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A. R. Merchant

Marion D. Arnold

Missouri University of Science and Technology

A. Herbert Harvey

Missouri University of Science and Technology

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A Technique for Improving Material Balance Accuracy in Reservoir Simulation Models

By

A. R. Merchant, Alcore, S.A.,
M. D. Arnold and A. Herbert Harvey, U. of Missouri at Rolla, Members AIME

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ABSTRACT

One criterion for determining the precision of pressures calculated by a simulation model is the accuracy of the material balance. A high degree of precision in the calculated pressures is a necessary but not sufficient condition for elimination of significant errors in the prediction of fluid migration in the reservoir. A new procedure, called error matrix technique, has been developed for reduction of material balance errors. This reduction is accomplished by adjusting potential gradients at every grid point in the simulation model so that the material balance is accurately maintained. Performance of a hypothetical oil reservoir was simulated both by conventional methods and by the error matrix technique. The new technique was found to be more accurate than the conventional simulation.

INTRODUCTION

A mathematical reservoir simulation model is based on sets of nonlinear partial differential equations that express pressure and saturations as functions of time and position. Two types of error should be considered in evaluating the accuracy of a simulation developed by such a model. One of them is machine round-off error. Accumulation of this error can lead to instability of the calculation scheme. If round-off error were not present, an exact
References and illustrations at end of paper.

solution of the finite-difference equations would be obtained. However, an exact solution of the difference equations usually is not an exact solution of the differential equations that they represent.

Exact finite-difference representation of the differential equations is not ordinarily achieved because the derivatives are approximated by truncated Taylor's series expansions. The discretization error that is caused by the truncation is another source of inaccuracy that can be anticipated when finite-difference techniques are employed. The magnitude of this error is reflected in the deviation of the calculated pressures from the true ones.

The use of imprecise pressures for saturation computations leads to the calculation of inaccurate saturation values. If these saturations are used to compute the mass (or volume) of fluids in the reservoir, these computations will also be somewhat in error. Therefore, the law of mass conservation provides a basis for evaluating the accuracy of a reservoir simulation model. This paper describes a method for improving the accuracy of the simulation by increasing the precision of the mass balance.

MATERIAL BALANCE ERRORS

For each phase, the fluid mass in the

reservoir should always equal the initial mass less net cumulative production. Any deviation from this equality represents an error in the material balance. Several mathematical expressions have been devised for the measurement of material balance error. Perhaps the most commonly used relationship is

$$\begin{aligned} &(\text{oil material balance}) = \\ &\frac{(\text{original oil in place}) - (\text{oil remaining})}{(\text{cumulative oil produced})} \end{aligned} \quad (1)$$

where volumes are expressed in terms of stock-tank conditions. Any deviation of Eq. 1 from unity represents an error in the material balance.

A similar technique has been employed by Merchant,¹ who determined the accuracy of the material balance for each time step in a simulation study. This equation is written

$$(\text{incremental oil material balance}) = \frac{\Delta(\text{OIP})}{q_o \Delta t}, \quad (2)$$

where q_o is the oil production rate during time increment (Δt) , and $\Delta(\text{OIP})$ is the calculated change of oil in place during the time increment. This equation should also have a value of unity.

It has been recognized for some time that an accurate material balance can be maintained only if accuracy is achieved in solving the system of equations that describe reservoir pressures. The relationship between material balance errors and accuracy of pressure calculations was investigated in some detail by Traylor and Sheffield.² These authors measured material balance error in terms of an absolute water volume error, which they showed to decrease as the average pressure residual error was reduced by iteration.

Although accuracy in the pressure calculations is essential, it has been noted that such accuracy does not always preclude error in the material balance. A technique for direct correction of material balance error was presented recently by Nolen and Berry.³ These authors included correction factors in the accumulation terms for both the wetting and nonwetting phase equations. These factors cancel the material balance errors for the previous time step. Thus, the procedure limits the cumulative errors to those made during the current time increment.

