Scattering of SH-Waves by Arbitrary Surface Topography

M. E. Manoogian
Loyola Marymount University, Los Angeles, California

V. W. Lee
University of Southern California, Los Angeles, California

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M.E. Manoogian
Assistant Professor of Civil Engineering, Loyola Marymount University, Los Angeles, California

V.W. Lee
Associate Professor, Civil Engineering, University of Southern California, Los Angeles, California

SYNOPSIS The weighted residual method was applied to the problem of scattering and diffraction of plane SH-waves by a shallow alluvial valley of arbitrary shape on the surface of a two-dimensional half-space. The formulation was also applied to the case of an shallow canyon. In order to demonstrate the versatility of the method, it was applied to shallow circular, shallow elliptical, and shallow rectangular canyons and alluvial valleys. Results obtained for the cases of a semi-cylindrical and a shallow semi-elliptical valleys and canyons match those obtained using closed form solutions. It was shown that significant ground motion amplifications, with respect to the amplitude of incident waves, occurred near and in the canyon or valley. Amplifications were dependent upon the shape and depth of the canyon or valley, the relative properties of the alluvium in the valley and the surrounding medium, and the frequency and angle of incidence of incoming waves. Amplification profiles for the lower frequency incident waves were simple near the canyon and valley on the surface of the half-space with peak amplifications that did not significantly vary from 2, the value expected on the surface of the half-space. Within a valley containing softer alluvium, the amplification profile is more complex with values larger than 2. Within the canyon, amplification profiles remained simple with peaks near 2. As the frequency of the incident waves are increased, the amplification profiles near the canyon and valley became more complicated with peak values exceeding 5 for rectangular shaped valleys. Within the canyon, the profiles were similar. On the surface of the valley, the amplification profiles are more complex with peak values exceeding 10 for many valley configurations.

INTRODUCTION

One of the many areas of earthquake engineering and seismological research has been the effect of local site conditions on ground motion. Among the local site topographies of interest are alluvial valleys and canyons. In this paper, the problem of the scattering and diffraction of incident SH-waves by an arbitrarily shaped alluvial valley or canyon on the surface of a two-dimensional half-space and the transmission of these waves into the valley is studied. A numerical solution for the problem is appropriate as the boundary between the material of the canyon or valley and the half-space may be irregular, making it difficult to describe the solution in closed form. The method of weighted residuals is implemented in order to study possible amplifications and de-amplifications of displacements on the surface of the half-space on and near the canyon or valley.

Solutions of various types have been implemented by many authors. Currently, closed form solutions exist only for a semi-cylindrical valley (Trifunac, 1971), a semi-elliptical valley (Wong and Trifunac, 1974a), a semi-cylindrical canyon (Trifunac, 1973) and a semi-elliptical canyon (Wong and Trifunac, 1974b). Analytical solutions for a shallow circular cylindrical alluvial valley (Todorovska and Lee, 1991) and a shallow circular cylindrical canyon (Cao and Lee, 1989) have also been developed. A closed form solution for the scattering and diffraction of P, SV, and SH-waves by a three-dimensional alluvial valley has also been developed (Lee, 1984).

Several approximate numerical approaches for the analysis and diffraction of elastic waves by irregularities on the surface of the half-space have been developed. Among these is a modification of the Aki-Larner method (Bouchon and Aki, 1977) was applied to the case of an irregularly shaped sedimentary valley, a finite difference approach applied by Boore, Aki and Larner (1971), Boore(1972), the finite element method (Joyner and Chen, 1975; Joyner, 1975), the ray method applied to the case of a sediment filled valley (Hong and Helmberger, 1977), and the Gaussian beam approach (Nowack and Aki, 1984). Boundary integral methods have been applied to a dipping layer of alluvium (Wong, et. al., 1977), in the form of an integral equation (Wong and Jennings, 1975), in the form of a Fredholm integral solution (Sills, 1978), an alluvial valley (Dravinski, 1983) and extended to three dimensions by Sanchez-Sesma (1983). Other boundary approaches have been applied by Sanchez-Sesma and Rosenbleuth (1979), Wong (1979), Sanchez-Sesma et. al. (1985), Moeen-Vaziri and Trifunac (1988), Diankui and Feng (1991), Manoogian (1992), and Lee and Wu (1993a, 1993b).

