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Finite Element Method for Love Wave Dispersion in Soils

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SYNOPSIS The secular equation of Love wave is conducted by using conbined finite element method and semi-infinite element method (or analytical method) in this paper. The formulation of calculating displacement distributions is also generated. The soils can be more complicate than analytical method. the results indicate that the presented method is more advantageous than analytical method. Some numerical examples are employed to illustrate its applicability and correrponding accuracy.

INRODUCTION

Chatterjee (1972) and Kazi (1978), have determined Love wave dispersion curves and displacement vectors by analytical method. The secular equation (developed by analytical method) of Love wave is difficult to solve (using iterative approximating method). The method is not suitable to nonhomogeneous soils. Recently, Vrettos (1990) presented the dispersion relation of Love waves for a linear-elastic isotropic half-space with constant density ρ and shear modulus G increasing with depth. But the method presented by Vrettos can not be used to calculate more complicate soils (such as nonhomogeneous intercalry soils).
Lysmer (1972) presented the dispersion secular equation of Love wave for layered soils with rigid bottom . The layered soils with a semi — infinite layer which can describe the behaviour of real soils is discussed in this paper. Based on the characteristics of Love wave in half-space, this paper presents two methods to calculate the boundary nodal force acting on layered soils due to the Love waves in semi-infinite layer. The first is semi-infinite method. The second is analytical method.

RECTANGULAR FINITE ELEMENT

The element mesh is shown in Fig. 1, where N — 1 is the number of sublayers in the discretized system (each sublayer is about $\frac{1}{10}\lambda$). The semi-infinite layer is not discretized. Displacement of Love waves can be given by

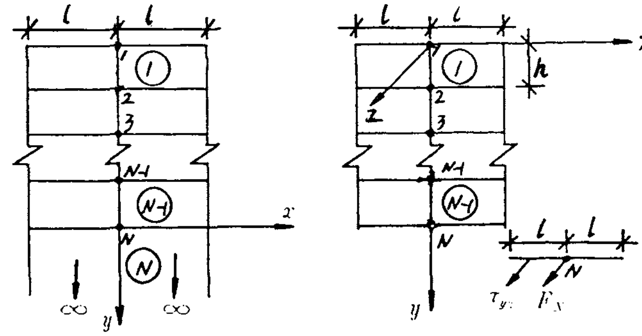
$$W=f(y)\cdot \exp[-ik(x-ct)] \tag{1}$$

where k is wave number. The condensation of stiffness matrixes in element 1 and 2 (see Fig. 1) is given by Lysmer (1972). It can be given as

$$[K]_e=\begin{bmatrix} 2k^2h+\frac{6}{h} & k^2h-\frac{6}{h} \\ k^2h-\frac{6}{h} & 2k^2h+\frac{6}{h} \end{bmatrix}\cdot\frac{1}{6}\beta^2\rho l \tag{2}$$

in which β is shear wave velocity, ρ is mass density, h is height of the element.
In the same way, the consistent mass matrix is

$$[M]_e=\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\cdot\frac{\rho h}{6}l \tag{3}$$



(a) (b)

Fig. 1 Element Mesh

SEMI-INFINITE ELEMENT

It is more suitable to use conbined finite element and semi-infinite method to calculate dynamic response problem. The accuracy of semi-infinite method depends on selection of decay functions. The different decay functions should he selected in different dynamic problem. Based on the characteristic of Love wave in a half-space, this paper will discuss the selection of decay functions of a semi-infinite element shown in Fig. 1a.

The motional equation of SH-wave in half-space is

$$W=A\cdot\exp(-aky)\cdot\exp[-ik(x-ct)] \quad (4)$$

where $a=\sqrt{1-c^2/\beta^2}$, c is phase velocity, A is coefficient. According to Eq. (4), the decay functions can be selected as

$$N_1=(1-\frac{x}{l})\cdot\exp(-aky),\quad N_2=\frac{x}{l}\cdot\exp(-aky) \quad (5)$$

The sitffness and mass matrices of a semi-infinite element can be expressed as

$$[K]_\infty=\int_0^\infty\int_0^l[B]^T[D][B]d_yd_x \quad (6)$$

$$[M]_\infty=\int_0^\infty\int_0^l[N]^T[N]d_yd_x \quad (7)$$

Similer to the rectangular finite element, the two semi-infinte elements can be condensed. The stiffness and mass matrices of the condensed semi—infinite element are

$$[K]_\infty=(a+\frac{1}{a})\cdot\frac{1}{2}\beta^2\rho kl \quad (8)$$

$$[M]_\infty=\frac{1}{ak}\cdot\frac{1}{2}\rho l \quad (9)$$

According to Eqs. (8) and (9), the condensed stiffness and mass matrices have not any unknown parameters. It is different from Rayleigh wave (Xia, 1991).