SOURCES OF MATERIAL BALANCE ERROR

The material balance error problem was investigated by simulating the performance of a hypothetical oil reservoir. The mathematical

model used for the study was a three-phase simulator capable of accounting for reservoir heterogeneity as well as the variation of fluid properties with respect to pressure. The material balance error was computed by Eq. 1.

Since material balance problems are frequently associated with water breakthrough and with reduction of pressure to a value near the bubble point, the problem was designed so these events would occur early in the study. Thus, the simulation study was not based on a realistic development program, but rather a hypothetical situation in which the material balance problem could be readily investigated. The grid system and well locations that were used for the problem are illustrated by Fig. 1; summaries of the data and mathematical relationships that were employed are presented in Tables 1 and 2.

One criterion for maintenance of an accurate mass balance is that the sum of the three-phase saturations must be unity. Since errors in the pressure calculations cause errors in computed saturations, the iterative technique described by Achmad⁴ was used to reduce residual pressure errors. For the example problem it was found that the error in the sum of saturations became negligible when the average absolute residual pressure error was reduced to 0.001 atm and the maximum absolute residual was limited to 0.01 atm. These criteria were used in this study to improve accuracy of the computed pressures.

The equations that describe potential distribution within the reservoir provide another opportunity for verification of accuracy of the material balance. For the oil phase, this equation may be written

$$\nabla \cdot T_o \nabla \Phi_o = q_o / (\Delta x \Delta y) + H \frac{\partial}{\partial t} \left[\frac{S_o \phi}{B_o} \right]. \quad (3)$$

If Eq. 3 is discretized and solved for Φ_o at every grid location, it can be shown that the law of mass conservation predicts

$$\sum_j \sum_i \Delta_{xy} T_{o,i,j} \Delta_{xy} \Phi_{o,i,j} = 0. \quad (4)$$

This test was made for each computer run during this study, and it was found that the summation was near zero in every instance. This result led to the conclusion that machine round-off errors were small and accurate solutions to the finite-difference equations were being obtained. Therefore, it seems likely that the errors that were observed arose primarily from nonlinearity in the equations or from discretization errors.

The simulation model employed for the study included the usual features which require that calculated saturations conform to logical constraints. For example, an oil saturation that was computed to be greater than $(1-S_{wc})$ was reduced to $(1-S_{wc})$. These small adjustments in S_o (generally less than 0.001) were found to occur rather frequently even though the sum of the three unadjusted phase saturations was essentially unity. It was also found that these adjustments consistently increased the incremental material balance error defined by Eq. 2. Therefore, it was concluded that these necessary adjustments in oil saturation were a significant source of material balance error.

The law of mass conservation is the basis of reservoir simulation, and material balance errors that violate this law presumably arise from inaccuracies in the calculated potential gradients. Hence, it was concluded that it is proper to adjust calculated potential gradients in order to reduce material balance errors. A technique that has been developed for making these adjustments is described below.

ERROR MATRIX TECHNIQUE

Eq. 3 describes potential and saturation distributions of the oil phase in a petroleum reservoir. Discretization of this equation, with slight rearrangement, yields the following set of difference equations.

$$\Delta t \Delta x_i \Delta y_j \Delta_{xy} \left[(T_o)_{i,j} \Delta_{xy} (\Phi_o)_{i,j} \right]^{n+1} - \Delta t q_{o,i,j} = (OIP)_{i,j}^{n+1} - (OIP)_{i,j}^n, \dots (5)$$

where

$$(OIP)_{i,j} = \left[\frac{S_o PV}{B_o} \right]_{i,j} \dots (6)$$

Eq. 5 is used as a basis for correction of mass balance errors by potential adjustment. This correction procedure will be designated as the error matrix technique.

