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ABSTRACT

Dynamic behavior of piles is strongly affected by the variation in the soil stiffness with depth and the interaction between the soil and the pile. The dynamic characteristics of the piles can be determined by using simple lumped mass models to complex finite element modeling. There is a need for experimental research, in particular large-scale field tests, to validate existing linear/nonlinear models, which are used to predict the dynamic stiffness and damping of the soil-pile system. The determination of dynamic characteristics of soil-pile system by full-scale lateral dynamic pile load testing is an important aspect in the design of pile foundations subjected to dynamic loads. This paper presents the results of field lateral dynamic load tests conducted at two different petrochemical complex sites in India and the measured dynamic constants of the soil-pile system. A three-dimensional finite element analysis is performed using ABAQUS to predict the nonlinear response of the soil-pile system under dynamic lateral loads. The lateral stiffness estimated from the FE analysis shows good agreement with stiffness measured from field tests. The paper also discusses on the dynamic analysis and design of Primary Air (PA) and Force-Draft (FD) fan foundations of a thermal power plant using finite-element analysis.

INTRODUCTION

Stiffness and damping of single piles can be computed using various techniques. Most of the modern techniques consider propagation of elastic waves through the soil, which modifies the stiffness of the piles and make it frequency dependent. Piles in a homogenous soil subjected to small amplitudes of vibrations are compatible with the assumptions of linearity (Novak and Sheta 1982). However, field experiments indicate that the assumption of soil homogeneity is not suitable for practical applications, as it results in overestimation of pile stiffness and damping. This is mainly due to the variation in the stiffness of the soil with depth. The analysis and design of piles subjected to dynamic lateral loads is complex due to the degradation of stiffness of the soil near the ground surface, compounded by the soil-pile separation. Additionally, the stiffness in the lateral direction is very low in comparison to the vertical stiffness and thus the lateral capacity/stiffness of the pile governs the design in most cases. The lateral capacity, stiffness and its bending behaviour are mainly dependent upon the characteristics of the top soil layers within a few meters below ground level, which mainly consists of weak deposits such as soft clay or loose sand, therefore the degree of nonlinearity that prevails during the lateral loading on piles is severe. Hence, evaluation of lateral stiffness of a single pile/pile group under dynamic lateral loading taking into account the strong nonlinearity and soil gapping becomes a crucial step in the satisfactory design and performance of pile-supported foundations subjected to dynamic loads.

In the recent past, many sophisticated linear and nonlinear models were proposed to study the lateral response of piles under dynamic loads (Novak and EI-Sharnouby 1983, Dobry and Gazetas 1988, Haldar and Bose 1990, Nogami et al. 1992, Badoni and Makris 1996, El-Naggar and Novak 1996, Mylonakis and Gazetas 1999, Mostafa and El-Naggar 2002, Kucukarslan and Banerjee 2003, Chau and Yang 2005, Assareh and Asgarian 2008), but there are only a very few full scale experimental data available to confirm the reliability of these models. The major full-scale field testing carried out on piles embedded in clay and sandy clay sites by various authors (Novak 1985, Blaney and O'Neill 1989, El-Marsafawi et al. 1992, Puri and Prakash 1992, Crouse et al. 1993, Anandarajah et al. 2001, Boominathan et al. 2002, Pak et al. 2003, Boominathan and Ayothiraman 2006) clearly demonstrate that the performance of existing linear and nonlinear models are highly dependent on in-situ soil nonlinearity and dynamic loading characteristics. Hence, conducting in-situ full-scale dynamic tests on piles in order to have a better understanding of nonlinear response of piles should accurately assess the dynamic characteristics of soil-pile system. This paper presents the results of full-scale field dynamic lateral pile load tests carried out at two sites in India (Chennai and Hazira) and a nonlinear three-dimensional finite element analysis of piles under dynamic lateral loads carried out using ABAQUS. This paper also presents the dynamic analysis and design of Primary Air (PA) and Force-Draft (FD) fan foundations of a thermal power plant in South India using finite-element analysis. The stiffness parameters determined using simplified approaches were used as an input for the dynamic analysis of the foundation.

DYNAMIC TESTING OF PILES

Large-scale steady state forced lateral vibration tests were conducted on driven cast-in-situ piles installed at two different sites in India (Site-I: Chennai, Tamil Nadu and Site-II: Hazira, Gujrat) as per the procedure recommended by Indian Standard code of practiceIS: 9716.

