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SYNOPSIS: Wave propagation due to shock loading on the surface of elastic inhomogeneous half-space is considered. The shock loading is applied over a circular area in the form of suddenly applied vertical and shear stress. The analysis makes use of semi-analytical finite element formulation and explicit time integration technique. Extrapolation algorithm is employed for modelling the transmitting boundaries. Details of numerical procedure adopted are given and results of typical analysis are presented and discussed.

INTRODUCTION

Solutions to problems of shock loading on the surface of an elastic half-space has a number of applications in practice ranging from bomb blasts to hammer foundations. Analytical solutions are difficult to obtain for such problems when the medium is inhomogeneous. Time domain finite element approach has distinct advantages in handling problems of this nature. Firstly time domain approach is ideally suited when the loading is transient in nature. It is well known that the finite element method is one of the popular numerical techniques for effectively handling spatial variation of mechanical properties and of complex geometries. However, modelling of soil medium under dynamic loading requires special consideration at the boundaries as otherwise the reflected waves from them would interfere with the response of computation resulting in the false physical behaviour of the system. In order to overcome this difficulty many investigators have developed special boundary conditions called transmitting boundaries. Many of such boundaries that could be used in time domain are summarized by Wolf (1988). Simplest and widely used transmitting boundary is the standard viscous boundary proposed by Lysmer and Kuhlemeyer (1969). This approach is straightforward when the role of surface waves is not significant. However, the dashpot characterisation becomes frequency dependent when the effects of surface waves need to be considered. This render the direct time domain approach cumbersome in such situations. Extrapolation algorithm was proposed by Liao and Wong (1984) for wave propagation study in linear elastic half-spaces under plane strain idealisation subjected to vertical transient and harmonic surface loads. Liao and Liu (1992) presented the numerical instabilities of this algorithm for one dimensional wave propagation analysis in a bar using frequency domain approach. The authors have found that this approach not only models the far field boundaries adequately but also has the added advantage of handling loads which have non-vanishing time average.

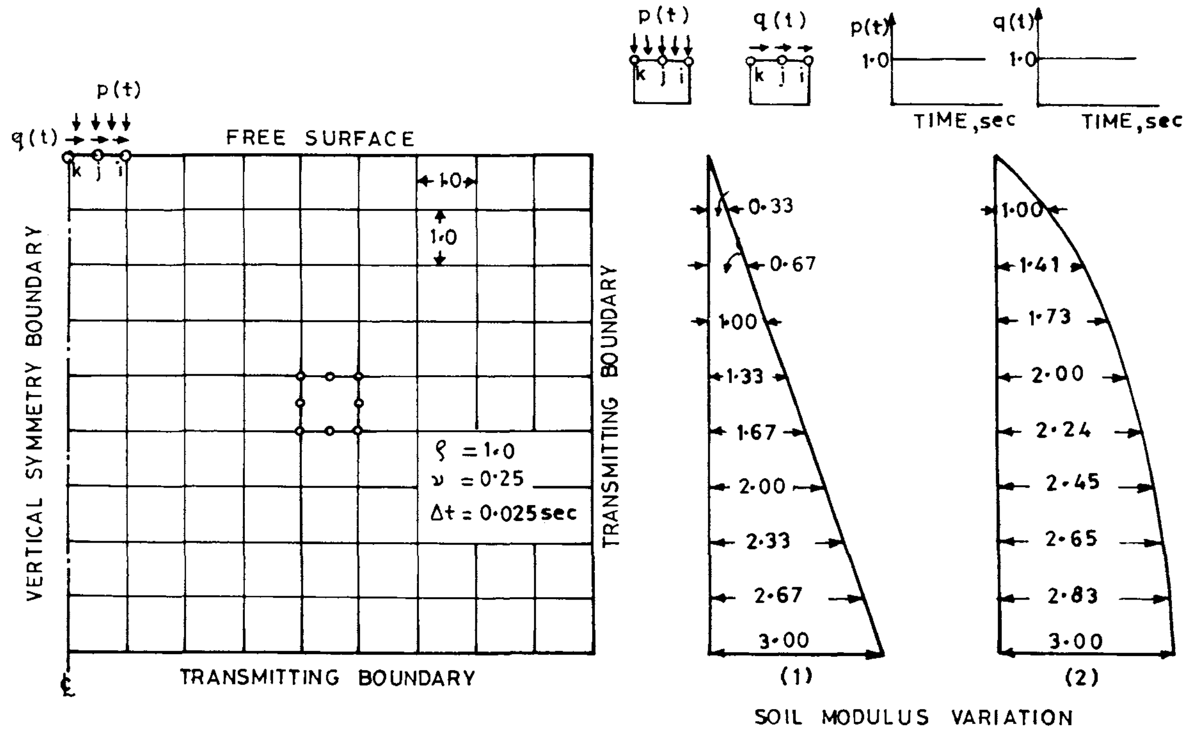
The present paper deals with analysis of wave propagation in inhomogeneous soil medium subjected to surface shock loading which is in the form of suddenly applied load at time t=0 and maintained constant thereafter. The shock loading is considered to act over a circular area on the surface of semi-infinite medium. Both vertical and shear loading are considered. The latter loading is tackled using semi-analytical formulation in the circumferential direction. Two types of elastic modulus variations with depth are considered. These are (i) elastic modulus varying in proportion to depth and (ii) elastic modulus varying in proportion to square root of depth. Solutions of equations of motion is obtained by using explicit method of time integration based on central difference approximation. Typical results of these studies are presented and discussed in the paper.

