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SYNOPSIS: The deformation behavior of sand under simple shear loading was studied. The relation between the shear stress ratio and fabric factor, and the relationship between the fabric and dilatancy rate of a sand were developed. Thus the stress-dilatancy relation for sand was derived. Adopting the concept that the volumetric changes of the sample induced by the cyclic shear stress were compensated by the volumetric changes due to vertical stress changes in the constant volume simple shear tests, the relation of shear stress ratio and vertical stress change was derived.

INTRODUCTION

It is the well-known fact that the application of cyclic loading, such as that induced by earthquakes, to sands results in a progressive decrease in volume. If drainage is unable to occur during the time span of the loading sequence, then the tendency for volume change during cyclic loading causes a corresponding pore water pressure change.

The cyclic simple shear tests can more closely simulate field conditions, such as rotations of the principal directions and may be used to study the deformation of sand during earthquake loading.

The cylindrical specimen was enclosed in a rubber membrane supported by the stacked thin teflon rings and placed in the pressure chamber in this research. Because the stacked teflon rings prohibited lateral deformations, the sample was consoliated by a vertical stress under Ko consoliation and later was sheared by a cyclic horizontal shear stress without lateral deformations. It hasnot proved possible to perform truly undrained tests in the simple shear devices and the undrained tests are normally investigated by performing drained tests at constant volume.The changes in vertical stress that are seen should correspond to the pore water pressure changes that would be meassured in an undrained test. The relationship between the stress-dilatancy relation in the drained test and corresponding pore water changes in the undrained test was established for the cyclic simple shear tests in this paper.

THE STRESS-DILATANCY RELATION

The deformation behavior of sand under shear loading had been studied based on a particulate approach considering the slip and non-slip contacts between grains on a microscopic shear plane(Lee,1987). The relationship between the shear stress ratio, which was the ratio of shear stress, τ, to normal stress, σn, on the shear plane and the fabric factor, Z, which depended on the distribution of contact angles between grains on a microscopic shear plane was:

For positive shear stress

$$\frac{\tau}{\sigma n} = \tan\phi_\mu + \sec^2\phi_\mu * Z \qquad ....(\text{1-a})$$

For negative shear stress

$$\frac{\tau}{\sigma n} = -\tan\phi_\mu - \sec^2\phi_\mu * Z \qquad ....(\text{1-b})$$

where ϕ_μ = basic friction angle between the sand grains. The relation between the fabric factor, Z, and average dilatancy rate, $(d\epsilon/d\gamma)$, of the sand mass was obtained by considering the displacement relationship of all sliding sand grains in contacts on the shear plane. Noted that the displacement wasnot contributed by the non-slip contacts. Hence

$$Z = \alpha(\theta) * \left(\frac{d\epsilon}{d\gamma}\right) \qquad ....(2)$$

where $\alpha(\theta)$ = a positive value which depended on number of slip and non-slip contacts; $d\epsilon$ and $d\gamma$ are normal strain increment and shear strain increment on the shear plane. Therefore the stress-dilatancy relation became:

For positive shear stress

$$\frac{\tau}{\sigma n} = \tan\phi_\mu + \sec^2\phi_\mu * \alpha(\theta) * \left(\frac{d\epsilon}{d\gamma}\right) ..(\text{3-a})$$

For negative shear stress

$$\frac{\tau}{\sigma n} = -\tan\phi_\mu - \sec^2\phi_\mu * \alpha(\theta) * \left(\frac{d\epsilon}{d\gamma}\right) ....(\text{3-b})$$

In considering the simple shear tests in which no lateral deformation was allowed, (Eq.3) on the horizontal shear plane can be expressed as:

$$\frac{\tau}{\sigma y} = \pm\tan\phi_\mu \pm \frac{\sec^2\phi_\mu * \alpha(\theta)}{V0} * (\frac{dV1}{d\gamma}) \quad ..(4)$$

where σy = vertical stress applied on the horizontal plane; τ =shear stress applied on the horizontal plane; V0=total sample volume; dV1=the volume change increment; $(dV1/d\gamma)$ = average dilatancy rate with dilation positive.

