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Variability Analysis of Undrained Shear Strength for Reliability-Based Design

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ABSTRACT

Description of the spatial variation of compositional and mechanical soil parameters is often paramount in site characterization of geotechnical design and analysis. The values of the parameters themselves largely depend on in-situ state factors which are related to spatial locations. Also, for large-scale engineering projects such as earth dams or highways, it is generally expected that heterogeneous site characteristics will be revealed by investigations at spatially distant locations. Currently, most Reliability-Based Design methods are based on the second-moment statistics (mean and variance). However, they are unable to adequately describe the spatial variation of soil properties. In this paper, spatial correlation analysis is developed to evaluate the undrained shear strength profile at a location in Warrensburg in Missouri, where undisturbed samples were taken from Shelby tubes and undrained shear strengths were determined through the unconsolidated undrained triaxial hear strength (UU) tests.

INTRODUCTION

The most prevalent techniques for the investigation of uncertainty/material variability involve the combined use of probability and statistics (Lumb, 1974; Rethati, 1988; Phoon and Kulhawy, 1999a, 1999b; Lacasse and Nadim, 1996; Duncan, 2000; Uzielli et al, 2007; Ang and Tang, 2007; DNV, 2007; Fenton and Griffiths, 2008). In these techniques, the parameters are modeled as random variables. A random (or independent) variable is a quantity that is not known due to its random nature. The procedure generally involves defining the material properties by their statistics (principally their first and second moments): the mean, μ; and the standard deviation, σ, which define the probability density function and combine to give the coefficient of variation, *COV*. The mean of a data set is the sum of the data points in the data set divided by the total number of data points in the data set. The standard deviation of a random variable is the square root of its variance. The variance of a random variable is the mean value of the square of the deviation of that variable from its expected value or mean. The mean is the most common measure for the center of a data set. The variance is a measure of dispersion about the mean value of a data set. High values of dispersion mean higher uncertainty. Conversely, low values of dispersion mean low uncertainty.

These second moment-based techniques for the characterization of uncertainty in geotechnical parameters discussed above do not take into account the spatial variability of the parameters. Geotechnical parameters are known to show dependence both laterally and with depth; geotechnical parameters vary spatially with a greater tendency for the properties to be similar in value at closely neighboring points than at widely spaced points (Lacasse and Nadim, 1996). This is the reason second moment statistics alone are not enough to characterize uncertainty in geotechnical parameters (Uzielli, et al., 2007).

Soil properties do not vary randomly in space; rather such variation is gradual and follows a pattern that can be quantified using spatial correlation structures, where soil properties are treated as random variables (Elkateb, et al., 2003). The knowledge of spatial behavior of soil properties is often paramount in geotechnical analysis and design for the following reasons (Uzielli, et al., 2007; Cherubini et al, 2007): geotechnical design is based on site characterization, which objective is the description of the spatial variation of compositional and mechanical parameters of soil; the values of the parameters themselves very often depend on the on the in-situ factors, state factors which are related to spatial locations; and for large scale engineering projects such as earth dams or highways, it is generally expected that heterogeneous site characteristics will be revealed by investigations at spatially distant locations.

Geotechnical performance is often governed by spatial average soil properties, such as average shear strength along a pile shaft, or average compressibility of a volume of soil beneath a footing in settlement calculation. The variability of soil properties averaged over a domain is less than that of their inherent variability or point properties (Vanmarcke, 1977). For instance, the variability of the average shear strength along the length of a pile, the average unit weight with in a volume of soil, and the average compressibility of a volume soil underneath a footing will all be less than their respective point values. This is known as the averaging effect in spatial variability. The spatial averaging effect results in a reduction in the variance used in reliability-based geotechnical design (RBD). Neglecting this spatial averaging effect will lead to an overestimate of variability and lead to a very conservative design. Hence, the spatial average soil property over an appropriate domain will be the soil parameter of primary interest.

