1970

Realistic impulsive P wave source in an infinite elastic medium

Joseph Hugo Hemmann

Department: Physics

REALISTIC IMPULSIVE P WAVE SOURCE
IN AN INFINITE ELASTIC MEDIUM

by

JOSEPH HUGO HEMMANN, 1941-

A DISSERTATION

Presented to the Faculty of the Graduate School of the

UNIVERSITY OF MISSOURI - ROLLA

In Partial Fulfillment of the Requirements for the Degree

DOCTOR OF PHILOSOPHY
in
PHYSICS

1970

Advisor

ABSTRACT

The generally assumed instantaneous impulsive pressure sources for an elastic P wave in an infinite medium lead to basic contradictions, a step function source yielding infinite displacements as the shear modulus goes to zero, and the total energy radiated by a delta function (or any derivative of the delta function) source being infinite. By noting that the character of the solution in the elastic region must match the acoustic case near the source, a realistic impulsive pressure source is postulated which has a finite time history. The elastic field produced by this source is then obtained in closed form. A number of numerical examples are considered.

ACKNOWLEDGEMENTS

I would like to thank Dr. J. Paul Wesley, my advisor, for suggesting this research problem and for his advice and patience during the course of the work.

Thanks also to Victor Pol, who assisted in the preparation of the drawings, and to the many others who contributed their enthusiasm and encouragement.

TABLE OF CONTENTS

LIST OF FIGURES

Figure Page

I. INTRODUCTION

There have been many presentations of the problem of elastic wave propagation in an infinite medium caused by a compressional wave (P wave) source (see MIKLOWITZ, [1960] for a general review). Much of the interest in this problem arises from its application to seismology, especially in the propagation of waves produced by an underground nuclear explosion. Experimental observations of underground nuclear explosions have been presented by ROMNEY [1959], ADAMS, et al. [1961], CARDER, et al. [1962], WEART [1962], WRIGHT, et al. [1962], WERTH and HERBST [1963], and TREMBLY and BERG [1966]. The treatment of elastic waves in half space [PINNEY, 1954; CAGNIARD, 1962] and in layered media [EWING, et al., 1957] requires the solution for the infinite medium, since the initial stage of the wave propagation occurs as in an infinite medium and the effects of reflections are included separately.

Various authors have treated this problem using a variety of pressure sources with the object of approximating an explosive source. A step function pressure source was considered by SELBERG [1952]. SHARPE [1942] and BLAKE [1952] treated a pressure source consisting of a decaying exponential term, $e^{-\alpha t}$, times a step function. They also considered the limit of α approaching zero, which gives just a step function. DUVALL [1953] and BERG and PAPAGEORGE [1964] considered a pressure function consisting of two decaying exponential terms of the form

$$P(t) = A(e^{-\alpha t} - e^{-\beta t})u(t)$$

where P(t) is the pressure and u(t) is the step function. Duvall presented results for several values of α and β while Berg and Papageorge chose the particular values,

$\alpha=0$ and $\beta=\infty$, which yield the step function. The delta function pulse of pressure, which gives rise to the Green's function formulation of the elastic wave problem [MORSE and FESHBACH, 1953], is frequently interpreted as an elementary impulsive source that could arise from an explosion. TSAY [1969] has treated wave propagation in a generalized Voight solid using three pressure sources; a delta function, a step function times a decaying exponential term, and a step function times two decaying exponential terms. A shock wave theory has been presented by PEET [1960], in which the pressure has the form

$$P(t) = u(t) P_m e^{-t/\Theta}$$

where the parameters P_m and Θ are determined by the shock behavior of the medium near the source.

A perfectly general instantaneous impulsive source is considered here. It is shown that any such instantaneous source cannot yield physically reasonable results. In particular, a pressure source established stepwise in time yields infinite particle displacements when the shear modulus, μ, goes to zero. Pressure sources represented by a delta function in time, or by any derivative of a delta function, give an infinite amount of radiated energy. As a consequence, a realistic pressure source must be specified over a finite time interval.

It is reasoned that the wave near the source will be essentially an acoustic wave since the shear stress will be much larger than the shear strength of the medium. The general character of the particle velocities is noted in this case. This then leads to a postulation of a realistic pressure source that is specified over a finite time history using error functions and gaussian time variation.

The elastic wave equations are solved in closed form using this realistic pressure source. It is found that this source specified over a finite time interval yields a finite radiated energy and gives physically meaningful results when the shear modulus goes to zero.

II. GENERAL THEORY

A. ELASTIC WAVE EQUATION

A point in the infinite elastic medium is specified by the coordinates x^1, x^2, and x^3. The symbols u_{ij}, e_{ij}, and τ_{ij} represent the covariant components of the displacement, strain, and stress, respectively. For small displacements the strain is given by

$$e_{ij} = (1/2)(u_{i,j} + u_{j,i}) \tag{1}$$

where the commas denote covariant differentiation [SOKOLNIKOFF, 1964]. For an isotropic, lossless medium the stress is determined by Hooke's law

$$\begin{aligned} \tau_{ij} &= \lambda g_{ij} e^s{}_s + 2\mu e_{ij} \\ &= \lambda g_{ij} u^s{}_{,s} + \mu(u_{i,j} + u_{j,i}) \end{aligned} \tag{2}$$

where g_{ij} is the covariant metric tensor and λ and μ are the Lamé constants, μ being the shear modulus and $\lambda + 2\mu/3$ the bulk modulus [LOVE, 1927].

The equation of motion of an element of volume according to Newton's second law is then

$$\tau_i{}^j{}_{,j} + F_i = \rho a_i \tag{3}$$

where F_i and a_i are the covariant components of the body force and the acceleration, respectively, and where ρ is the density. For small displacements the equation of motion can be linearized by writing

$$a_i = \frac{\partial^2 u_i}{\partial t^2} = \ddot{u}_i \tag{4}$$

Combining equations 2, 3, and 4 then gives the elastic wave equation

$$(\lambda + \mu)\frac{\partial}{\partial x^i}(u^s{}_{,s}) + \mu\nabla^2 u_i + F_i = \rho\ddot{u}_i$$

which may be written in vector notation as

$$\mu\nabla\times(\nabla\times\underline{u}) - (\lambda + 2\mu)\nabla(\nabla\cdot\underline{u}) - \underline{F} + \rho\ddot{\underline{u}} = 0 \tag{5}$$

This equation may be separated by expressing $\underline{u}$ and $\underline{F}$ as sums of scalar and vector potentials

$$\begin{aligned} \underline{u} &= \nabla\phi + \nabla\times\underline{A} \\ \underline{F} &= \nabla f + \nabla\times\underline{G} \end{aligned} \tag{6}$$

Substitution of equations 6 into equation 5 gives the two wave equations

$$\begin{aligned} (\lambda + 2\mu)\nabla^2\phi - \rho\ddot{\phi} + f &= 0 \\ \mu\nabla(\nabla\cdot\underline{A}) - \mu\nabla\times(\nabla\times\underline{A}) - \rho\ddot{\underline{A}} + \underline{G} &= 0 \end{aligned} \tag{7}$$

The solution to the first of equations 7 gives rise to the P wave, or compressional wave; the solution to the second is the S or shear wave.

B. THE SOURCE

The source may be specified by prescribing the uniform pressure over the surface of a spherical cavity of radius R_c. Because of the spherical symmetry, the relation between the pressure and the displacement on the surface of the sphere is given by [LOVE, 1927]

$$\tau_{RR} = -P = \frac{(\lambda + 2\mu)}{R^2} \frac{\partial}{\partial R}(R^2 u_R) - (4\mu/R)u_R \tag{8}$$

where P is the prescribed pressure, u_R the radial displacement, and R the radial distance from the origin, evaluated on the sphere $R=R_c$. From spherical symmetry only the radial displacement is non-zero. Thus, the vector potential $\underline{A}$ may be set equal to zero and only the first of equations 7, the one for P waves, need be considered. With the body force potential f zero equation 8 yields

$$P = -\rho\ddot{\phi} + (4\mu/R)\frac{\partial\phi}{\partial R} \tag{9}$$

where $R=R_c$.

The scalar potential ϕ is a function of R and t only and the general solution to the scalar wave equation 7 is

$$\phi(R,t) = F(t - R/\alpha) + G(t + R/\alpha) \tag{10}$$

where α is the compressional wave velocity given by

$$\alpha = [(\lambda + 2\mu)/\rho]^{\frac{1}{2}} \tag{11}$$

Assuming that the source causes only outgoing waves, the function G in equation 10 may be set equal to zero. Substitution of equation 10 with G=0 into equation 9 gives

$$-R^3\alpha P = \alpha\rho R^2 F'' + 4\mu R F' + 4\mu\alpha F \tag{12}$$

where the primes on the F's denote differentiation with respect to the argument $(t-R/\alpha)$, and where $R=R_c$.

C. INTEGRAL SOLUTION USING LAPLACE TRANSFORMS

If it is assumed that the pressure on the cavity wall is zero prior to the time $t=R_c/\alpha$, then F and F' may be assumed to be zero for $t<R_c/\alpha$. Functions which vanish for negative arguments can be conveniently analyzed using Laplace transforms. Later it will be found necessary to introduce a pressure source that does not vanish for $t<R_c/\alpha$. In that case Fourier transforms must be used.

Defining the retarded time t' by

$$t' = t - R/\alpha \tag{13}$$

the Laplace transform of equation 12 is

$$\begin{aligned}(R^3/\rho)\mathcal{P}(s) = &- (R^2s^2 + bRs + b\alpha)\mathcal{F}(s) \\ &+ R(Rs + b)F(0) + R^2F'(0)\end{aligned} \tag{14}$$

where s is the transform variable,

$$b = 4\mu/\alpha\rho \tag{15}$$

and where the functions $\mathcal{P}(s)$ and $\mathcal{F}(s)$ are defined by

$$\begin{aligned}\mathcal{P}(s) &= \int_0^\infty e^{-st'}P(t')dt' \\ \mathcal{F}(s) &= \int_0^\infty e^{-st'}F(t')dt'\end{aligned} \tag{16}$$

Solving equation 14 for $\mathcal{F}(s)$ and then taking the inverse transform gives F as a function of the prescribed pressure on the sphere $R=R_c$ and of the initial values of F and F', for $t=R_c/\alpha$.

$$F(t-R/\alpha) = \frac{1}{2\pi i}\int_{-i\infty+\gamma}^{+i\infty+\gamma} \frac{e^{s(t-R/\alpha)}}{R_c^2 s^2 + bR_c s + b\alpha}[R_c^2 F'(0) + (R_c s+b)R_c F(0) - (R_c^3/\rho)P(s)]ds \tag{17}$$

The result, equation 17, is more general than that obtained by CAGNIARD [1962], who only considers functions F such that F and F' are zero at $t=R_c/\alpha$. Since the precise physical events are not known, it is unwise to discard F(0) and F'(0). Here the field produced by nonvanishing values of F(0) and F'(0) is considered.