This paper presents a new application of the weighted residual approach used to determine the scattering and diffraction of SH-waves by a canyon or alluvial valley of arbitrary shape and size. This approach is used to evaluate boundary conditions and is a special case of the method of moments (Harrington; 1967, 1968). This approach has been applied to electromagnetic wave fields (Harrington, 1967), acoustic radiation fields (Fenlon, 1969), elastic inclusions, canyons, cavities, and alluvial valleys of irregular shape (Manoogian, 1992). Use of this method results in a matrix equation from which the unknown coefficients are determined and used to develop a series solution for the scattering, diffraction, and transmission of waves by the valley. The method is applied to valleys of many shapes and the character of amplifications on and near the valley are studied. This paper presents a new and alternative method for the evaluation of displacements in and near the valley caused by incident SH-waves.

MODEL, EXCITATION AND SOLUTION

The cross section of the model to be studied is shown in Figure 1. It represents a shallow alluvial valley of circular shape situated on the surface of the half-space. Although the approach is derived using a shallow circular alluvial valley, the resulting equations may be used for a valley of any size or shape. The origin is on the surface of the half-space, centered with respect to the canyon edges. The half-space is assumed to consist of an elastic, homogeneous, isotropic, material with rigidity μ and shear wave velocity c_β. The valley is assumed to consist of an elastic, homogeneous isotropic material with rigidity μ_v and shear wave velocity $c_{\beta v}$. For a canyon, the rigidity and density would be zero. Two coordinate systems are required; a Cartesian coordinate system and a cylindrical coordinate system as shown in Figure 1. The z-axis may be assumed to be perpendicular to the plane defined by these coordinate systems.

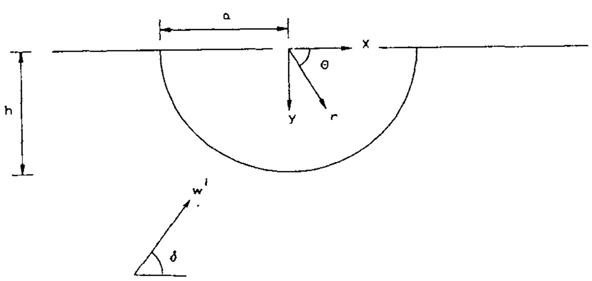

Figure 1. Alluvial Valley Model

Initially, define the excitation, $w^{(i)}$, as shown below:

$$w^{(i)}=\exp(-i\omega t)\exp\left(i\omega\left(\frac{x}{c_x}-\frac{y}{c_y}\right)\right) \tag{1}$$

This corresponds to a wave with incidence angle δ, amplitude of 1, excitation frequency ω, and wavelength $\lambda=2\pi/k$, where $k=\omega/c_\beta$, c_x and c_y are the components of the phase velocity in the direction of the coordinate axes. In the absence of the valley, the incident SH-wave is reflected by the free surface (y=0) and defined as shown below:

$$w^{(r)}=\exp(-i\omega t)\exp\left(i\omega\left(\frac{x}{c_x}+\frac{y}{c_y}\right)\right) \tag{2}$$

Omit $\exp(-i\omega t)$ from later expressions. The following relations are applied to equations (1) and (2) to convert from the Cartesian to the polar (r,θ) coordinate system.

$$\begin{gathered} x=r\cos\theta \\ y=r\sin\theta \\ c_x=c_\beta/\cos\delta \\ c_y=c_\beta/\sin\delta \\ k=\omega/c_\beta \end{gathered} \tag{3}$$

The incident and reflected waves are combined into the expression shown below:

$$\begin{aligned} w^{(i+r)}=&\exp(ikr\cos(\theta+\delta)) \\ &+\exp(ikr\cos(\theta-\delta)) \end{aligned} \tag{4}$$

Due to the presence of the valley, the waves are scattered and diffracted within the half-space and transmitted into the valley. Within the half-space, the result is a sum of the incident, reflected, scattered, and diffracted waves. Within the valley, the result consists of the transmitted waves. These must satisfy the wave equation as defined below:

$$\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}=\frac{1}{c_\beta^2}\frac{\partial^2 w}{\partial t^2} \tag{5}$$