A slightly different form of Eq. 3 results from expansion of the time derivative. This relationship may be written

$$\nabla \cdot T_o \nabla \Phi_o = q_o / (\Delta x \Delta y) + \frac{H \Phi}{B_o} \left[S_o C_r - \frac{S_o}{B_o} \frac{dB_o}{dP} \right] \frac{\partial P}{\partial t} + \frac{H \Phi}{B_o} \frac{\partial S_o}{\partial t} \dots (7)$$

In the following discussion, S_o will be defined as the oil saturation computed from a finite-difference expansion of Eq. 7. The symbol S_o' will be used to denote the corrected oil saturation that has been adjusted to conform to all constraints. Similarly, we will define OIP' as oil in place (at standard conditions) based on S_o' . Since S_o' is considered to be more accurate than S_o , the difference between OIP and OIP' represents an error in the oil in place computed from the finite difference form of Eq. 7; this oil error will be represented by the symbol OE . Thus,

$$(OIP)^{n+1} = (OIP')^{n+1} + OE \dots (8)$$

Using this terminology, we may rewrite Eq. 5 as

$$\Delta t \Delta x_i \Delta y_j \Delta_{xy} \left[(T_o)_{i,j}^{n+1} \Delta_{xy} (\Phi_o)_{i,j}^{n+1} \right] - \Delta t q_{o,i,j} = (OIP')_{i,j}^{n+1} - (OIP)_{i,j}^n + OE_{i,j}, \dots (9)$$

where

$$(OIP')_{i,j}^{n+1} = (S_o')_{i,j}^{n+1} \left[\frac{PV}{B_o} \right]_{i,j}^{n+1} \dots (10)$$

The oil error (OE) exists because the pressure equation, which was solved before the saturation equations, contained an error that allowed the calculated saturations to exist outside the constraints. The oil error may be forced to zero by defining a change in potential $\delta \Phi_o$ and a new potential, $\bar{\Phi}_o$:

$$\bar{\Phi}_o = \Phi_o + \delta \Phi_o, \dots (11)$$

where $\bar{\Phi}_o$ is a corrected value of the oil potential function. Since gravity and capillary pressure effects remain essentially constant during a time step, the total correction may be applied to pressure as

$$\bar{P} = P_o + \delta P_o, \dots (12)$$

where \bar{P} is the new oil-phase pressure and δP_o is the correction necessary to calculate \bar{P} from the noncorrected pressures computed for the new time level. Then Eq. 9 becomes

$$\Delta t \Delta x_i \Delta y_j \Delta_{xy} \left[(T_o)_{i,j}^{n+1} \Delta_{xy} (\bar{\Phi}_o)_{i,j}^{n+1} \right] - \Delta t q_{o,i,j} = (\bar{OIP})_{i,j}^{n+1} - (OIP)_{i,j}^n, \dots (13)$$

$$\text{where } (\overline{OIP})_{i,j}^{n+1} = (S'_O)_{i,j}^{n+1} \left(\frac{\overline{PV}}{B_O} \right)_{i,j}^{n+1} \dots (14)$$

and \overline{PV} and \overline{B}_O are defined at \overline{P} . In order to simplify the notation, the symbols P_O and δP_O will be replaced by P and δP in the following discussion.

Note that all the variables that are substantially pressure-sensitive in Eq. 9 have been revised in Eq. 13. The transmissibility, T_O , is only slightly sensitive to pressure, and it was, therefore, left unchanged.

Since $\delta \Phi_O$ and δP are considered to be equal, we may subtract Eq. 9 from Eq. 13 and obtain

$$\Delta t \Delta x_i \Delta y_j \Delta_{xy} [(T_O)_{i,j} \Delta_{xy} (\delta P)_{i,j}]^{n+1} \\ = (\overline{OIP})_{i,j}^{n+1} - (OIP')_{i,j}^{n+1} - (OE)_{i,j} \dots (15)$$

Substitution of Eqs. 10 and 14 for the first two terms on the right side of Eq. 15 yields

$$(\overline{OIP})_{i,j}^{n+1} - (OIP')_{i,j}^{n+1} = \\ (S'_O)_{i,j}^{n+1} \left[\frac{\overline{PV}}{B_O} - \frac{PV}{B_O} \right]_{i,j}^{n+1} \dots (16)$$