D. GENERAL INSTANTANEOUS IMPULSIVE SOURCE

It is now necessary to specify the pressure P over the spherical surface $R=R_c$ and then to obtain its Laplace transform. The present problem involves the consideration of an instantaneous impulsive source, defined here as any source of pressure that varies rapidly over a vanishingly short interval of time, the pressure being initially constant and finally constant. Such a source may be approximately represented by a series consisting of the step function and its derivatives. In particular, the ideal impulsive source of pressure over the sphere may be written as

$$P(t^*) = \sum_{n=o}^{\infty} a_n u^{(n)}(t^*) \tag{18}$$

where $t^*=t-R_c/\alpha$, the a_n's are appropriate coefficients, and $u^{(n)}(t^*)$ is the nth derivative of the step function

$$u(t) = \begin{cases} 1, & t>0 \\ 0, & t<0 \end{cases} \tag{19}$$

In order to see that equation 18 does in fact represent the most general impulsive source, it may be noted that

$$u^{(n)}(t^*) = (1/2\pi i) \int_{-i\infty+\gamma}^{+i\infty+\gamma} s^{n-1} e^{st^*} ds \qquad (20)$$

where γ is an arbitrarily small real constant, so that

$$P(t^*) = (1/2\pi i) \int_{-i\infty+\gamma}^{+i\infty+\gamma} \sum_{n=o}^{\infty} a_n s^{n-1} e^{st^*} ds \qquad (21)$$

Equation 21 yields the inverse Laplace transform of any function $P(s)$ that can be expressed as a power series in s

$$P(s) = a_0/s + a_1 + a_2 s + a_3 s^2 + \cdots \qquad (22)$$

thereby indicating the completely general character of $P(t^*)$.

E. SOLUTION FOR GENERAL INSTANTANEOUS IMPULSIVE SOURCE

After substituting the series representation of $P(s)$, equation 22, into equation 17, the solution for $F(t-R/\alpha)$ may be obtained by integration. The integral in equation 17 may be evaluated by expanding the algebraic part of the integrand into partial fractions. It is convenient to express the solution in the form

$$F(t-R/\alpha) = F_i + F_0 + F_1 + F_2 + \cdots \qquad (23)$$

where F_i is the result of specifying the initial values $F(0)$ and $F'(0)$. The terms F_0, F_1, F_2, ..., arise from the terms containing a_0, a_1, a_2, ..., respectively, in the expansion for $P(s)$. Thus F_0 arises from a step function source of pressure only, F_1 from a delta function source of pressure, F_2 from the pressure specified by the derivative of a delta function, and similarly for higher terms in equation 23.

The value of F_i in terms of the initial values of F and F' is given by

$$F_i(t-R/\alpha) = -\frac{1}{2\pi b}\left(\frac{\mu}{\lambda+\mu}\right)^{\frac{1}{2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} e^{s(t-R/\alpha)} \times\{(s+a)^{-1}[R_c F'(0) + (b- aR_c)F(0)] - (s+a^*)^{-1}[R_c F'(0) + (b-a^*R_c)F(0)]\}ds \tag{24}$$

where

$$a = \xi - i\eta = (b/2R_c)[1 - i(1 + \lambda/\mu)^{\frac{1}{2}}] \tag{25}$$

where b is defined by equation 15, and a* is the complex conjugate of a. The poles of the integrand in equation 24 lie in the left half of the complex s-plane. For $t-R/\alpha>0$ the contour of integration may be closed in the left half plane, yielding the residues from the poles. For $t-R/\alpha<0$ the contour may be closed in the right half plane, yielding zero. After a little manipulation the result becomes

$$F_i(t-R/\alpha) = e^{-\xi(t-R/\alpha)}\{F(0)\cos\eta(t-R/\alpha) + (1/\eta)[F'(0) + \xi F(0)]\sin\eta(t-R/\alpha)\}u(t-R/\alpha) \tag{26}$$

where

$$\xi = \frac{2}{R_c}\left[\left(\frac{\mu}{\lambda+2\mu}\right)\left(\frac{\mu}{\rho}\right)\right]^{\frac{1}{2}} \tag{27}$$

and

$$\eta = \frac{2}{R_c}\left[\left(\frac{\lambda+\mu}{\lambda+2\mu}\right)\left(\frac{\mu}{\rho}\right)\right]^{\frac{1}{2}} \tag{28}$$

The term F_0 due to a step function pressure source is obtained by substituting the term a_0/s for $P(s)$ in equation 17, which yields

$$F_0(t-R/\alpha) = \frac{R_c^{\,3} a_0}{16\pi\eta\mu} \int_{-i\infty+\gamma}^{+i\infty+\gamma} e^{s(t-R/\alpha)} \times \left\{ \frac{2i\eta}{s} - \frac{a^*}{s+a} + \frac{a}{s+a^*} \right\} ds \tag{29}$$

Performing the integration in the complex s-plane gives the result [CAGNIARD, 1962; Eq. 2-A-6. p, 11]

$$F_0(t-R/\alpha) = (R_c^{\,3} a_0/4\mu)\left\{-1 + e^{-\xi(t-R/\alpha)}[\cos\eta(t-R/\alpha) + (\xi/\eta)\sin\eta(t-R/\alpha)]\right\}u(t-R/\alpha) \tag{30}$$

Since a_0 is arbitrary, $F_i(t-R/\alpha)$ is not completely independent of $F_0(t-R/\alpha)$. In particular, if only a step function source occurs, then equations 26 and 30 must be compatible. In this case it is possible to choose

$$F_0(0) = R_c^{\,3}/4\mu \quad \text{and} \quad F'(0) = 0 \tag{31}$$

which gives the following result

$$F(t-R/\alpha) = F_i(t-R/\alpha) - F(0)u(t-R/\alpha) \tag{32}$$

It is important to note that the solution for the step function pressure source fails to represent the physical situation for the case of the shear modulus, μ, going to zero. In this limit equation 30 reduces to

$$F_0(t-R/\alpha) = -(R_c a_0/2\rho)(t-R/\alpha)^2 u(t-R/\alpha) \tag{33}$$

Since the displacement u_R is given in general by

$$u_R = -F(t-R/\alpha)/R^2 - F'(t-R/\alpha)/\alpha R \tag{34}$$

the result of equation 33 is that the displacement u_R grows indefinitely with time, which violates the original assumption and observed fact that the displacements must remain small.

The term F_1 for a delta function pressure source is obtained by using the term a_1 for $P(s)$ in equation 17. The result after performing the integration is

$$F_1(t-R/\alpha) = -a_1(R_c/\eta\rho)e^{-\xi(t-R/\alpha)}[\sin\eta(t-R/\alpha)u(t-R/\alpha)] \tag{35}$$

Comparing this result to equation 26, it is apparent that the delta function pressure source is equivalent to specifying the initial conditions

$$F(0) = 0 \quad \text{and} \quad F'(0) = -R_c a_1/\rho \tag{36}$$

It may be noted that the nth power of s in the expansion for $P(s)$, equation 22, may be generated in equation 17 by taking the nth derivative with respect to time t; thus

$$F_n(t-R/\alpha) = -\frac{R_c^3\rho}{2\pi i}a_n\frac{\partial^n}{\partial t^n}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s(t-R/\alpha)}\,ds}{s(R_c^2s^2+bR_cs+b\alpha)} \tag{37}$$

This equation may be written as

$$F_n = \left(\frac{a_n}{a_0}\right)\frac{\partial^n F_0}{\partial t^n} \tag{38}$$

where F_0 is given by equation 30, and where the delta function and its derivatives are to be evaluated at $t-R/\alpha=0$. It may be seen that F_1 satisfies this relation. The solution F_2 for a pressure pulse varying as the first derivative of the delta function is

$$F_2(t') = -(R_c/\rho)a_2e^{-\xi t'}[\cos\eta t' - (\xi/\eta)\sin\eta t']u(t') \quad (39)$$

where $t'=t-R/\alpha$. Similarly the term F_3 is given by

$$F_3(t') = (R_c/\rho)a_3\{-\delta(t') + e^{-\xi t'}[2\xi\cos\eta t' - (1/\eta)(\xi^2-\eta^2)\sin\eta t']u(t')\} \quad (40)$$

The solution for the general instantaneous impulsive source is

$$F(t-R/\alpha) = F_i(t-R/\alpha) + (1/a_0)\sum_{n=o}^{\infty} a_n\frac{\partial^n}{\partial t^n} F_0(t-R/\alpha) \quad (41)$$

where F_i is given by equation 24 and F_0 by equation 30.

F. ENERGY RADIATED FROM AN INSTANTANEOUS IMPULSIVE P WAVE SOURCE

It is of interest to calculate the total energy radiated from an instantaneous impulsive P wave source to ascertain whether this idealized source can, in fact, approximate actual explosive sources. The energy flux, $\underline{S}$, in an elastic medium is given by

$$S_i = -\tau_{ij}\dot{u}^j \quad (42)$$

and the total energy density, U, by

$$U = (1/2)[\rho\dot{u}^i\dot{u}_i + \tau^{ij}e_{ij}] \quad (43)$$

[SOKOLNIKOFF, 1956]. Using equations 1 and 2, the energy flux may be expressed as

$$S_i = -\lambda u^j{}_{,j}\dot{u}_i - \mu(u_{i,j} + u_{j,i})u^j$$

which for the present spherically symmetric case reduces to

$$S_R = -(\lambda+2\mu)\dot{u}_R \frac{\partial u_R}{\partial R} - (2\lambda/R)u_R\dot{u}_R \quad (44)$$

The total energy, Q, radiated from the source, may be obtained by integrating S_R over any spherical surface and over all time; thus

$$\begin{aligned} Q &= 4\pi R^2 \int_{-\infty}^{\infty} S_R dt \\ &= -4\pi\lambda R u_R{}^2 \Big|_{t=-\infty}^{t=+\infty} - 4\pi R^2(\lambda+2\mu) \int_{-\infty}^{\infty} u_R \frac{\partial u_R}{\partial R} dt \end{aligned} \quad (45)$$

Using equation 34, u_R and $\frac{\partial u_R}{\partial R}$ can be expressed in terms of F, F', and F''. Now at t=-∞, long before the source has been turned on, and at t=+∞, long after the source has been turned off; F and all of its derivatives are zero. This results in the following expression for Q.

$$Q = (4\pi\rho/\alpha) \int_{-\infty}^{\infty} F''^2 dt \quad (46)$$

It may be noted that this result is independent of the radius R, as it should be for a lossless medium. HASKELL

[1967] obtains a result similar to equation 46, but he includes an additional term that represents static elastic strain energy stored in the medium. This requires a permanent displacement, which would only be possible in an ideal elastic medium if the pressure was maintained indefinitely, which is not the case of interest here.

Substituting the solution, equation 41, for the general instantaneous impulsive source into equation 46 yields an infinite amount of energy radiated. This occurs because of the presence in the integrand of products of the delta function and/or its derivatives multiplied by functions which are nonvanishing at $t-R/\alpha=0$ [BREMERMANN, 1965]. Thus in general the assumption of a completely instantaneous impulsive pressure source cannot approximate the actual physical situation. Pressure functions that have a finite spread in time must be used in lieu of the delta function and its derivatives, in order to have a finite amount of energy radiated.