SEMI-ANALYTICAL FINITE ELEMENT FORMULATION

Consider the axisymmetric idealisation of the half-space shown in Fig. 1. If the external loads are symmetrical about $\theta = 0$ plane, then the displacements may be expressed in the form of finite Fourier series. The radial, vertical and circumferential components of displacements for eight noded quadrilateral element may be represented as,

$$\begin{aligned} u(r,z,\theta) &= \sum_{n=1}^{L} \sum_{i=1}^{8} N_i \, \bar{u}_{ni} \cos n\theta \\ v(r,z,\theta) &= \sum_{n=1}^{L} \sum_{i=1}^{8} N_i \, \bar{v}_{ni} \sin n\theta \\ w(r,z,\theta) &= \sum_{n=1}^{L} \sum_{i=1}^{8} N_i \, \bar{w}_{ni} \cos n\theta \end{aligned} \qquad (1)$$

where u, v and w are the displacements in r, z and θ directions respectively, $\bar{u}_{ni}$, $\bar{v}_{ni}$ and $\bar{w}_{ni}$ are the amplitudes of displacements for the nth harmonic at node i, N_i is the shape function for the node i and L is the number of harmonics required for representing the loading. For uniform density and elastic properties in the circumferential direction, the orthogonality of trigonometric functions can be exploited to represent the general



FIG·1 FINITE ELEMENT IDEALISATION OF INHOMOGENEOUS SOIL MEDIUM

three dimensional problem into a series of uncoupled two dimensional problems. The complete solution is then simply the superposed solution of all individual uncoupled two dimensional finite element solutions. In the present problem of wave propagation under the action of uniformly distributed surface vertical and shear stress over a circular area requires consideration of harmonics n=0 and n=1. Element stiffness matrix for n=0 and n = 1 may be derived for eight noded rectangular element (Cook et al. (1989)). Lumped element mass matrix is derived based on a special lumping scheme suggested by Hinton et al. (1976). The idea behind this procedure is to use only the diagonal terms of the consistent element mass matrix, but to scale them in such a way that the total mass of the element is preserved. This procedure is known to yield satisfactory results in dynamic finite element analysis.

