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# The Role of Rotational Shear in Site Response Analyses

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**SYNOPSIS** Rotational shear is a class of loading under which the second invariant of the deviatoric stress tensor,  $J$ , is kept constant during shear. A limited number of undrained tests on saturated loose sand have shown that rotational shear yields more pore pressure than other shear paths of the same  $J$  magnitude. By intuition, this experimental finding seems to suggest that soil subjected to multi-directional earthquake loading, which bears the characteristics of the rotational shear, has lower liquefaction resistance than that under unidirectional shaking. However, in contrast to intuition, a preliminary but careful examination of the field stress and boundary conditions indicates that rotation shear seems to have very little impact on the seismic response of natural soil deposits. This paper presents the theoretical evidence that supports the preliminary conclusion.

## INTRODUCTION

When a soil deposit is subjected to an upward propagating seismic shake, the soil in the deposit experiences cyclic simple shear in two horizontal directions, thus resulting in the change of the resultant shear stress in both magnitude and direction. As a vector, the resultant shear yields irregular stress paths in its component plane. The characteristics of the stress paths is similar to that of the horizontal acceleration trajectories. Fig. 1 shows such an acceleration trajectory which was recorded in Lotung, Taiwan, in 1986, during an earthquake of  $M=7.0$ . It is evident that the resultant shear exhibited a widespread change in both magnitude and direction.

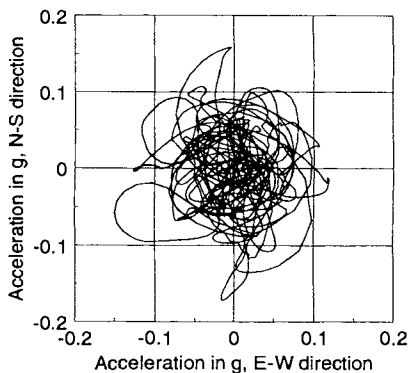


Fig. 1 Trajectory of Acceleration Recorded in Lotung, Taiwan, Nov., 1986

Earlier studies on soil behavior under seismic loading were essentially based on the results of unidirectional cyclic shear tests, and the influences of change in shear direction were simply ignored. In view of this, Ishihara and Yamazaki (1980) conducted a series of undrained tests on sand using a two directional simple shear apparatus. The apparatus had two mutually perpendicular horizontal actuators for cyclic shear applications. The soil samples were initially consolidated at an isotropic effective confining pressure of 200 kpa and during shear the total confining pressure was kept unchanged. By combining two

shear components, a variety of shearing schemes characterized by the loading paths in the horizontal plane could be achieved. The investigators studied two types of shearing schemes: the **Rotational Simple Shear** and the **Alternate Simple Shear**. For the rotational simple shear tests, two sinusoidal shears with  $90^\circ$  out of phase to each other were applied simultaneously in two horizontal directions. Depending on the relative amplitudes of the two shear components, the resultant loading path could be either circular or elliptic. For the tests of the alternate simple shear, cyclic simple shear was applied alternately in two horizontal directions cycle by cycle, and the amplitudes of the two components could also be different. The typical stress paths of those tests are shown in Fig. 2. The test results of the two types of shearing

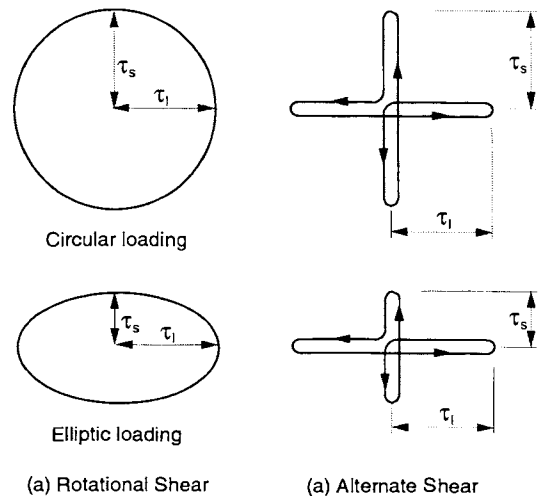


Fig. 2 Patterns of Bi-directional Simple Shear (After Ishihara and Yamazaki, 1980)

schemes are reproduced and shown in Figs. 3a and 3b. It is obvious that when  $\tau_s/\tau_l=1$  the stress pattern is a circle, when  $\tau_s/\tau_l=0$ , the shear loading becomes unidirectional. From Fig. 3a it can be seen that the

