05 Apr 1995, 1:30 pm - 3:30 pm

Dynamic Behavior of Beijing Sandy Clay and the Dynamic Responses of Beijing Subway to Traffic Vibrations

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Proceedings: Third International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, April 2-7, 1995, Volume II, St. Louis, Missouri

Dynamic Behavior of Beijing Sandy Clay and the Dynamic Responses of Beijing Subway to Traffic Vibrations

Paper No. 11.24

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SYNOPSIS Experiments were carried out for the Beijing sandy clay using a triaxial apparatus Model DTC—240 (JAPAN) with the dynamic characteristics of the soil obtained and formulated. The test data were applied to the 2—D and 3—D finite element analyses for the dynamic responses of Beijing Subway to traffic vibrations. Resulte acquired from the numerical analyses happened to be in good coincidence with the measured responses.

OBJECTIVE

The project sponsored by the Beijing Subway Company is aimed at acquiring necessary data to verify the feasibility of the construction of a new subway line from the environmental considerations. This paper is to present some unpublished results concerning the dynamic behavior of Beijing sandy clay and the dynamic responses of the subway to traffic vibrations in addition to the former studies (Pan, 1989).

DYNAMIC BEHAVIOR OF BEIJING SANDY CLAY

Samples of sandy clay were collected at the Xidan station site of Beijing subway and transported to Lanzhou for the laboratory tests carried out on the servo—controlled triaxial testing apparatus Model DTC — 240. The physical characteristics of the undisturbed soil samples being as follows: specific gravity g = 2. 70; dry weight γ_d=14. 4kN/m^3; unit weight γ=19. 0kN/m^3; natural water content w=33. 3%; porosity ratio e = 0. 89; plastic limit W_p=19. 5%; liquid limit W_l= 27. 9; plastic index I_p=8. 4.

Normally consolidated (K_c = 1. 0) samples under undrained condition were loaded by multicyclic axial loads so that the normal pressure σ_1=100,150,200kPa respectively with 10 cycles (equivalent to an M7 earthquake) of 1Hz sinusoidal wave vibrations being adopted.

In Fig. 1 are presented the ε_d versus E_d, ε_d versus $1/E_d$ and ε_d versus σ_d curves under different normal pressures σ_1 and corresponding confine pressures $\sigma_3 = \sigma_1$. Regression curves (Fig. 2) were obtained by assuming a hyperbolic model $\sigma_d = \varepsilon_d/(\tilde{a} + \tilde{b}\varepsilon_d)$ and they were in good coincidence with the test curves.

From these figures it is observed that the sandy clay exhibits a dynamically strain hardening character; the dynamic moduli E_d decrease with the increase of the dynamic strains and increase with the increase of nor-

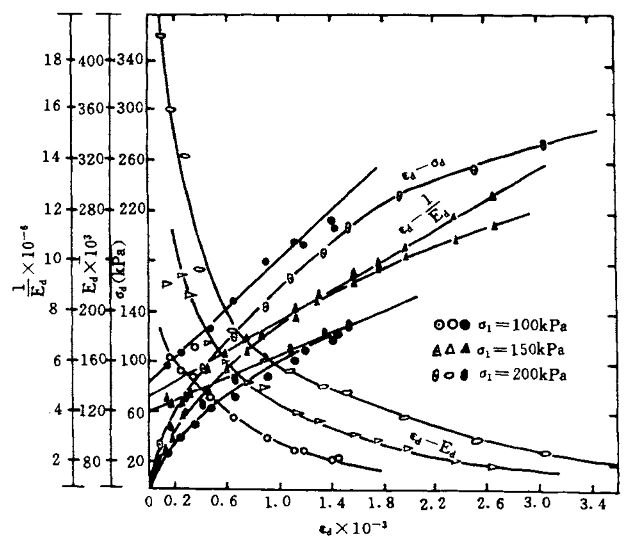

Fig. 1

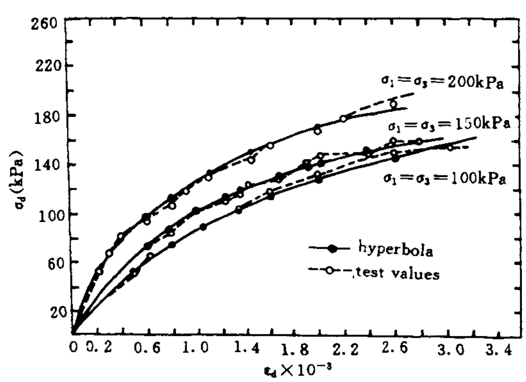

Fig. 2

mal consolidating pressure σ_1 when the dynamic strain keeps constant. With $E_{d,max}=1/a$ and $G_{d,max}=E_{d,max}/2(1+\mu)$, from these ourves we obtain $G_{d,max}$=72296—93985kPa, while by using the regression values $\tilde{a}$, $G_{d,max}$=70071—121101kPa.