EXPLORATORY AND LABORATORY PROGRAM

Data for this paper were obtained principally from a very carefully planned Missouri wide field exploration and laboratory investigation program. Field exploration for soil samples by means of continuous sampling and cone penetrometer (with porewater pressure measurements) (CPTu) soundings, and a suite of laboratory tests were carried out under this program.

A special protocol, which largely followed best practice in geotechnical exploration and testing (Ladd and DeGroot, 2003), was implemented to reduce disturbance to the samples/specimens at all stages: Exploration and sampling (continuous sampling); Transportation and storage; Extrusion and trimming; and Testing. The data for this paper is the undrained shear strength, $Su_{(UU)}$ from the unconsolidated undrained triaxial shear strength test (UU) from one exploratory borehole (W-BH2) from Warrensburg (Johnson County) in the Western Plain geological region of Missouri.

ANALYSIS

The spatial variability of a parameter can be described statistically by the mean, variance, and the scale of fluctuation, $\boldsymbol{\theta}$ (Vanmarcke, 1977). The scale of fluctuation defines the distances over which there is significant correlation of material property values (Vanmarcke, 1977). Analysis of spatial variability of geotechnical parameters has involved the use of: random field theory and time series analysis (Lumb, 1974; Vanmarcke, 1977); and geostatistics, an approach proposed by Matheron (1965). The geostatistics approach is used in this paper.

Geostatistics is based on regionalized variables that have properties that are partly random and partly spatial and that have continuity from point to point (Clark, 1979). One of the basic statistical measures of geostatistics is the semivariogram, which is used to quantify the degree of spatial dependence between samples along a specific orientation, and so presents the degree of continuity of the property in question.

Even though a regionalized variable is spatially continuous, its values can only be determined from samples taken from a population. Thus, in practice, the experimental semivariogram, γ^*, is estimated from the available data using standard relationships and a continuous theoretical semivariogram model fitted to the experimental semivariogram (Clark, 1979). There are many theoretical semivariogram models available; the most common of these being the spherical model (Kelkar and Perez, 2002; Meek, 2001).

The experimental semivariogram is characterized by three parameters (Jaksa et al, 1997): (a) the nugget effect, Co, which is due to microvariabilities of the mineralization, and measurement errors; (b) the sill, $C + Co$, which measures, on the average, the squared difference between data pairs; and (c) the range of influence, a, which is the distance at which samples become independent of one another. A typical theoretical semivariogram is presented in Figure 1.

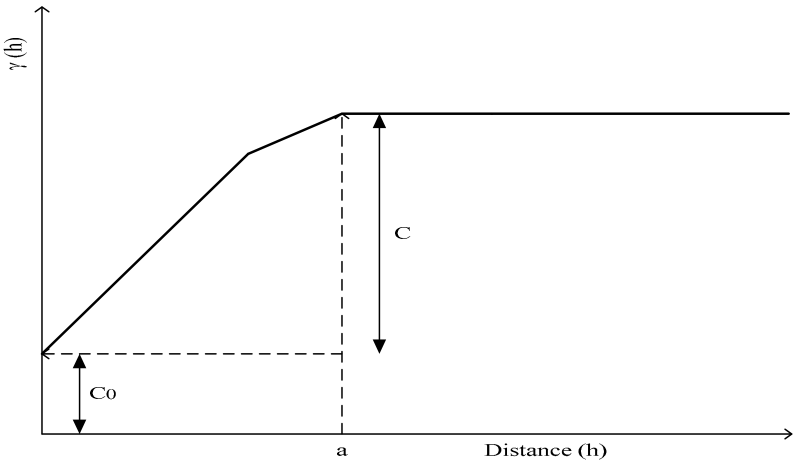

Figure 1. Typical experimental and theoretical variogram

The experimental semivariogram γ^* is defined by (Clark, 1979):

$$\gamma^*\,(h) = \frac{1}{2n}\sum[g(x) - g(x+h)]^2 \qquad (1)$$

Where g stands for $Su_{(UU)}$, x denotes the position of one sample in the pair and $x + h$ the position of the other, and n is the number of pairs.