Soil and Pile Properties

Site-I: Chennai, Tamil Nadu. The test pile is 450 mm in diameter and extends 20.15 m and 18.25 m below the ground level and the below cut off level respectively. The dynamic Young's modulus(Ep) of the M30 grade concrete pile is 37000 MPa. The soil profile of Site-I and soil properties obtained from the borehole data are presented in Table 1. It can be observed from Table 1, that the Site-I is characterized by four layers with a 5m thick very soft silty clay layer at the top. The shear wave velocity of different layers up to a depth of 12.5m in the site was determined from MASW test and the shear wave velocity of soil layers below 12.5 m depth was estimated using Eq. 1 based on the average SPT-N value (Imai, 1977). The maximum dynamic shear modulus required for the finite element analysis is determined based on the shear wave velocity using Eq. 2 (Prakash and Puri, 1987) and the values are reported in Table 1.

$$V_s = 91\, N^{0.337} \qquad (1)$$

$$G_{max} = V_s^2 \rho \qquad (2)$$

Site-II: Hazira, Gujarat. The 500 mm diameter test pile extends up to 15.41m below the cut off level. Pile is made of M30 grade concrete with a corresponding dynamic Young's modulus (Ep) of 37000 MPa. The typical soil profile of Site-II obtained from the borehole record is presented in Table 2. It can be observed from the table that Site-II has six layers, wherein the top three layers are characterized by medium stiff silty-clay / stiff marine clay to a depth of 8.5 m and is followed by a 1.5m thick loose sandy layer. Since direct measurement of shear wave velocity were not carried out at this site, shear wave velocity of layers was estimated using Eq. 1 based on the average SPT-N value. The maximum dynamic shear modulus was determined based on the shear wave velocity. The values of shear wave velocity and the shear modulus of various layers are summarized in Table 2.

TABLE 1. Properties of soil at Site-I

Depth (m)	Thick (m)	Soil Type	*Avg. SPT* (N_{avg})	V_s (m/s)	G_{max} (MPa)
2.0-7.0	5	Soft silty clay	0	75*	10.1
7.0-12.5	5.5	Fine sand	40	170*	54.9
12.5-18.0	5.5	Silty Clay	60	362#	262.1
18.0-20.2	2.2	Yellow Silty Clay	>100	430#	388.3

*based on MASW test; #based on SPT 'N' value (Look, 2007)

TABLE 2. Properties of soil at Site-II

Depth (m)	Thick (m)	Description	*Avg. SPT* (N_{avg})	V_s (m/s)	G_{max} (MPa)
0.0-5.5	5.5	High plastic silty clay	5	157	44.4
5.5-8.5	3.0	Marine clay	8	183	56.9
8.5-10.0	1.5	Loose silty sand	17	236	105.8
10.0-15.5	5.5	Dense silty sand	25	269	144.7
15.5-18.0	2.5	Stiff silty clay	21	254	135.5
18.0-20.5	2.5	Silty coarse sand	32	293	180.3

Test Setup and Procedure

Typical layout of a forced vibration test is shown in Figure 1. A steady state sinusoidal force was generated with a mechanical oscillator of 5-tonne capacity. The speed of the oscillator was controlled by a DC motor and a speed control unit. The forced vibration response of the piles were measured using two acceleration transducers fixed at the mid height of the pile cap, and at the pile cut off level as shown in Figure 1.

A Data Acquisition System (DAS) consisting of a multi-channel carrier-frequency amplifier system and a digital storage oscilloscope was used to monitor and record the time history of response of the pile measured by accelerometers. Each acceleration transducer was calibrated before and after conducting the tests. After every steady state lateral vibration test, the eccentricity of the oscillator was increased to raise the dynamic force and the test was repeated to cover a wide range of lateral displacements expected during a dynamic loading. The tests were repeated for five levels of eccentricities. More information about interpretation of test data is presented in Boominathan and Ayothiraman (2006).

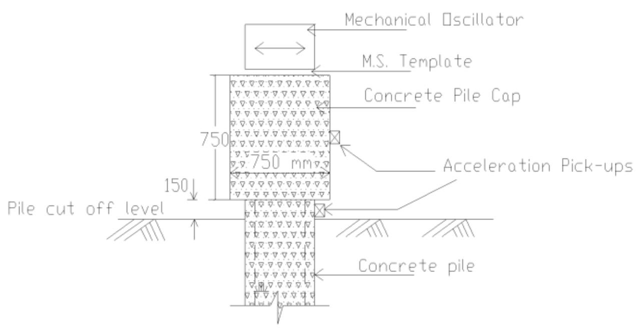


Fig. 1. Typical forced vibration test layout (after Boominathan and Ayothiraman, 2006)

Test Results

The displacement amplitude of vibration, Ax, was computed from the measured acceleration using Eq. 3.

$$A_x = \frac{a_x}{4\pi^2 f^2} \tag{3}$$

where, a_x= measured horizontal acceleration of vibration (mm/s^2) at a particular frequency , f (Hz).