The one case of a step function pressure source does give a finite radiated energy, since using equation 30 gives

$$F''(t') = -(R_c a_0/\rho)e^{-\xi t'}[\cos\eta t' - (\xi/\eta)\sin\eta t']u(t') \qquad (47)$$

The total energy radiated is then given by equation 46 as

$$Q = (4\ R_c^{\ 2} a_0^{\ 2}/\alpha\rho) \int_0^\infty e^{-2\xi t}[\cos\eta t - (\xi/\eta)\sin\eta t]^2 dt \qquad (48)$$

Performing the indicated integration and using equations 11 and 27, gives for the total elastic energy radiated by a step function pressure applied to a spherical surface, $R=R_c$,

$$Q = \pi R_c^{\ 3} a_0^{\ 2}/2\mu \qquad (49)$$

where a_0 is the pressure on the surface for $t-R_c/\alpha>0$. Note that for the shear modulus μ equal to zero, equation 49 again predicts an infinite radiated energy. For a lossless medium the energy radiated must equal the energy supplied by the source. Any physical source, such as an explosion, will supply a definite energy input that is independent of the characteristics of the medium. The energy radiated should not depend on the shear modulus, and thus the step function pressure source does not give a realistic result.

III. REALISTIC IMPULSIVE SOURCE FOR AN EXPLOSION

It is possible to determine the general chacter of the pressure that must be assumed to act on a spherical cavity in order to simulate an explosive source. This is done by considering the general nature of the pressure that must be assumed to obtain a reasonable solution in the hydrodynamic case of no shear, $\mu=0$, in the region of weak shock, or acoustic region. The pressure for the acoustic case is then used as the source for the elastic region where $\mu\neq 0$. The justification for this procedure lies in the fact that near the source an explosion generates a shock wave. The energy of compression is many times larger than that of shear so the problem becomes a hydrodynamic problem. As the shock moves out from the source it is attenuated due to the spherical geometry, eventually becoming sufficiently weak so that the square of the velocity may be neglected, the differential equations becoming linear in this acoustic limit. To match the solution near the source the shear is still neglected.

The velocity v_R for the case $\mu=0$ is given in the acoustic limit by

$$v_R = \frac{\partial u_R}{\partial t} = -F'(t-R/\alpha)/R^2 - F''(t-R/\alpha)/\alpha R \qquad (50)$$

where here $\alpha=(\lambda/\rho)^{\frac{1}{2}}$ is the acoustic wave velocity. In the region near the source the term varying as $1/R$ may be neglected compared to the term varying as $1/R^2$. From the nature of the shock wave travelling through an acoustic medium the velocity ahead of the shock is zero. In the shock front the velocity rises rapidly to a maximum. The velocity falls off behind the shock front because of the spherical geometry and after a sufficient time has elapsed the velocity finally becomes zero [COURANT and FRIEDRICHS, 1948]. An expression for $F'(t-R/\alpha)$ that incorporates these features taking place over a finite time interval is

$$F'(t') = -A(t')e^{-\beta t'}\mathrm{Erfc}(t'/\varepsilon) \tag{51}$$

where $t' = t-R/\alpha$ and

$$\begin{aligned}\mathrm{Erfc}(t'/\varepsilon) &= (1/2)[1 + \mathrm{erf}(t'/\varepsilon)] \\ &= \pi^{-\frac{1}{2}} \int_{-\infty}^{t'/\varepsilon} e^{-x^2}dx\end{aligned} \tag{52}$$

which reduces to the step function $u(t')$ for ε approaching zero. The function $A(t')$ remains unspecified; in the simplest case it may be assumed that $A(t')$ is a constant. From equation 12 with $\mu=0$, the pressure that gives rise to the function $F'(t')$ given by equation 51 is

$$P(t,R_c) = (\rho/R_c)\frac{d}{dt^*}[A(t^*)e^{-\beta t^*}\mathrm{Erfc}(t^*/\varepsilon)] \tag{53}$$

where $t^*=t-R_c/\alpha$. Thus, considering the region of weak hydrodynamic shocks near the source, it is seen that only certain pressures can be prescribed that will correspond to an explosion. The pressure $P(t,R_c)$ is presented qualitatively in Figure 1, for the simplest case of $A(t')$ a constant. The negative pressures, which are merely pressures less than the ambient pressure in the medium, are at variance with the usual pressures assumed when no matching to the hydrodynamic or acoustic case is attempted.

A. ELASTIC FIELD PRODUCED BY AN EXPLOSIVE SOURCE

The field produced for the elastic case of $\mu\neq 0$ will be considered for the simplest possible pressure given by

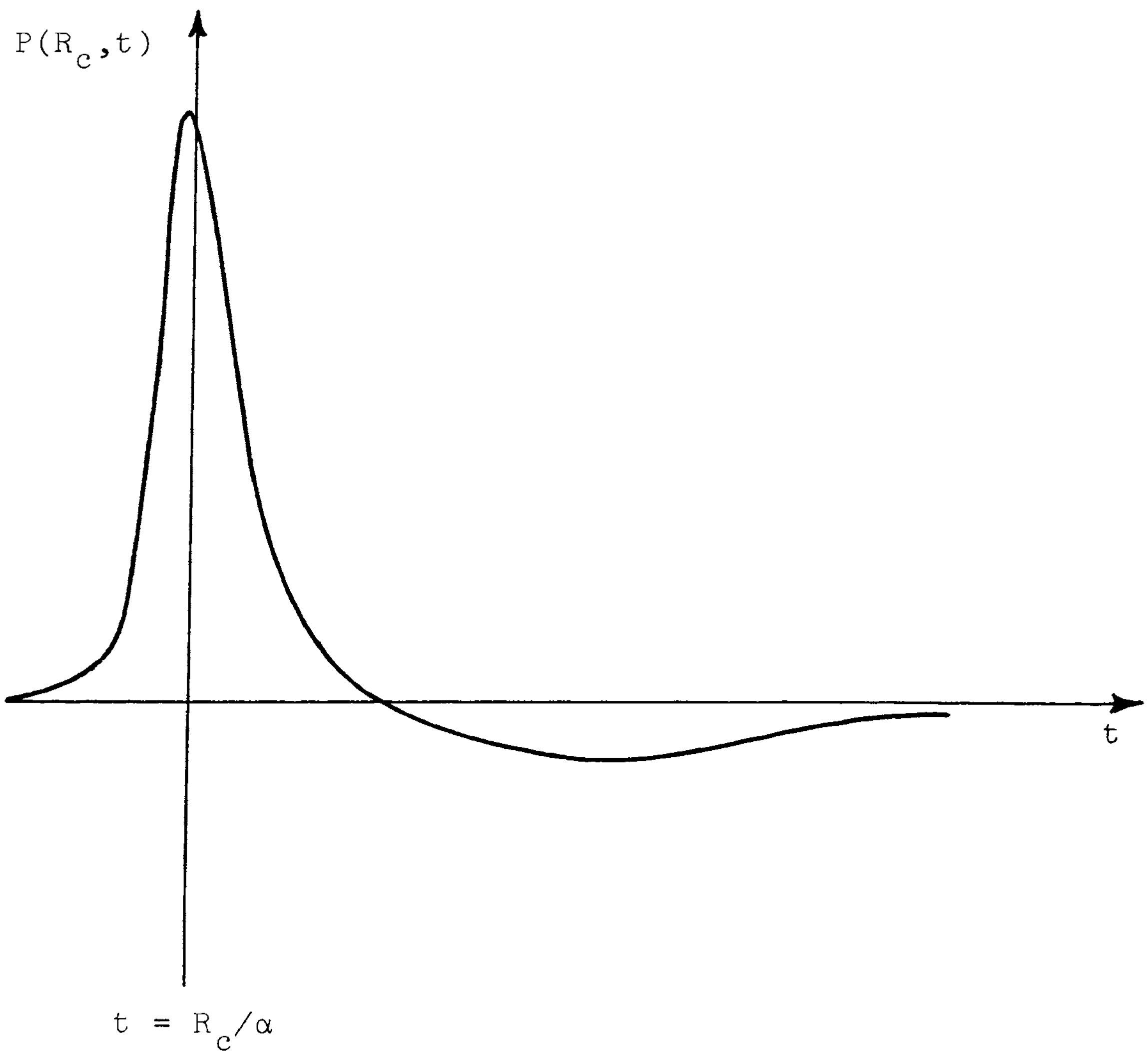

Figure 1. Sketch of simplest pressure source that produces proper acoustic velocities near the source

equation 53; for the case of $A(t')=A$, a constant. Since the pressure is given for all time, Laplace transforms are not applicable and Fourier transforms must be used. Taking the Fourier transform of equation 12, solving for the transform of F, and then taking the inverse Fourier transform yields

$$F(t-R/\alpha) = \frac{R_c}{2\pi\rho}\int_{-\infty}^{\infty} \frac{e^{i\omega(t-R/\alpha)}\ P(\omega)\,d\omega}{\omega^2-(4i\mu/\alpha R_c)\omega-(4\mu/\rho R_c^{\ 2})} \qquad (54)$$

where $P(\omega)$ is the Fourier transform of $P(t,R_c)$, given by

$$P(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t}\{(\rho A/R_c)\frac{d}{dt}[e^{-\beta t}\mathrm{Erfc}\ (t/\varepsilon)\}dt \qquad (55)$$

Integrating by parts once gives

$$P(\omega) = (iA\omega\rho/R_c)\int_{-\infty}^{\infty} e^{-t(\beta+i\omega)}\mathrm{Erfc}(t/\varepsilon)dt \qquad (56)$$

Integrating by parts again, noting that

$$\frac{d}{dt}\mathrm{Erfc}(t/\varepsilon) = (1/\varepsilon\pi^{\frac{1}{2}})e^{-(t/\varepsilon)^2} \qquad (57)$$

and then performing the final integration, results in

$$P(\omega) = (iA\rho/R_c)\omega(\beta+i\omega)^{-1}e^{\varepsilon^2(\beta+i\omega)^2/4} \qquad (58)$$

Substitution of equation 58 into equation 54 yields

$$F(t') = (A/2\pi)\frac{d}{dt'}\int_{-\infty}^{\infty}\frac{e^{[i\omega t'+\varepsilon^2(\beta+i\omega)^2/4]}}{(\beta+i\omega)(\omega-b)(\omega+b^*)}d\omega \tag{59}$$

where

$$b = \eta + i\xi \quad \text{and} \quad b^* = \eta - i\xi \tag{60}$$

where η and ξ are defined by equations 27 and 28. From the relation

$$e^{-\beta t}\mathrm{Erfc}(t/\varepsilon) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{e^{[i\omega t+\varepsilon^2(\beta+i\omega)^2/4]}}{\beta + i\omega}\,d\omega \tag{61}$$

which is derived similarly to equations 56 and 58, the integral in equation 59 may be evaluated, yielding

$$\begin{aligned} F(t') = -\frac{A}{2\eta}\frac{d}{dt'}\Big\{&[(b-i\beta)^{-1}+(b^*+i\beta)^{-1}]e^{-\beta t'}\mathrm{Erfc}(t'/\varepsilon) \\ &-(b-i\beta)^{-1}e^{[ibx+\varepsilon^2(\beta^2+b^2)/4]}\,\mathrm{Erfc}(x/\varepsilon) \\ &-(b^*+i\beta)^{-1}e^{[-ib^*y+\varepsilon^2(\beta^2+b^2)/4]}\,\mathrm{Erfc}(y/\varepsilon)\Big\} \end{aligned} \tag{62}$$

where

$$\begin{aligned} x &= t' + \varepsilon^2(\beta + ib)/2 \\ y &= t' + \varepsilon^2(\beta - ib)/2 \end{aligned} \tag{63}$$

