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SYNOPSIS: In this paper, according to the principle of fuzzy mathematics, five principal factors are considered, earthquake intensity I, the thickness of overburden layer of non-liquefacted cohesive soil Ho, SPT blow count (China) N, the depth of point of SPT ds, and groundwater level dw, at the same time, the subordinate functions of them are established. The rank of liquefaction is classified into four sorts: non-liquefaction, probable-non-liquefaction, probable-liquefaction and liquefaction. The fuzzy multifactorial evaluation is presented for predicting site earthquake liquefaction of sandy soil, and the operation of M (•,+) is applied to calculation of fuzzy matrix. At last, a comparison between the results of predicting with the procedure and the fact of the cases of Haicheng and Tangshan earthquake, the effects are satisfactory.

INTRODUCTION

The method in evaluation of liquefaction of sand is mainly divided into two types at present, i.e., experienced method and theoretic method. The methods are based on study of the relation between density of soil and shear stress of earthquake.

The objective evaluation of liquefaction of sand is on important step in earthquake resistant measures of buildings. There is much fuzzy nature on the problem of liquefaction of saturated sand because of more influence factors at liquefaction potential of sand, i.e., its extension and intension is fuzzy. In a general way, concept of liquefaction and non-liquefaction of sand itself is fuzzy, therefore, it is suitable that liquefaction of sand can be assessed using fuzzy mathematics.

FUZZY NATURE AND SETS OF LIQUEFACTION OF SAND

Fuzzy nature of liquefaction potential of sand is not only fuzzy of evaluation criterion of liquefaction (influence factors) but complicated relations among influence factors. Consequently, it well not be suitable for evaluation of liquefaction only considering one determined influence factor, for example, N or Vs etc.

In the paper, five principal factors are considered, i.e.,

1. earthquake intensity I
2. the thickness of overburden layer of non-liquefaction cohesive soil Ho
3. SPT blow count (China) N
4. the depth of point of SPT ds
5. groundwater level dw

Therefore, the set of influence factors can be described as follows:

$$U = (I,\ Ho,\ N,\ ds,\ dw) = (u_1,\ u_2,\ u_3,\ u_4,\ u_5) \qquad (1)$$

here, the influence factors ui (i=1,2,...5) may be fuzzy or non-fuzzy, but the relation between ui and U is only ui $\tilde{\in}$ U, or ui $\in$ U.

Based on the needs of practical engineering, the rank of liquefaction is classified into four sorts: Non-liquefaction, probable-non-liquefaction, probable-liquefaction and liquefaction. Thus, fuzzy sets of the rank of liquefaction is shown:

$$V = (v_1,\ v_2,\ v_3,\ v_4) \qquad (2)$$

Obviously, vj (j=1,2,3,4) represent the final evaluated results. In fact, purpose of the fuzzy multifactorial evaluation is to obtain a perfect assessed result based on the comprehensive consideration of influence factors (or principal factors) ui.

ESTABLISHMENT OF SET OF WEIGHT $\underset{\sim}{A}$ AND THE SUBORDINATE FUNCTIONS

Generally, level of influence of each factor for the evaluated results is not identical, for the sake of representing the level of influence of each factor, appropriate weight of each factor ai (i=1,2,...5) should be given, thus, set of weight $\underset{\sim}{A}$ is established.

$$\underset{\sim}{A} = (a_1,\ a_2,\ a_3,\ a_4,\ a_5) \qquad (3)$$

Evidently, $\underset{\sim}{A}$ is part set of fuzzy sets of U, and it is indicated:

$$\underset{\sim}{A} = \frac{a_1}{u_1} + \frac{a_2}{u_2} + \frac{a_3}{u_3} + \frac{a_4}{u_4} + \frac{a_5}{u_5} \qquad (4)$$

In the five principal factors (I, Ho, N, ds, dw), effect of N is first, then Ho, dw. Therefore, $\underset{\sim}{A}$ is determined in this paper as follows:

$$\underset{\sim}{A} = (0.1,\ 0.25,\ 0.3,\ 0.1,\ 0.25) \qquad (5)$$

The key of treating question with fuzzy mathematics (fuzzy multifactorial evaluation) is building the appropriate subordinate functions of the influence factors. Establishment of them are decided by nature of the problem and level we understand the problem, usually, determination of them depend on experence.

According to analysing data of the sites (liquefied sites and non-liquefied sites), curves of the sub-ordinate functions of the factors is shown Fig. 1 - Fig. 5.