ANALYTICAL METHOD

The boundary force (acts on the rectangular element N—1) of semi-infinite layer can be calculated by analytical method. According to Eq. (4), the shear stress is

$$\tau_{yz}=G\cdot\frac{\partial W}{\partial y}=-akGW \quad (10)$$

It shows that the shear stress τ_{yz} in semi-infinite layer has relation to its displacement. Therefore, the houndary force in Fig. 1b can be calculated as

$$F_N=\frac{1}{2}\int_{-l}^{l}\tau_{yz}d_x \quad (11)$$

According to Eq. (4), the nodal point displacement of semi-infinite boundary in Fig. 1b can be given as

$$W=\exp(-ikx)\cdot W_N \quad (12)$$

Substituting Eqs. (12) and (10) into Eq. (11) yields after $l\to 0$

$$F_N=-akGl \quad (13)$$

By using $G=\beta^2\rho$, Eq. (13) can be expressed as

$$F_N=-ak\beta^2\rho l \quad (14)$$

in which the force is boundary force acting on the finite element N—1 by semi-infinite layer. According to Eq. (8) and Eq. (9). the force acting on the finite element N—1 by semi-infinite element N is

$$\bar{F}_N=-([K]_\infty-\omega^2[M]_\infty)=-ak\beta^2\rho l \quad (15)$$

we can find Eq. (14) and Eq. (15) are the same.

SECULAR EQUATION AND DISPLACEMENT CALCULATING

We have obtained the condensed stiffness and mass matrixes for rectangular finite element and semi-infinte element (or nodal point force Eq. (14) acting on the finite element). Therefore, the global stiffness matrix [K] and mass matrix [M] can be assembled. The secular equation of Love wave is given by

$$|[K]-\omega^2[M]|=0 \quad (16)$$

Because of the special structure of Eq. (16), it can be rearranged as

$$|[A]k^2+[B]k+[C]|=0 \quad (17)$$

There is no unknown k in matrixes [A]、[B] and [C]. Choosing a value for c, Eq. (17) is a quadratic eigenvalue problem and can be solved as a linear eigenvalue of problem. The eigenvectors for wave number k can be calculated from Eq. (17). It is displacement distribution of Love wave in soils. The nodal point displacement of semi-infinite layer can be calculated from

$$W=\exp(-aky)\cdot W_N \quad (18)$$

NUMERICAL EXAMPLES

Three numerical examples will be given in this paer. The first one is a layered soil with two homogeneous layers (one is half-space). It will be compared with analytical method (Eringen, 1975). The second one is a layered nonhomogenous soil with a harder layer. The third one 13 a layered nonhomogeneous with a softer layer.

Example 1.
Material properties of the layered soil are shown in Fig. 2. The dispersion curves of the first and second mode waves are given in Fig. 2. From Fig. 2, it is knwon that the results calculated

by the presented method are very good agreement with analytical results. The first mode wave velocity c approaches the shear wave velocity of the surface layer soil as wave length λ decreases and approaches the shear wave velocity of semi-infinite soil as wave length λ increases.

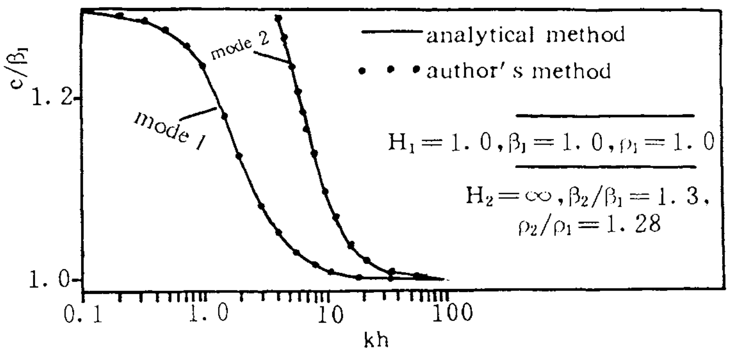


Fig. 2 Dispersion curves of Love wave

Example 2.