With the theoretical semivariogram model was fitted to the data, the parameters of the variogram are determined and used in computing the scale of fluctuation, θ. The scale of fluctuation, θ is defined by Cressie (1993):

$$\theta \;=\; 2\int_0^\infty \rho(h)\,dh \qquad (2a)$$

Where

$$\rho(h) = 1 - \;(\gamma(h) - c_0)/c \;\; for\; an\; experimental\; semivariogram \qquad (2b)$$

$$\hat{\rho}(h) = 1 - \;(g(h) - c_0)/c \;\; for\; a\; semivariogram\; model \qquad (2c)$$

Meek (2001) provides alternative relationships for the determination of scale of fluctuation, θ, for various theoretical semivariogram models based on the range of influence, a.

RESULTS

The data of the UU tests and the undrained shear strength, $Su_{(UU)}$ profile for W-BH2 are presented in Table 1 and Figure 2 respectively. The UU tests were conducted at about 2 to 3 feet intervals; hence, there is a paucity of data in this case compared to the data yield of a CPT. The summary statistics for the undrained shear strength, $Su_{(UU)}$ are presented in Table 2.

Table 1. UU test data

W - BH 2							
Depth	Su	Depth	Su	Depth	Su	Depth	Su
6.3	506.0	16.2	1272.0	26.3	1276.0	26.3	1276.0
6.7	698.0	16.5	594.0	26.6	1074.0	26.6	1074.0
8.7	561.6	18.6	1440.0	28.3	907.2	28.3	907.2
9.0	1008.0	18.9	2059.2	28.6	1094.4	28.6	1094.4
11.2	444.0	20.9	1035.0	31.3	805.0	31.3	805.0
11.6	442.0	21.2	606.0	31.7	565.0	31.7	565.0
13.5	403.2	23.6	2102.4	33.8	1296.0	33.8	1296.0
13.9	187.2	23.9	2779.2	34.1	1195.2	34.1	1195.2

Su = Undrained Shear Strength (psf)

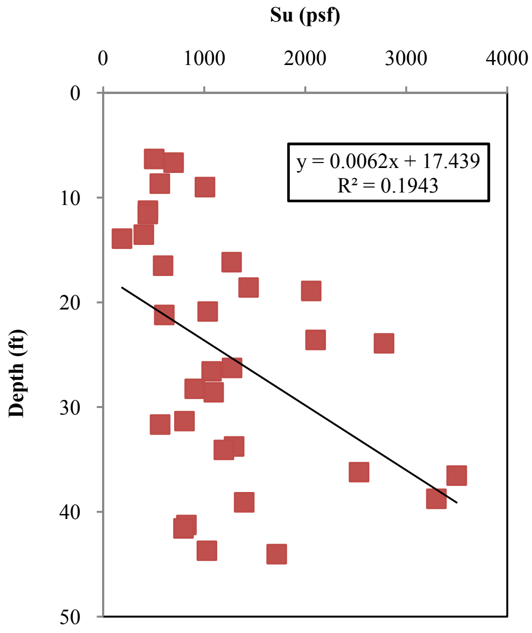

Figure 2. The undrained shear strength, $Su_{(UU)}$ profile for W-BH2

Based on laboratory tests, the soil within the limits of the exploration depth was classified as CL according to the Unified Soil Classification System (USCS). This was verified with the soil behavior type from the CPT profiles. Consequently, for purposes of analyses, the soil is considered as a uniform soil type (CL). The $Su_{(UU)}$ profile in Figure 2 shows a linear trend line with very low coefficient of correlation, R^2 of 0.1943. In the almost absence of a deterministic trend, the W-BH2 data can be considered as stationary. Hence data transformation will not be required. In geostatistics, nonstationary data [data with trend(s)] are transformed to a stationary one by removing the deterministic, low-order polynomial trend, usually no higher than a quadratic, using the method of Ordinary Least Squares, OLS (Jaksa et al, 1997).