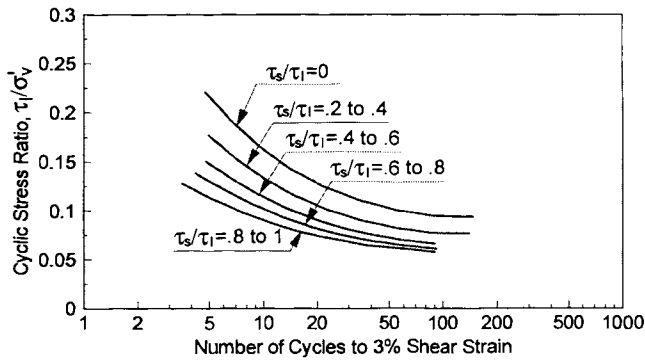


Fig. 3a Rotational Simple Shear Results  
(After Ishihara and Yamazaki, 1980)

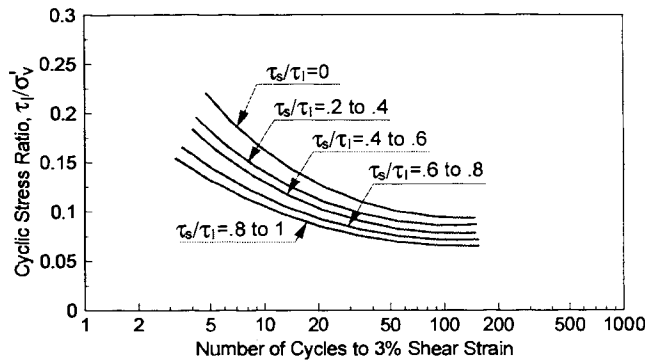


Fig. 3b Alternate Simple Shear Results  
(After Ishihara and Yamazaki, 1980)

resistance of the tested soil to failure was gradually reduced from unidirectional shear to circular shear. The investigators concluded that for the saturated loose sand being tested, under the circular loading path the cyclic stress ratio  $\tau_s/\sigma'_v$  causing failure (defined as 3% of shear strain) at a given number of loading cycles was approximately 65% of that obtained under the corresponding unidirectional shear loading. The alternate simple shear results shown in Fig. 3b also evidenced that under the condition of changing shear directions the soil's resistance to failure was less than that under unidirectional shear condition. Tests aimed at examining the behavior of sand under multi-directional cyclic shear have also been performed on true triaxial and torsional shear devices (Yamada and Ishihara, 1981, 1982, 1983; Towhata and Ishihara, 1985), the typical results are replotted in Fig. 4. It is apparent

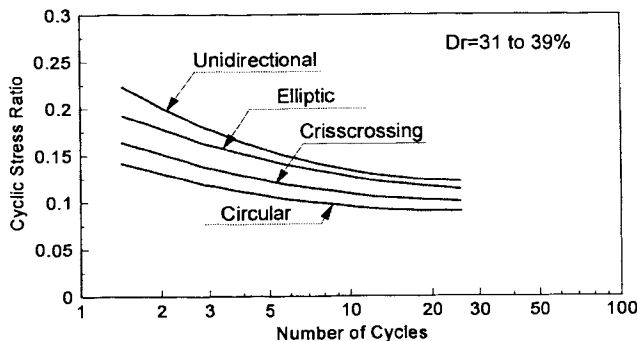


Fig. 4 Failure Resistance Averaged for Different Paths  
(After Yamada and Ishihara)

that the trend of the influence of loading pattern is very similar to that observed in the multi-directional simple shear tests. Based on these laboratory results, one may argue that any seismic response analysis without taking into consideration the rotational shear effects may lead to an underestimate of the liquefaction potential of loose sand deposits.

Aimed at exploring the influences of rotational shear in a more rigorous way, a multi-directional numerical procedure for ground response problems was developed by the authors (Li, 1990; Li et al., 1992). This fully-coupled effective stress finite element program takes the rotational shear effects into consideration through a sophisticated hypoplasticity soil model (Wang, 1990; Wang et al, 1990). In contrast to classical plasticity, the model yields plastic deformation for a stress increment in any possible direction, even along neutral loading paths. Because of this, the model can simulate the behavior of granular materials under rotational shear which has been rigorously defined in theory as a class of loading under which the second invariant of the deviatoric stress tensor,  $J$ , is kept constant. The circular shear stress path as shown in Fig. 2 obviously falls into this category. Other non-unidirectional stress paths such as those induced by seismic events also bear the resemblance of this type of loading. The model's capability to account for the effect of rotational shear is clearly shown in Figs. 5a and b – the typical behavior of the model responding to unidirectional shear

