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Multimedia dispersion relation for surface electromagnetic waves*

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We have derived a general, n-media dispersion relation for surface electromagnetic waves propagating on isotropic layers with complex dielectric functions. The equation is presented in a convenient, compact form for ease of application.

INTRODUCTION

Recently the need has evolved to accurately describe the dispersion of surface electromagnetic waves (SEW) propagating in multilayer systems. The dispersion of SEW on an interface between two media has been studied with[1] and without[2] damping. In addition, the same problem has been examined for systems of three media. Mills and Maradudin[3] have studied this case, neglecting damping; Bell *et al.*[4] included damping in theoretical studies of three media and developed approximate solutions for the special case of infrared SEW propagating on a metal with a dielectric slab against a third weakly absorbing medium.

Developments in catalytic studies,[5] integrated optics, and other fields which need nondestructive testing techniques now require equations which describe SEW dispersion for systems of more than three media. Previous work in this area[6] has resulted in equations which are quite involved, recursive, or neglect retardation. We have developed a general, n-media dispersion relation for isotropic materials which includes damping, uses no approximations, is simple to use and program, and accurately describes SEW propagation on any system which can be modeled as a series of layers, each characterized by its own (complex) dielectric function.

METHOD OF DERIVATION

Figure 1 gives a pictorial representation of the system under consideration. Each layer m is described by a complex dielectric function ϵ_m and a thickness d_m. The wave is propagating in the x direction, and the decay in the z direction is either exponential or oscillatory.

The total field in each layer may be written as

$$E_m = \xi_{1m} e^{ik_x x} e^{k_{mz} z} \left(1, 0, \frac{-ik_x}{k_{mz}}\right) + \xi_{2m} e^{ik_x x} e^{-k_{mz} z} \left(1, 0, \frac{ik_x}{k_{mz}}\right). \tag{1}$$

If the boundary conditions $E_{\parallel}$ and $D_{\perp}$ continuous are applied at each interface, a system of equations results which may be solved by a determinant method to yield the dispersion relation for the number of layers under consideration.

The aforementioned technique was applied to systems of up to five media, and the expressions obtained are shown in Fig. 2. As one can see, a definite pattern begins to emerge. As n, the number of layers, increases, the cases of lower n appear as subsets of the equation for n layers. Each equation is made up of terms composed of multiplicative two-media solutions (with sign alternations) and an exponential multiplier. The equations (up to $n = 5$) were checked for degeneration by letting one or more of the thicknesses approach zero, and the proper lower forms were obtained.

THE GENERAL EQUATION

The evolving form just described established the possibility of one equation describing any number of layers. A general equation was deduced and checked by a prediction of the six-media solution, which was then independently derived. The general dispersion relation for SEW propagating on n isotropic media is

$$\sum_{p=1}^{2^{n-2}} e^{-2(\Sigma_{m=1}^{n-1} A_{p,m} \tilde{\alpha}_m d_m)} \prod_{m=1}^{n-1} \left(\frac{\tilde{\epsilon}_m}{\tilde{\alpha}_m} + (-1)^{Z_{p,m}} \frac{\tilde{\epsilon}_{m+1}}{\tilde{\alpha}_{m+1}} \right) = 0, \tag{2}$$

where each term p (see Fig. 2) is composed of an exponential multiplying a series of factors over m. The variables $\tilde{\alpha}_m$, $\tilde{\epsilon}_m$, and d_m are defined in the captions of Figs. 1 and 2. $A_{p,m}$ and $Z_{p,m}$ are sine functions which determine the existence of an exponential term and the sign within a factor, respectively. Their values may be 0 or 1, and they are defined as follows:

$$A_{p,m} = \sin^2\left(\tfrac{1}{2}\pi \gamma_{p,m}\right)$$

with

$$\gamma_{p,m} = \tfrac{1}{2} r_{p,m-1} + (p-1)\,\delta_m,$$
$$r_{p,m-1} = \gamma_{p,m-1} - \tfrac{1}{2}\left[1 - (-1)^{r_{p,m-2}}\right], \tag{3}$$

where

$$\delta_m = 2,\; m = 1$$
$$= 0,\; m \neq 1,$$
$$r_{p,-1} = 0, \text{ and } \gamma_{p,0} = 0,$$