Material properties of the layered soils are shown in Fig. 3. Shear wave velocity of the first layer and half-space has following equation (Gibbson soil)

$$\beta=\beta_0(1+By)^{\frac{1}{2}} \tag{19}$$

in which β_0 is shear wave velocity of surface soil. B(=0. 1) is coefficient and its unit is m^{-1}. The second layer is homogen soil. The dispersion curves of the first mode wave is given in Fig. 3. Phase velocity c approaches the shear wave velocity of the surface soil as wave length λ decreases and c increases as wave length λ increases.

The displacements of the first and second mode waves are given in Fig. 4. From Fig. 4, it is known that the propagational depth of the first mode is about 1. 5 λ_1.

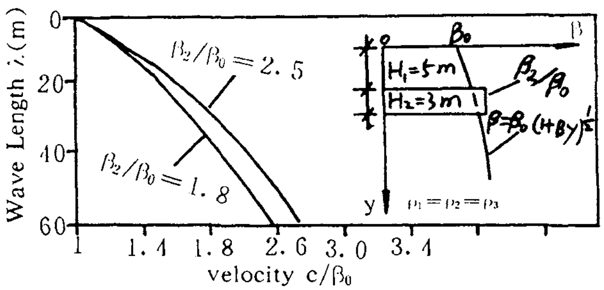


Fig. 3 Dispersion curves of Love wave

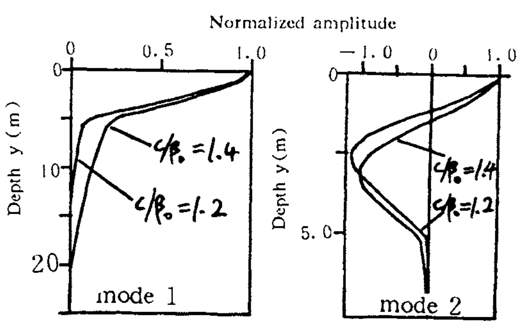


Fig. 4 Displacement distributions of Love wave

Example 3.

Soil stratification is shown in Fig. 5. It represents a soil system with a homogeneous softer layer trapped between a Gibbson harder surface layer and a Gibbson half-space. The dispersion curves of Love wave are given in Fig. 5. The first mode approaches the shear wave velocity of the solfter layer as frequency increases. It is different from examples 1 and 2. The displacement vector of Gibbson half-space with B=0. 3 is given in Fig. 6.