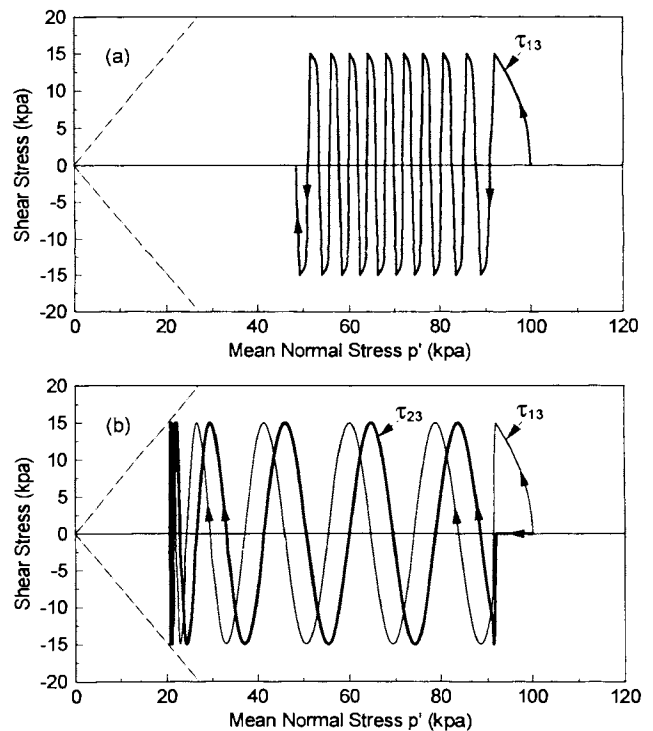


Fig. 5 Typical Response of the Hypoplasticity Model to  
(a) Unidirectional Simple Shear and (b) Rotational Simple Shear

and rotational shear, respectively. It can be seen from the figures that the rate of pore pressure buildup under the rotational shear is significantly higher than that induced by the unidirectional shear of the same amplitude until the stress ratio ( $\tau/p'$ ) hits a characteristic line. In agreement with the experimental observation (Ishihara and Yamazaki, 1980), rotational shear prevents pore pressure from equating the initial confining pressure because of the continuous dilative tendency.

Interestingly, however, the ground response analyses using this model showed a rather different picture. The significantly faster pore

pressure buildup as expected under circular motion never materialized. Fig. 6 shows the calculated excess pore pressure buildup of a liquefiable element under three different input motions. The element was centered at 3.5 m below the ground surface of a hypothetical sand deposit, which is 20m thick, uniform, and fully saturated. The model parameters that produce the cyclic behavior as shown in Figs. 5a and b were used to characterize the sand, and the initial value of  $K_0$  was assumed 0.6. All the three input motions were 1 Hz in frequency and were directly applied at the bottom boundary of the deposit. The unidirectional input motion was a sinusoidal wave of 0.1g p-p applied in only one of the horizontal directions; the circular input motion comprised two sinusoidal waves in two perpendicular horizontal directions, the two waves were equal in amplitude (0.1g p-p) and 90° out of phase; and the in-phase bi-directional input motion was actually a unidirectional sinusoidal motion but its magnitude had been increased to 0.14g p-p. It can be seen from Fig. 6 that the rate of pore pressure buildup induced by the unidirectional shaking is almost the same as that under the circular motion, and the rate under the in-phase bi-directional shaking is higher than that under the 90° out-of-phase shaking (circular motion). The results seem to indicate that the rotational shear or the change in shear directions has no significant impact on site response analyses for earthquake loading. The "discrepancy" between the rotational shear effect at element level and that in site response analyses demands an explanation. This paper presents some preliminary findings to this question.

