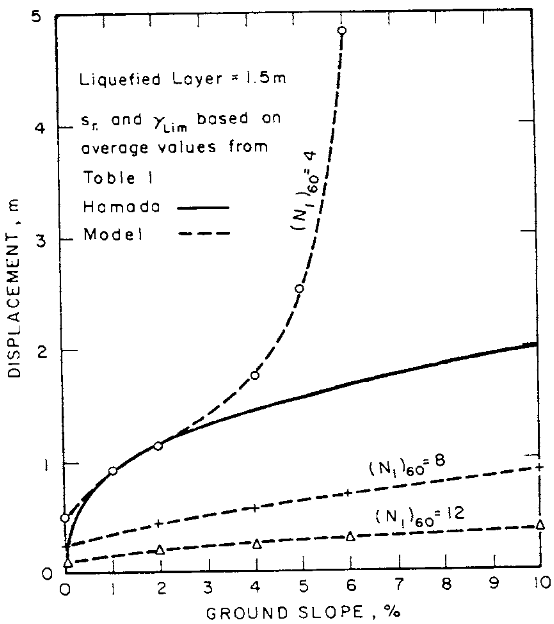

Fig. 14. Comparison of Model Liquefaction Induced Dispalcements; and Hamada's Equation.

The results indicate that the predictions for $(N_1)_{60}$ = 4 are in close agreement with Hamada's equation for ground slopes up to about 3%. For steeper slopes the prediction is significantly above Hamada's. The sharply increasing predicted displacement with increased ground slope occurs because the driving stress is approaching the residual strength and a flow slide condition. A flow slide is predicted when the driving stresses reach the residual strength, and the proposed procedure predicts the onset of such a situation. The predicted displacements for $(N_1)_{60}$ values of 8 and 12 are also shown and lie well below Hamada's predictions.

Predicted displacements as a function of $(N_1)_{60}$ value are shown in Fig. 15 and compared with Hamada's equation. The displacements are for a slope of 2%. s_r and γ_{Lim} values are based on $(N_1)_{60}$ from Table 1. Both the average condition as well as the lower bound values of s_r and the upper bound on γ_{Lim} are considered. This latter condition leads to a lower bound value of modulus and the largest predicted displacements.

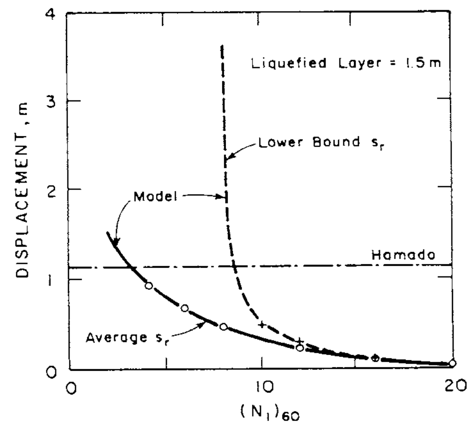

Fig. 15. Liquefaction Induced Displacement versus $(N_1)_{60}$ Value; Model and Hamada's Equation. Slope = 2%.

The proposed model predictions of liquefaction induced displacements (Fig. 15) are strongly dependent on $(N_1)_{60}$ value. For the "average" condition the model displacements are in excess of 1.2 m for $(N_1)_{60}$ <3. For $(N_1)_{60}$ >12, the model displacements are <.2 m. For the lower bound s_r conditions, the model displacements are very large and correspond to flow slide conditions for $(N_1)_{60}$ <8. However, even for the lower bound condition the model predicts displacements <0.3 m for $(N_1)_{60}$ = 12. Hamada's equation predicts a displacement of about 1.2 m which is independent of $(N_1)_{60}$ values.