The computed values of amplitude of displacement corresponding to the pile cut-off level at each frequency for different eccentricities of the oscillator were plotted as frequency response curves. Typical frequency response curves obtained for Site-I and Site-II are presented in Figures 2 and 3, respectively.

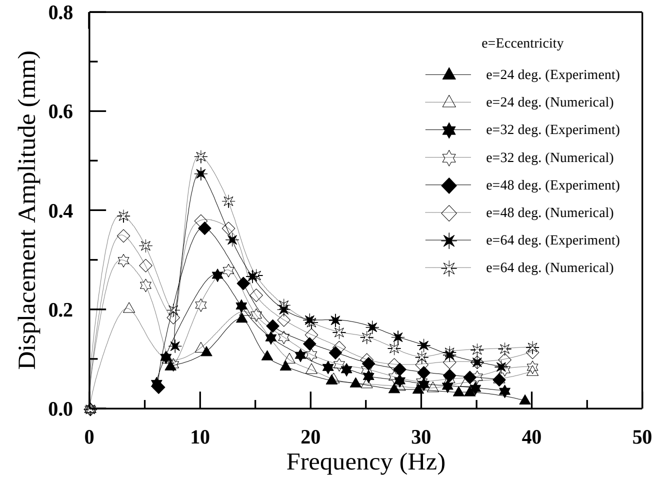


Fig. 2 Dynamic amplitude vs. frequency for Site-I

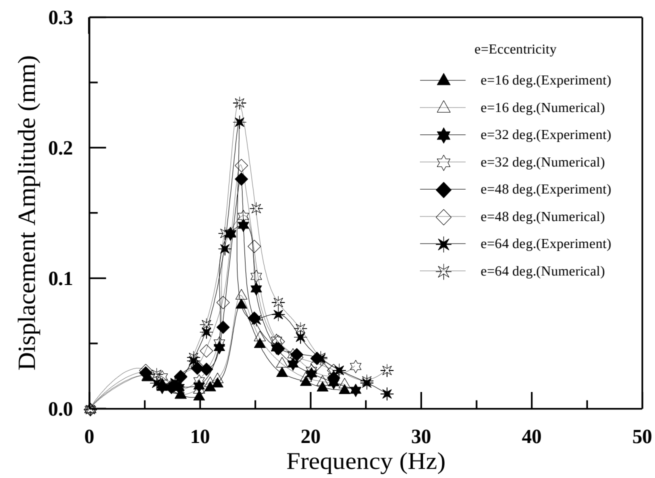


Fig. 3 Dynamic amplitude vs. frequency for Site-II

It can be noticed from Fig. 2 that the resonant frequency of soil-pile system measured from the experimental data for Site-I ranges from 10.5 to 14 Hz. The resonant frequency reduces from 14 Hz to 10.5 Hz as the magnitude of the dynamic force increases, indicating a non-linear response of the soil-pile system due to the degradation of soil stiffness. This observation is consistent for almost all the test piles at Site-I. The observed non-linearity could be due to the presence of a 5m thick very soft silty clay layer at the top with a SPT-N value of zero. It is also observed from Fig. 3 that the resonant frequency of soil-pile system measured from experimental data for Site-II ranges from 13 Hz to 14 Hz, i.e., the resonant frequency remains practically same irrespective of different magnitude of the dynamic force levels. This indicates that the degree of nonlinearity for Site-II is significantly less, which is due to presence of relatively medium stiff to stiff clay layers in the top 8.5 m depth below the ground surface.

MODELING IN ABAQUS

A full 3D finite element analysis of the pile-soil is performed using ABAQUS to simulate the pile load test. The non-linear response of piles under lateral loading is strongly influenced by the variation in the stiffness of the soil with depth and the interaction between the soil and the pile. In the present study, the pile is idealized as an elastic linear isotropic material. The solid circular pile is modeled to the same scale as in the field conditions. The pile is discretized using solid tetrahedral elements. The pile is assumed to be pinned at the base and is free at the top simulating a free head condition similar to the field conditions. The non-linear stress-strain behavior of soil is modelled using an elasto-plastic Drucker-Prager model. The soil profile in the FE analysis is modeled identical to the field conditions. The input parameters, such as the Young's modulus that is determined from the shear modulus, are presented in Table 1 and 2. The properties of cohesion and friction angle were adopted based on their correlations with SPT 'N' value (Look, 2007) and are presented in Table 1 and 2, respectively. The soil matrix of size 20D is adopted, where D is the diameter of the pile. Absorbing boundaries were simulated along the sides to prevent reflection of stress waves on the boundaries and the boundaries are restrained in the vertical direction. The base of the FE model is fixed in all directions and the top of the soil mass is not restrained. The discretized soil–pile model is presented in Figure 4.