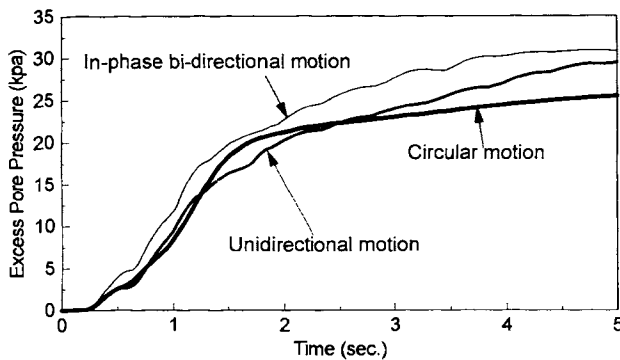


Fig. 6 Variation of Pore Pressure Response Caused by Different Motion Patterns

### ISOTROPIC AND $K_0$ STRESS CONDITIONS

It is well known that soils in level grounds are under  $K_0$  conditions where no lateral deformation is allowed to develop. Ideally,  $K_0$  conditions should also be preserved when laboratory tests are conducted to evaluate soil behavior under seismic loading conditions. Unfortunately, due to the use of flexible membranes,  $K_0$  condition could not be imposed during the bi-directional simple shear tests (Ishihara and Yamazaki, 1980). Instead, the reported bi-directional simple shear results were limited to only isotropically confined loose sand.

One of the differences between isotropic and  $K_0$  stress states is that under the former condition the deviatoric part of the stress does not depend on normal stresses at all because of their isotropic attribute. However, under  $K_0$  state, the deviatoric stress depends not only on the simple shear components but also on the differences among the normal stresses. Thus, under  $K_0$  condition there exist two sources contributing to the variation of the deviatoric stress, one comes from the simple shear and the other from the changes of the differences among the

normal stresses. Experiments have already shown that for a soil under  $K_0$  condition, as cyclic simple shear loading is applied the lateral total stresses will gradually increase and finally approach the isotropic normal stress state if the initial value of  $K_0$  is less than 1 (Pyke, 1973; Youd and Craven, 1975; Seed et al., 1977). In other words, the  $K_0$  value will successively increase towards unity, which will in turn reduce the magnitude of the deviatoric stress, or the value of the second invariant of the deviatoric stress tensor. Such a phenomenon can be elaborated by plasticity theory. Fig. 7 shows a deviatoric stress space composed of the

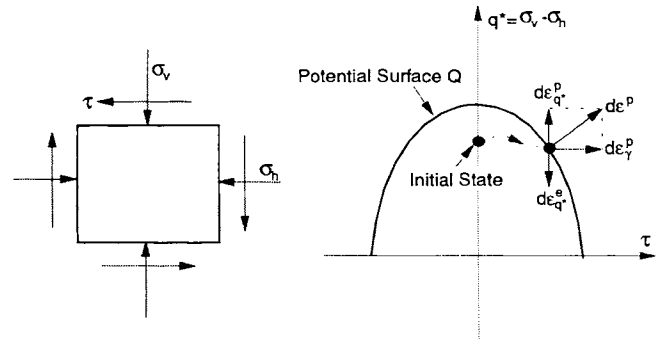


Fig. 7 Elastic and Plastic Strain Components in  $q^*$  and  $\tau$  space

difference between vertical and horizontal normal stresses,  $q^*$ , and the simple shear stress  $\tau$ . In the figure  $de^p$  represents a plastic strain increment which can be decomposed into two components:  $de^p_{q^*}$  and  $de^p_{\gamma}$ , in which  $de^p_{q^*}$  is proportional to  $de^p_{vertical} - de^p_{horizontal}$ , the difference between vertical and lateral incremental plastic strains, and  $de^p_{\gamma}$  is the plastic strain increment in simple shear direction. When simple shear (either rotational or unidirectional) starts from a  $K_0$  value less than 1, due to the convexity of plastic potential surface Q, as illustrated in Fig. 7, one can expect a positive  $de^p_{q^*}$ . However, under the condition of no drainage and no lateral strains, both vertical and lateral total strains are absent,  $de_{q^*} = de^e_{q^*} + de^p_{q^*} = de^e_{vertical} - de^e_{horizontal} = 0$ , thus, the elastic strain increment  $de^e_{q^*} = de^e_{vertical} - de^e_{horizontal} = -de^p_{q^*} < 0$ . As stress increment is proportional to elastic strain increment,  $dq^*$  will also be negative, i.e.,  $d\sigma_v - d\sigma_h < 0$ . Since the vertical total stress is virtually unchanged during earthquake, the lateral total stress must increase which results in an increase in  $K_0$  value. Fig. 8 shows the calculated  $p$ - $q^*$  plot of the soil element used previously to demonstrate the pore pressure changes in Fig. 6. While quantitatively the rate of the  $K_0$  change depends on the shape of the plastic potential function which needs to be further calibrated by experimental data on 'undisturbed' samples (natural soil