In order to determine the dynamic shear modulus of the soil (G_0) an impirical formula proposed by Hardin (1972) is adopted and rewritten in the form:

$$G_0 = A_g \frac{(2.973-e)^2}{1+e}(\sigma_m)^{1/2}(K_c)^k \qquad (1)$$

Where A_g is a constant to be obtained inversely from the test data. With $e=0.89$, $\sigma_m=100, 150, 200$kPa, $K_c=1.0$, $k=0.07$ (for $I=8.4$), G_0 obtained as the average of the above-mentioned $G_{d,max}$ values, we obtain $A_g=3204$, which is very close to the Hardin's coefficient 3260.

From the test curves λ versus $\log \gamma_d$ in Fig. 3, it is obvious that the damping ratio of the sandy clay ranges from 0.12 to 0.20.

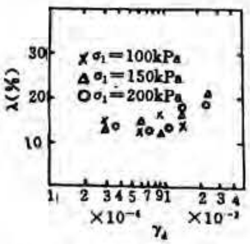

Fig. 3 $\lambda-\log \gamma_d$ graph

To obtain the dynamic strength of the soil in the laboratory 9 samples under different consolidating stresses (75, 100, 150kPa) were used to obtain ε_d versus log N curves, where N is the number of cycles of the dynamic load. With 5% set as the failure strain the failure stress σ_{df} versus N curves were obtained as shown in Fig. 4 and for $N_c=10$ the corresponding failure stresses were acquired. With these data the dynamic cohesion c_d and friction angle φ_d were obtained graphically by Mohr's circles (Fig. 5) and by method of least squares which gave rise to $c_d=17.6$kPa and $\varphi_d=24°28'$ for OCK=1.

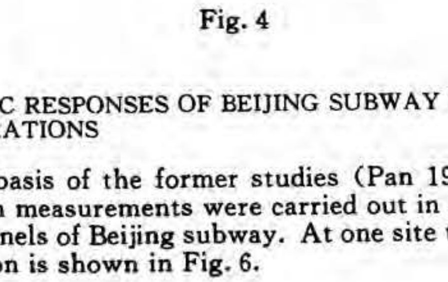

Fig. 4

DYNAMIC RESPONSES OF BEIJING SUBWAY TO TRAFFIC VIBRATIONS

On the basis of the former studies (Pan 1989), new vibration measurements were carried out in three running tunnels of Beijing subway. At one site the instrumentation is shown in Fig. 6.

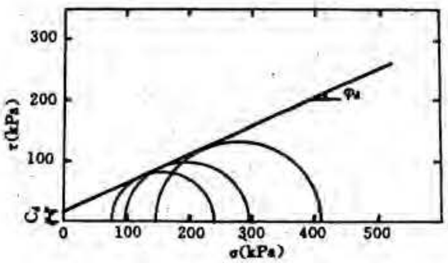

Fig. 5 Mohr circles

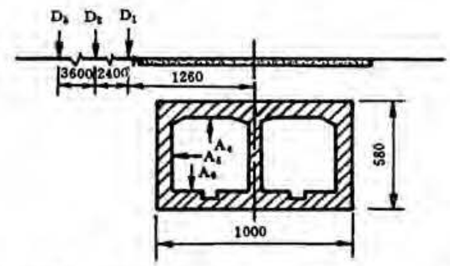

Fig. 6 Instrumentation (Units in cm)

Dynamic signals were accepted by a multifunction signal proccessor Model INV-303 at a sampling frequency of 512 Hz.

Vertical acceleration history and the simulated train loading acquired in the same way as in ref. (Pan, 1989) are shown in Fig. 7.

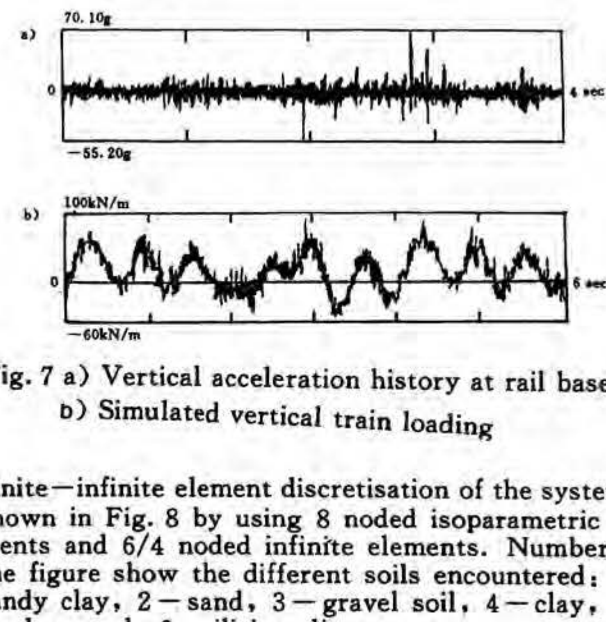

Fig. 7 a) Vertical acceleration history at rail base
b) Simulated vertical train loading

Finite-infinite element discretisation of the system is shown in Fig. 8 by using 8 noded isoparametric elements and 6/4 noded infinite elements. Numbers in the figure show the different soils encountered: 1-sandy clay, 2-sand, 3-gravel soil, 4-clay, 5-sandy gravel, 6-silicious limestone.