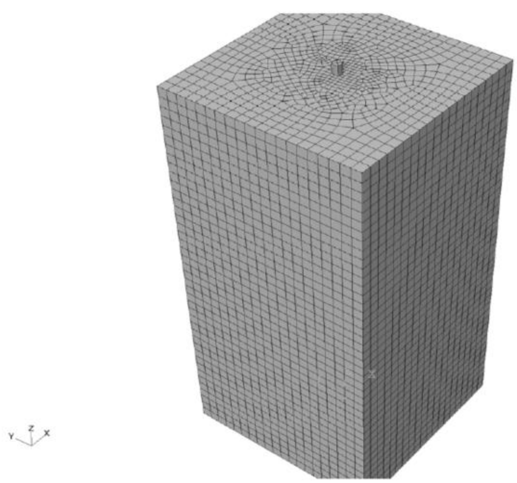


Fig. 4 The discretized FE model

The modeling of the pile-soil interface is an important aspect in capturing the response of piles to dynamic lateral loading. The interface between the pile and the soil was modeled using surface-to-surface contact elements. The pile is considered as the contact surface and the soil as the target surface, to allow for the interpenetration of pile nodes into the soil surface. Penalty contact method with small sliding was utilized to simulate the normal and tangential contact behavior. The separation of contact under reversal of loading (tension) was also simulated. A cyclic ramped load similar to that applied in the field was applied at the top of the pile at 0.75 m above the ground level. The acceleration, stresses, and displacements were recorded throughout the soil-pile system. The typical displacement contour obtained from finite element analysis is shown in Figure 5.

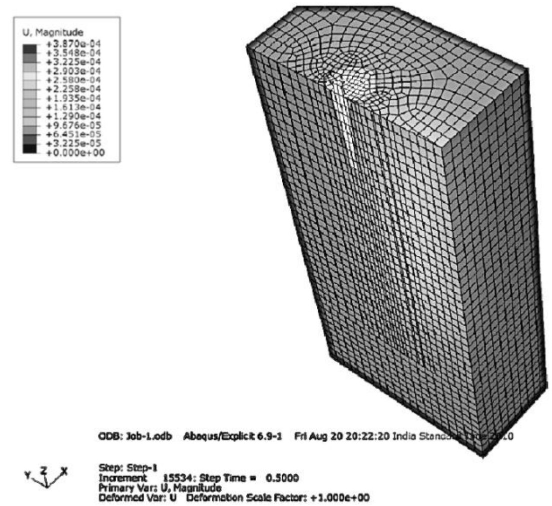


Fig. 5 A typical displacement contour obtained in FE simulation

The dynamic amplitude versus frequency response predicted from the finite element model for Site-I and Site-II are presented in Fig. 2 and 3. It can be observed from Fig. 2 and 3 that the FE simulations reasonably predict the lateral dynamic response of piles as measured in field tests. Closer obeservation of Figures 2 and 3 reveals that the FE simulation over predicts the amplitude by about 5 to 10%. The over-estimation increases to 10% at higher eccentricities and at near resonant frequency. The FE simulation overpredicts the resonant frequency by about 5% in most cases. A smaller peak is also observed at a frequency of about 5Hz. To understand better the effect of resonance, a modal analysis or frequency extraction analysis was performed. About 20 modes were extracted using Block Lanczos methodolgy (Montgomery, 1995) and it was observed that the frequencies were in the range of 0.0783 to 5.5Hz. The lateral vibration of pile is the fourth mode of vibration with a frequency of 4.485Hz, which was the apparent reason for occurance of a small peak at about 5Hz. A typical mode shape obtained is presented in Figure 6. The lateral field vibration test captures the combined response of the soil-pile system. In order to understand the response of pile alone, a frequency extraction analysis was carried out for the pile without the surrounding soil. The results of the modal analysis are presented in Figure 7. It can be observed from Fig. 7 that the first mode of the pile is lateral with frequency of around 5 Hz. However, vertical resonance condition is achieved only at a frequency of 58 Hz usually over the 10^{th}mode of vibration. Hence lateral resonance is more predominate at 5 Hz and at around 15 Hz for piles of length 16.5 to 18.5m.