The model was also used to predict the shaking table tests of Towhata et al., 1988, and shown in Fig. 6. These tests were for relative densities generally ranging between 45 and 66%. Based on Skempton (1986), these densities would correspond with $(N_1)_{60}$ in the range 7-15. Model predictions were made using a liquefied layer thickness, T_L = 0.2, a crust T_C= 0 and s_r and γ_{Lim} based on $(N_1)_{60}$ values of 7 and 15, with "average" conditions from Table 1. The predicted displacements ranged between 0.9 and 3.1 cm compared with the observed range of 0.3 to 2.5 cm. This agreement is quite remarkable. Hamada's equation predicts 57 cm.

The proposed model provides a simple procedure for estimating liquefaction induced displacements. The predicted displacements are in good agreement with both field and laboratory test data. The results suggest that the Hamada equation is appropriate for loose sands having $(N_1)_{60}$ values of less than about 4. For denser sands which are triggered to liquefy, the resulting displacements are likely to be significantly less.

The model predicts both static displacements due to the softened stress-strain response as well as dynamic displacements due to inertia effects. The displacement is the sum of these two values. In general the predicted displacement is 2 to 3 times the static displacement but can be much larger when τ_{st} is either much smaller than s_r in which case the V_0 term in Eq. 10 dominates, or when τ_{st} approaches s_r in which case the work done by the static forces is very large and causes very large dynamic displacements.

The proposed analysis procedure can be extended to 2-D slopes in a manner similar to that used by Newmark (1965) when modelling a 2-D slope as a single degree of freedom system. The static driving stress, τ_{st}, for use in Eq. 9, can be computed from a limit equilibrium analysis as follows:

$$\tau_{st} = s_r/F \qquad (16)$$

where s_r is the appropriate average strength along the failure surface and F is the factor of safety computed from limit equilibrium analysis. The other parameters, such as the crust thickness and the thickness of the liquefied layer, should be based on the geometry and soil conditions present.

A more accurate 2-D analysis can be obtained from a pseudo-static finite element analysis in which the post-cyclic stress-strain curves are used together with an appropriate horizontal seismic coefficient applied such that an energy balance is achieved following the concepts of Fig. 12. This is presently being incorporated into the computer code SOILSTRESS, Byrne and Janzen (1981).

SUMMARY

Experience during past earthquakes has shown that very large horizontal displacements can be induced by the triggering of liquefaction. The Hamada empirical equation provides a good fit to this data but does not include a parameter which reflects the density of the soil. Laboratory data indicates that the post liquefacion behaviour of sand is highly dependent on the density of the sand.

Complex dynamic analyses could be used to determine the complete response of the soil. However, the displacements that occur prior to liquefaction are very small, less than 0.5%. Most displacement is caused by the loss in strength and stiffness of the soil which occurs after liquefaction is triggered.

A simple analysis procedure based upon a single-degree-of-freedom system is presented. The model is similar to that proposed by Newmark (1965) except that a nonlinear spring representing the stiffness of the liquefied layer as well as its residual strength is incorporated, rather than the rigid plastic spring considered by Newmark. Upon liquefaction a loss in stiffness occurs and results in a static displacement caused by the static driving stresses acting on the softened soil. The abrupt nature of the loss in stiffness will lead to a dynamic displacement because the driving stresses will be initially unbalanced and lead to acceleration of the mass. In addition, there will be dynamic displacements due to the velocity of the mass at the instant of liquefaction.

The key parameter for the model are the residual strength and the limiting strains upon liquefaction. Based upon experimental data, these values are strongly dependent on relative density.

The model predicts displacements that are in good agreement with both the observed field and laboratory shaking table values. The predicted displacements for denser sands are very much less, and suggest that the use of Hamada's equation for slope comprised of denser sands could be very conservative.

ACKNOWLEDGEMENT

The author acknowledges the contributions of Drs. D.L. Anderson and N.D. Nathan in reviewing the work-energy aspects of the model, and Dr. Y.P. vaid for discussions regarding the post-liquefaction response of soil. The author is grateful to Ms. G. Breig for typing and presentation of the paper, and Mr. R. Brun for drawings. The financial suport of NSERC, Canada, is appreciated.