Material parameters of the soils used in the analysis were determined by the above-mentioned laboratory test and from other resourses.

A Newmark implicit time integration algorithm was employed to solve the equation of motion of the system:

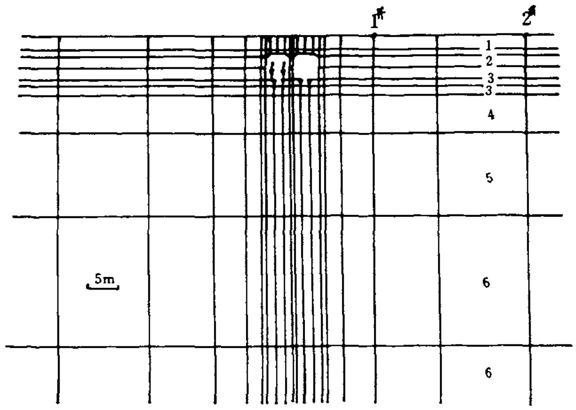

Fig. 8 Finite/infinite element discretisation

$$\underline{M}\,\ddot{\underline{U}}+\underline{C}\,\dot{\underline{U}}+\underline{K}\underline{U}=\underline{F}(t) \qquad (2)$$

Where $\underline{M}$, $\underline{C}$, and $\underline{K}$ are the global mass matrix, damping matrix and stiffness matrix of the system; $\ddot{\underline{U}}$, $\dot{\underline{U}}$ and $\underline{U}$ are acceleration, velocity and displacement vectors; $\underline{F}(t)$ is the vertical train loading acting on two rails only for one line of the tunnel. A Rayleigh damping is adopted: $\underline{C}=\alpha\underline{M}+\beta\underline{K}$, with $\alpha=0.03$, $\beta=0.01$. The Newmark constants being $\delta=0.5$, $\gamma=0.25$.

Free vibration characteristics of the system are shown in Table 1.

Table 1

No. of mode shape	ω (rad/sec)	f (Hz)	T (sec)
1	10. 207	1. 6245	0. 6156
2	10. 495	1. 6703	0. 5987
3	14. 513	2. 3098	0. 4329
4	15. 380	2. 4478	0. 4085
5	16. 622	2. 6455	0. 3780

The time step length adoped in the 9 sec time integration process is taken as $\Delta t=0.006$sec in which 6 seconds are assumed as required for the train to pass through the section under consideration while during the rest 3 sec the system will undergo free vibration.

Fig. 9 indicates the acceleration history of the system. Fig. 10 shows the vertical displacement history at some specified points of the system. A maximum settlement of 0. 5mm on the ground surface due to the subway traffic is predicted. Fig. 11 shows the principal stress history of the invert. A maximum tensile stress of 474 kN/m^2(4. 74 kgf/cm^2) is predicted.

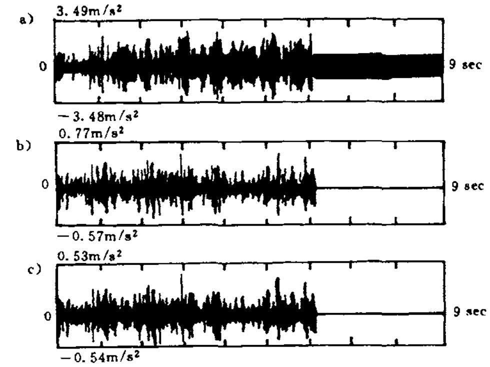

Fig. 9 Acceleration history

a) Vertical component at rail base
b) Horizontal component at wall center
c) Vertical component at roof center

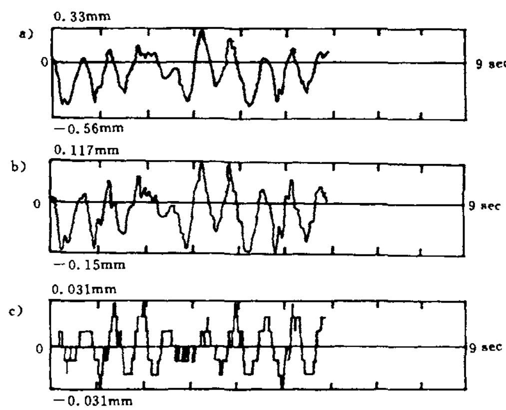

Fig. 10 Vertical displacement history
a) At roof center;
b) At pt. No. 1;
c) At pt. No. 2

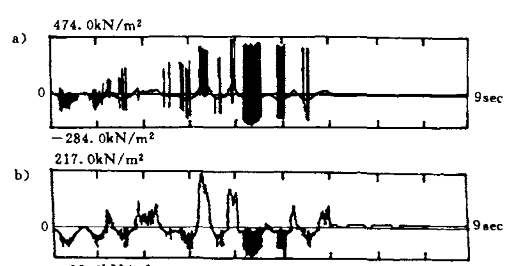

Fig. 11 Principal stress history of the invert
a) Near the top fibers;
b) Near the bottom fibers

CONCLUDING REMARKS

The data obtained in this work can be referred to for the design and exploitation of subways with similiar conditions.