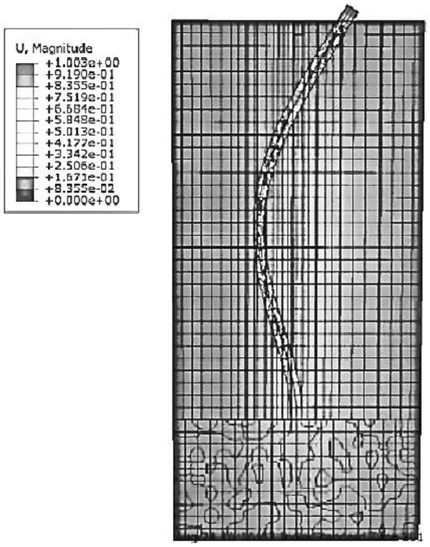


Fig. 6. Mode 4 of soil-pile system

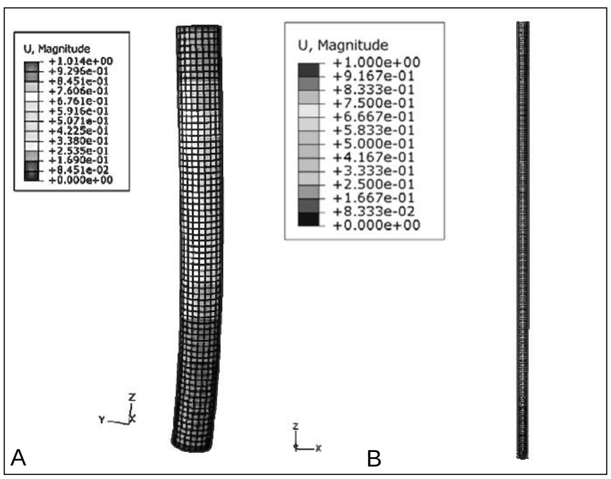


Fig. 7. Modal analysis of piles
(A) Mode 1 @ 4.75Hz, (B) Mode 12 @ 58.3 Hz

The dynamic stiffness of the soil-pile system is typically obtained from a plot of dynamic force versus equivalent static amplitude using the procedure described in Boominathan and Ayothiraman (2006) and as per IS 9716 (1981). The variation of static amplitude with dynamic force measured from field tests and that predicted from FE simulation for both sites are presented in Figures 8 and 9 respectively.

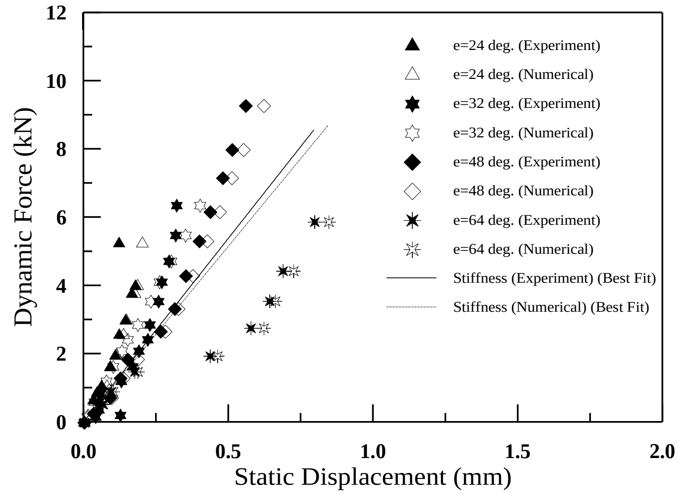


Fig. 8 Force vs. Static amplitude plot for Site-I

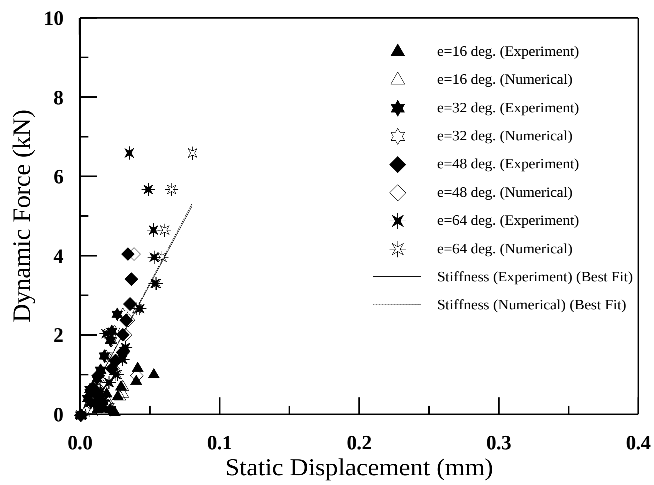


Fig. 9 Force vs. Static amplitude plot for Site-II

It can be observed from Fig. 8 and 9 that the simulated response matches remarkably well with that of the response observed in Site-I. For Site-II, though the simulated response matches fairly well with the measured response at low to moderate force levels, the simulated response at higher force level is 30% more than that measured in the field.