Let the uniformly distributed vertical load intensity p act on the element surface (Fig. 1) which represents harmonic load term when n=0. Equating the virtual work done by the distributed forces of the surface loading the equivalent nodal force vector for the three nodes in radial,vertical and circumferential directions may be derived as,

$$\{P\} = \frac{\pi p (r_i - r_k)}{3}\{0,\ r_i,\ 0,\ 0,\ 4r_j,\ 0,\ 0\ ,\ r_k,\ 0\}^T \qquad (2)$$

Similarly, the equivalent nodal force vector for uniformly distributed shear stress intensity q over a circular area (Fig. 1) which corresponds to harmonic load term when n=1 may be derived as,

$$\{Q\} = \frac{\pi q(r_i - r_k)}{6}\{r_i, 0, -r_i, 4r_j, 0, -4r_j, r_k, 0, -r_k\}^T \qquad (3)$$

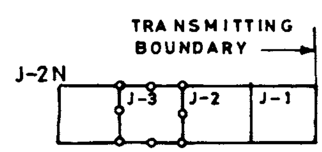
TRANSMITTING BOUNDARIES USING EXTRAPOLATION ALGORITHM

The method is based on the concept that the displacements on the artificial boundary at a given time step are extrapolated based on the displacements at earlier times along a line normal to the artificial boundary in the region's interior, thus allowing free transmission of waves across the boundary. Using quadratic interpolation in the time domain, the transmitting formula for the first order extrapolation may be written as (Liao and Liu (1992)).

$$\{U_J^{p+1}\} = [T_1]\{u_1\} \qquad (4)$$

where $\{U_J^{p+1}\}$ = Displacement of the boundary node J at time t
= (p+1) Δt, p is an integer and Δt is the time step.

$\{u_1\}$ = Displacement vector along the line normal to the node J (Fig. 2)



FIG· 2 TRANSMITTING BOUNDARY AND NODE POINTS

$$\{ u_1 \} = \{ u^p_J, u^p_{J-1}, u^p_{J-2} \}$$

$$[T_1] = [t_1, t_2, t_3] \quad (5)$$

$$\text{where } t_1 = \frac{(2-s)(1-s)}{2} \; ; t_2 = s(2-s) \; ; t_3 = \frac{s(s-1)}{2}$$

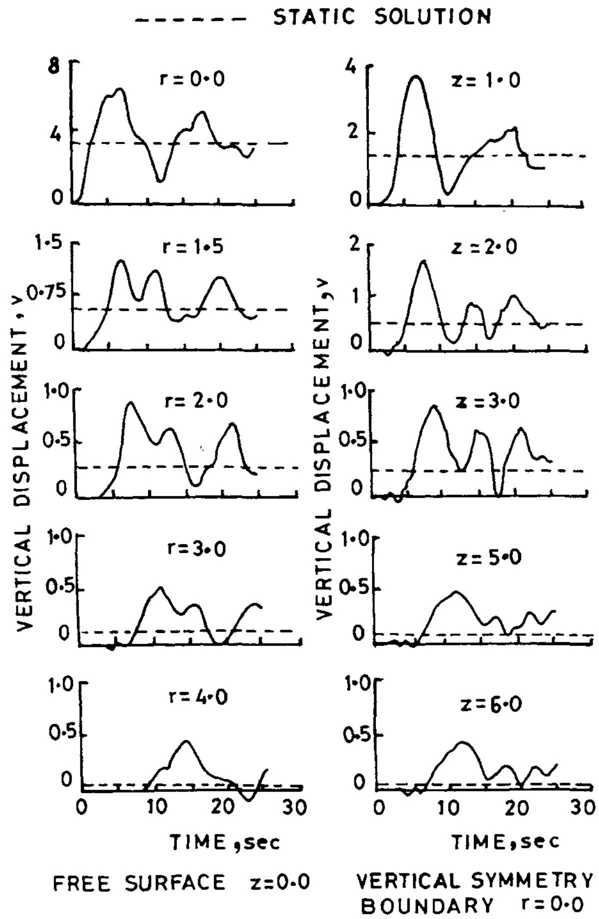
$$\text{and } s = C_a \frac{\Delta t}{\Delta x} \quad (6)$$

Where C_a is the apparent wave velocity and Δx is the discrete step in space.