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Proceedings: Third International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, April 2–7, 1995, Volume II, St. Louis, Missouri

Dynamics of Massless Rigid Foundation Resting on Elastic Half Space

Paper No. 12.01

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SYNOPSIS In this paper, a numerical procedure is presented for the analysis of the dynamic response of a rectangular foundation resting upon an homogeneous, isotropic elastic half-space. The method of analysis simplifies the mechanical behaviour of the soil-footing contact surface by assuming a relaxed boundary. The contact interface of the foundation and soil medium is discretized into rectangular subregions. Simple numerical integration is conducted to evaluate the compliance coefficients. The consistency of the proposed solution is verified by a comparison with the reported literature. The effect of the aspect ratio upon the dynamic impedance functions for the foundation is investigated. The results show that the suggested approach is economical and convenient for practical engineering purposes.

INTRODUCTION

The major step in the study of the dynamic interaction between structures and the supporting medium is the evaluation of the frequency dependent compliance matrix for a massless foundation supported on the soil.

Of particular interest is the response to a harmonic excitation. Once the harmonic response of a massless plate is known, the response of any linear structure with mass superposed on the plate may be evaluated. Applying the method of Fourier transform it is also possible to evaluate the transient response of the structure to an arbitrary excitation. Problems of this type have been investigated extensively. Lamb's solutions (1904) for a periodic point load acting on the surface of a homogeneous elastic half-space form the basis of all the modern procedures. Elorduy et al. (1967) introduced a numerical technique for calculating the dynamic response of a massless rectangular base. They avoided the singularity of the displacement under the point load by obtaining the influence at an arbitrary corner of the subregion. This procedure is somewhat improved by Hamidzadeh and Grootenhuis (1981) who determined the flexibility coefficients at every corner of the rectangular subregions.

For a uniformly distributed load over a small area of the surface of an elastic half-space, Thomson and Kobori (1963) derived expressions for the displacements of the center of the loaded area. Wong (1975) extended this work by determining the displacements at points outside the loaded area. Using these expressions Wong and Luco (1976) have obtained results for surface foundations of arbitrary shape on an elastic half-space. The evaluation of these integrals requires a considerable amount of computer time.

A simpler approach was followed by Kitamura and Sakurai (1979, 1982), using much simpler expressions, attributed to Tajimi (1959), which provided excellent results at low values of frequencies. The expressions for the displacements are still singular under point loads. Thus, they evaluate the displacement influence value under the loaded subregion by considering a uniform stress distribution. The resulting integrals based on the approximations to Lamb's solutions (1904) are simpler, but they still suffer from a singularity at zero frequency.

In recent years, the boundary element method (BEM) has become an efficient and viable numerical technique for solving a wide class of engineering problems. Dominguez and Roesset (1978) was the first to use the BEM to obtain the dynamic response of a rigid foundation in the frequency domain. They used the infinite-space Green's function to formulate the radiation condition. Israil and Banerjee (1990) investigated the effects of geometrical and material properties on the vertical vibration of three-dimensional foundations. The elastic half-space has been represented by isoparametric quadratic boundary elements.

In this paper, a simplified numerical procedure, attributed to Kitamura and Sakurai (1979), is described to obtain the impedance functions for rigid rectangular foundations with different length to width ratios. This approach is economical and it is simple to use. Comparisons are made with the work of Israil and Banerjee (1990) for rigid square foundation. The agreement between the two approaches is very well.

BASIC EQUATIONS

The response of an infinite elastic half-space subjected to a vertical point load has been studied by many authors. The original formulation of the problem is due to Lamb (1904) who provided an integral expression for the harmonic displacement of the surface of an elastic half-space as follows:

$$W_{zz} = \frac{Q e^{i\omega t}}{2\pi G_s} \left(\frac{\alpha}{r}\right) \int_0^\infty \frac{\zeta \sqrt{\zeta^2 - \mu^2}}{f(\zeta)} J_0(\zeta \alpha)\, d\zeta \qquad (1)$$

where :

$$f(\zeta) = (2\zeta^2 - 1)^2 - 4\zeta^2 \sqrt{\zeta^2 - \mu^2} \sqrt{\zeta^2 - 1}$$

$$\mu = \sqrt{\frac{(1 - 2\nu_s)}{2(1 - \nu_s)}}$$

$$a = \frac{\omega r}{V_s}$$

r = Distance from load to point considered
Q = Amplitude of dynamic force
G_s = Shear modulus of soil
V_s = Propagation velocity of shear wave
ν_s = Poisson's ratio for soil
ω = Circular frequency
t = Time
i = $\sqrt{-1}$
ζ = The integration variable and
$J_0(\zeta\alpha)$ = Zero order of Bessel's function of the first kind