The lateral stiffness of soil-pile system predicted by the finite element model and that measured from field tests for both the sites are presented in Table 3.

TABLE 3. Measured and Simulated Lateral Stiffness

SITE	Lateral Stiffness (MN/m)	
	Field Test	ABAQUS
Site-I	10.75	10.28
Site-II	56.98	52.85

It can be observed from Table 3 that the FE simulation matches well with the results obtained from the field tests. Site-I characterized by soft clay deposits at the top with a low shear wave velocity (V_s) of 75 m/s has lower stiffness in contrast to Site-II. Hence, it is evident that the site conditions especially within the top 1/3rd length of the pile plays an improtant role in the lateral stiffness of the pile, although the resonant frequency remains practically unchanged for piles of approximately same dimensions in different strata. FE simulation is found to underpredict the response in the field by about 4% in Site-I and by about 7% in case of stiffer soil site (Site-II).

DYNAMIC ANALYSIS AND DESIGN OF PILE FOUNDATIONS

The Primary Air (PA) and Force Draft (FD) fan foundations are to be designed for a thermal powerplant in South India. The PA and FD fans are to be founded at a rocky site. The top 3.0m of the site is characterised by Medium Rock with RQD increasing from 12.5% to 55%. A core recovery of 60 to 90% was recorded in this layer. It is followed by a 5m thick highly weathered rock layer, with a core recovery of <27%. This layer is follwed by a layer of Fractured Rock with a core recovery of about 70% and a RQD of 27%. Cross-hole test was carried out to determine the variation in the shear wave velocity with depth. The results of the cross-hole test are presented in Table 4. The Shear wave velocity increases from about 500 m/s at a depth of 3 to 1500 m/s at a depth of 6 m. The shear wave velocity varies between 1100 m/s and 1500 m/s within a depth of 8 to 10 m.

The PA/FD Fan foundations are 11.75m x 3.5m in size and are to be founded at a depth of 1.5m below ground level. The operating frequncy of the PA and FD fans is between 16Hz to 25Hz. The frequency of the fan soil-foundation system should be separated from the operating frequency of the PA and FD fans.

TABLE 4. Shear Wave Velocity and Maximum Shear Modulus at PA/FD Site

Type of Soil / Rock	Depth	Time (ms)	Vs (m/s)	G_{max} (MPa)
Weathered Rock	3.0	6.0	500	525
Weathered Rock	4.5	3.4	882	1713
Fractured Rock	6.0	2.0	15000	5175
Fractured Rock	8.0	2.7	11111	2840
Fractured Rock	10.0	2.0	15000	5175

Initial design calculations revealed that a low tuned foundation is feasible to meet the dynamic criteria. It was found that the foundation resting on the composite medium consisting of top 4.5m and 5.0m well-compacted fill for PA and FD fan foundation on the existing weathered rock to be a viable option.

The stiffness for the PA and FD Fan foundations was evaluated using advance DYNA 5.4 software developed by Geotechnical Research Centre of University of Western Ontario (Canada). The stiffness of the soil foundation system was evaluated considering a rigid surface footing resting on the composite soil strata consisting of a layer overlying a half space as shown in Figure 10.

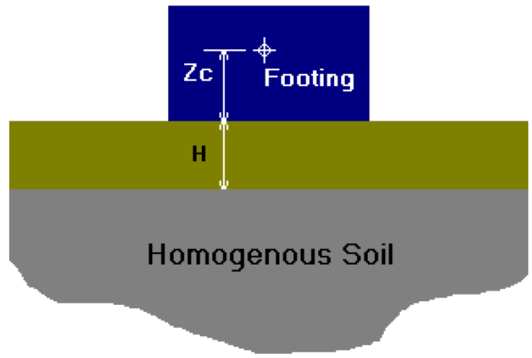


Fig. 10 Rigid footing resting on composite strata

To achieve the required stiffness, it was recommended to replace the existing fill / weathered rock with well graded gravel (GW) compacted to 95% relative density to achieve a shear wave velocity of about 300 m/s at a depth of 5.0m below the base of the PA and FD fan foundations.