This result may be written in the form

$$F(t') = -\frac{A}{2\eta[\eta^2+(\xi-\beta)^2]}\frac{d}{dt'}\Big[\!\Big[2\eta e^{-\beta t'}\mathrm{Erfc}(t'/\varepsilon) - \exp\{-\xi t'-\varepsilon^2[\eta^2-(\xi-\beta)^2]/4\} \times \{[\eta-i(\xi-\beta)]e^{i\eta z}\ \mathrm{Erfc}(z/\varepsilon+i\varepsilon\eta/2) + [\eta+i(\xi-\beta)]e^{-i\eta z}\mathrm{Erfc}(z/\varepsilon-i\varepsilon\eta/2)\}\Big]\!\Big] \qquad (64)$$

where

$$z = t' - \varepsilon^2(\xi-\beta)/2 \qquad (65)$$

Carrying out the differentiation and simplifying, equations 64 and 65 yield

$$F(t') = A[\eta^2+(\xi-\beta)^2]^{-1}\Big[\!\Big[e^{-\beta t'}[\ \mathrm{Erfc}(t'/\varepsilon)-(1/\varepsilon\pi^{\frac{1}{2}})e^{-t'^2/\varepsilon^2}] + \exp\{-\xi t'-\varepsilon^2[\eta^2-(\xi-\beta)^2]/4\} \times \mathrm{Re}\{[1-i(\xi-\beta)/\eta]e^{i\eta z}[(-\xi+i\eta)\mathrm{Erfc}(w)+(1/\varepsilon\pi^{\frac{1}{2}})e^{-w^2}]\}\Big]\!\Big] \qquad (66)$$

where

$$w = z/\varepsilon - i\varepsilon\eta/2 \qquad (67)$$

and where Re denotes the real part. The derivative of F is given by

$$F'(t') = A[\eta^2+(\xi-\beta)^2]^{-1}\Big[\!\Big[e^{-\beta t'}[-\beta^2\mathrm{Erfc}(t'/\varepsilon) + (2/\varepsilon\pi^{\frac{1}{2}})(\beta-t'/\varepsilon^2)e^{-t'^2/\varepsilon^2}] + \exp\{-\xi t'-\varepsilon^2[\eta^2-(\xi-\beta)^2]/4\}\mathrm{Re}\{[1-i(\xi-\beta)/\eta]e^{i\eta z} \times[(-\xi+i\eta)^2\mathrm{Erfc}(w)+(2/\varepsilon\pi^{\frac{1}{2}})(-\xi+i\eta-w/\varepsilon)e^{-w^2}]\}\Big]\!\Big] \qquad (68)$$

When ε and β are zero, equation 64 reduces to the time derivative of the solution for the step function source, equation 30, with $A=a_0R_c/\rho$. This is correct since for this case the pressure given by equation 53 with $A(t')$ a constant is just the derivative of the step function. As a further check on the validity of equation 64 it may be noted that for $\mu=0$ this equation becomes

$$F(t') = -\frac{A}{\beta^2}\frac{d}{dt'}\Big\{e^{-\beta t'}\mathrm{Erfc}(t'/\varepsilon) - e^{\varepsilon^2\beta^2/4}$$

$$[\mathrm{Erfc}(t'/\varepsilon+\varepsilon\beta/2)-(t'+\varepsilon^2\beta^2/2)\mathrm{Erfc}(t'/\varepsilon+\varepsilon\beta/2) \quad (69)$$

$$-(\varepsilon\beta/2\pi^{\frac{1}{2}})e^{-(t'/\varepsilon+\varepsilon\beta/2)^2}]\Big\}$$

Performing the differentiation gives

$$F(t') = (A/\beta)[e^{-\beta t'}\ \mathrm{Erfc}(t'/\varepsilon)-e^{\varepsilon^2\beta^2/4}\mathrm{Erfc}(t'/\varepsilon+\varepsilon\beta/2)] \quad (70)$$

which when differentiated again agrees with equation 51, as it should.

For the interesting case when ε may be regarded as small, equation 64 reduces to

$$F(t') = \frac{A}{\eta^2+(\xi-\beta)^2}\frac{d}{dt'}\Big[\Big[\Big\{-e^{-\beta t'}+ e^{-\xi t'}[\cos\eta t' + (\xi-\beta)\eta^{-1}\sin\eta t']\Big\}\mathrm{Erfc}(t'/\varepsilon)\Big]\Big] \quad (71)$$

This result may also be written in the form

$$F(t') = (1/\varepsilon\pi^{\frac{1}{2}})e^{-t'^2/\varepsilon^2}B(t') + \mathrm{Erfc}(t'/\varepsilon)B'(t') \quad (72)$$

where

$$B(t') = A[\eta^2+(\xi-\beta)^2]^{-1}\Big\{-e^{-\beta t'}+e^{-\xi t'}[\cos\eta t'+(\xi-\beta)\eta^{-1}\sin\eta t']\Big\} \quad (73)$$

and where $B'=\frac{dB}{dt'}$.

The value of the constant A in the expression for the pressure, equation 53, can be expressed in terms of the peak pressure that acts on the sphere $R=R_c$. Differentiating equation 53 and equating the result to zero gives

$$\frac{dP}{dx} = 0 = (\rho A/R_c \varepsilon \pi^{\frac{1}{2}}) e^{-\beta x} [\pi^{\frac{1}{2}} \varepsilon^2 \beta^2 \mathrm{Erfc}(x) - 2(\varepsilon\beta + x) e^{-x^2}] \tag{74}$$

where $x=t^*/\varepsilon$. Neglecting $\varepsilon^2\beta^2$ the solution to this equation is just $x=-\varepsilon\beta$. Substituting $t^*=-\beta\varepsilon^2$ into equation 53 gives the maximum pressure for small ε and β as

$$P_{max} = P_0 = \rho A/\pi^{\frac{1}{2}} \varepsilon R_c \tag{75}$$

B. DISPLACEMENT AND VELOCITY FOR LARGE DISTANCES

The radial displacement u_R is given by equation 34 as

$$u_R = - F(t')/R^2 - F'(t')/\alpha R$$

At large distances from the source the term involving $1/R^2$ can be neglected and the displacement is given by

$$u_R = - F'(t')/\alpha R \tag{76}$$

where F(t') is given by equation 64, which for small values of ε reduces to equations 72 and 73. By differentiating equation 72, for small values of ε,

$$F'(t') = (1/\pi^{\frac{1}{2}} \varepsilon^3) e^{-t'^2/\varepsilon^2} [-2t'B(t') + 2\varepsilon^2 B'(t')] + \mathrm{Erfc}(t'/\varepsilon) B''(t') \tag{77}$$

The derivatives of the function B(t') can be obtained by successive differentiation of equation 73.

$$B'(t') = A[\eta^2+(\xi-\beta)^2]^{-1}\{\beta e^{-\beta t'} + e^{-\xi t'}[\beta\cos\eta t' + \eta^{-1}(\eta^2+\xi^2-\xi\beta)\sin\eta t']\}$$

$$B''(t') = A[\eta^2+(\xi-\beta)^2]^{-1}\{-\beta^2 e^{-\beta t'} - e^{-\xi t'}[(\eta^2+\xi^2-2\xi\beta)\cos\eta t' + \eta^{-1}(\eta^2\xi+\eta^2\beta+\xi^3-\xi^2\beta)\sin\eta t']\} \quad (78)$$

$$B'''(t') = A[\eta^2+(\xi-\beta)^2]^{-1}\{\beta^3 e^{-\beta t'} + e^{-\xi t'}[(2\eta^2\xi+\eta^2\beta+2\xi^3 -3\xi^2\beta)\cos\eta t' + \eta^{-1}(\eta^4-3\eta^2\xi\beta-\xi^4+\xi^3\beta)\sin\eta t']\}$$

The maximum displacement is given by $[-F'(t')/\alpha R]_{max}$. For ε sufficiently small the maximum of $[-F'(t')]$ occurs for $\beta t'<<1$, $\eta t'<<1$, and $t'/\varepsilon>>1$; i.e., for $t'\simeq 0$. Since $B(t')$ and $B'(t')$ are both zero for $t'=0$, the maximum displacement for sufficiently small ε is

$$u_{Rmax} \simeq [-B''(0)\mathrm{Erfc}(0)/\alpha R] = A/2\alpha R \quad (79)$$

This may be written in terms of the peak pressure P_0 as

$$u_{Rmax} = \pi^{\frac{1}{2}}\varepsilon P_0 R_c/2\alpha\rho R \quad (80)$$

The velocity v_R is determined from equation 34 as

$$v_R = \frac{\partial u_R}{\partial t} = - F'(t')/R^2 - F''(t')/\alpha R \quad (81)$$

Again for large distances the term involving $1/R^2$ can be neglected, giving

$$v_R = - F''(t')/\alpha R \quad (82)$$

where from equation 77

$$F''(t') = (1/\pi^{\frac{1}{2}}\varepsilon^5)e^{-t'^2/\varepsilon^2}[(4t'^2-2\varepsilon^2)B(t') -6t'\varepsilon^2B'(t')+3\varepsilon^4B''(t')]+\mathrm{Erfc}(t'/\varepsilon)B'''(t') \qquad (83)$$

For ε sufficiently small the maximum of $[-F''(t')]$ also occurs for very small t'. Using equation 78 gives

$$[-F''(t')]_{max} \simeq -3B''(0)/\pi^{\frac{1}{2}}\varepsilon = 3A/\pi^{\frac{1}{2}}\varepsilon \qquad (84)$$

since for small ε the term containing $B'''(0)$ can be neglected. Thus the maximum velocity is

$$v_{Rmax} = [-F''(t')/\alpha R]_{max} = 3A/\pi^{\frac{1}{2}}\varepsilon\alpha R \qquad (85)$$

which in terms of the peak pressure P_0 is

$$v_{Rmax} = 3P_0R_c/\rho R \qquad (86)$$

C. TOTAL ELASTIC ENERGY RADIATED FROM THE SOURCE

For small ε, $F''(t')$ is given by equation 83 and the total radiated energy is found from equation 46 to be

$$Q = (4\pi\rho/\alpha)\int_{-\infty}^{\infty}\Big\{(1/\pi\varepsilon^{10})e^{-2t^2/\varepsilon^2}[2(2t^2-\varepsilon^2)B(t) -6\varepsilon^2tB'(t)+3\varepsilon^4B''(t)]^2 +(2/\pi^{\frac{1}{2}}\varepsilon^5)e^{-t^2/\varepsilon^2}[2(2t^2-\varepsilon^2)B(t)-6\varepsilon^2tB'(t) +3\varepsilon^4B''(t)]B'''(t)\mathrm{Erfc}(t/\varepsilon) +[\mathrm{Erfc}(t/\varepsilon)B'''(t)]^2\Big\}dt \qquad (87)$$

