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SYNOPSIS: The parameters determined in laboratory tests for use in dynamic analyses are usually the secant modulus and the area of the hysteresis loop. The accuracy of this approach was studied for highly non-linear behaviour through analytical solutions and numerical simulations of a single degree of freedom system. Only the effect of the shape of the hysteresis loop was studied, i.e. the stiffness and the area of the loop were kept constant independent of amplitude. The study shows that irregular hysteresis may cause a significant increase in the response of the system in higher frequencies compared to what would be expected from visco-elastic and standard non-linear models. The area of the hysteresis loop provides a good measure of the damping also for high degrees of non-linearity.

INTRODUCTION

Highly non-linear soils are often represented by visco-elastic or hysteretic models in analytical procedures. In such models the stiffness is usually set equal to the secant modulus and the damping is obtained from the area of the hysteresis loop based on results from cyclic or dynamic laboratory tests. Non-linear soil behaviour, such as the change in the secant modulus and damping value with amplitude (strain), has been studied quite extensively in the past (e.g. Seed and Idriss, 1970; Hardin and Drnevich, 1972; Roesset, 1989). Effects of the shape of the hysteresis loop seems, on the other hand, to have received little attention.

This study investigates the effect of the shape of the non-linear hysteresis loop on the dynamic response of a single degree-of-freedom (1DOF) system. The influence of the other non-linear effects are excluded by keeping the area of the hysteresis loop and the secant stiffness constant independent of amplitude.

NON-LINEAR ELASTIC SPRINGS

Theoretical resonance values were obtained for free vibration of a 1DOF system with non-linear elastic spring. The equation of motion is:

$$M \cdot \ddot{u} + K(u) \cdot u = 0 \qquad (1)$$

where: M = mass
K(u) = non-linear spring
u = displacement
ü = acceleration

This equation may be solved by expanding the displacement as a sum of harmonic motions. In order to determine the resonance frequency for different forms of the spring, the equation of motion for free vibrations is rewritten as:

$$K(u) = -M \cdot \ddot{u}/u \qquad (2)$$

A simple solution is obtained by using only two terms in the series expansion for the displacement:

$$u = A_1 \sin\omega t + A_3 \sin 3\omega t \qquad (3)$$

Choosing M = 1 for simplicity gives:

$$K(u) = \omega^2(A_1\sin\omega t + 9A_3\sin3\omega t)/(A_1\sin\omega t + A_3\sin3\omega t) \qquad (4)$$

Setting K(u) and u equal to 1 for $\omega t = \pi/2$ gives:

$$u_{max} = A_1 - A_3 = 1 \qquad (5)$$

$$K(u{=}1) = \omega^2(A_1 - 9A_3) = 1 \qquad (6)$$

The resonance frequency, ω, for the system in free vibration is thus obtained:

$$\omega^2 = 1./(A_1 - 9A_3) = 1./(1. - 8A_3) \qquad (7)$$

This means that the resonance frequency may be smaller or greater than one, depending on the value of A_3. The resonance frequency based on the secant stiffness would of course be equal to one (M = K = 1) independent of the value of A_3.

Plot of the elastic restoring force, $K(u) \cdot u$, versus the displacement, u, is shown on Fig. 1 for ω = 0.8, 0.9, 1.0, 1.1, and 1.2.

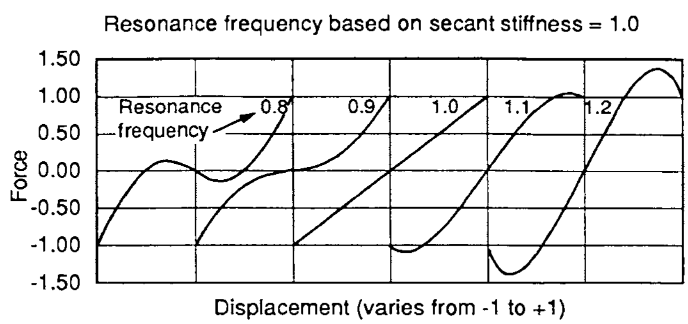

Fig. 1 Non-linear elastic force-displacement curves with different resonance frequencies

HYSTERETIC SPRINGS

The response of a system with a hysteretic spring was studied through a numerical simulation where the amplification and damping of different non-linear springs were compared with the theoretical solutions for visco-elastic and hysteretic systems. Figure 2 shows the hysteretic shapes that were used in the study:

- curve A is typical of a standard hysteretic or visco-elastic material
- curve B represents the shape often used in non-linear models (Ramberg-Osgood, hyperbolic or Iwan models)
- curve C is a bi-linear curve often used in non-linear models
- curve D is a laboratory curve from a highly non-linear soil
- curve E is a mirror image of curve D with the same Fourier spectrum

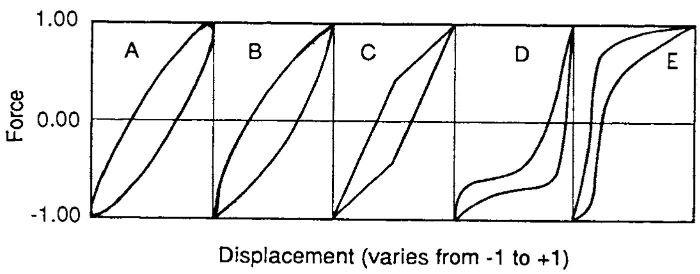

Fig. 2 Hysteretic shapes used in the study

The parameters and effects studied were the amplification, resonance frequencies based on both forced and free vibrations, damping ratio from the maximum amplification and the logarithmic decrement, and the spreading of energy.

Numerical Analysis

A computer program which determines the dynamic response of a 1DOF system with non-linear spring was developed. The program solves the following differential equation:

$$M \cdot \ddot{u} + C \cdot \dot{u} + K(u) \cdot u = P \cdot \sin\omega t \quad (8)$$

where the damping parameter C may be a constant (viscous damping), zero, or inversely proportional to frequency (hysteretic damping). The stiffness K(u) is specified as a table of force-displacement values and the force P may be a constant (harmonic excitation) or zero (free vibration).

The solution is based on direct integration of Newton's second law (explicit method). The time step used in the integration to obtain a stable solution is about 1/1000 of the period. Iterations on each half cycle are used to obtain the desired shape of the force-displacement relationships.

The accuracy of the program was tested and found acceptable by comparisons with theoretical amplification values for a visco-elastic and a hysteretic material (Selnes and Nadim, 1989), and with the theoretical values for the resonance of the non-linear elastic springs described earlier. The latter results are presented in Table 1. It may be seen that the program gives reasonable agreement with theory even for the shapes with negative stiffness in part of the loop.

Table 1 Comparison between resonance values from theory and numerical analysis for the non-linear elastic springs shown on Fig. 1

	Resonance Frequency				
Theoretical	.8	.9	1.	1.1	1.2
Computer prog.	.827	.89	1.	1.1	1.18

Damping and Resonance

Results of the response computations for the different hysteretic shapes are shown in Table 2. The damping in free vibration (logarithmic decrement), resonance frequency and amplification factors are listed. All shapes have hysteretic area corresponding to critical damping ratio equal to 0.1. This corresponds to amplification values of 5.01 and 5.0, and resonance frequencies of .99 and 1.0 for visco-elastic and hysteretic materials, respectively.

Table 2 Amplification at resonance, resonance frequency determined from free vibrations and damping determined from logarithmic decrement and from maximum amplitude

		Hysteretic Shape (Fig. 2)				
		A	B	C	D	E
Amplification		4.95	5.0	5.07	4.75	4.85
Resonance		.99	1.0	1.0	.76	.91
Damping	log. dec.	.1	.1	.1	.13	.125
	max amp.	.1	.1	.1	.105	.103

It may be seen from the table that the shapes often used in non-linear analyses (B and C) are not able to represent the behaviour of highly non-linear materials. The shapes D and E cause a significant shift in resonance frequency relative to the secant value. They also have somewhat higher damping in free vibration. The reason for the higher damping in free vibration is probably the reduction in amplitude coupled with the asymmetric loading and unloading cycle.

Amplification curves and attenuation in free vibrations are shown in Figs. 3 through 5. The shapes A and B give values close to the theoretical values for visco-elastic and hysteretic materials, while the highly non-linear shape D gives a significant shift in the resonance frequency.