At pressures above bubble point the following relationship holds, since compressibilities are small:

$$\left(\frac{\overline{PV}}{B_O} \right)^{n+1} = \left(\frac{PV}{B_O} \right)^{n+1} \left[\frac{1+C_r(\delta P)}{1-C_O(\delta P)} \right] \dots (17)$$

Since $C_O(\delta P)$ is small,

$$\{1-C_O(\delta P)\}^{-1} \approx 1 + C_O(\delta P).$$

Therefore,

$$\left[\frac{\overline{PV}}{B_O} \right]^{n+1} \\ = \left[\frac{PV}{B_O} \right]^{n+1} \{ [1+C_r(\delta P)] [1+C_O(\delta P)] \} \\ = \left(\frac{PV}{B_O} \right)^{n+1} [1+(C_r+C_O)(\delta P)] \dots (18)$$

neglecting the term $C_r C_O (\delta P)^2$. Eq. 16 then

simplifies to

$$(\overline{OIP})_{i,j}^{n+1} - (OIP')_{i,j}^{n+1} =$$

$$(S'_O)_{i,j}^{n+1} \left(\frac{PV}{B_O} \right)_{i,j}^{n+1} [(C_r+C_O)(\delta P)]_{i,j}$$

or,

$$(\overline{OIP})_{i,j}^{n+1} - (OIP')_{i,j}^{n+1} =$$

$$(OIP')_{i,j}^{n+1} (C)(\delta P)_{i,j} \dots (19)$$

where

$$C = C_r + C_O \text{ when } P > BP. \dots (20)$$

Substitution of Eq. 19 into Eq. 15 yields

$$\Delta t \Delta x_i \Delta y_j \Delta_{xy} [(T_O)_{i,j} \Delta_{xy} (\delta P)_{i,j}]^{n+1} \\ = (OIP')_{i,j}^{n+1} (C)(\delta P)_{i,j} - (OE)_{i,j} \\ \text{or,} \\ (OE)_{i,j} = \\ (OIP')_{i,j}^{n+1} (C)(\delta P)_{i,j} \\ - \Delta t \Delta x_i \Delta y_j \Delta_{xy} [(T_O)_{i,j} \Delta_{xy} (\delta P)_{i,j}]^{n+1} \dots (21)$$

For pressures below the bubble point, the equation that corresponds to Eq. 17 may be written

$$\left[\frac{\overline{PV}}{B_O} \right]^{n+1} = \left[\frac{PV}{B_O} \right]^{n+1} \frac{1+C_r(\delta P)}{1+(\frac{\partial B_O}{\partial P}) \delta P} \dots (22)$$

However, since this expression would introduce an undesirable nonlinearity into the computation, Eq. 22 is replaced by the approximation

$$\left[\frac{\overline{PV}}{B_O} \right]^{n+1} \sim \left[\frac{PV}{B_O} \right]^{n+1} (1+C_r \delta P), \\ P < BP \dots (23)$$

If we define

$$C = C_r \text{ when } P < BP, \dots (24)$$

we can combine Eqs. 23, 16 and 15 as before. The result is Eq. 21 with C defined according to Eq. 24. Thus, Eq. 21 may be employed both above and below the bubble point.

The approximation expressed by Eq. 23 has no effect on the final values in the δP array, since the value of C becomes trivial as δP converges to zero. It is desirable, however, to assign a positive value to C in order to assure strict diagonal dominance in the matrix of δP coefficients.