The undamped equation of motion for the half-space may be written as,

$$[M]\{\ddot{U}\} + [K]\{U\} = \{F(t)\} \quad (7)$$

where $[M]$ is the mass matrix $[K]$ the stiffness matrix, $\{\ddot{U}\}$ the acceleration vector, $\{U\}$ the displacement vector and $\{F\}$ is the time varying external load vector. The transmitting boundaries in radial and vertical directions (Fig 1) of the half-space are modelled using first order extrapolation algorithm. The solutions of equations of motion is obtained by using central difference method of explicit time integration in direct time domain. However, it may be noted that the value of the apparent wave velocity C_a to be used in equation (6) is the least value of S-wave velocity in inhomogeneous half-space.



FIG·3 RESPONSE OF VERTICAL DISPLACEMENTS

RESULTS OF ANALYSES OF PROPAGATION DUE TO SHOCK LOADING

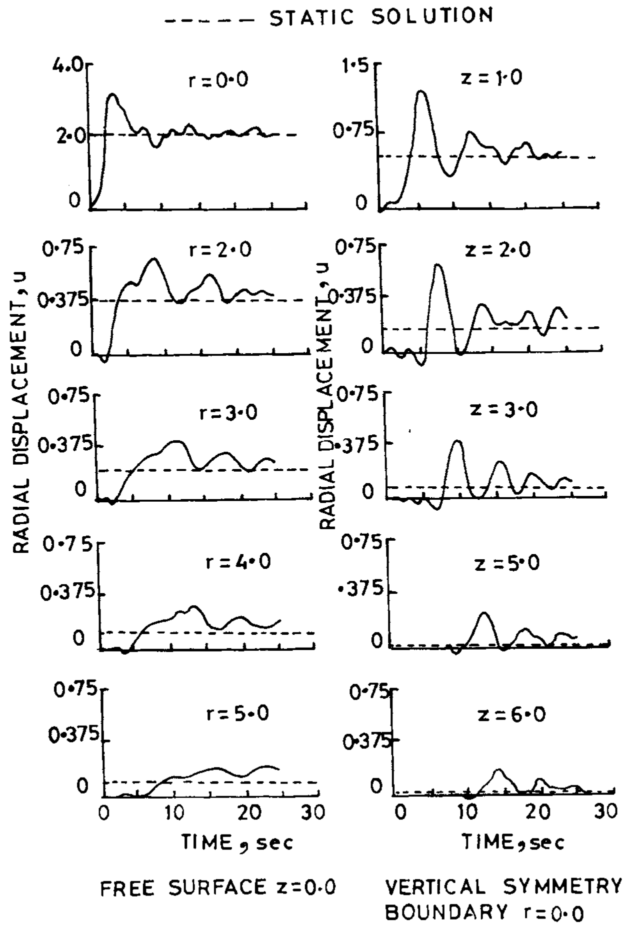
Consider the finite element discretisation of the inhomogeneous medium having constant values of mass density and Poisson's ratio. Soil inhomogenity in the vertical direction is considered to take into account elastic modulus variations with depth. Two types of elastic modulus variations with depth are considered and they are (i) elastic modulus varying proportional to depth and (ii) elastic modulus varying proportional to square root of depth. The inhomogeneous media are subjected to uniformly distributed vertical (p) and horizontal shear (q) shock loading over a circular area on the surface. These loadings excite n=0 and n=1 harmonics respectively. The shock loading is considered in the form of suddenly applied loading at time t=0 and thereafter maintained constant.