and

$$Z_{p,m} = \sin^2\left(\tfrac{1}{2}\pi \beta_{p,m}\right),$$

with

$$\beta_{p,m} = \tfrac{1}{2}\beta_{p,m-1} + \tfrac{1}{4}\left[(-1)^{q_{p,m-2}} - (-1)^{q_{p,m-1}}\right] + (p-1)\delta_m,$$
$$q_{p,m-1} = \beta_{p,m-1} - \tfrac{1}{2}\left[1 - (-1)^{q_{p,m-2}}\right] \tag{4}$$

where

$$\delta_m = 1,\; m = 1$$
$$= 0,\; m \neq 1,$$

$q_{p,-1} = 0$, and $\beta_{p,0} = 0$.

Thus one can choose any n and write down the full dispersion relation for SEW using the preceding formulas.

It should be mentioned that Eqs. (2)–(4) are of forms which easily lend themselves to programming of the general equation on a computer. This allows one to vary the number of layers at will.

METHODS OF SOLUTION

In systems of more than two layers, it becomes more and more difficult to predict the types of solutions expected. Two media produce just one solution to the dispersion equation. However, the three-media approximations mentioned previously show multiple modes for the $n=3$ case. Higher order systems should produce a corresponding increase in the number of solutions.

We have developed a computer method which numerically gives solutions for the three-media case, and they agree with the approximation results[4] to within 1%. We are currently applying the same technique to obtain numerical results for systems of more than three layers.

APPLICATIONS

The chief characteristic of SEW investigations lies in the nondestructive nature of the technique. Surface electromagnetic wave spectroscopy (SEWS)[4] is currently being used as a tool for the study of catalytic reactions. The substrate, the thin layer of reactants, and the gas form a three-layer system; however, the inclusion of a contaminant layer extends the situation to a four-media problem.

It has been observed that under the proper conditions, SEW may be guided on a metal surface by a two-dimensional optics system.[7] The technique depends on being able to vary the thickness and/or composition of the optical components. Any desired effective index of refraction for the optical components may be obtained by multimedia layering. Our equation allows the removal of the middle layer thickness restrictions imposed by the three media approximations, so SEW dispersion calculations can be made for any thickness within the semi-infinite limit imposed by the wave decay normal to the direction of propagation.

Other possible applications are the investigation of layered paints on metals, laminated plastics studies, depletion layer studies, and biomedical investigations.

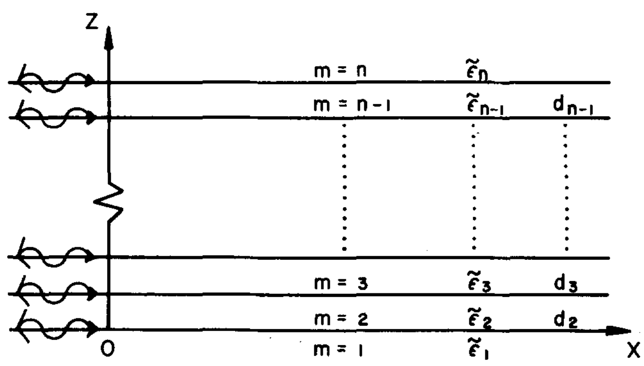

FIG. 1. Surface electromagnetic waves propagating on a system of n layers. The wave is propagating in the x direction. Each layer, m, has an associated complex dielectric function $\tilde{\epsilon}_m = \epsilon_{m1} + i\epsilon_{m2}$ and thickness d_m.

2 MEDIA
$$\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}\right)=0$$

3 MEDIA
$$\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}\right)\left(\frac{\epsilon_2}{\alpha_2}+\frac{\epsilon_3}{\alpha_3}\right)+e^{-2\alpha_2 d_2}\left(\frac{\epsilon_1}{\alpha_1}-\frac{\epsilon_2}{\alpha_2}\right)\left(\frac{\epsilon_2}{\alpha_2}-\frac{\epsilon_3}{\alpha_3}\right)=0$$