The stiffness of the foundation-soil system is evaluated assuming the footing to be resting on the 4.5m thick well graded gravel layer with shear wave velocity of 300m/s above an elastic half-space of weathered rock with shear wave velocity of 700m/s. The total vertical and horizontal stiffness of the PA Fan foundation was found to be $6.29x10^5$ kN/m and $1.66x10^5$ kN/m respectively. A FE analysis was performed on the foundation with the spring constants obtained by assuming foundation resting on a composite layer above elastic half-space. A modal analysis was performed extracting the first 20 modes using Block Lanczos Technique. It was observed that the frequency of the foundations are well separated from the operating frequency of the PA and FD fans. The displacement of the PA foundation resting on sand in it's first modal frequency is presented in Figure 11.

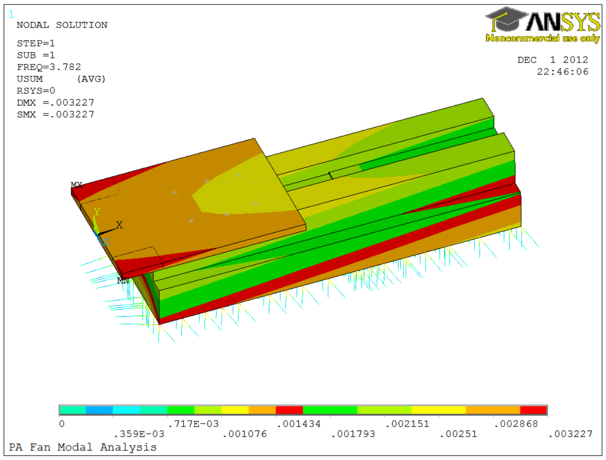


Fig.11. Nodal displacement in the first mode – PA fan foundation on sand

Although PA/FD Fan resting on a dense fill was able to separate the frequency of the foundation from the operating frequency, it was considered impractical for the given site conditions. Fan foundation supported on rock socketed piles to be a viable solution.To evaluate the vertical stiffness, the pile is assumed as an end bearing pile carrying a vertical load. The horizontal stiffness is obtained assuming the pile is free from the surrounding soil and fixed at the top and bottom over a length (Srinivasulu and Vaidyanathan 1977). The M25 grade concrete pile of 0.45m diameter and 11.5m in length is assumed to be socketed for a length of 1.5m below 10.0m GL (ground level). 15 Piles were used to support the PA/FD Fan, each supporting a load of 192 kN. The effective length of the pile is 10.0m. The vertical and horizontal stiffness of the pile is computed as 7.1 x 10^6 kN/m and 2.7 x10^4 kN/m, respectively. The bearing capacity is calculated assuming the pile is socketed to weathered rock at 10m below GL with RQD of 0.1 (Bowles 2001). Assuming the pile to be end bearing the allowable load carrying capacity was estimated as 450 kN. The lateral capacity of the pile subjected to 5mm lateral displacement is estimated as 3.6kN. The pile was found to be safe against buckling failure. Ayothiraman et al. (2012) observed that the site conditions, and in particular, the properties of the topsoil layers greatly govern the degree of non-linearity and the dynamic lateral stiffness of the soil-pile system. To achieve the required stiffness, 800mm diameter bore hole is to be drilled up to the surface of medium rock (i.e. upto a depth of 10m from FGL or EGL) following which 450mm diameter boring would be drilled for a length of 1.5m. Thus the rock-socketed pile would have a freestanding length of 10m.

A Finite Element analysis of the PA and FD fan foundations was performed assuming the piles to be linear springs with one end fixed and other end carrying a load. The fan foundations were modelled using SOLID elements and the 15 piles were modelled as springs using COMBIN14 elements. Both horizontal and vertical stiffness were accounted for using 2 horizontal and 1 vertical spring elements for each pile. The FE mesh with the springs is shown in Figure 12.