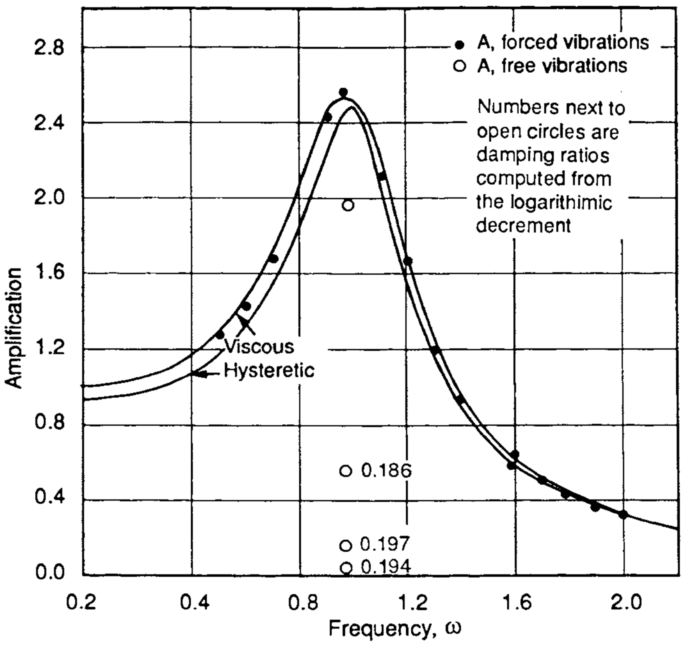

Fig. 3 Comparison between theoretical amplification values and values computed numerically for Type A hysteresis loop with 20% of critical damping

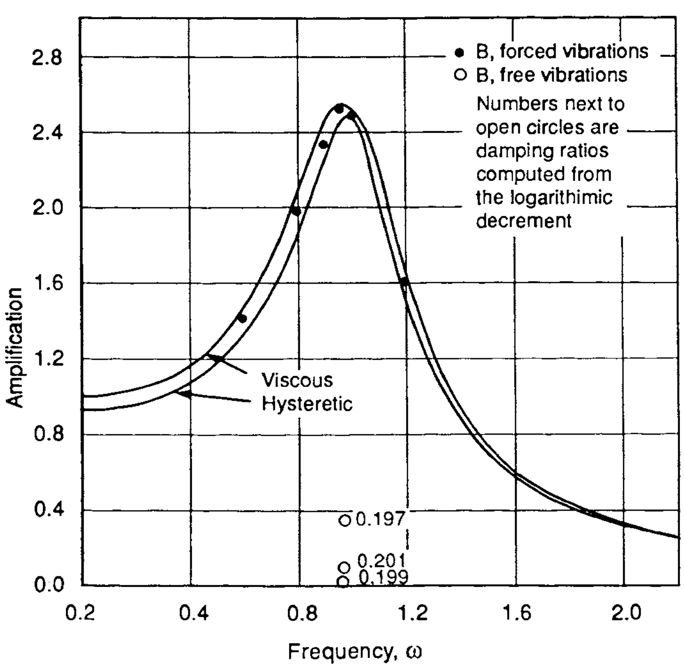

Fig. 4 Amplification and logarithmic decrement for Type B hysteresis loop with 20% of critical damping

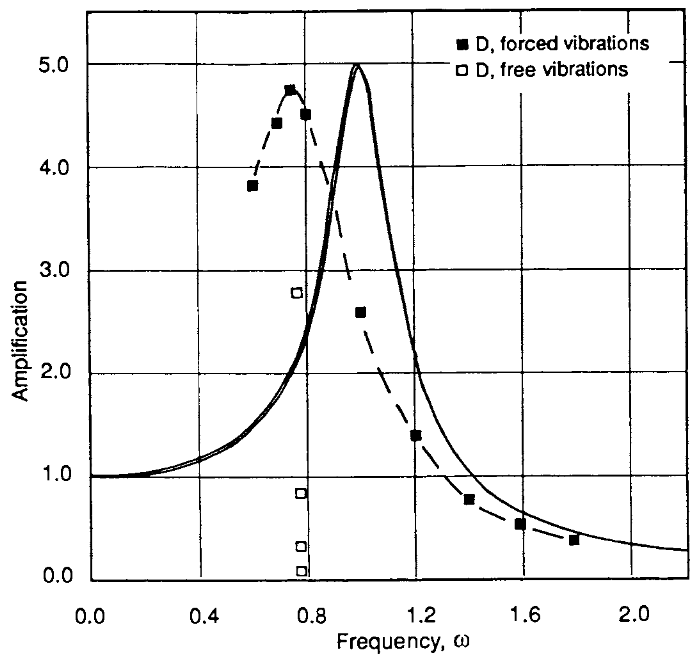

Fig. 5 Amplification and attenuation for Type D hysteresis loop with 10% of critical damping