Table 2. Summary statistics for the undrained shear strength, $Su_{(UU)}$

Minimum	568.0
Maximum	1089.0
Mean	797.5
Variance	25983.0
Std. Dev.	834.6
Skewness	1.4
Kurtosis	1.4
CV (%)	67.7
n	32
Std. Dev. = Standard Deviation	

To estimate the experimental semivariogram, the pairs ($Su_{(UU)}$ values) at the various lags were collected. In this study, the maximum lag considered was half the depth of exploration and only the lags that had 5 pairs and above were used to plot the experimental semivariogram. The number of pairs and experimental semivariogram at each lag are presented in Table 3. The experimental semivariogram is presented in Figure 3.

Table 3. Lag, number of pairs and experimental semivariogram

Lag (ft)	No. of Pairs	Experimental Variogram (psf)^2	Theoretical Variogram (psf)^2
0.3	11	287074	104650
2.1	5	341857	614950
2.3	6	261133	647280
2.4	10	1254941	660800
2.6	5	618679	682163
2.7	10	804110	689850
4.7	9	974601	700000
4.9	7	643394	700000
5	10	689646	700000
5.4	5	653866	700000
7.4	10	626813	700000
7.5	8	781285	700000
7.7	7	543281	700000
7.8	6	514424	700000
9.9	10	642638	700000
10.2	6	870172	700000
12.5	5	560290	700000
Theoretical Variogram = Spherical; $a = 3$, $C = 700000$, $C_0 = 0$			

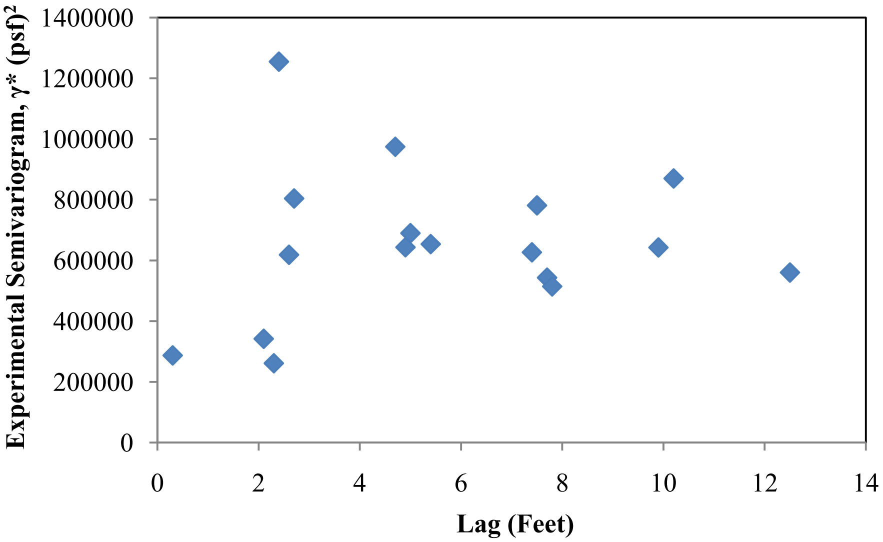

Figure 3. Experimental semivariogram

For an efficient experimental variogram, Kelkar and Perez (2002) suggest a maximum lag of half the maximum possible distance, and a minimum of 10 pairs for each lag within a region of stationarity. Lags that had 5 pairs and above were used in this case due to insufficient of data arising from the fact that the UU tests were conducted at about 2 to 3 feet intervals. Variogram analysis is better suited to CPT data which are continuous and are at close intervals. When tests are conducted at close intervals, the continuous sampling method used in this study is capable of providing data for variogram analysis.

Among the theoretical semivariogram models available (Kelkar and Perez, 2002; Meek, 2001), the spherical theoretical semivariogram model provided the best fit and was fitted to the data. The spherical theoretical semivariogram, γ was computed from the following relationship (Clark, 1979):

$$\gamma(h) = C_0 + C\left(\frac{3}{2}\frac{h}{a} - \frac{1}{2}\frac{h^3}{a^3}\right) \; where \; 0 < h \leq a \qquad (3a)$$

$$\gamma(h) = C_0 \; + C \qquad where \; h \; \geq a \qquad (3b)$$

Where a is the range of influence of the specimen - the distance at which specimens become independent of one another, $C_0 + C$ is the sill of the variogram - the value of γ at which the plot levels off, and C_0 is the nugget effect.