The integral's which appear in these solutions are very complicated. Therefore, equation (1) can be rewritten as:

$$W_{zz} = \frac{1 - \nu_s}{2\pi G_s} \frac{Q e^{i\omega t}}{r} [f_1 + i f_2]$$

$$W_{zz} = \frac{1 - \nu_s}{2\pi G_s} \frac{Q e^{i\omega t}}{r} \sqrt{f_1^2 + i f_2^2}\; e^{-\phi} \qquad (2)$$

where $\phi = -\tan^{-1}(f_2/f_1)$

The absolute value $\sqrt{f_1^2 + f_2^2}$ and the phase angle ϕ in the above equation are expressed as functions of the value of $\omega r/V_s$. However, Tajimi (1959) has proposed a simple approximate formula given by equation (3) and (4) instead of equation (2), assuming the absolute value $\sqrt{f_1^2 + f_2^2}$ to be unity and the phase angle ϕ to be linearly increasing with the dimensionless frequency ($\omega r/V_s$). The formulae are

$$W_{zz} = \frac{1 - \nu_s}{2\pi G_s} \frac{e^{-i\psi r}}{r} Q e^{i\omega t} \qquad (3)$$

$$\psi = \beta \omega / V_s \qquad (4)$$

The constant β introduced in equation (4) is taken as 1.2. Tajimi's (1959) approximate formula gives high accuracy on the condition that the value of $\omega r/V_s$ is close to zero, and it is obvious that this formula gives Boussinsq's solution when the circular frequency, ω, is equal to zero.

INFLUENCE FUNCTION

By considering a rectangular contact area (2B * 2L) of a footing resting on the surface of a semi-infinite medium, the contact area is divided into subregions as shown in Fig. 1. The assumption of frictionless forces acting on the contact surface is introduced in this study.

The off diagonal coefficients of the compliance matrix can be determined by using equation (3) as follows :

$$F_{SL,ij} = \frac{\Gamma_{ij}}{R b} e^{-i\Psi R} \qquad (5)$$

where:

$F_{SL,ij}$ = The off diagonal of the compliance matrix
Γ_{ij} = $(1 - \nu_s)\, \lambda / 2\pi G_s$
λ = b/a
Ψ = $\psi\ a = 1.2\,(\omega a / V_s)$
R = Distance

Difficulties can arise in the evaluation of equation (3) when the point of excitation and response coincide. For this case, the diagonal coefficients of compliance are undefined. Kitamura and Sakurai (1979) overcome this difficulty by integrating equation (3) over the entire area of the element. A typical diagonal coefficient of the compliance matrix is:

$$F_{SL,SL} = \frac{4\Gamma_{SL}}{\Psi \lambda b} \Big[-i\left(\tan^{-1}\lambda + \tan^{-1}\frac{1}{\lambda}\right) + \int_0^{\tan^{-1}\lambda} \left\{ \sin\left(\frac{\Psi}{2\cos\theta}\right) + i\cos\left(\frac{\Psi}{2\cos\theta}\right) \right\} d\theta + \int_0^{\tan^{-1}(1/\lambda)} \left\{ \sin\left(\frac{\Psi\lambda}{2\cos\theta}\right) + i\cos\left(\frac{\Psi\lambda}{2\cos\theta}\right) \right\} d\theta \Big] \qquad (6)$$

It can be seen that the equation (6) is still singular when the frequency is set equal to zero. Therefore, the integration of the Boussinesq's formula can be used for a point load over the area of the rectangle when the frequency is equal to zero.

For a half-space situation, the off-diagonal coefficients of flexibility given by Boussinesq's equation are as follows:

$$F_{SL,ij} = \frac{(1 - \nu_s)}{2\pi G_s r_{SL\,ij}} \qquad (7)$$

The diagonal coefficient of the flexibility matrix can be obtained by the integration of equation (7) over the area (a*b) as shown in Fig.2.

$$F_{SL,SL} = \int_{\eta=0}^{\eta=\frac{\bar{a}}{2}} \int_{\xi=0}^{\xi=\frac{b}{2}} \frac{(1-v_s)}{\bar{a}\, b\, \pi\, G_s \sqrt{\eta^2+\xi^2}}\, d\eta\, d\xi \qquad (8)$$