Assume a scattered wave in the form shown below:

$$\begin{gathered} w^{(s)}=\sum_{n=0}^{\infty}A_n H_n^{(1)}(kr(\theta))\cos n\theta \\ n=0,1,2,\ldots \end{gathered} \tag{6}$$

The transmitted wave is defined as shown below:

$$\begin{gathered} w^{(v)}=\sum_{n=0}^{\infty}C_n J_n(k_v r(\theta))\cos n\theta \\ n=0,1,2,\ldots \end{gathered} \tag{7}$$

Define $k_v=\omega/c_{\beta v}$ as the wave number in the valley. In addition to the stress free boundary conditions at the free surface of the half-space, displacement and stress continuity conditions between the half-space and the valley must be used. These may be defined as shown below:

$$\begin{gathered} 0=\frac{\mu}{r}\frac{\partial w}{\partial\theta} \\ 0=\frac{\mu_v}{r}\frac{\partial w^{(v)}}{\partial\theta} \\ \theta=0,\pi \end{gathered} \tag{8}$$

at the surface of the half-space, and

$$0=w^{(i+r)}+w^{(s)}-w^{(v)} \tag{9}$$

$$\begin{aligned} 0=\mu\Big(&\frac{\partial w^{(i+r)}}{\partial r}n_r+\frac{1}{r}\frac{\partial w^{(i+r)}}{\partial\theta}n_\theta+\frac{\partial w^{(s)}}{\partial r}n_r+ \\ &\frac{1}{r}\frac{\partial w^{(s)}}{\partial\theta}n_\theta\Big) \\ &-\mu_v\left(\frac{\partial w^{(v)}}{\partial r}+\frac{1}{r}\frac{\partial w^{(v)}}{\partial\theta}n_\theta\right) \end{aligned} \tag{10}$$

Here at the interface between the valley and the half-space, n_r and n_θ are the unit normals in the r and θ directions. The free surface boundary condition is automatically satisfied by $w^{(s)}$ and $w^{(v)}$. The displacement continuity condition, equation (9), is satisfied by substituting equations (4), (6), and (7). Assemble (4), (6), and (7) into equation (10) in order to satisfy the stress boundary condition. Assemble the resulting equations into the weighted residual forms shown below:

$$0=\int_{\theta_1}^{\theta_N} W_m(r(\theta),\theta)\,(w^{(i+r)}+w^{(s)}-w^{(v)})\,d\theta \qquad m=0,1,2,\ldots \tag{11}$$

$$0=\int_{\theta_1}^{\theta_N} W_m(r(\theta),\theta)\,(\tau^{(i+r)}+\tau^{(s)}-\tau^{(v)})\,d\theta \qquad m=0,1,2,\ldots \tag{12}$$

Define $W_m(r(\theta),\theta)$ as the weight function and $\tau^{(i+r)}$, $\tau^{(s)}$, and $\tau^{(v)}$ as stresses due to the incident and reflected waves, scattered waves, and transmitted waves. The weight function used in this case was cosmθ. Since convergence was achieved and solutions matched closed form solutions, others were not used. Weight functions using Bessel and Hankel functions were also tested. Solutions resulting from the use of Hankel functions were successful. It was found that the use of Bessel functions resulted in a ill conditioned coefficient matrix. The weighted residual forms are assembled into the matrix form shown below:

$$\left[C_{mn}\right]\begin{bmatrix}A_n\\C_n\end{bmatrix}=\left[b_m\right] \qquad m,n=0,1,2,\ldots \tag{13}$$

Denote the matrix $[C_{mn}]$ as the matrix of coefficients from the scattered and transmitted wave weighted residual expressions, equations (11) and (12). The vector $[b_m]$ consists of the coefficients from incident and reflected wave weighted residual expressions. Constants A_n and C_n are determined and substituted into equations (6) and (7). The transmitted wave amplitudes are defined by equation (7). The scattered wave amplitudes are added to equation (4) to obtain amplitudes in the half-space.