In matrix notation Eq. 21 has the form,

$$[E] \delta P = OE, \dots \dots \dots (25)$$

where $[E]$ is the matrix of the coefficients of $\delta P_{i,j}$. Eq. 25 may be solved for $\delta P_{i,j}$ at each mesh point; in this study the system of equations was solved by the SIP technique.⁵ The adjusted pressures were computed from

$$\bar{P}_{i,j} = P_{i,j}^{n+1} + \delta P_{i,j} \dots \dots \dots (26)$$

Saturations were then recalculated according to the adjusted pressures, and the computation was iterated until the material balance error was reduced to a low level. In this study a material balance tolerance of ± 1 percent was selected, where the material balance was defined by Eq. 1. This accuracy was usually achieved after one iteration. Reduction of the material balance error stabilized the computation sufficiently to permit crossing of the bubble-point pressure without a reduction in time-step size.

The example problem was studied by two computational techniques that will be designated Case I and Case II. Case I, which served as a basis for comparison, was a conventional simulation conducted without the material balance error correction that was developed in this study. Case II was a simulation modified by error matrix technique so the material balance error was reduced to less than 1 percent. The sequence of computations employed in this study is illustrated by Fig. 2. Implicit matrix coefficients were used for both cases.

The effectiveness of error matrix technique is illustrated by a comparison of the material balance curves for the two cases (Fig. 3). For Case II, the curve showed a maximum value of 1.008 and did not oscillate. It stabilized at a constant value of approximately 1.001. For Case I, the material balance curve started at a relatively high value (1.033) and declined to a value of approximately 1.002.

Although no attempt was made to correct

the water material balance, Case II showed improvement over the conventional simulation (Case I) when the potential gradients, based on oil material balance error, were adjusted. This comparison can be observed in Fig. 4. Both Cases I and II gave the same water material balance up to approximately 600 days. The curve of Case II then remained near unity, whereas that of Case I showed some deviation. After 5 years Case I had a water material balance of 0.988 and Case II had a corresponding value of 0.992.

When the error matrix technique was applied to correct any material balance error, the maximum adjustment necessary in any grid block was 1.5 psi. This correction, though too small to show an appreciable change in average reservoir pressure (Fig. 5), affects the pressure gradients sufficiently to account for all the material balance error that was observed.

The conventional type of simulation (Case I) may produce a satisfactory history match even though the material balance is in error. Therefore, although the simulated producing characteristics of a field may be correct during early stages of production regardless of small material balance errors, the discrepancy will become evident in predicted performance. Incorrectly calculated fluid volumes will ultimately result in erroneous predicted production rates. Computed ultimate recovery will also be in error.

In this study, the reservoir performance curves (Figs. 5-9) are similar for the two cases during the 5 years of simulated production history. However, some differences are observed in the graphs. The difference in the average reservoir pressures computed for the two cases was approximately 10 to 15 psi. This difference in calculated pressures is scarcely noticeable in Fig. 5 because of the scale used. Small deviations in total oil produced and oil production rate are shown by Figs. 6 and 8.

The additional computing time required to solve the error equations for Case II was slight, since only 1 or 2 SIP iterations were adequate to obtain the necessary convergence when material balance correction was required. The iterations were terminated when the difference in the computed pressures between two consecutive iterations was less than 5×10^{-5} atm. The average computer run time per time step for the 120 blocks used in this study was 50.51 seconds for Case I and 51.70 seconds for Case II using an IBM 360/50 computer.

CONCLUSION

The error matrix technique that has been described will reduce errors in mass conservation that arise in petroleum reservoir

simulation models. This reduction of material balance errors can be expected to improve stability and increase the accuracy of the simulator in predicting reservoir performance.