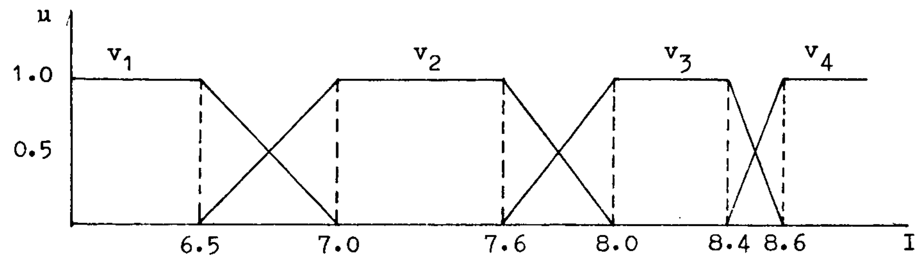

Fig. 1

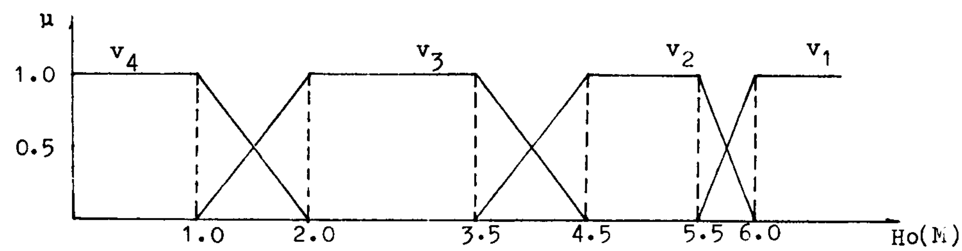

Fig. 2

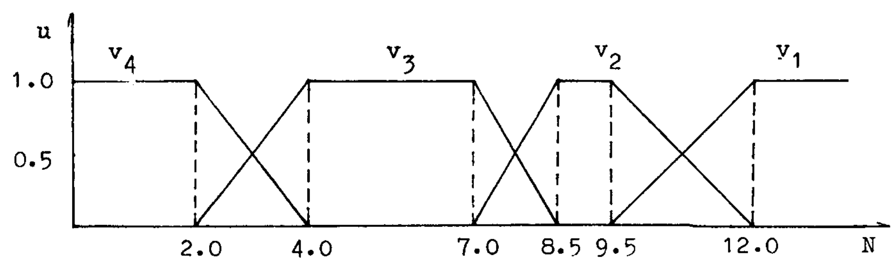

Fig. 3

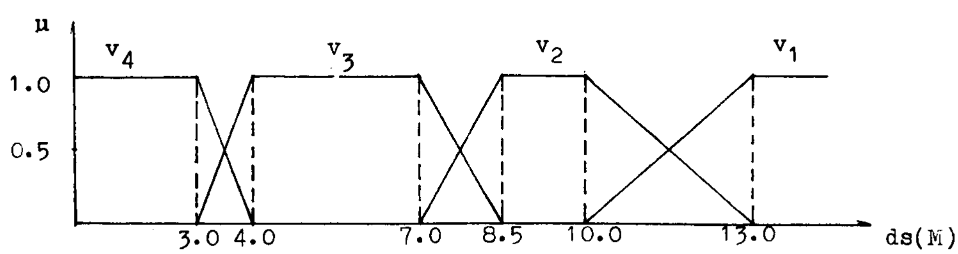

Fig. 4

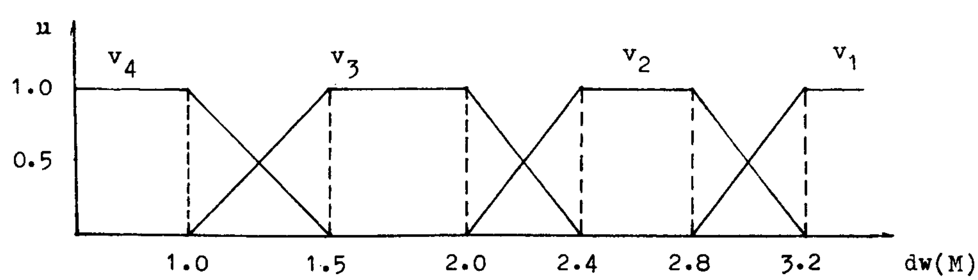

Fig. 5

FUZZY MULTIFACTORIAL EVALUATION

Fuzzy multifactorial evaluation is realized by way of compound operation, it can be shown that

$$\underset{\sim}{B} = \underset{\sim}{A} \cdot \underset{\sim}{R} = (b_1, b_2, b_3, b_4) \qquad (6)$$

where $\underset{\sim}{R}$ is a fuzzy relation (or fuzzy relation matrix) between the set U and V, it decide a fuzzy reflection. $\underset{\sim}{A}$ is the set of weight of factors, and original image of fuzzy reflection; $\underset{\sim}{B}$ is reflected image of fuzzy reflection, i.e., assessed result.