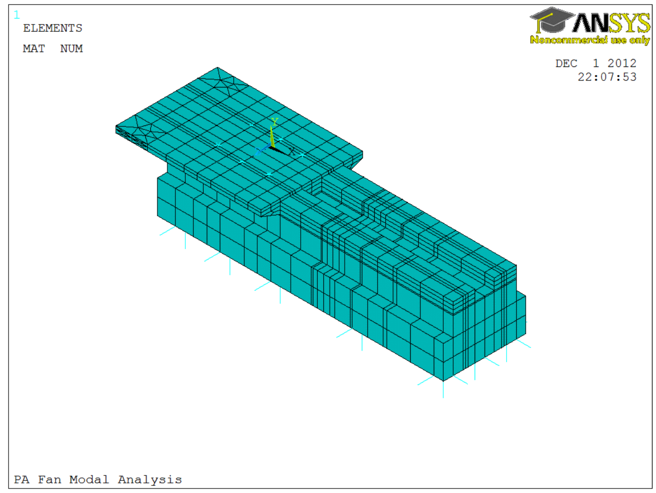
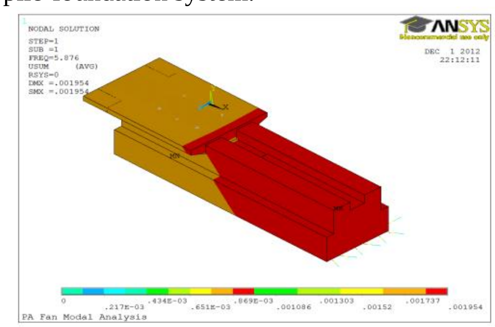
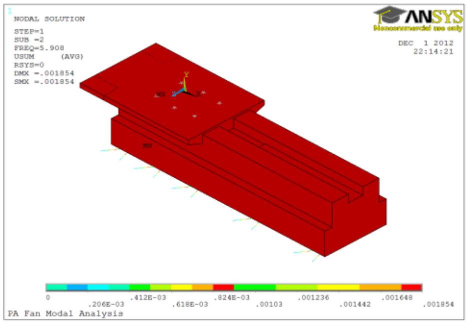


Fig.12. Mesh of PA Fan foundation with Piles modelled as spring elements

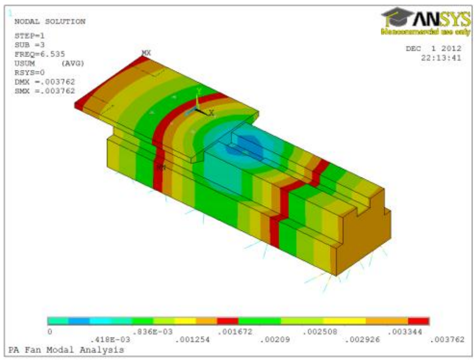
Modal analysis was performed to estimate the natural frequencies of the PA and FD fan foundation pile system. 20 modes were extracted using Block Lanczos approach. All the modes were found to be well separated from the operating frequencies of the PA and FD fans (16 to 25Hz). The displacement profiles of the PA fan in the first four modes are presented in Figure 13. It can be observed that the fan foundations are undertuned and the natural frequencies are well separated from the operating frequncies of the fans. First mode corresponds to rocking displacement and the second mode corresponds to translational displacement. The lateral displacement observed in the second mode is well within the allowable displacement and the lateral capacity of pile. Thus a simple approach used to determine the stiffness of the pile is coupled with FE analysis to determine the natural frequency of the soil-pile-foundation system.



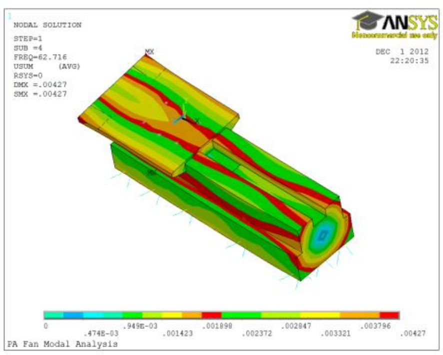
(a) Mode 1: 5.876Hz, max disp. of 1.95mm



(b) Mode 2: 5.9Hz, max disp. of 1.85mm



(c) Mode 3: 6.5Hz, max disp. of 3.76mm



(d) Mode 4: 62.72Hz, max disp. of 4.2mm

Fig. 13. First four modes of vibration for PA fan foundation on piles

SUMMARY AND CONCLUSIONS

Based on the lateral dynamic pile loads tests carried out on full scale single piles at two different sites in India, it is found that the site conditions, and in particular, the properties of the top soil layers greatly governs degree of nonlinearity and the dynamic lateral stiffness of the soil-pile system. It is found that the piles in medium stiff to stiff clay have about 6 times higher lateral stiffness compared to the piles in very soft clays. The finite element analysis is able to predict well the dynamic lateral response of pile for soft as well as stiff soil sites, thus signifying the efficiency of non-linear FE models in simulating the different degrees of nonlinearity of soil and pile separation. Fan foundations are to be designed for a thermal powerplant in South India founded at a rocky site. Althougth Rock socketed piles supporting the PA/FD Fans was able to separate the frequency of the foundation from the operating frequency. Modal analysis of the PA and FD fan foundation pile system revealed that the natural frequency of the pile-foundation system is well separated from the operating frequencies of the fan foundations.

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