RELATIONSHIP BETWEEN VERTICAL STRESS RATE IN CONSTANT VOLUME TESTS AND PORE WATER PRESSURE CHANGES IN UNDRAINED TESTS

Consider a sample subjected to a shear in a constant load simple shear drained test. Let dV1 =the volume change increment due to slip at grains contacts under shearing. It has been shown that dV1 is a function of magnitude of shear stress, relative density and the number of cycles. Consider now the same shear stress as given previously was applied to the same sample in the constant volume condition. Assume that intergranular contact forces induced in the constant volume shearing were similar to those induced in the constant load drained shearing. Therefore the slip at grain contacts resulting in the volume change increment,dV1, must also occur. The vertical stress changes, $d\sigma y$, had been found from the constant volume test data. The vertical stess changes no doubt resulted in the volume changes of sand structure. No vertical stress was carried by the water in the constant volume tests. Hence the compatibility condition of no changing volume under shearing gived :

change in volume of voids = net change in volume of sand structure

$$dV1 = V0 * d\sigma y / D1 \quad ....(5)$$

where $D1 = d\sigma y/(dV1/V0)$ = one dimensional bulk modulus of the sand structure; $d\sigma y$ = vertical stress changes due to constant volume shearing. Consider the same condition as given previously was applied under undrained shearing. The relationship between dV1 and pore water pressure change,du, had been derived by Martin et al(1975) . It was

$$dV1 = V0 *(-du) / D2 \quad ....(6)$$

D2 had the same value as D1 if the anistropy of the sample was neglected. Then

$$du = -d\sigma y \quad ....(7)$$

Therefore it would seem resonable to assume that the vertical stress change with the constant volume tests should correspond to the pore water change measured in the undrained tests. Hence substituting (Eq.5) into (Eq.4) gived:

$$\frac{\tau}{\sigma y} = \pm\tan\phi_\mu \pm \frac{\sec^2\phi_\mu * \alpha(\theta)}{D1} * (\frac{d\sigma y}{d\gamma})$$

$$\frac{\tau}{\sigma y} = \pm\tan\phi_\mu \pm \frac{A}{D1} * (\frac{d\sigma y}{d\gamma}) \quad ....(8)$$

where $A= \sec^2\phi_\mu * \alpha(\theta)$; $(d\sigma y/d\gamma)$=vertical stress rate

The insufficient frame stiffness may caused the inevitable vertical movement of vertical load ram and caused the volume changes,dV2, with dilation positive during constant volume tests. dV2 can be measured with two LVDTs which were mounted at vertical ram throughout the tests. In order to correct the error induced by the insufficient frame stiffness, (Eq.5) may be rewritten as :

$$dV1 = V0 * \frac{d\sigma y}{D1} - dV2 \quad ....(9)$$

Substituting (Eq.9) into (Eq.4) gived:

$$\frac{\tau}{\sigma y} = \pm\tan\phi_\mu \pm \frac{A}{D1} * (\frac{d\sigma y}{d\gamma} - \frac{dV2/V0*D1}{d\gamma}) \quad ..(10)$$

EVALUATION OF THE STRESS-DILATANCY RELATION

The slow constant load drained cyclic simple shear tests (deformation rate = 0.075mm/sec) were performed on Ottawa sand and Fulung sand of various densities and under different normal stresses.

Ottawa sand was a commercially available sand, consisting of rounded hard quartz grains, having a specific gravity of 2.63 and maximun and minimum void ratios of 0.77 and 0.49, respectively. Fulung sand was a fine beach sand, composed mainly of subangular quartz grains, having a specific gravity of 2.66 and maximum and minimum void ratios of 1.01 and 0.68, respectively. These two sands were sieved to pass the No.40(425μm) sieve and retained on the No.100 (149μm) sieve.

The test specimens were prepared using dry pluviating method. The oven-dried sand was poured through air from a pre-determined height to form the specimens with the desired density. The dimensions of the specimens were 10 cm in diameter and 2.5cm in height.

Fig.1 and Fig.2 were the typical results of $(\tau/\sigma y)$ versus $((dV1/V0)/d\gamma)$ for Ottawa sand and Fulung sand of various densities under different normal stresses . In these figures, the shear stress , τ , was considered to be positive clockwise, the increment of shear strain,$d\gamma$,was always positive regardless of the deformation direction, the increment of vertical strain was positive with increasing sample thickness, and dilatancy rate, $((dV1/V0)/d\gamma)$, was positive with volume increase. It appears that the experimental data can be represented fairly well by the relation in (Eq.4). Scattering of data was observed at the beginning of the shear stress reversal which may be due to difficulty in measuring the extremely small deformation responding to a rather larger stress change during shear stress reversal. Based on the test results, $\sec^2\phi_\mu * \alpha(\theta)$ can be considered virtually a constant for each sand.