In matrix $\underset{\sim}{R}$, $\underset{\sim}{R}i$ =(ri1, ri2, ri3, ri4) is single-factor evaluation of ui, and part set of fuzzy sets of V. bj is evaluation target,

$$bj = \sum_{i=1}^{5} ai \cdot rij \qquad (7)$$

(7) represents calculation of fuzzy matrix, i.e., the operation of M (·,+). The model not only considers the effect of the factors, but reserves all information of single-factor evaluation of ui.

Provided a set of data Uk =$(u_{1k}, u_{2k}, \ldots u_{5k})$ is imported, the relevant $\underset{\sim}{R}k$ is obtained by way of the subordinate functions, and assessed result $\underset{\sim}{B}k$ is obtained,

$$\underset{\sim}{R}_k = (rij) \qquad i = 1, 2, \ldots 5 \qquad (8)$$
$$j = 1, 2, 3, 4$$

where rij is the subordinate level of "i" of the influence factors for "j" of the rank of liquefaction in this set of data U_k,

$$rij = \mu_{\underset{\sim}{R}k}(ui, vj) \qquad (9)$$

eventually, evaluation target $\underset{\sim}{B}_k=(b_{1k}, b_{2k}, b_{3k}, b_{4k})$ is obtained.

If sets of practical data are obtained for any field sandy layer, correspondingly, evaluation targets $\underset{\sim}{B}_k$ are obtained. In the paper, we have the presumption as follows

if $\sum(b_1+b_2) > \sum(b_3+b_4)$, the sandy layer would not be liquefaction, conversely, the sandy layer would be liquefaction.

EXAMPLES OF COMPUTATION USING THE METHOD

There is the practical information of Tangshan earthquake (1976), its geological column is shown in Fig. 6, and four sorts of data are given as follows:

elevation depth(M)	geological column	name of soil
4.1		loam
9.2		silty sand
13.7		loam

Fig. 6

(7, 4.1, 14, 4.8, 1.0)
(7, 4.1, 24, 5.8, 1.0)
(7, 4.1, 25, 6.8, 1.0)
(7, 4.1, 24, 8.3, 1.0)

The fuzzy relation matrix of the first data can be obtained according to the subordinate functions (Fig. 1 --- Fig. 5), so we have

$$\underset{\sim}{R}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0.6 & 0.4 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $\underset{\sim}{A}$ = (0.10, 0.25, 0.30, 0.10, 0.25)

thus,

$$\underset{\sim}{B}_1 = \underset{\sim}{A} \cdot \underset{\sim}{R}_1 = (0.1, 0.25, 0.3, 0.1, 0.25) \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0.6 & 0.4 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$= (0.30, 0.25, 0.20, 0.25)$$

similarly, $\underset{\sim}{B}_2$, $\underset{\sim}{B}_3$, $\underset{\sim}{B}_4$ can be obtained,

$$\underset{\sim}{B}_2 = (0.30, 0.25, 0.20, 0.25)$$
$$\underset{\sim}{B}_3 = (0.30, 0.25, 0.20, 0.25)$$
$$\underset{\sim}{B}_4 = (0.30, 0.34, 0.11, 0.25)$$

Finally, the evaluation result is $\sum(b_1+b_2)$= 2.29$>\sum(b_3+b_4)$=1.71, therefore, this sandy layer would not be liquefaction, the result corresponds with the fact.

In order to verify the reliability of proposed procedure, the cases of Haicheng earthquake (1975) and Tangshan earthquake (1976) (there are 64 liquefaction sites and 46 non-liquefaction sites) have been judged again with the fuzzy multifactorial evaluation, all results can be seen Table 1. It is proved that the correct rate is about 84% for liquefaction and 82% for non-liquefaction.

CONCLUSIONS

The conclusions obtained in this study are summerized as follows:

a. Establishment of the subordinate function is difficult, generally, its distributed type can be assumed in advance, then it is determined according to checking computations until it is considered suitable, the subordinate function of each influence factor in the method is obtained based on analysing firsthand information.

b. The suggested procedure is a effective and convenient according to the Table 1.

c. It is suitable for the fuzzy problems such as liquefaction with the method of fuzzy mathematics, and it will be vast vistas.

Tab. 1 Results of Multifactorial Evaluation

intensity	numbers of sites		level of evaluation		correct rate	
	liquefied	non-liquefied	liquefied	non-liquefied	liquefied	non-liquefied
7	24	15	3	2	87.8	86.7
8	20	17	3	3	85	82.3
9	15	12	4	3	73.3	75
10	5	2	0	0	100	100

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