Since the exponentials in the integral fall off so rapidly for small ε, the major contribution to the integral will occur for small t. For small t the following expressions may be obtained from equations 73 and 78.

$$\begin{aligned} B(t) &= A(gt^3/6 - t^2/2) \\ B'(t) &= A(gt^2/2 - t) \\ B''(t) &= A(gt - 1) \\ B'''(t) &= Ag \end{aligned} \tag{88}$$

where

$$g = (\beta^3+\beta\eta^2+2\xi\eta^2-3\beta\xi^2+2\xi^3)/[\eta^2+(\xi-\beta)^2] \tag{89}$$

Assuming that ε is chosen sufficiently small so that $\varepsilon g<<1$, $\varepsilon\xi<<1$, and $\varepsilon\eta<<1$; equation 87 reduces to

$$\begin{aligned} Q &= (4\rho A^2/\alpha\varepsilon) \int_{-\infty}^{\infty} e^{-2x^2}(-3+7x^2-2x^4)\,dx \\ &= (321/16)(\pi/2)^{\frac{1}{2}}\rho A^2/\alpha\varepsilon \qquad (90) \\ &= 24.9\rho A^2/\alpha\varepsilon \end{aligned}$$

The total energy radiated may be expressed in terms of the peak pressure P_0 as

$$Q = [321\pi(\pi/2)^{\frac{1}{2}}/16]P_0^{\,2}\varepsilon R_c^{\,2}/\alpha\rho = 78.2P_0^{\,2}\varepsilon R_c^{\,2}/\alpha\rho \tag{91}$$

Since P_0 and ε are supposed to be characteristics of the shock wave when it becomes weak, they depend on the yield of the source and are independent of the nature of the medium. Because of the assumption that the medium is perfectly elastic no losses occur and the energy supplied by the source to the medium at radius R_c must equal the total energy radiated. Then from equation 91 the yield of the explosion, Y, is given by

$$Y = 78.2P_0{}^2\varepsilon R_c{}^2/\alpha\rho \tag{92}$$

This result means that the energy that is supplied to the elastic medium may be regarded as being in a thin spherical shell of radius R_c; i.e., in a shock front of radius R_c; thus

$$Y = 4\pi P_0 R_c{}^2 \delta R_c \tag{93}$$

where the thickness of the spherical shell δR_c is given by

$$\delta R_c = 321\ (\pi/2)^{1/2} P_0 \varepsilon/64\alpha\rho \tag{94}$$

which may be regarded as a characteristic of the medium for the minimal shock wave that generates the elastic field.

It may be noted from equation 90 that the radiated energy is finite when $\mu=0$. However ε cannot be zero in this expression for a realistic total energy flux. This confirms the original assumption that the pressure has to be specified over a finite time interval and cannot be represented by an instantaneous impulsive source.

Equations 80, 85, and 92 yield the result that the maximum displacement and the maximum velocity vary as the square root of the yield.

$$u_{Rmax} = (2/\pi)^{\frac{1}{4}}(4\varepsilon/321\alpha\rho)^{\frac{1}{2}}\ Y^{\frac{1}{2}}/R$$
$$v_{Rmax} = (2/\pi)^{\frac{1}{4}}(54\alpha/321\varepsilon\pi)^{\frac{1}{4}}Y^{\frac{1}{2}}/R \tag{95}$$

Theories in which the maximum displacement varies at a higher power of the yield have been proposed by LATTER, MARTINELLI, and TELLER [1959] and by PEET [1960]. However to produce maximum displacements varying as some power higher than one half of the yield would require propagation in a half space or in a layered medium. An infinite medium cannot give rise to such a field in a lossless medium without dispersion. Even with dispersion it is doubtful that any significant increase in the power of the yield above the one half power could be obtained.

IV. NUMERICAL RESULTS

Reasonable values of the basic parameters α, σ, and R_c (compressional wave velocity, Poisson's ratio, and radius upon which the source pressure is specified, respectively), and the two adjustable parameters ε and β which determine the time history of the source function, may be chosen to fit the theoretical results to experimental observations. The values of α, R_c, and σ chosen here are

$$\alpha = 2{,}500 \text{ meters/sec}$$

$$R_c = 500 \text{ meters}$$

$$\sigma = .3$$

Since Poisson's ratio, σ, is defined by

$$\sigma = \lambda/2(\lambda+\mu) \tag{96}$$

λ and μ being the Lamé constants; the parameters ξ and η, which appear in the theoretical results, are given from equations 27, 28, and 96 as

$$\begin{aligned} \xi &= (\alpha/R_c)[(1-2\sigma)/(1-\sigma)] = 2.857 \text{ sec}^{-1} \\ \eta &= (\alpha/R_c)[(1-2\sigma)^{\frac{1}{2}}/(1-\sigma)] = 4.517 \text{ sec}^{-1} \end{aligned} \tag{97}$$

The radial displacement u_R was calculated as a function of time at a distance of R=550 meters for three values each of the parameters ε and β, using the exact expressions, equations 66 and 68, for F and F'. Figure 2 shows the quantity $(\alpha R/A)u_R$ as a function of the reduced time t' for β=.25 sec^{-1} and for three values of ε; .05, .1, and .2 sec. Figure 3 shows $(\alpha R/A)u_R$ as a function of t' for