The subgrade compliance matrix, $[C_{sub}]$, is then evaluated using equations (6) and (8) for the diagonal coefficients and equation (5) for all others. The full, complex, non- singular and symmetric compliance matrix may be evaluated directly without the need to resort to expensive numerical integration procedures. Thus, this proposed procedure is economical to use. The complex impedance matrix of the subgrade , $[K_{sub}]$, may be obtained by inversion of the compliance matrix, i.e. :

$$[K_{sub}] = [C_{sub}]^{-1} \qquad (9)$$

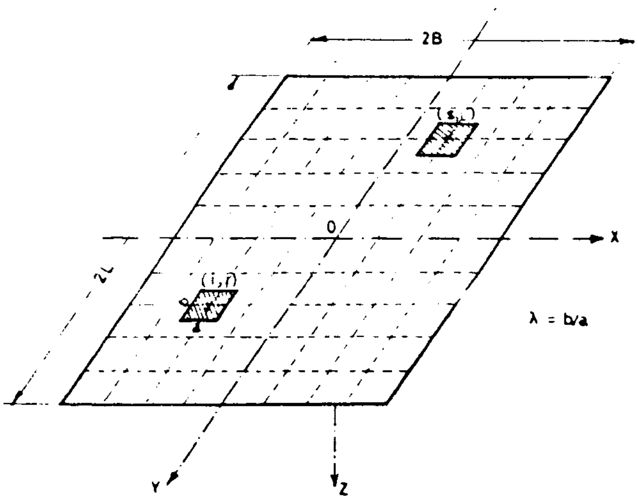


Fig. 1 Subdivision of Contact Plan

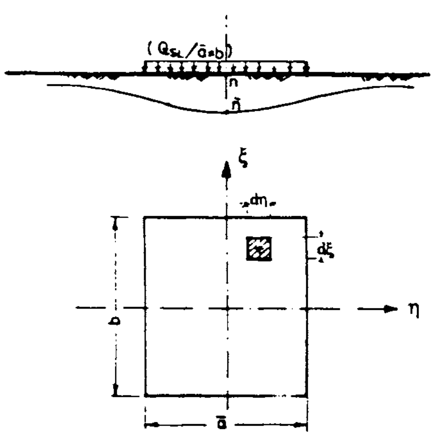


Fig. 2 Vertical Relative Displacement due to Uniformly Loaded Rectangular Area on Isotropic Half Space

VERIFICATION OF THE PROPOSED APPROACH

The square contact area of the subgrade soil is divided into 64 subdivisions. Then, the impedance matrix can be formulated using the current approach. The resulting subgrade impedance can be multiplied by a unit displacement vector (rigid body motion or rigid displacement) and the resulting values gave the complex contact forces. Then, the complex compliance functions for the vertical mode of vibration for a massless, rigid foundation may be defined as:

$$C_v(a_0) = B\, G_s \left(\frac{\Delta}{\Sigma \Re}\right) \qquad (10)$$

where

C_v = The compliance function

a_0 =The dimensionless frequency ratio = $B\omega / V_s$

B = Half width of footing

ω = Frequency of excitation

V_s = Shear wave velocity for soil

G_s = Shear modulus of soil

$\Re$ = The complex contact forces and

Δ = Unit vector of displacement

The results obtained from the program were checked by comparing them with the results from the solution obtained by Israil and Banerjee (1990). In their analysis, a full-space Green function is formulated by using isoparametric quadratic boundary elements. From Fig. 3, it can be seen that the results obtained by the Israil and Banerjee approach and the present simplified method are in close agreement.