REFERENCES

Byrne, P.M. and Janzen, W., "SOILSTRESS: A Computer Program for Nonlinear Analysis of Stresses and Deformations in Soil," Soil Mechanics Series No. 52, Department of Civil Engineering, Univ. of B.C., Vancouver, B.C., December 1981.

Byrne, P., Vaid, Y. and Stuckert, B., "Model Tests on Earthquake Stability of Tailings Slopes," ASCE National Convention, Specialty Session No. 67 on Physical Modeling of Soil, Dynamic Problems, Las Vegas, Nevada, April 1982.

Byrne, P.M. "LIQDISP; A Computer Program to Predict Liquefaction Induced Displacements of Slopes," Soil Mechanics Series No. 147, Dept. of Civil Engineering, University of British Columbia, Sept., 1990.

Finn, W.D. Liam, Byrne, P.M. and Martin, G.R., "Seismic Response and Liquefaction of Sands," Journal of the Geotechnical Eng. Division, ASCE, No. GT8, August 1976.

Finn, W.D. Liam, Yogendrakumar, M., Yoshida, N. and Yoshida, H., "TARA-3: A Program to Compute the Response of 2-D Embankments and Soil-Structure Interaction Systems to Seismic Loadings," Dept. of Civil Engineering, University of British Columbia, Vancouver, B.C., 1986.

Hamada, M., Towhata, I., Yasuda, S. and Isoyama, R., "Study on Permanent Ground Displacement Induced by Seismic Liquefaction," Computers and Geotechnics 4, 1987, pp. 197-220.

Kuerbis, R.H., "Effect of Gradation and Fines Content on the Undrained Response of Sand," Ph.D. Thesis, Department of Civil Engineering, University of British Columbia, Vancouver, B.C., June 1989.

Martin, G.R., Finn, W.D. Liam and Seed, H.B., "Fundamentals of Liquefaction Under Cyclic Loading," Journal of the Geotechnical Eng. Division, ASCE, Vol. 101, No. GT5, May 1975.

Newmark, N.M., 1965, "Effects of Earthquakes on Dams and Embankments," Geotechnique, Vol. 15, No. 2, pp. 139-160.

Prevost, J.H., "DYNA-FLOW: A Nonlinear Transient Finite Element Analysis Program," Report No. 81-SM-1, Department of Civil Engineering, Princeton University, Princeton, N.J., 1981.

Seed, H.B., Tokimatsu, K., Harder, L. and Chung, R., "The Influence of SPT Procedures in Soil Liquefaction Resistance Evaluations," Report No. UCB/EERC-84/15, College of Engineering, University of California, Berkeley, California, 1984.

Seed, H.B., Harder, L.F., "SPT-Based Analysis of Cyclic Pore Pressure Generation and Undrained Residual Strength," H. Bolton Seed, Memdorial Symposium Proceedings, Vol. 2, 1990.

Skempton, A.W., "Standard Penetration Test Procedures and the Effects in Sands of Overburden Pressure, Selective Density, Particle Size, Ageing and Overconsolidation," Geotechnique 36, No. 3, 1986, pp. 425-447.

Towhata, I., Yasuda, S., Ohtomo, K., and Yamada, K., "Experimental Studies on Liquefaction Induced Permanent Ground Displacement," Proceedings, First Japan-U.S. Workshop on Liquefaction, Large Ground Deformation and Their Effects on Lifeline Facilities, Tokyo, Japan, November 1988.

Vaid, Y., "Data on Residual Strength of Loose Sand," Pers. communication, August 1990.

Youd, T., and Bartlett, S., "US Case Histories of Liquefaction-induced Ground Displacements," Proceedings, First Japan-U.S. Workshop on Liquefaction, Large Ground Deformation and Their Effects on Lifeline Facilities, Tokyo, Japan, November 1988.