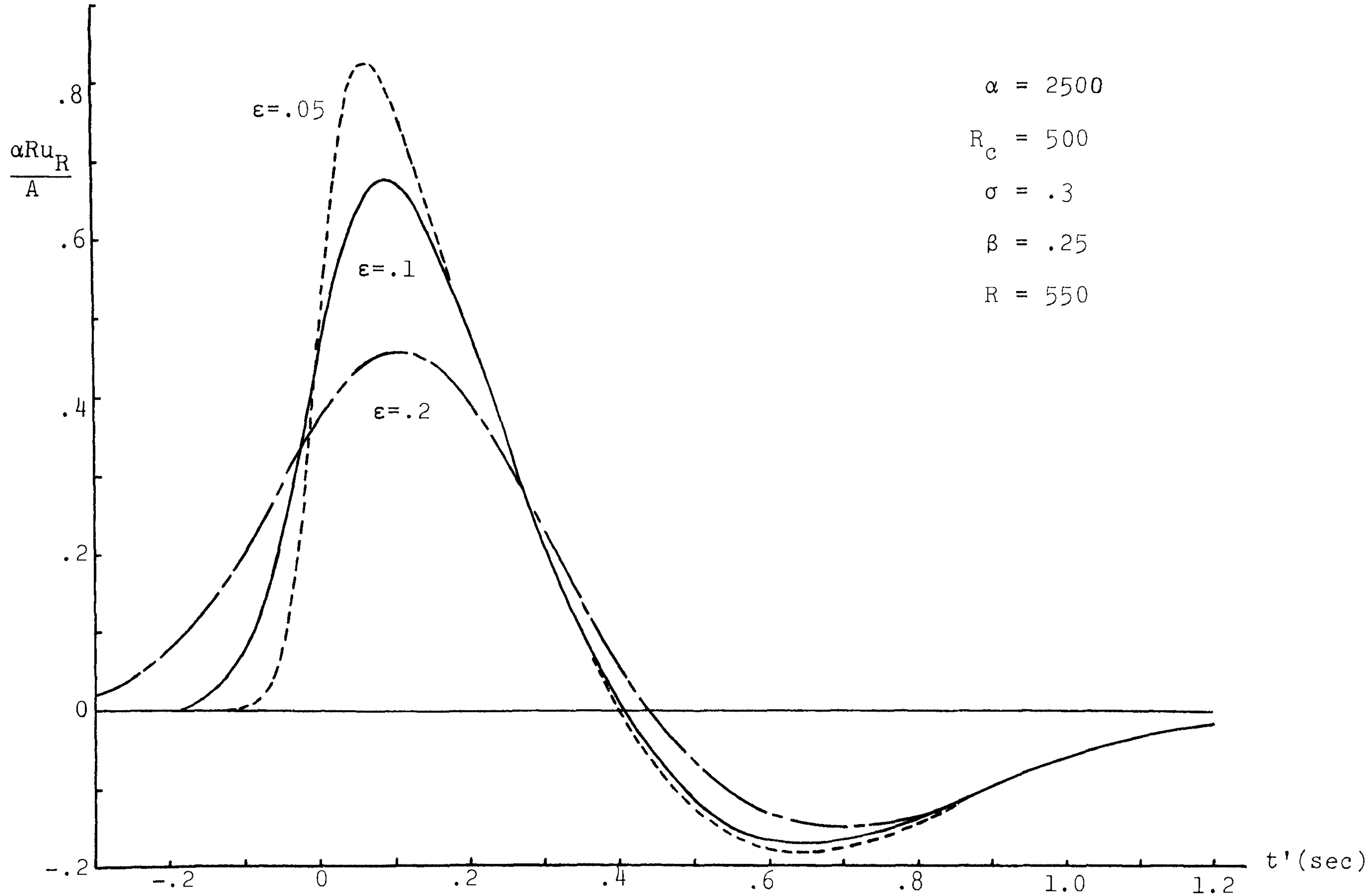

Figure 2. Displacement versus t', with ε as parameter

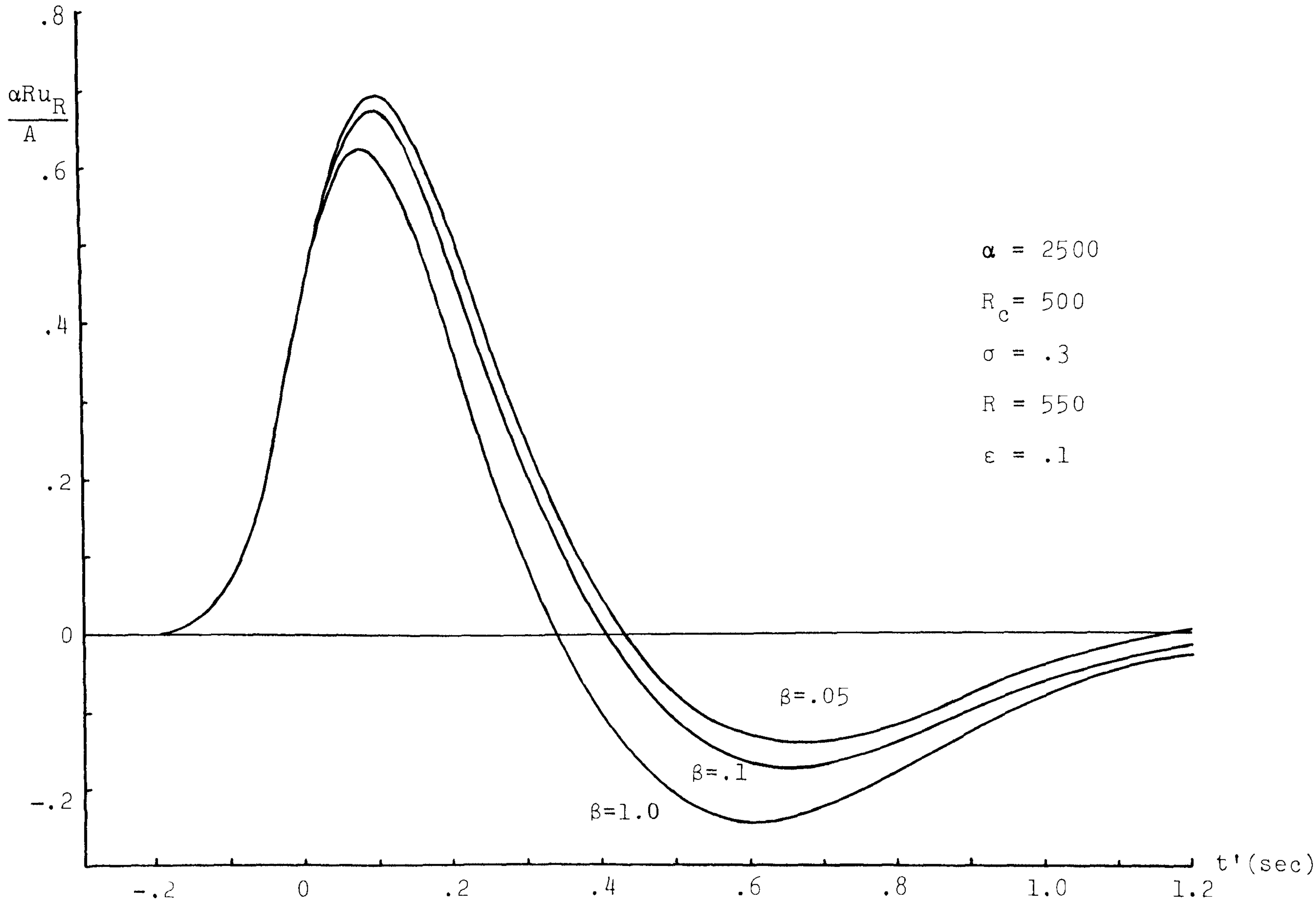

Figure 3. Displacement versus t', with β as parameter

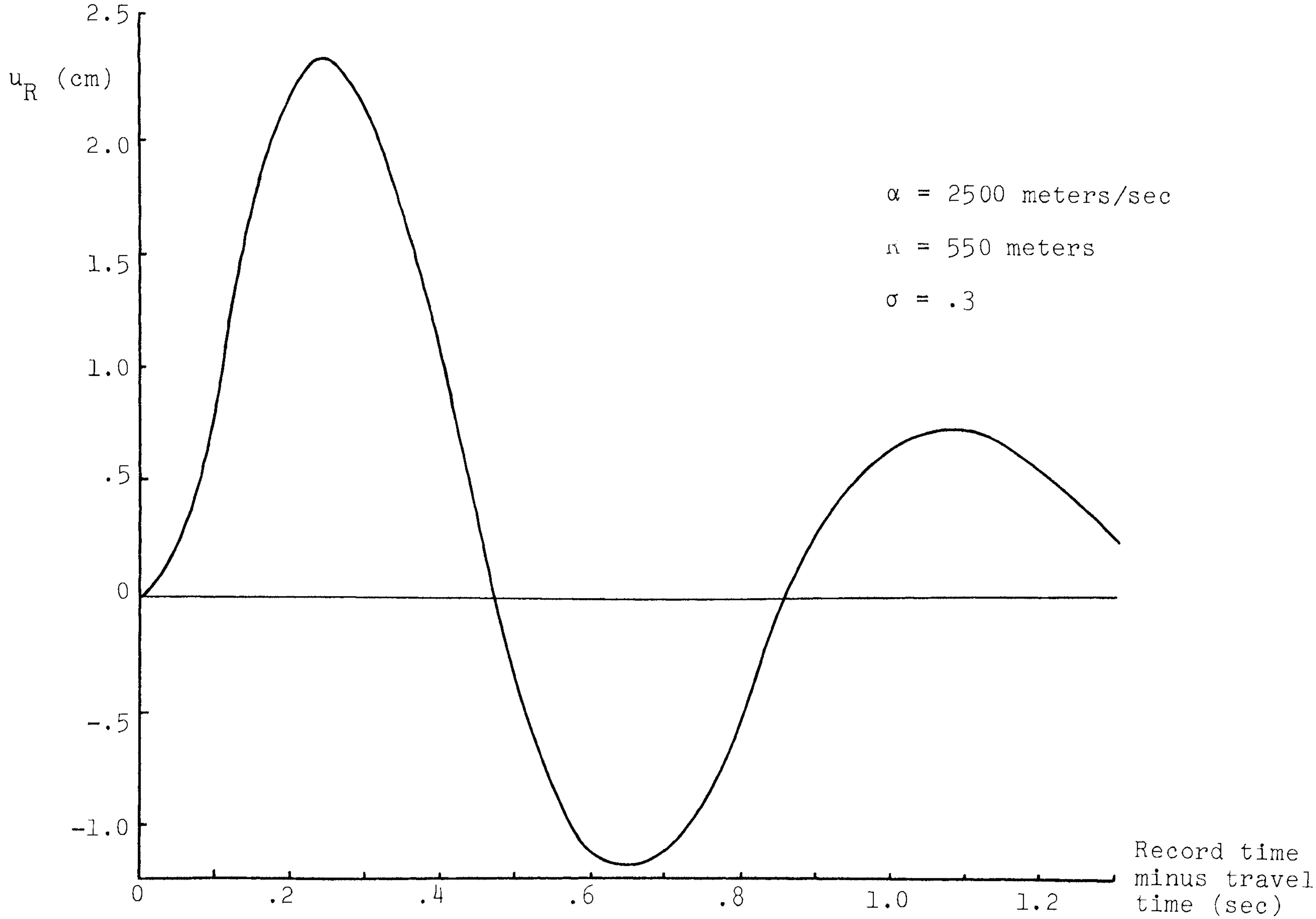

Figure 4. Observed displacement for Haymaker explosion

$\varepsilon=.1$ sec and for three balues of β; .05, .25, and 1.0 sec^{-1}.

The parameter ε principally affects the initial slope and the peak value of the displacement while β affects the peak value and the decaying part. Very small values of ε, approximating a delta function pressure source, give a very steep initial slope which does not agree with experimental observations. The choice of $\varepsilon=.1$ sec and $\beta=.25$ sec^{-1} is in approximate agreement with the experimental displacement observed by TREMBLY and BERG [1966] for the Haymaker underground nuclear explosion, which is shown in Figure 4. No attempt was made to exactly fit the theoretical curve to the experimental data since the theoretical model is for an infinite medium and cannot treat reflections, which are obviously present for larger values of time in the observed displacements. Also the values of the parameters α, σ, and R_c are not accurately known. Thus the travel time is uncertain, as indicated by the difference in the time scales for the theoretical curves, Figures 2 and 3, and the experimental observations, Figure 4.

The quantity $(\alpha R/A)u_R$, given by

$$(\alpha R/A)u_R = -(F' + \alpha F/R) \tag{98}$$

is plotted versus t' in figure 5; for three values of distance R; 550, 1000, and 4000 meters. This shows the affect of the two terms, F and F'. For relatively small values of R there is a significant contribution from the term in equation 98 containing F, while for large distances $(\alpha R/A)u_R$ is equal to F'.

The quantity $(\alpha R/A)v_R$, where v_R is the radial velocity defined by equation 81, is plotted versus t' in Figure 6; for two values of R; 550 and 4000 meters. Figure 7 shows the quantity $(\alpha R/A)a_R$, a_R being the radial acceleration defined by

$$a_R = -\frac{\partial v_R}{\partial t} = -F''(t')/R^2 - F'''(t')/\alpha R \qquad (99)$$

as a function of t' for R equal 550 and 4000 meters. Both $(\alpha R/A)v_R$ and $(\alpha R/A)a_R$ are practically independent of the distance R.

The quantity $(\alpha R/A)v_{Rmax}$, where v_{Rmax} is the peak velocity defined by

$$v_{Rmax} = [-(F''/\alpha R + F'/R^2)]_{max} \qquad (100)$$

is plotted versus distance R on a Log-Log scale in Figure 8. For large values of R the peak velocity is proportional to 1/R as given by equations 82 and 95. At smaller values of R the term involving F' in equation 100 causes a slight departure from a 1/R dependence.

The pressure, $(R_c/\rho A)P$, calculated from equation 53, is plotted as a function of time t, for ε=.1 sec and β=.25 sec^{-1}, in Figure 9.