The experimental semivariogram, γ^* showing the spherical theoretical semivariogram model fitted to the data is presented in Figure 4. From Figure 4, the range of influence, a = 3 feet, $C_0 = 0$ and the sill, C = 700000 (psf)^2.

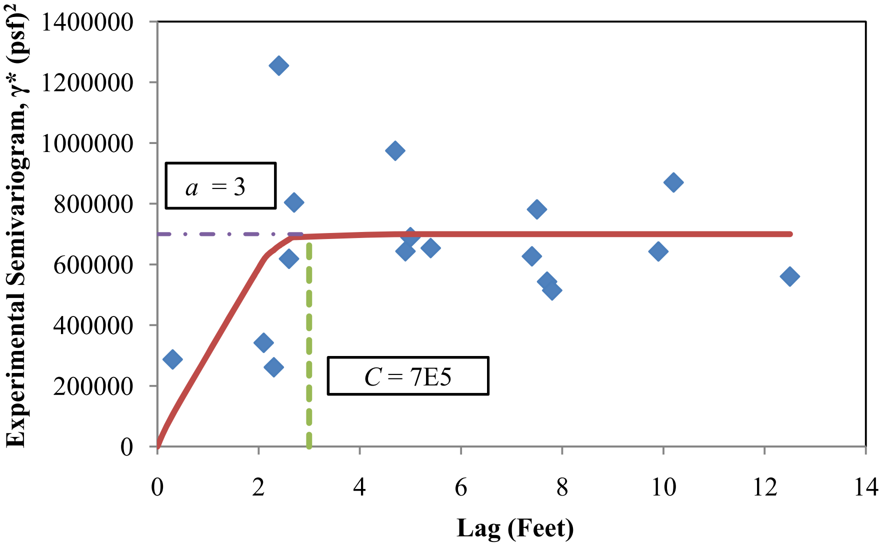

Figure 4. Experimental and theoretical variogram

For the spherical theoretical semivariogram model used in this study, the scale of fluctuation can be determined from Equation 2; where $g(h)$ can be determined from Equation 3. The scale of fluctuation, θ for the spherical theoretical semivariogram model is also defined by Meek (2001) as:

$$\theta = 3a/4 \tag{4}$$

The scale of fluctuation, θ for the spherical theoretical semivariogram model was determined from Equation 4,

$$\theta = 3 * 3/4 = 2.25\, feet$$

The determination of the scale of fluctuation, *θ*, is important in RBD. The scale of fluctuation is used in the spatial averaging of geotechnical properties to reduce their point variance. Vanmarcke (1977) defined a variance reduction factor, $\Gamma^2(\mathrm{L})$ as the ratio of the point variance, σ_i^2 and the variance of the spatially averaged property, σ_L^2.

$$\Gamma^2(\mathrm{L}) = \frac{\sigma_L^2}{\sigma_i^2} \tag{5}$$

Vanmarcke (1983) has suggested a variance reduction factor defined solely in terms of scale of fluctuation, θ and the averaging distance (that is, the distance over which a geotechnical property is averaged).

$$\Gamma^2(L) = \left[\frac{\theta}{L}\left(1 - \frac{\theta}{4L}\right)\right]^{\frac{1}{2}} \qquad for\ L/\theta > 1/2 \tag{6a}$$

$$\Gamma^2(L) = 1 \qquad for\ L/\theta \le 1/2 \tag{6b}$$

Once the variance of the geotechnical property is updated, accounting for its spatial variation as stated above, this is used in the limit state function under consideration, alongside the associated mean value, and the values of any other probabilistic and, or deterministic parameters.