NOMENCLATURE

B = formation volume factor, res vol/std vol
BP = bubble-point pressure, psia
C = term defined by Eqs. 20 and 24
C_o = oil compressibility, atm⁻¹
C_r = rock compressibility, atm⁻¹, (1/φ) dφ/dP
H = thickness, cm
k = absolute permeability, darcy
k_r = relative permeability, fraction
OIP = oil in place, std cc
P = oil-phase pressure, atm
P_c = capillary pressure between oil and water, atm
PV = pore volume, cc
q = producing rate, std cc/sec
R_s = gas solubility in oil, std vol/std vol
S = saturation, fraction
S_{gc} = equilibrium gas saturation, fraction
S_{or} = residual oil saturation, fraction
S_{wc} = irreducible water saturation, fraction
t = time, sec
T = (kHk_r)/Bμ
x = coordinate direction
Y = coordinate direction
Z = depth to the top of formation from a datum elevation, cm
Δ = difference
δ = increment
Δ_{xy} = first difference along x and y directions
Δ_{xy}Δ_{xy} = second difference along x and y directions
μ = viscosity, cp
ρ = density, gm/cc
φ = porosity, fraction
Φ = potential, atm

Φ_o = potential defined by Eq. 11, atm

$\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y}$ the nabla differential operator

Subscripts

i = coordinate position on x axis
j = coordinate position on y axis
o = oil
s = standard conditions
w = water

Superscripts

n = previous time level
n+1 = new time level

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TABLE 1 - DATA USED FOR SIMULATION STUDY

Depth top productive zone, northernmost grid blocks	-8000 ft	Bubble point pres.	3800 psia
Easterly structural dip i=3,4,...,12	792 ft/mi	ρ_{os}	0.822 gm/cc
Boundaries	closed	ρ_{ws}	1.037 gm/cc
Reservoir thickness	30 ft	ρ_{gs}	9.1×10^{-5} gm/cc
Initial water/oil contact	-7985 ft	μ_w	0.395 cp
Initial S_w below WOC	1.0	Time step size	0.5 to 70.0 days
Initial S_w above WOC	0.2	Water injection rate	6000 STB/well/mo
Initial S_o above WOC	0.8	Max. liquid pdn.rate	7200 STB/well/mo
Initial S_g	0.0	Initial oil pdn.rate	3600 STB/well/mo
S_{or}	0.3	Porosity	0.15
S_{wc}	0.2	Permeability	100 md
S_{gc}	0.05	Grid block sizes:	
Initial pressure at -8000 ft	4000 psi	Δx_1	5000 ft
		Δx_2	2000 ft
		$\Delta x_i, i=3,4,...,12$	200 ft
		$\Delta y_j, \text{all } j$	200 ft

TABLE 2 - MATHEMATICAL RELATIONSHIPS USED IN SIMULATION STUDY

$$\begin{aligned}
 R_s (\text{SCF/SCF}) &= 0.2707 + 0.03147P, & P < BP \\
 R_s (\text{SCF/SCF}) &= 119.8567 & P > BP \\
 B_g &= 1.0 / (0.1936 + 0.05045P) \\
 \rho_g &= \rho_{gs} / B_g \\
 B_w &= \exp(C_w [14.7 - P]) \\
 \rho_w &= \rho_{ws} / B_w \\
 B_o &= 1.0785 + 1.005 \times 10^{-4}P, & P < BP \\
 B_o &= (B_o \text{ at BP}) \exp(C_o [BP - P]), & P > BP \\
 \rho_o &= (\rho_{os} + R_s \rho_{gs}) / B_o \\
 \mu_g &= 0.0096 + 4.44 \times 10^{-6} \text{ cp} \\
 \mu_o &= 1.523 - 2.43 \times 10^{-4}P \text{ cp}, & P < BP \\
 \mu_o &= 1.523 - 2.43 \times 10^{-4}BP \text{ cp}, & P > BP \\
 P_c^{-2} &= 1.0 - [99(1 - S_w)] / (1 - S_{wc}) \\
 k_{rw} &= (S_w - S_{wc})^3 / (1 - S_{wc})^3 \\
 k_{ro} &= (S_o - S_{or})^2 / (1 - S_{or} - S_{wc})^2 \\
 k_{rg} &= (S_g - S_{gc})^3 / (1 - S_{gc} - S_{wc} - S_{or})^3
 \end{aligned}$$

Note: In this Table the symbol P represents pressure in psia.