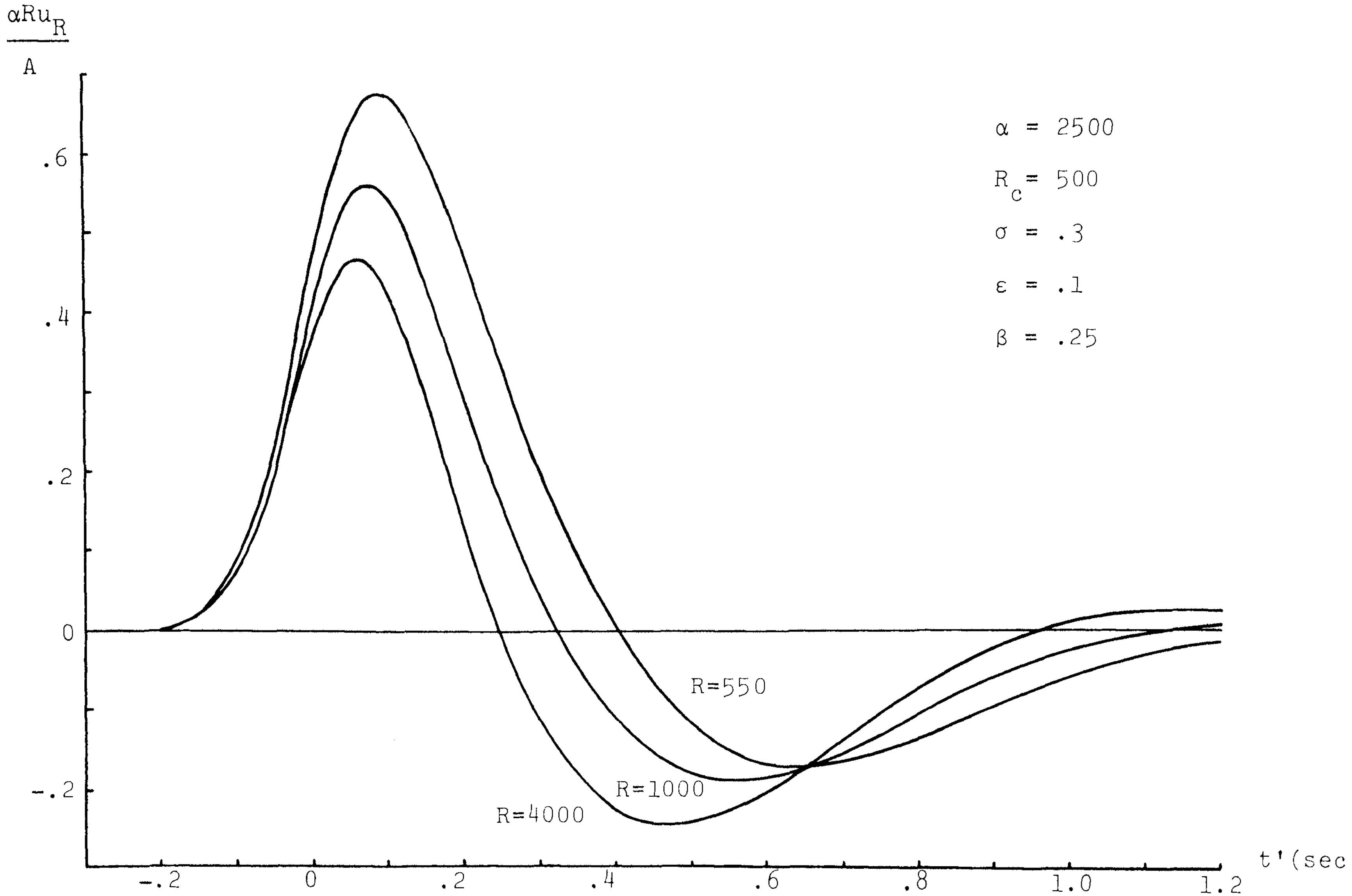

Figure 5. Displacement versus t' for 3 values of distance

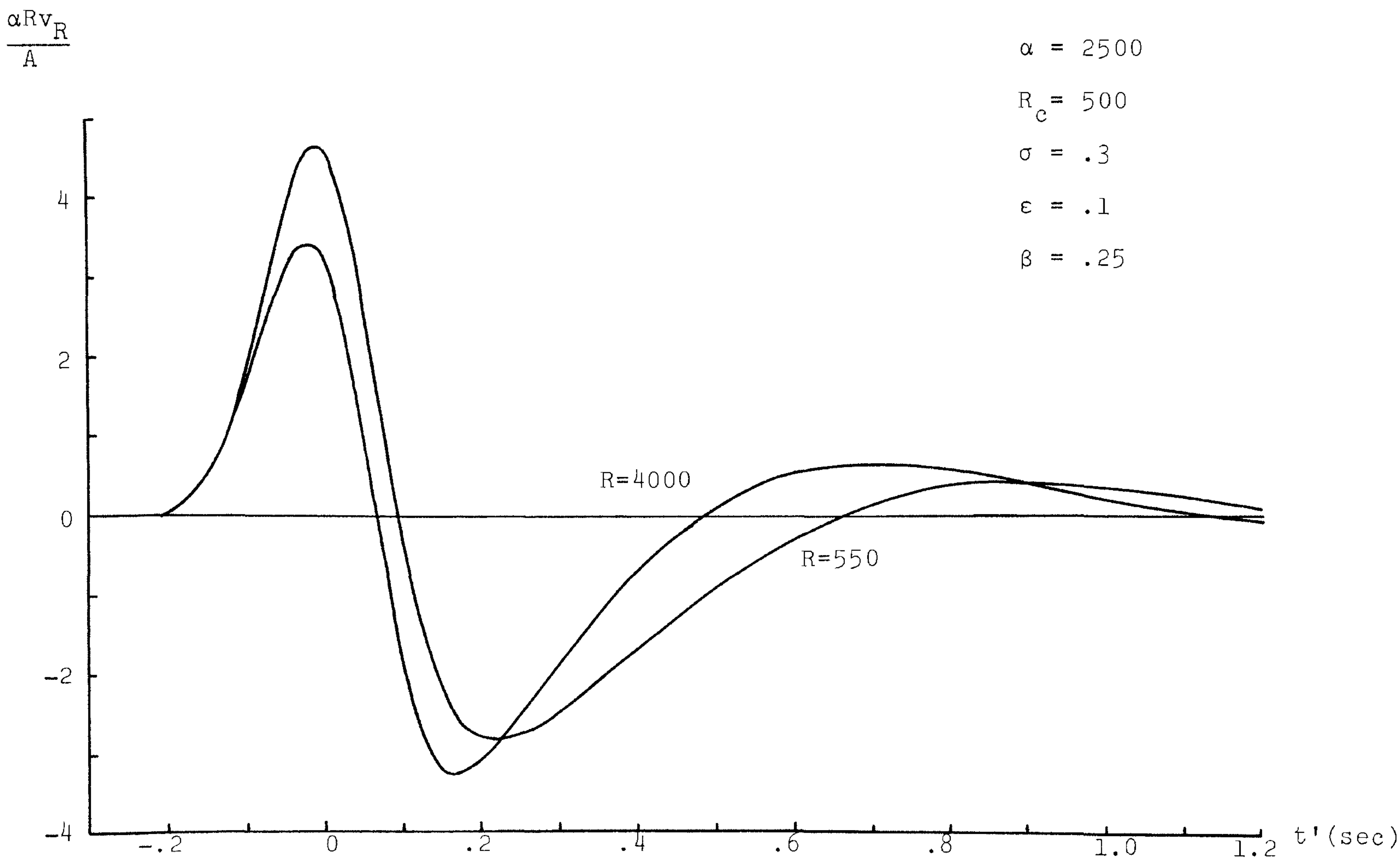

Figure 6. Velocity versus t' for 2 values of distance

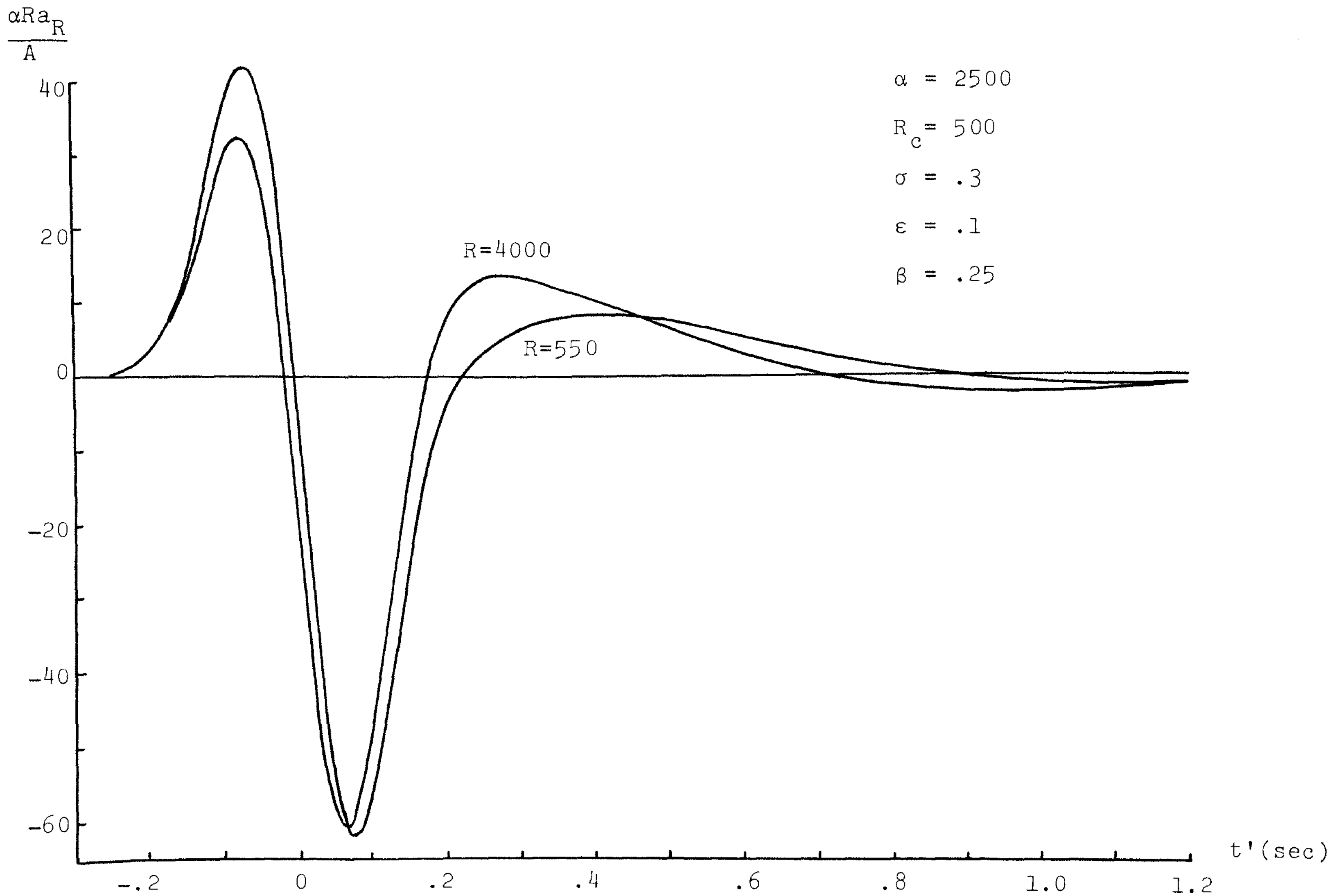

Figure 7. Acceleration versus t' for 2 values of distance

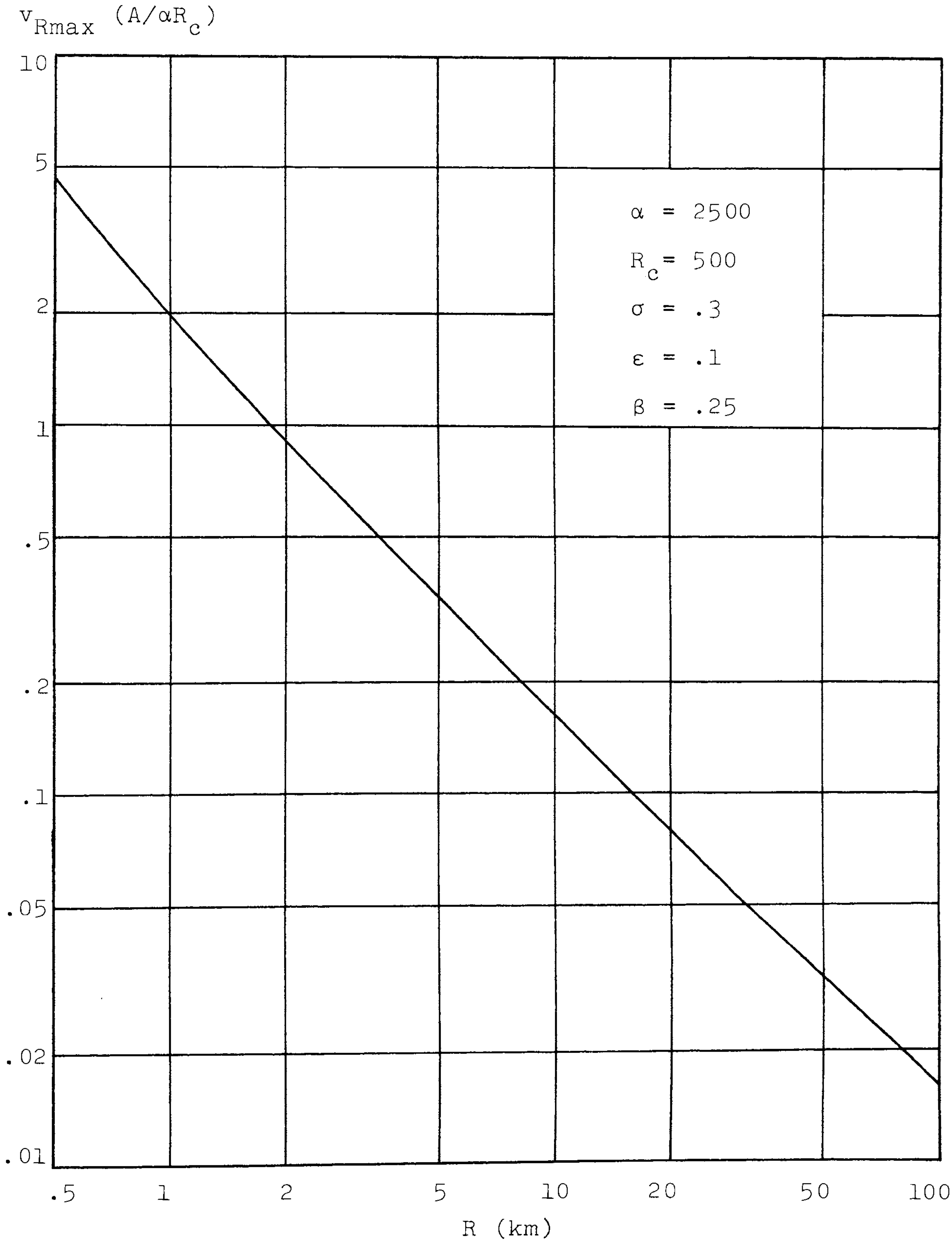

Figure 8. Peak velocity as a function of distance

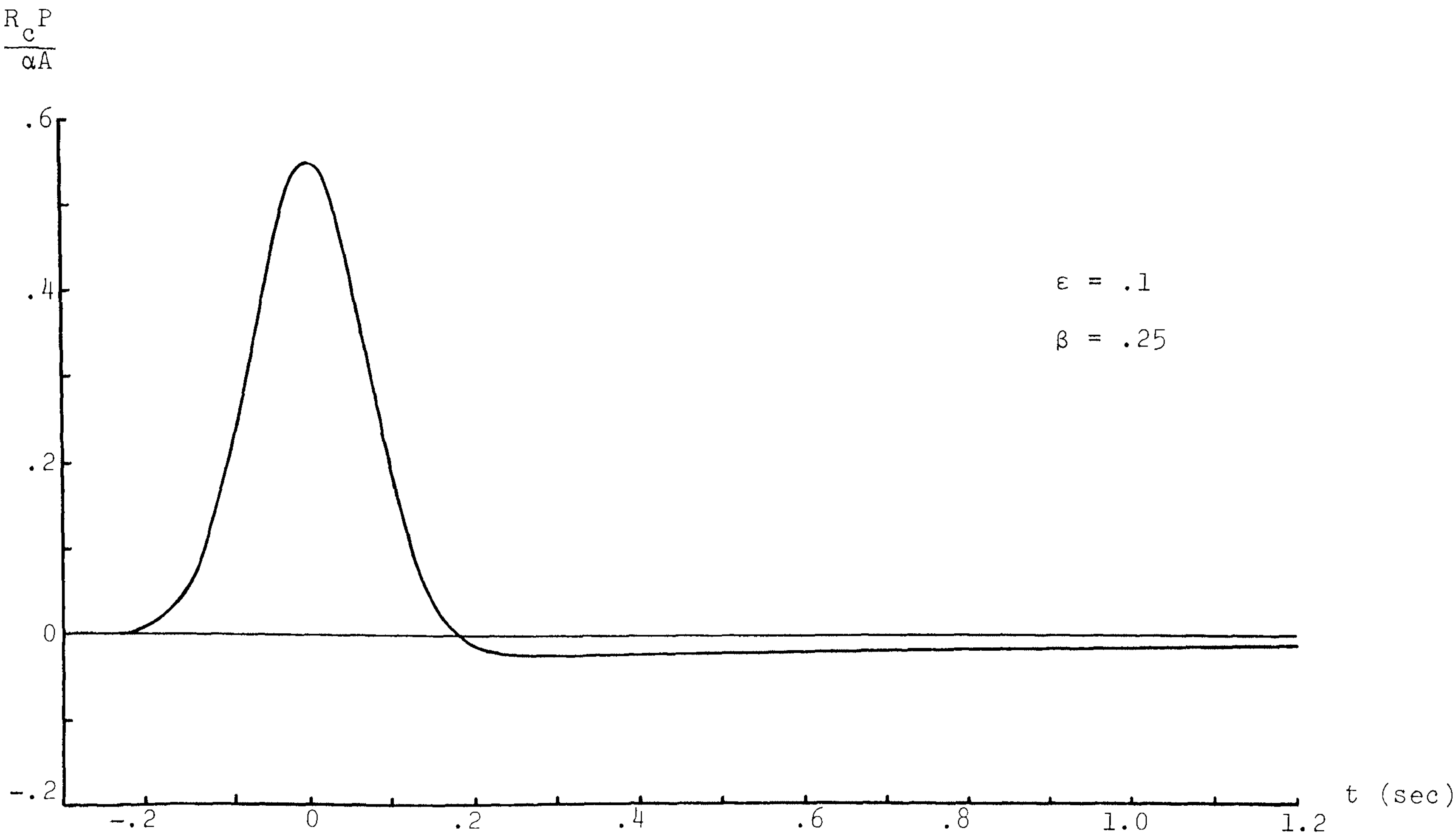

Figure 9. Pressure as a function of time

V. CONCLUSION

It has been shown that instantaneous impulsive pressure sources are not physically realistic. Sources consisting of the delta function or its derivatives yield an infinite amount of radiated energy. The step function source fails when the shear modulus μ goes to zero, predicting displacements which grow indefinitely large. Also the energy radiated by a step function source depends on the shear modulus of the medium, becoming infinite when $\mu=0$. The conclusion is that a source must be used that has a finite time history.

Such a source has been derived here by matching the acoustic wave near the source to the elastic region further out. It yields a finite amount of radiated energy and gives physically sensible results when $\mu=0$. The parameters in the source function can be chosen to make the theoretical results agree satisfactorily with experimental observations.

VI. BIBLIOGRAPHY

ADAMS, W.M., PRESTON, R.G., FLANDERS, P.L., SACHS, D.C., and PERRET, W.P. [1962] Summary report of strong motion measurements, underground nuclear detonations, J. Geophys. Res. 66, p. 903-942.

BERG, J.W. and PAPAGEORGE, G.E. [1964] Elastic displacements of primary waves from explosive sources, Bull. Seismol. Soc. Am. 54, p. 947-959.

BLAKE, F.G. [1952] Spherical wave propagation in solid media, J. Acoust. Soc. Am. 7, p. 211-215.

BREMERMANN, H. [1965] Distributions, complex variables, and fourier transforms, Addison Wesley, Reading, Mass., p. 138-139.

CAGNIARD, L. [1962] as translated and revised by Flinn, E.A. and Dix, C.H., Reflection and refraction of progressive seismic waves, Mc.Graw-Hill, New York, 282 p.

CARDER, D.S., CLOUD, W.K., MICKEY, W.V., JORDAN, J.N., and GORDON, D.W. [1962] Seismic waves from an underground explosion in a salt bed, U.S.A.E.C. Plowshare Program, Project Gnome, Final Rept. PNE150F, 60 p.

DUVALL, W.I. [1953] Strain wave shapes in rock near explosions, Geophysics 18, p. 310-323.

EWING, W.M., JARDETSKY, W.S., and PRESS, F. [1957] Elastic waves in layered media, Mc.Graw-Hill, New York, 378 p.

HASKELL, N.A. [1967] Analytic approximation for the elastic radiation from contained underground explosions, J. Geophys. Res. 72, p. 2583-2586.