5 MEDIA (m →; n = 2, n = 3, n = 4, n = 5; p ↓)
$$\begin{aligned}
&\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}\right)\left(\frac{\epsilon_2}{\alpha_2}+\frac{\epsilon_3}{\alpha_3}\right)\left(\frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_4}{\alpha_4}\right)\left(\frac{\epsilon_4}{\alpha_4}+\frac{\epsilon_5}{\alpha_5}\right)\\
&+e^{-2\alpha_2 d_2}\left(\frac{\epsilon_1}{\alpha_1}-\frac{\epsilon_2}{\alpha_2}\right)\left(\frac{\epsilon_2}{\alpha_2}-\frac{\epsilon_3}{\alpha_3}\right)\left(\frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_4}{\alpha_4}\right)\left(\frac{\epsilon_4}{\alpha_4}+\frac{\epsilon_5}{\alpha_5}\right)\\
&+e^{-2\alpha_3 d_3}\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}\right)\left(\frac{\epsilon_2}{\alpha_2}-\frac{\epsilon_3}{\alpha_3}\right)\left(\frac{\epsilon_3}{\alpha_3}-\frac{\epsilon_4}{\alpha_4}\right)\left(\frac{\epsilon_4}{\alpha_4}+\frac{\epsilon_5}{\alpha_5}\right)\\
&+e^{-2(\alpha_2 d_2+\alpha_3 d_3)}\left(\frac{\epsilon_1}{\alpha_1}-\frac{\epsilon_2}{\alpha_2}\right)\left(\frac{\epsilon_2}{\alpha_2}+\frac{\epsilon_3}{\alpha_3}\right)\left(\frac{\epsilon_3}{\alpha_3}-\frac{\epsilon_4}{\alpha_4}\right)\left(\frac{\epsilon_4}{\alpha_4}+\frac{\epsilon_5}{\alpha_5}\right)\\
&+e^{-2\alpha_4 d_4}\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}\right)\left(\frac{\epsilon_2}{\alpha_2}+\frac{\epsilon_3}{\alpha_3}\right)\left(\frac{\epsilon_3}{\alpha_3}-\frac{\epsilon_4}{\alpha_4}\right)\left(\frac{\epsilon_4}{\alpha_4}-\frac{\epsilon_5}{\alpha_5}\right)\\
&+e^{-2(\alpha_2 d_2+\alpha_4 d_4)}\left(\frac{\epsilon_1}{\alpha_1}-\frac{\epsilon_2}{\alpha_2}\right)\left(\frac{\epsilon_2}{\alpha_2}-\frac{\epsilon_3}{\alpha_3}\right)\left(\frac{\epsilon_3}{\alpha_3}-\frac{\epsilon_4}{\alpha_4}\right)\left(\frac{\epsilon_4}{\alpha_4}-\frac{\epsilon_5}{\alpha_5}\right)\\
&+e^{-2(\alpha_3 d_3+\alpha_4 d_4)}\left(\frac{\epsilon_1}{\alpha_1}+\frac{\epsilon_2}{\alpha_2}\right)\left(\frac{\epsilon_2}{\alpha_2}-\frac{\epsilon_3}{\alpha_3}\right)\left(\frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_4}{\alpha_4}\right)\left(\frac{\epsilon_4}{\alpha_4}-\frac{\epsilon_5}{\alpha_5}\right)\\
&+e^{-2(\alpha_2 d_2+\alpha_3 d_3+\alpha_4 d_4)}\left(\frac{\epsilon_1}{\alpha_1}-\frac{\epsilon_2}{\alpha_2}\right)\left(\frac{\epsilon_2}{\alpha_2}+\frac{\epsilon_3}{\alpha_3}\right)\left(\frac{\epsilon_3}{\alpha_3}+\frac{\epsilon_4}{\alpha_4}\right)\left(\frac{\epsilon_4}{\alpha_4}-\frac{\epsilon_5}{\alpha_5}\right)=0
\end{aligned}$$

FIG. 2. Surface electromagnetic wave dispersion relations for two to five media. The solid lines in the $n=5$ equation show the lower-n equations appearing as subsets. The complex variable $\tilde{\alpha}_m$ is defined as $\tilde{\alpha}_m = {}^c\sqrt{\tilde{k}_x^2 - \tilde{\epsilon}_m(\omega/c)^2} = \tilde{k}_{mz}$, where ω/c is the frequency in wavenumbers and $\tilde{k}_{mz}$ is the wave vector in the z direction.

CONCLUSION

It may be seen from the preceding discussion that the SEW n-media dispersion relation presented can be used as a tool in SEW investigations of systems of several isotropic layers, each characterized by its own complex dielectric function. The compactness of the form and the elimination of previously imposed restrictions implies relatively easy, accurate usage.

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