SURFACE DISPLACEMENTS

Of particular interest are the displacement amplitudes on the surface of the half-space near the valley and on the valley surface. If the amplitude of the incident plane SH-waves is 1, the responses shown defines amplification and de-amplification factors. The resultant motion is defined by the modulus as shown below:

$$amplitude=(Re^2(w)+Im^2(w))^{1/2} \tag{14}$$

In the absence of the valley, for a uniform half-space, the modulus of the ground displacement is 2. Due to the existence of the valley, incident waves are scattered and diffracted into the half-space and transmitted into the valley. As a result, the moduli may differ significantly from 2. Displacements were calculated for a discrete set of dimensionless frequencies, η, at intervals of 0.25 ranging from 0.25 to 2. The dimensionless frequency, η, is defined as shown below:

$$\eta=\frac{2a}{\lambda}=\frac{ka}{\pi}=\frac{\omega a}{\pi\beta} \tag{15}$$

Let a be the half-width of the valley, the distance between the edges of the valley at the surface. Figures that follow show the displacement amplitudes on the surface of the half-space and the valley. All displacements are plotted with respect to the dimensionless distance x/a. It should be noted that convergence was attained using 18 terms or less for the cases presented below.

Figure 2 shows the surface displacement amplitudes on and near a shallow circular valley (h/a=0.5) for a frequency range from 0.25 to 2 for an incident wave with an angle of incidence of 30°, $\mu_v/\mu=1/6$, and $\rho_v/\rho=2/3$. Lower frequency incident waves produce simple surface amplification profiles near the canyon. Higher frequency incident waves result in amplification profiles which are more complex with higher peak values. On the surface of the alluvium, since it is a softer, less dense material, are complicated at all frequencies with peak values reaching 10. On the surface of the half-space, near the valley at x<-a are more complex with values reaching 3. On the other side of the valley at x>a a shadow zone exists with simpler amplification profiles with values closer to 2. Convergence was difficult for the shallowest valleys.

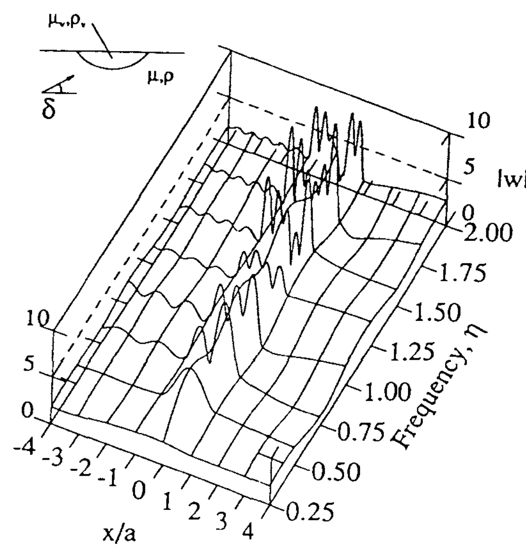

Figure 2. Shallow Semi-Cylindrical Valley, Incident SH-Wave, h/a=0.5, δ=30°, $\mu_v/\mu=1/6$, $\rho_v/\rho=2/3$

Figure 3 shows the surface displacement amplitudes for a semi-elliptical valley with a depth to width ratio of 0.35 for a frequency range from 0.25 to 2 for an incident wave with an angle of incidence of 60°, $\mu_v/\mu=1/6$, $\rho_v/\rho=2/3$. These solutions match those of the closed form solution (Wong and Trifunac, 1974). Amplitude profiles on the half-space become more prominent and complex for x/a<-1. On the other side of the valley, x/a>1, the amplitudes are smoother and tend towards 2. Within the soft valley, responses are significantly amplified and have greater complexity with respect to the half-space with peaks approaching 10. It was difficult to obtain adequate convergence for the shallowest canyons.