For the case where the loading is suddenly applied vertical load and the elastic modulus is proportional to depth (Fig. 1(1)), the response of vertical displacements at different locations are shown in Fig. 3. It is observed from the response of vertical displacements at varying radial distances (r=0.0,1.5,2.0,3.0 and 4.0) on the free surface (z=0.0) that they attain peak amplitudes and later tend towards the respective static solutions. Also, their peak amplitude decreases with increase in the radial distances. Similar behaviour of vertical displacements is seen on the vertical symmetry boundary (r=0.0) at different depths (z=1.0,2.0,3.0,5.0 and 6.0). No reflections of the waves from the boundaries can be detected in the response of vertical displacements (Fig. 3).

The response of radial displacements are shown in Fig. 4 for the inhomogeneous medium having elastic modulus proportional to square root of depth (Fig. 1(2)) due to suddenly applied surface shear loading over a circular area. In this case the response of radial displacements on the free surface (z=0.0) with varying radial distances (r=0.0,2.0,3.0,4.0 and 5.0) indicates that the response tend towards the corresponding static solutions after reaching peak amplitudes. The peak amplitudes tend to decay with increase in the radial distances. Similar behaviour of response of radial displacements may be seen on the vertical symmetry boundary (r=0.0) at different depths(z=1.0,2.0,3.0,5.0 and 6.0). It is seen that the response of radial displacements do not indicate wave reflections from the boundaries (Fig. 4). These observations clearly demonstrate the effects of shear loading and radiation damping.

CONCLUSIONS

Wave propagation analysis in inhomogeneous soil medium due



FIG·4 RESPONSE OF RADIAL DISPLACEMENTS

to shock loading on the surface is studied in time domain using semi-analytical finite element formulation combined with extrapolation algorithm for modelling the transmitting boundaries. The results of the analysis indicate that the displacements after attaining peak amplitude tend towards static solutions. The proposed procedure is also useful in tackling problems involving loads having non-vanishing time average. The procedure can not only take into account any variation of material property with depth but also different types of transient loading.

REFERENCES

Cook, R.D., Malkus, D.S. and Plesha, M.E. (1989), "Concepts and Applications of Finite Element Analysis", John Wiley and Sons Inc.

Hinton, E., Rock, T. and Zienkiewicz, O.C. (1976) , "A Note on Mass Lumping and Related Processes in the Finite Element Method", Journal of Earthquake Engng. Struct. Dyn., 4(3), 245-249.

Liao, Z.P. and Wong , H.L. (1984) ,"A Transmitting Boundary for Numerical Simulation of Elastic Wave Propagation", Soil Dyn. Earthquake Engng., 3, 174-183.

Liao, Z.P. and Liu, J.B. (1992) ,"Numerical Instabilities of a Local Transmitting Boundary", Journal of Earthquake Engng. Struct. Dyn. 26, 65-77.

Lysmer, J. and Kuhlemeyer, R.L. (1969), "Finite Dynamic Model for Infinite Media", Journal of Engng. Mech. Div., ASCE, 95, 859-877.

Wolf, J.P. (1988), "Soil Structure Interaction Analysis in Time Domain", Prentice-Hall Inc., Englewood Cliffs, NJ.

NOTATIONS

u, v and w : Displacements in radial, vertical and circumferential directions respectively

$\overline{u}_{ni}$, $\overline{v}_{ni}$ and $\overline{w}_{ni}$: Amplitudes of radial, vertical and circumferential displacements respectively for the n^{th} harmonic of node i

N_i :Shape function for node i

{ P }: Equivalent nodal load vector for uniformly distributed vertical loading

{ Q } :Equivalent nodal load vector for uniformly distributed shear loading

C_a : Apparent wave velocity

[M] : Mass matrix

[K] : Stiffness matrix

{ U } and $\{\ddot{U}\}$: Displacement and acceleration vector respectively

{ F (t) } : External load vector varying with time