When the shear stress increased,

$$\frac{\tau}{\sigma_y} = 0.42+1.68*(\frac{dV1/V0}{d\gamma}) \text{for Ottawa sand.} \quad (11\text{-}a)$$

$$\frac{\tau}{\sigma_y} = 0.45+1.815*(\frac{dV1/V0}{d\gamma}) \text{for Fulung sand.} \quad (12\text{-}a)$$

On the other hand, when the shear stress decreased,

$$\frac{\tau}{\sigma_y} =-0.42-1.68*(\frac{dV1/V0}{d\gamma}) \text{for Ottawa sand} \quad (11\text{-}b)$$

$$\frac{\tau}{\sigma_y} =-0.45-1.815*(\frac{dV1/V0}{d\gamma}) \text{for Fulung sand} \quad (12\text{-}b)$$

These equations did not appear to be affected by the initial density, normal stress or number of stress cycles.

EVALUATION OF THE SHEAR STRESS RATIO AND VERTICAL STRESS RATE

The slow constant volume cyclic simple shear tests (deformation rate = 0.075mm/sec) were performed on Ottawa sand and Fulung sand of various densities and under the different normal stress levels as well. After the samples had been consoliated at the predetermined normal stress, the vertical load ram was locked before the horizontal cyclic shear was applied. The values of dV2, the vertical movement of the vertical load ram, were continuously monitored during the horizontal cyclic shear stress applied. D1 can be calculated after performing one dimensional consoliation tests at the corresponding strss level. Hence (Eq.8) can be rewritten as:

When the shear stress increased,

$$\frac{\tau}{\sigma_y} = 0.42 + \frac{1.68}{D1}(\frac{d\sigma_y}{d\gamma}) \text{ for Ottawa sand} \quad (13\text{-}a)$$

$$\frac{\tau}{\sigma_y} =0.45 + \frac{1.815}{D1}(\frac{d\sigma_y}{d\gamma}) \text{for Fulung sand.} \quad (14\text{-}a)$$

When the shear stress decreased

$$\frac{\tau}{\sigma_y} = -0.42-\frac{1.68}{D1}(\frac{d\sigma_y}{d\gamma}) \text{ for Ottawa sand} \quad (13\text{-}b)$$

$$\frac{\tau}{\sigma_y} = -0.45-\frac{1.815}{D1}(\frac{d\sigma_y}{d\gamma}) \text{for Fulung sand} \quad (14\text{-}b)$$

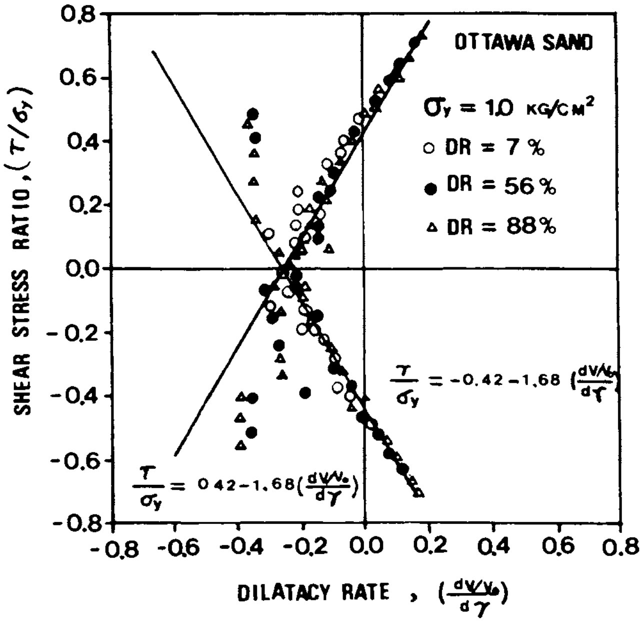

Fig.1 Shear stress ratio versus dilatancy rate in constant load cyclic drained simple shear tests on Ottawa sand

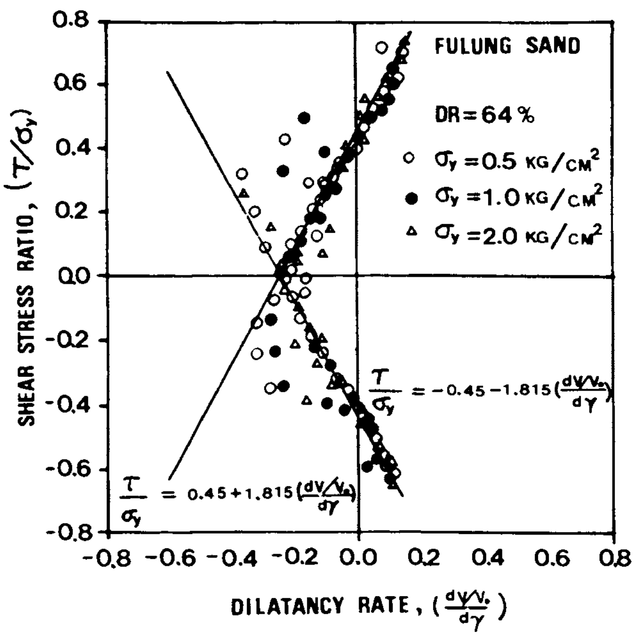

Fig.2 Shear stress ratio versus dilatancy rate in constant load cyclic drained simple shear tests on Fulung sand