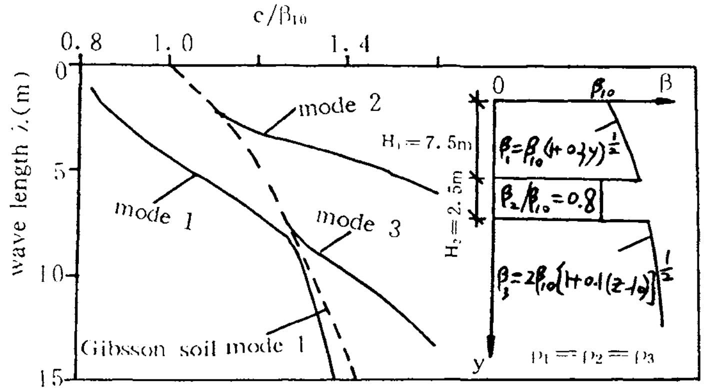


Fig. 5 Dispersion curves of Love wave

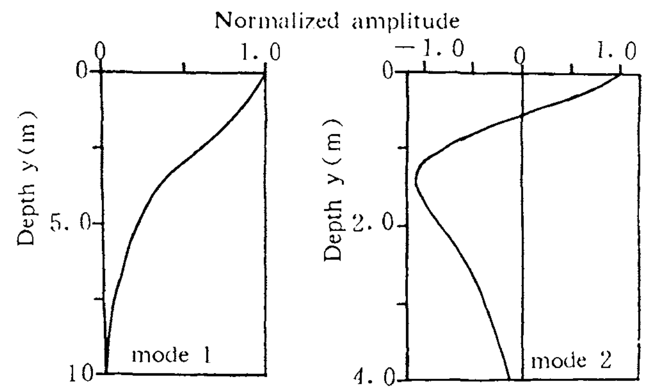


Fig. 6 Displacement in a Gibbson half-space with B=0. 3

Figs. 7, 8 and 9 present the displacement vectors ofFig. 5 system for different phase velocity c. The shape of mode 1 for a bigger velocity c is similar to the shape in Gibbson half-space (see Fig. 6) and mode 2 has bigger value in softer layer. The shape of mode 2 is very similar to the shape of mode 1 in Gibbson half-space for $c/\beta_{10}=1.2$. Both mode 1 and mode 3 have bigger value in softer layer and mode 3 is similar to mode 2 of Gibbson half-space. The shapes of mode 2 and mode 3 are very similar to the mode 1 of Gibbson half-space for $c/\beta_{10}=1.15$. The shape of higher mode is similar to the shape of mode 1 of Gibbson half-space as frequency increases (or phase velocity decreases). We can consider that the higher mode is the mode 1 of surface Gibbson layer. For a bigger frequency (or shorter wave length), Love surface wave cannot propagate into softer layer because the propagational depth of Love wave is about 1. 5λ. The Love surface wave can not be influenced by softer layer.

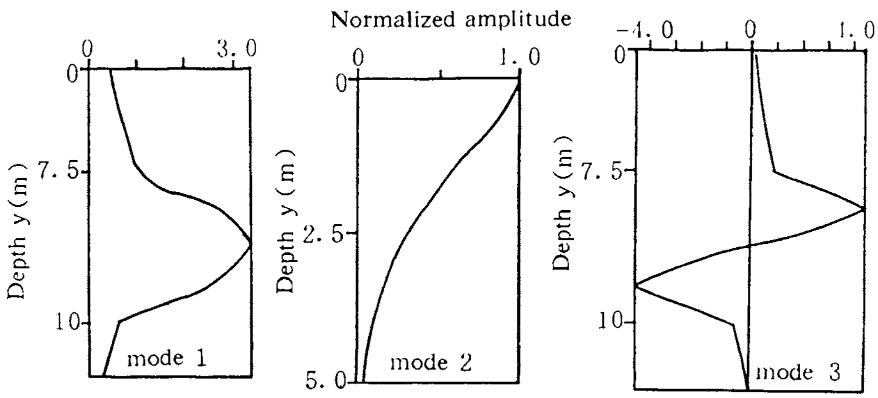


Fig. 8 Displacement distribution for $c/\beta_{10}=1.20$

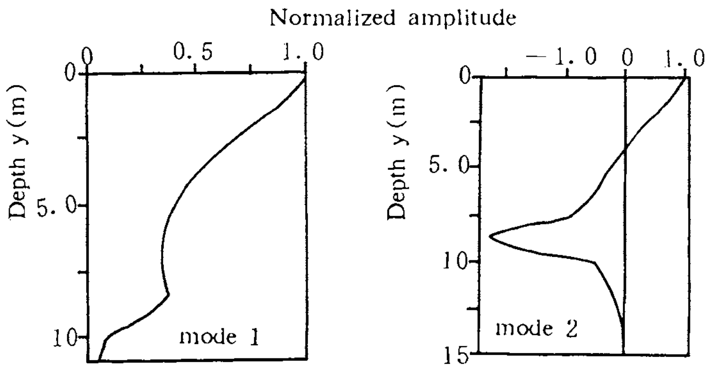


Fig. 7 Displacement distribution for $c/\beta_{10}=1.35$

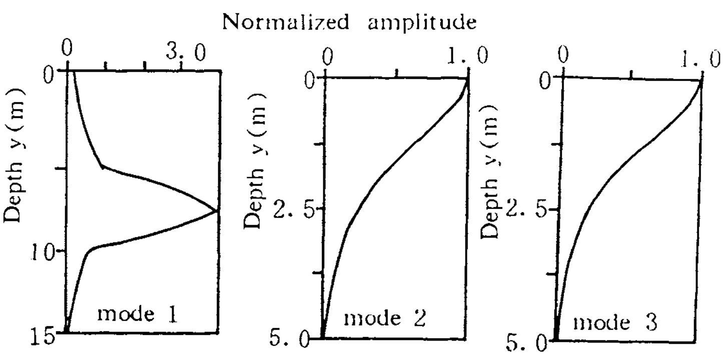


Fig. 9 Displacement distribution for $c/\beta_{10}=1.15$

CONCLUSIONS

The mechod presented in this paper can be used to calculate dispersion curves and displacement distributions of all mode Love waves. The accuracy of the presented method is good. It is not only suitable to the layered soils whose shear wave velocity increases with depth and intercalary soils but also suitable to the vertical nonhmonogeneous soils whose shear wave velocity gradually increases with depth (such as Gibbson soil).

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