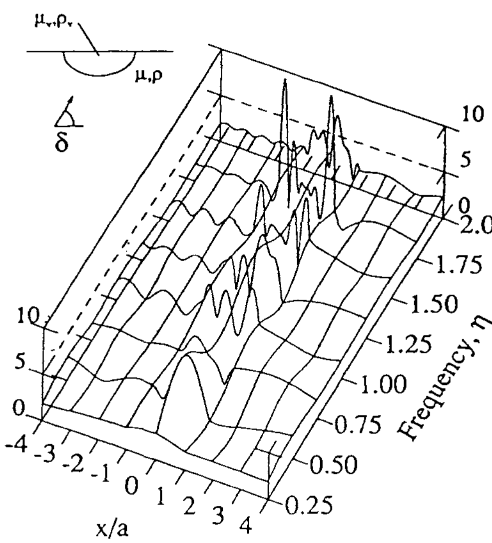

Figure 3. Shallow Semi-Elliptical Valley, Incident SH-Wave, D/W=0.35, δ=60° μ_v/μ=1/6, ρ_v/ρ=2/3

Figure 4 shows the surface amplitudes for a rectangular shaped valley for a dimensionless frequency range from 0.25 to 2 for incident waves with an angle of incidence of 90°, h/a=1, μ_v/μ=1/6, and ρ_v/ρ=2/3. Amplitudes are symmetric for this case in which the angle of incidence is vertical. Within the valley, amplifications are significantly larger with respect to those on the half-space with some cases exceeding 15. It should be noted that amplitudes for the rectangular valleys are not plotted on the sides of the valleys. Jumps in the amplitude plots at x=±a represent the range of the amplitudes on the sides of the valleys.

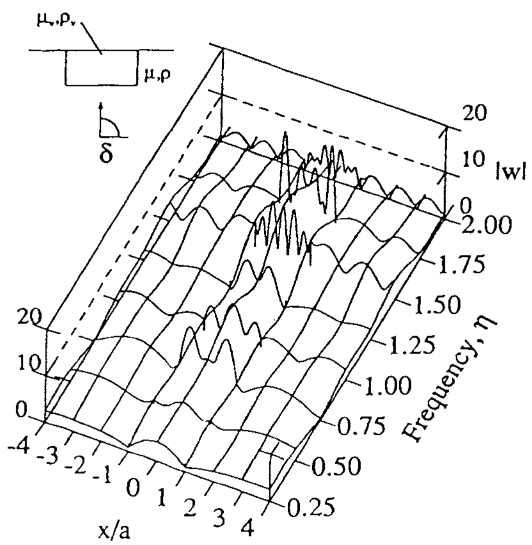

Figure 4. Shallow Rectangular Valley, Incident SH-Wave, h/a=0.5, δ=90° μ_v/μ=1/6, ρ_v/ρ=2/3

Figures that follow show the displacement amplitudes plotted on the surface of the half-space and the canyon. All displacements are plotted with respect to the dimensionless distance x/a.

Figure 5 shows surface displacement amplitudes on and near a shallow cylindrical canyon (h/a=0.5) for incident waves with an angle of incidence of 30°. Results shown indicate that the canyon produced significant scattering and diffraction of the incident waves. Lower frequency incident waves resulted in amplification profiles which were simple with peak amplitudes exceeding 2. Higher frequency incident waves resulted in a more complicated amplification profile with peaks reaching 3. Displacement amplitudes for x/a<0 tended to be larger and more complex with peak amplitudes slightly larger than 3. For x/a>0, a shadow zone resulted as displacement profiles were smoother with values that tended toward 2. Some difficulties were encountered with convergence for very shallow canyons.