Fig.3 and Fig.4 showed that the typical results of (τ / σ_y) versue ($d\sigma_y/d\gamma$) for Ottawa sand and Fulung sand . The solid lines in these figures represented (Eq.13) and (Eq.14), respectively. The hollow circles represented the measured data with the constant volume tests and the solid circles represented the predicted results which were corrected by (Eq.10).
Comparing the measured data (hollow circles) with the predicted data (solid circles), it showed good agreement for the first cyclic shearing. The discrepancy between the measured

and the predicted data gradually increased with the increase of number of shearing cycles. This may be due that the values of D1 and the fabric of sand were changed when the samples were subjected to the cyclic simple shearing.

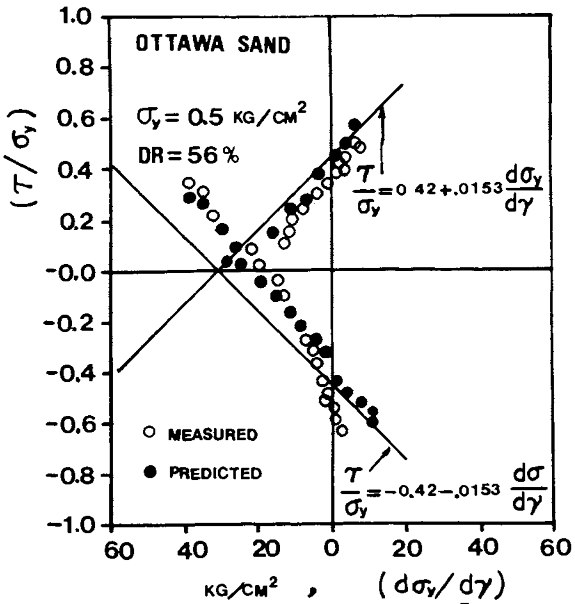

Fig.3 Shear stress ratio versus vertical stress rate in constant volume cyclic simple shear tests on Ottawa sand

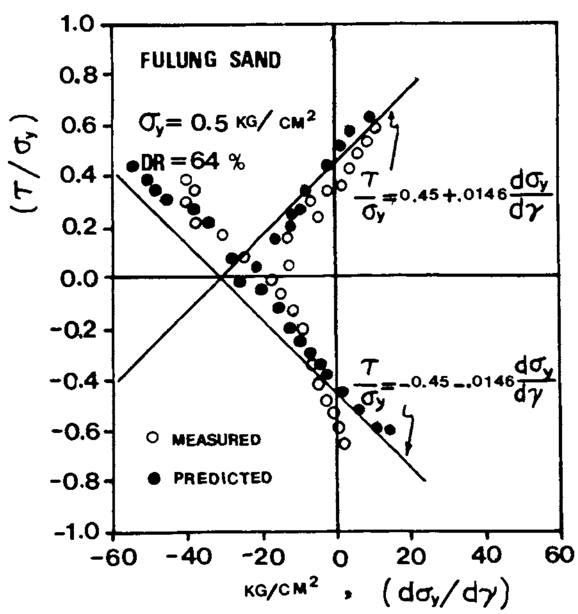

Fig.4 Shear stress ratio versus vertical stress rate in constant volume cyclic simple shear tests on Fulung sand

CONCLUSIONS

The deformation behavior of sand under shear loading was studied based on the particulate approach considering slip and non-slip contacts between grains on a macroscopic shear plane. The relationship was developed between the shear stress ratio and fabric factor for a sand in a state of equilibrium. The relation between the fabric and the average dilatancy rate of the sand was obtained by considering the displacement relationship of sliding sand grains in contact. Thus the stress-dilatancy relation for sand was derived accordingly.
Adopting the concept that the volumetric changes of the sample induced by the cyclic shear stress were compensated by the volumetric changes due to vertical stress changes in the constant volume simple shear tests, the relation of shear stress ratio and vertical stress change was derived. The results of slow cyclic simple shear tests on Ottawa sand and Fulung sand agreed reasonablly well with the theorectical stress-dilatancy relation in the constant load simple shear tests and theorectical shear stress ratio-vertical stress change relation in the constant volume simple shear tests.

ACKNOWLEDGMENT

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