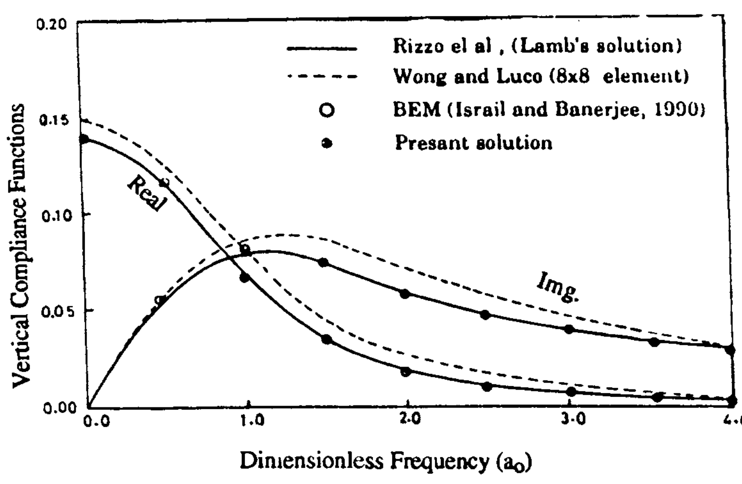


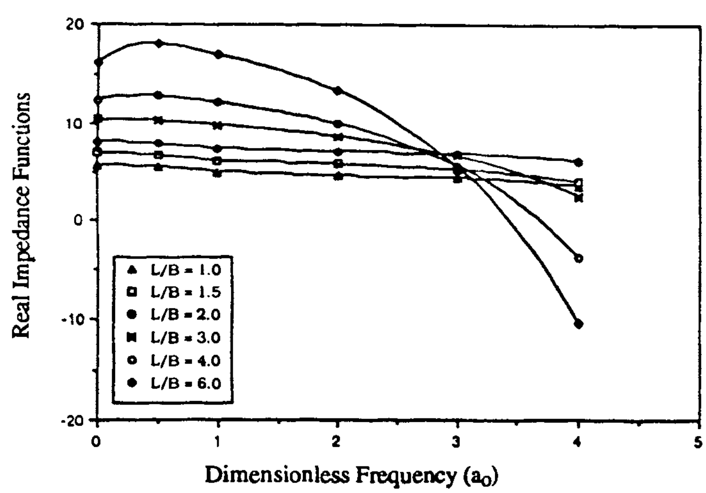
Fig. 3 Comparison of Vertical Compliances for Rigid Square Foundation on Elastic Half Space (v_s = 0.33)

DISCUSSION OF RESULTS

The investigation of the aspect ratio influence on the dynamic impedance functions of a massless rectangular foundation is conducted using the results obtained from the developed computer program. The shortest half-width of the foundation, B, is set equal to 1.0 and the other one took values from 1.0 to 6.0. All foundations are studied using the same number of elements, 64, at each mesh.

Figs. 4 to 6 present the relationships between the real and imaginary parts of the impedances and the dimensionless frequency, a_o, ranging between 0.0 to 4.0. Also the aspect ratio varied between 1.0 to 6.0 and the values of Poisson's ratio, v_s, which associate these figures are 0.15, 0.333 and 0.45, respectively. From these figures it can be observed that the real part of impedance, which represent the effect of the static stiffness and inertia of subgrade soil, generally decreases with increasing the dimensionless frequency, a_o, and they varied nearly along a straight line with a_o for the low values of aspect ratio, $1.0 \leq L/B \leq 2.0$. Whereas, the real impedances of large values of aspect ratio, $L/B \geq 4.0$, reach negative values at high frequencies. This trend is sim.lar to the behaviour of a strip foundation in two dimension analysis. The increase of the real impedance function as the length/width ratio of foundation increases is also obvious, where the impedance ratios at zero frequency are 1.0, 1.23, 1.442, 1.846, 2.209 and 2.873 for corresponding aspect ratios, L/B = 1.0, 1.5, 3.0, 4.0 and 6.0, respectively.

The imaginary part of impedance, which represents the geometrical damping dissipated into semi-infinite medium, is proportion with the dimensionless frequency, a_o. This is true for all the aspect ratios studied. Foundations with large value of aspect ratio, L/B = 6.0, show the highest rate of increase of the imaginary part with dimensionless frequency, a_o. This relation indicates the reduction in the radiation energy into the elastic half-space in case of large values of aspect ratio.



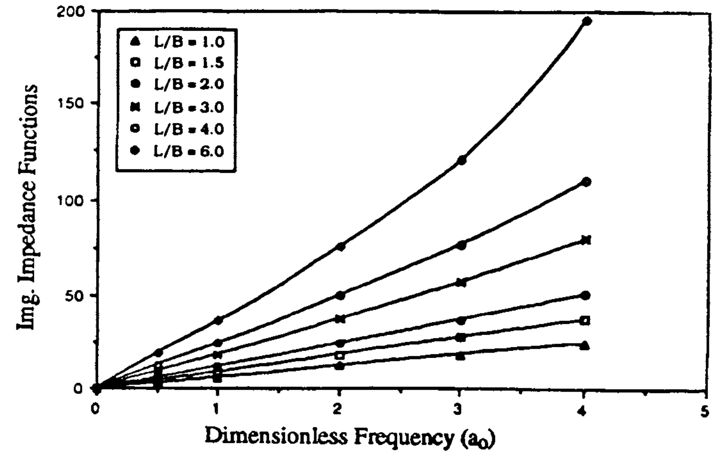


Fig. 4 Real and Imaginary Parts of Impedance Functions Versus Dimensionless Frequency for Different Aspect Ratio ($v_s = 0.15$)