The relationship between the variance reduction factor, $\Gamma^2(L)$ and the averaging length, *L* for undrained shear strength, $Su_{(UU)}$ profile for W-BH2 is presented in Figure 5. From Figure 5, it can be seen that the longer the averaging distance, *L* the lower the variance reduction factor and vice versa.

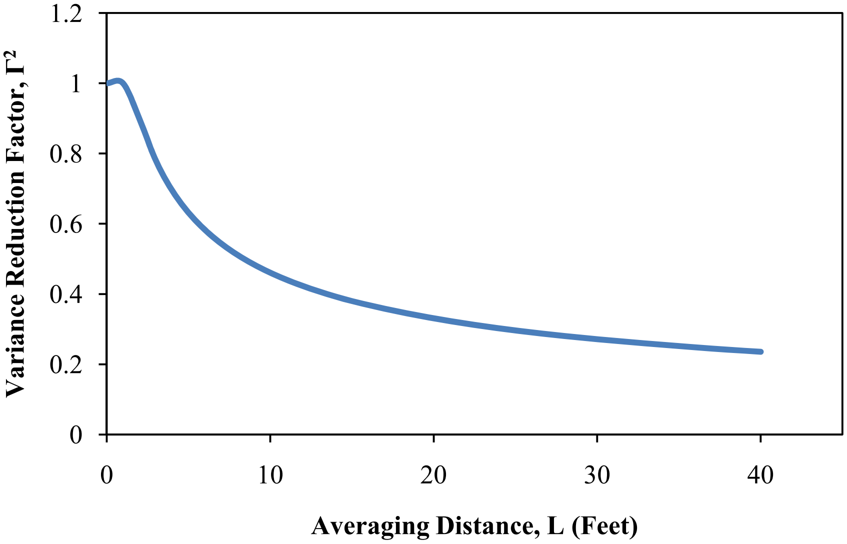

Figure 5. Variance reduction factor for undrained shear strength

To illustrate the spatial averaging effect, consider: (a) shallow footing; and (b) a pile foundation having averaging distances, *L* of 4 feet and 30 feet respectively. The averaging distance maybe the depth of the zone of influence in footing and pile foundation design or the length of a potential failure plane in slope stability analysis. From Figure 5, the variance reduction factor, $\Gamma^2(L)$ for the shallow footing ($L = 4$ ft) and the pile ($L = 30$ ft) are 0.70 and 0.27 respectively. Hence, the variance of the spatially averaged property, σ_L^2 (undrained shear strength in this case), which will be used in RBD, computed from Equation 5 will be 18188 and 7015 for the shallow footing ($L = 4$ ft) and the pile ($L = 30$ ft) respectively. This represents reductions in the point variance (= 25983 from Table 2) of 30% for the shallow footing and 75% for the pile foundation.

CONCLUSION

In this paper, spatial correlation analysis, determined in terms of the scale of fluctuation, θ, following the geostatistics approach was developed to evaluate the spatial variability of undrained shear strength profile at a location in Warrensburg in Missouri, where undisturbed samples were taken from Shelby tubes, obtained by continuous sampling, and undrained shear strengths were determined through the unconsolidated undrained triaxial shear strength (UU) tests conducted at about 2 to 3 feet centers.

Based on the analyses presented in this paper, the following conclusions were reached.

- The scale of fluctuation, θ of the undrained shear strength profile at the location of interest in Warrensburg in Missouri was determined, following the geostatistics approach, to be 2.25 feet.
- The reduction in the variance of geotechnical properties used in reliability-based geotechnical design (RBD) due to the spatial averaging effect was demonstrated an illustration where for a 30 feet averaging distance, L, the variance was reduced by about 75% from 25983 to 7015. Hence, neglecting this spatial averaging effect will lead to an overestimate of variability and will most likely lead to a very conservative design.
- The continuous sampling method used in this study is capable of providing data for variogram analysis when tests are conducted at close intervals, say, at 2 to 3 feet centers (about 2 determinations per 2.5 feet Shelby tube).

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