Spreading of Energy

A non-linear system under harmonic excitation does not only respond at the forcing frequency. It also transfers energy to other, mainly higher frequencies. Figure 6 shows the Fourier spectra of the computed accelerations for hysteretic shapes D and E compared to shape A for forcing frequency equal to 1. The values are normalized to the same maximum acceleration. It may be seen that the highly non-linear shapes transfer energy to higher frequencies in distinct frequency bands. Figures 7 and 8 show the normalized response spectra for the same motions. It may be seen that the response from the irregular shapes D and E are different from the harmonic shape A, and that shapes with the same energy spreading characteristics (i.e. Fourier spectra) have different responses.

Energy spreading will cause a frequency-dependent apparent damping with low or even negative values for high frequencies, and increased damping for low frequencies. Apparent damping due to inhomogeneities was described by Menke et al. (1986). This work indicates that apparent damping may also be caused by non-linear behaviour.

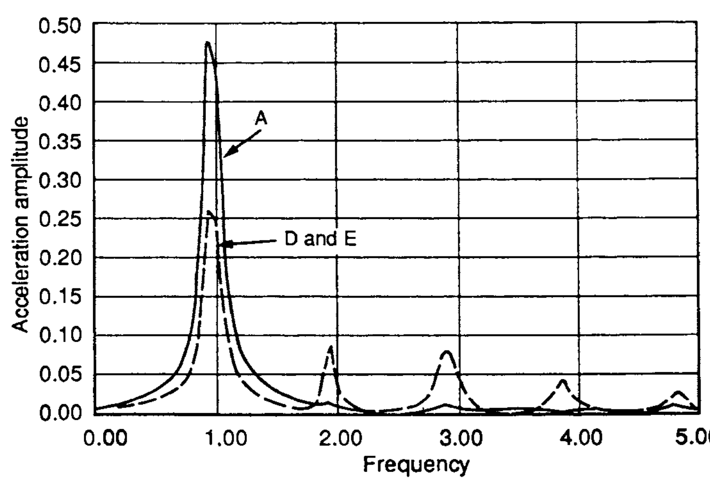

Fig. 6 Fourier spectra for driving force frequency equal to the resonance frequency for Shape A

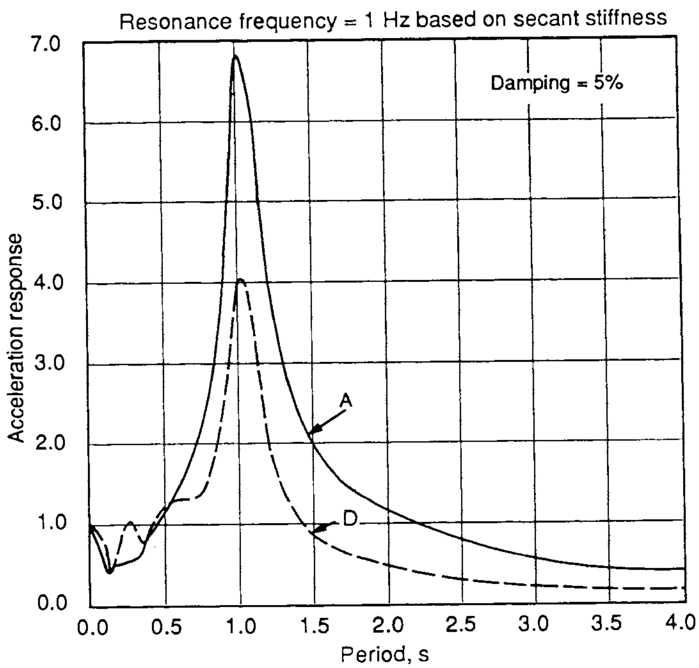

Fig. 7 Normalized response spectra for Shapes A and D for input harmonic motion at resonance frequency for Shape A

CONCLUSIONS

The results of the study may be summarized as follows:

- Damping values determined from the area of the hysteresis loop and from the maximum amplification ar in good agreement even for materials with highly non-linear behaviour, while damping values from the logarithmic decrement may be in less accord.
- High non-linearity may change the resonance frequency by up to 25-30% compared to what would be expected from the secant stiffness.
- Non-linearity causes spreading of energy towards higher frequencies, which may give significant frequency-dependent apparent damping.
- Hysteretic shapes often used in non-linear analyses give a dynamic response similar to that of a visco-elastic material - i.e. standard non-linear analyses do not provide realistic modelling of highly non-linear materials.