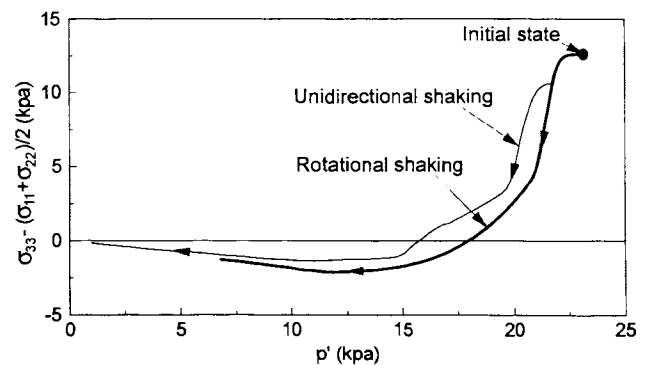


Fig. 8 Calculated  $p$ - $q^*$  relationship of a soil element under periodic shaking

may possess a plastic potential surface not centered around the hydrostatic axis), Fig. 8 does demonstrate, at least qualitatively, the characteristics of the earthquake induced lateral stress change.

Under undrained condition the accumulated excess pore pressure caused by shear is directly associated with the plastic volumetric strain developed in soil. When a changing deviatoric stress forces the soil to yield, a notable amount of plastic volumetric strain will be produced. For frictional materials like sand the yield criterion in deviatoric space is more related to the ratio of the deviatoric stress over the mean normal stress rather than to the deviatoric stress alone, thus it is sometimes more convenient to use a normalized yield surface (in terms of the stress ratio) to describe the yield criterion. For saturated sand subjected to undrained shear the change of the normalized yield surface is due to either an increase of the deviatoric stress or a reduction of the mean normal stress. For cyclic tests with constant shear amplitudes, the expansion of the yield surface is mainly due to the reduction of the effective mean normal pressure. As stated earlier, for isotropically confined soils the only deviatoric stress components are the simple shear stresses. At the beginning of a test, because of the absence of the deviatoric components in loading history, the size of the normalized yield surface would be in general zero or very small. Beginning with the application of the cyclic simple shear, the yield surface gradually expands. In the case of unidirectional shear, a shear cycle (except the first half cycle) reaches and then pushes the yield surface outwards only when the shear cycle approaches its peaks. However, in the case of circular shear, because of its constant magnitude, the yield surface may be touched and continuously pushed outwards all the time, resulting in the possibility that the pore pressure build up under circular shear is faster than that under unidirectional shear.

On the contrary, as shown in Fig. 8, for a soil under  $K_0$  state, even before the application of the simple shear cycles, the yield surface has already been pushed far away from the hydrostatic axis by the anisotropic confining stresses. As the cyclic simple shear proceeds, the value of  $K_0$  gradually increases which tends to reduce the magnitude of the resultant deviatoric stress and to lead the stress paths towards the hydrostatic axis. Therefore, no matter whether the shear is unidirectional or circular, the resultant deviatoric stress will no longer stay on the yield surface all the time, thus, the condition under which circular shear produces faster excess pore pressure now disappears.

Needless to say the mechanism discussed above needs to be verified by experimental results which are currently unavailable. However, it is another example showing that in order to examine soil behavior under field situation through laboratory tests, the actual loading condition should be reproduced as closely as possible. Any discrepancy between laboratory condition and field condition should be carefully identified and analyzed. In the case of site response, the  $K_0$  condition existing in the field could have made the influence of the rotational simple shear much less significant than that revealed in the laboratory on isotropically confined specimens. It should also be pointed out that our reservation towards the significance of the rotational simple shear under  $K_0$  condition does not imply that taking multi-directional shaking into consideration is unnecessary. To compare with unidirectional shaking, multi-directional earthquake loading substantially increases the resultant shear magnitude.

## SUMMARY

The rate of pore pressure built up in liquefiable soil during earthquake is an important factor being considered in site response analyses. Laboratory tests showed that change in shear direction has significant impact on the rate of pore pressure increase, but how to interpret those

results is not very clear. This paper briefly describes those results and points out the difference between the laboratory and the field conditions. With the help of a hypoplasticity sand model and a multi-directional site response analysis program, the authors have shown, at least in theory, that the impact of rotational shear on site response problems may not be as significant as previously thought.

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