LATTER, A.L., MARTINELLI, E.A., and TELLER, E. [1959] Seismic scaling law for underground explosions, Physics of Fluids 2, p. 280-282.

LOVE, A.E.H. [1927] A treatise on the mathematical theory of elasticity, 4th ed., Dover, New York, 643 p.

MIKLOWITZ, J. [1960] Recent developments in elastic wave propagation, Appl. Mech. Rev. 13, p. 865-878.

MORSE, P.M. and FESHBACH, H. [1953] Methods of theoretical physics, Mc.Graw-Hill, New York, p. 1779-1791.

PEET, W.E. [1960] A shock wave theory for the generation of the seismic signal around a spherical shot hole, Geophysical Prospecting 8, p. 509-533.

PINNEY, E. [1954] Surface motion due to a point source in a semi-infinite medium, Bull. Seismol. Soc. Am. 44, p. 571-596.

ROMNEY, C. [1959] Amplitudes of seismic body waves from underground nuclear explosions, J. Geophys. Res. 64, p. 1489-1498.

SELBERG, H.L. [1952] Transient compressional waves from spherical and cylindrical cavities, Arkiv. Fysik 5, p. 97-108.

SHARPE, J.A. [1942] The production of elastic waves by explosive pressures, Geophysics 7, p. 144-154.

SOKOLNIKOFF, I.S. [1956] Mathematical theory of elasticity, Mc.Graw-Hill, New York, p. 83-91.

SOKOLNIKOFF, I.S. [1964] Tensor analysis theory and applications to geometry and mechanics of continua, Wiley, New York, 361 p.

TREMBLY, L.D. and BERG, J.W., JR. [1966] Amplitudes of primary seismic waves near the Hardhat, Haymaker, and Shoal nuclear explosions, Bull. Seismol. Soc. Am. 56, p. 643-653.

TSAY, L. [1969] A five parameter study of spherical transient waves in a generalized Voight model. Dissertation, University of Missouri at Rolla. 169 p. (With 81 figures)

WEART, W.D. [1962] Particle motion near a nuclear detonation in halite, Bull. Seismol. Soc. Am. 52, p. 981-1005.

WERTH, G.C. and HERBST, R.F. [1963] Comparison of seismic waves from nuclear explosions in four mediums, J. Geophys. Res. 68, p. 1463-1475.

WRIGHT, J.K., CARPENTER, E.W., and SAVILL, R.A. [1962] Some studies of the P waves from underground nuclear explosions, J. Geophys. Res. 67, p. 1155-1160.

VITA

Joseph Hugo Hemmann was born on April 6, 1941, in St. Louis, Missouri. He received his primary education in St. Louis and his secondary education in St. Clair, Missouri. He received a Bachelor of Science Degree in Physics from the Missouri School of Mines and Metallurgy, in Rolla, Missouri, in August 1962.

He has been enrolled in the Graduate School of the University of Missouri-Rolla, in Rolla, Missouri, since September 1962. During the course of his studies he has held graduate teaching assistantships, a N.A.S.A. Fellowship, and a research assistantship. He received a Master of Science Degree in Physics in May 1969.