Figure 9 shows that high degrees of non-linearity may cause responses that are quite different from visco-elastic or standard non-linear models. The figure shows acceleration responses for the motions computed from a visco-elastic model and from a model with highly non-linear stiffness. As seen from the figure, the response for high frequencies is in this case up to 5 - 6 times higher for the non-linear spring than for a visco-elastic system with the same damping and the same secant stiffness. The visco-elastic model is obviously not able to represent this type of non-linear behaviour. High non-linearity may give considerably higher response in high frequency structures and structure components than would be expected from visco-elastic or standard non-linear models.

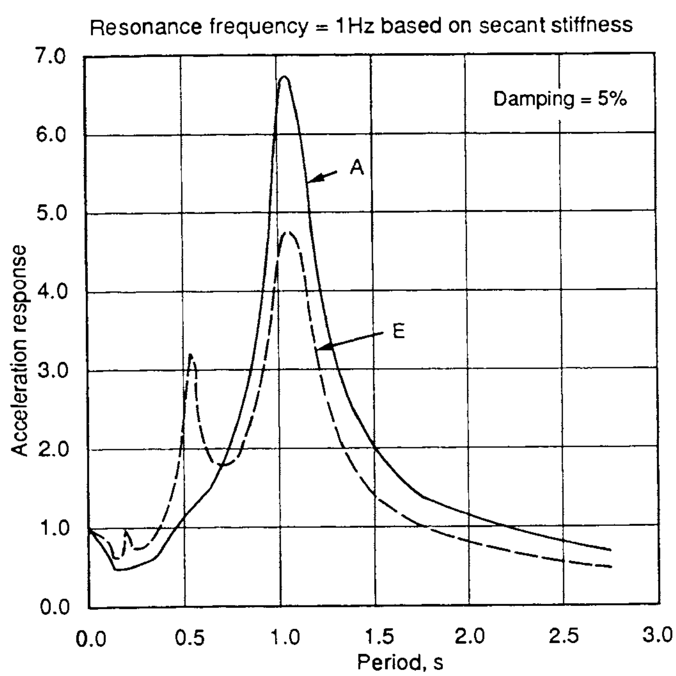

Fig. 8 Normalized response spectra for Shapes A and E for input harmonic motion at resonance frequency for Shape A

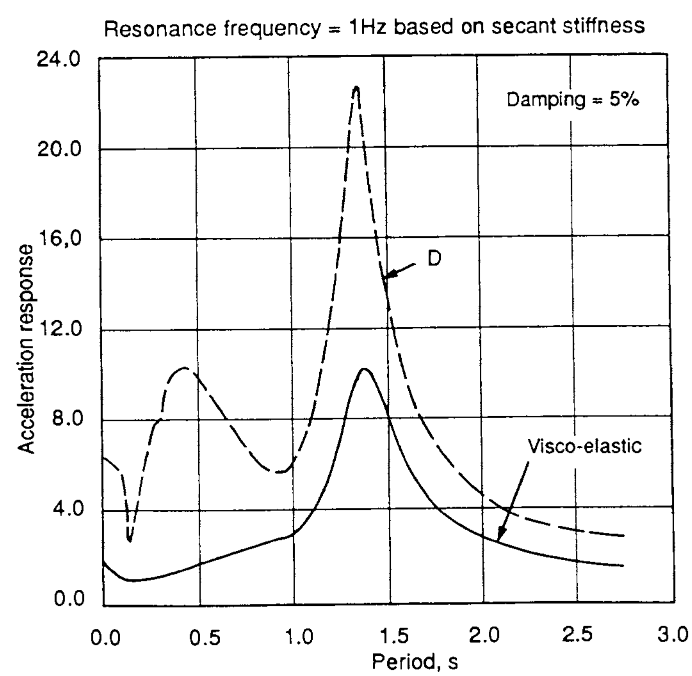

Fig. 9 Response spectra for Shapes A (visco-elastic) and D for input harmonic motion close to resonance frequency for Shape D

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