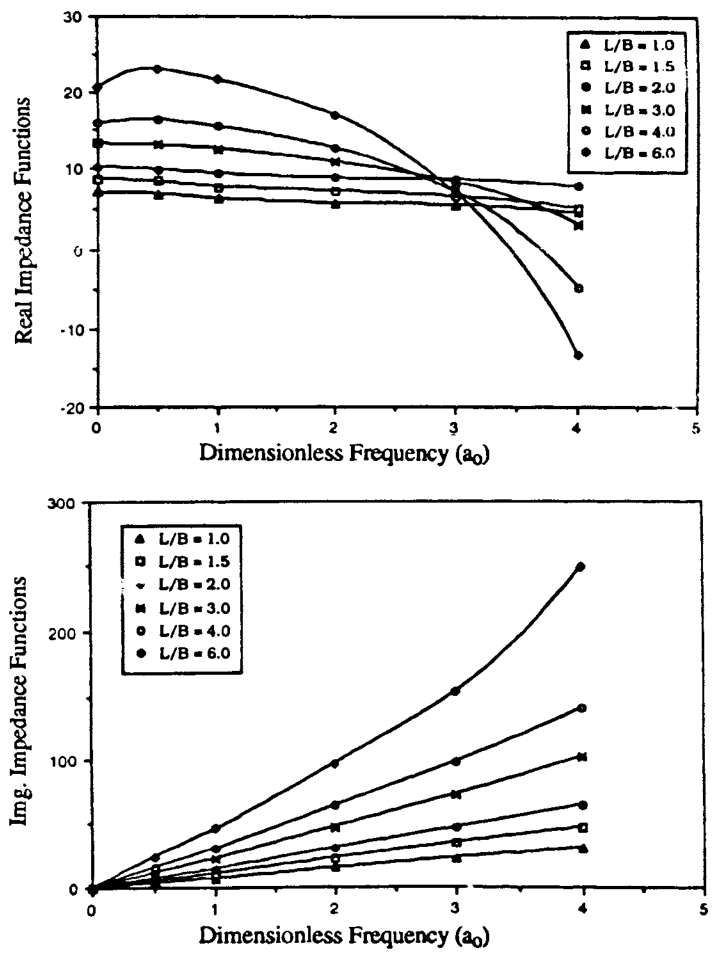


Fig. 5 Real and Imaginary Parts of Impedance Functions Versus Dimensionless Frequency for Different Aspect Ratio ($v_s = 0.33$)

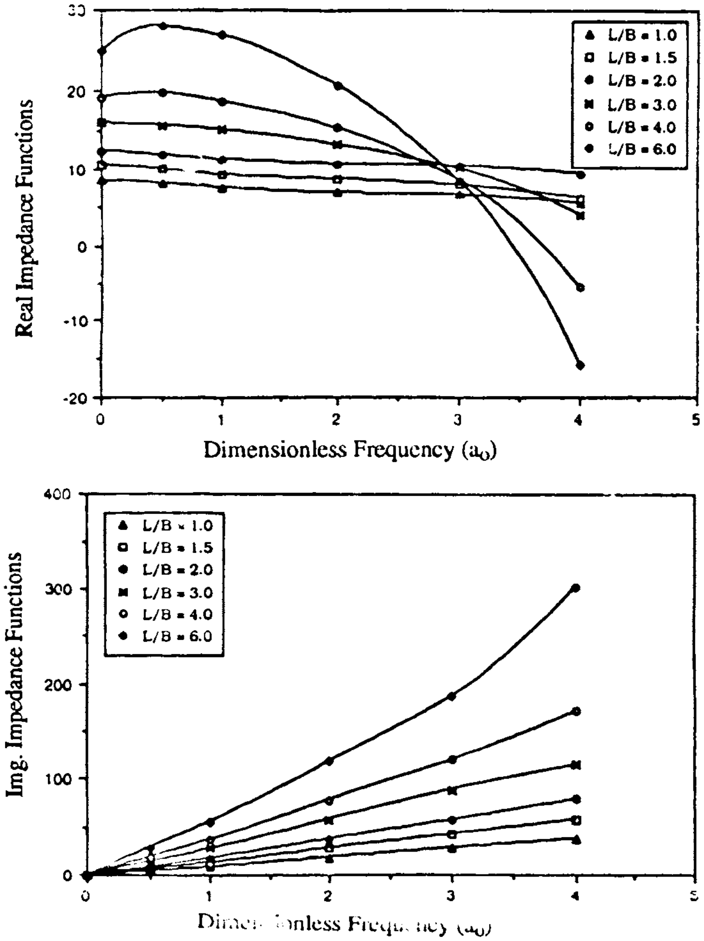


Fig. 6 Real and Imaginary Parts of Impedance Functions Versus Dimensionless Frequency for Different Aspect Ratio ($\nu_s = 0.45$)

CONCLUSIONS

A numerical procedure is described to compute the complex, frequency-dependent impedance matrix of soil as an elastic half-space. The proposed method yields results which are in good agreement with those obtained in previous studies. The impedance matrix and its inverse, the compliance matrix, depend on the frequency of the excitation, the plane configuration of the foundation and the characteristics of the underlying soil.

The dimensionless plots for the impedance functions provide considerable insight into the mechanics of the stiffness and inertia, and the radiation damping phenomena. They can also be used directly to calculate the dynamic response of a massive foundations excited by harmonic load.

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