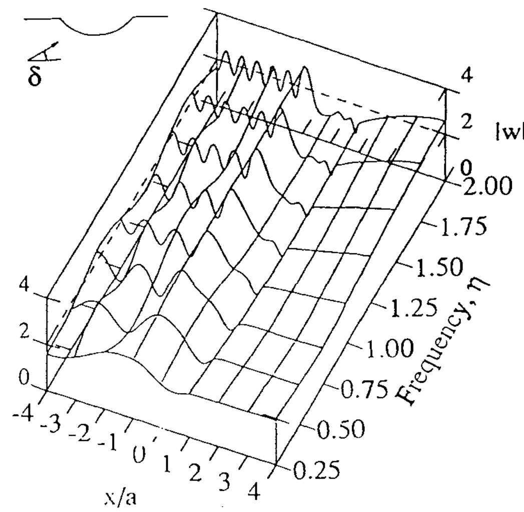

Figure 5. Shallow Semi-Cylindrical Canyon, Incident SH-Wave, h/a=0.5, δ=30°

Figure 6 shows the surface displacement amplitudes on and near a semi-elliptical canyon (D/W=0.35) for incident SH-waves with an angle of incidence of 60°. Here a=W is the radius of the major axis and D that of the minor axis. Results are consistent with results obtained by Wong and Trifunac (1974a) on and near the canyon. Displacement amplitude patterns for x/a<0 are complex and have peak values of nearly 4. On the other side of the canyon, x/a>0, a shadow zone exists with smoother amplitude patterns with amplitudes which tend toward 2. Some difficulties were encountered with convergence for very shallow elliptical canyons.

Figure 7 shows displacement amplitudes in and near a shallow rectangular shaped canyon for incident SH-waves with an orientation of 0°. Here a is the horizontal half-width of the rectangular canyon and h is the corresponding depth. Displacement amplitude patterns shown are similar to those encountered in canyons with other shapes as discussed above.

Amplitude patterns are more complex with magnitudes that tend toward 4 for x/a<0 and smoother with magnitudes tending towards 2 for x/a>0. Amplitudes are not plotted for the sides of the canyons. Jumps in the amplitude plots at ±x/a represent the range of amplitudes on the sides of the canyons. It should be noted that these conditions occur at the canyon edge.

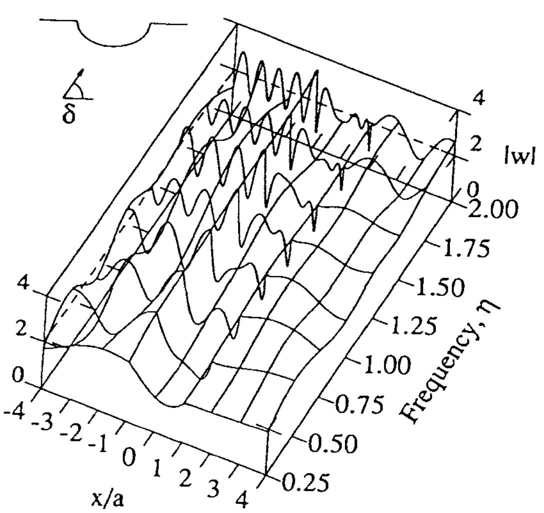

Figure 6. Shallow Semi-Elliptical Canyon, Incident SH-Wave, D/W=0.35, δ=60°

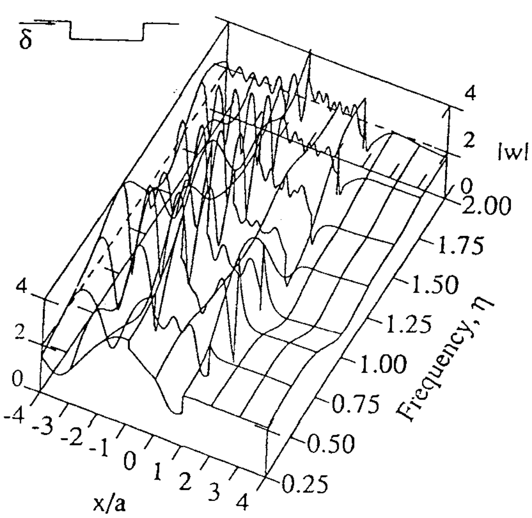

Figure 7. Shallow Rectangular Canyon, Incident SH-Wave, h/a=0.5, δ=0°

CONCLUSIONS

A number of observations result from the work presented in this chapter.

1. The weighted residual approach approximation for the scattering, diffraction, and transmission of SH-waves by a shallow alluvial valley or canyon yields solutions which match the known closed form solutions.
2. The weighted residual approach is suitable for valleys and canyons of arbitrary shape though convergence for the shallowest valleys is difficult to obtain.
3. Ground surface amplitudes on the half-space outside the valleys and canyons may be significantly larger than 2 and depend on the angle of incidence, dimensionless frequency, shape of the valley, and the relative properties of the valley.
4. Ground surface amplitudes on the half-space within the valleys may also be significantly larger than 2 and may differ from those on the half-space outside the valley and depend on the same factors and the properties of the valley relative to the half-space.

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