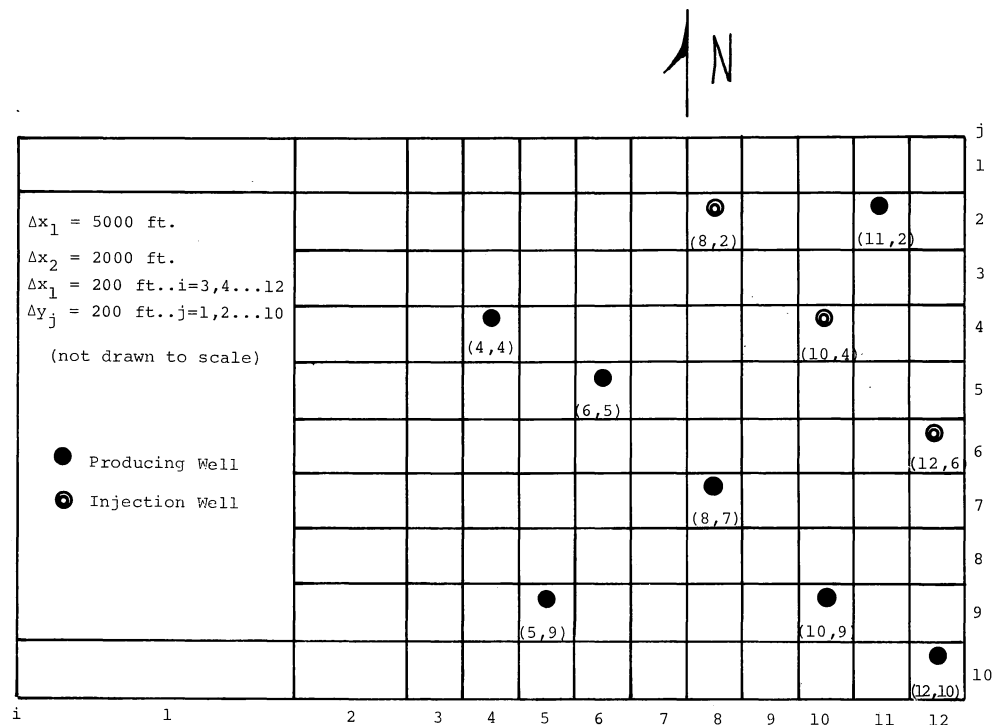


Fig. 1 - Field grid and well locations.

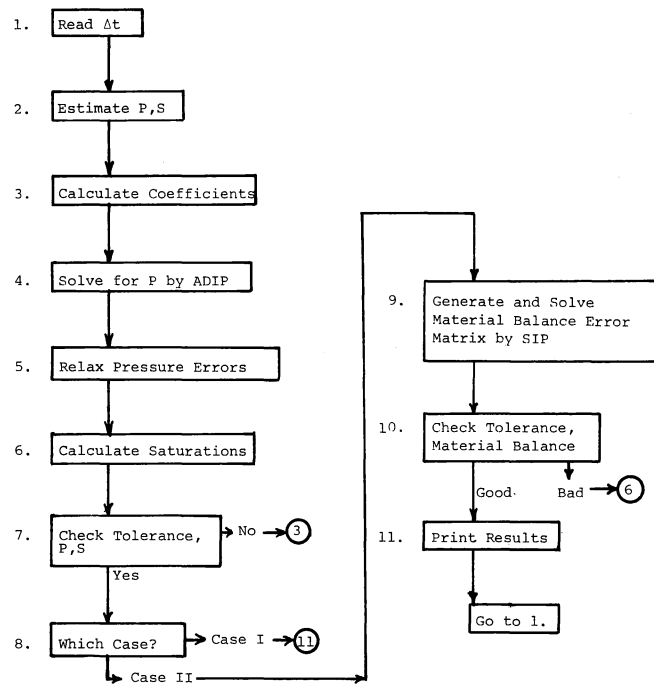


Fig. 2 - Flow chart of computation sequence.

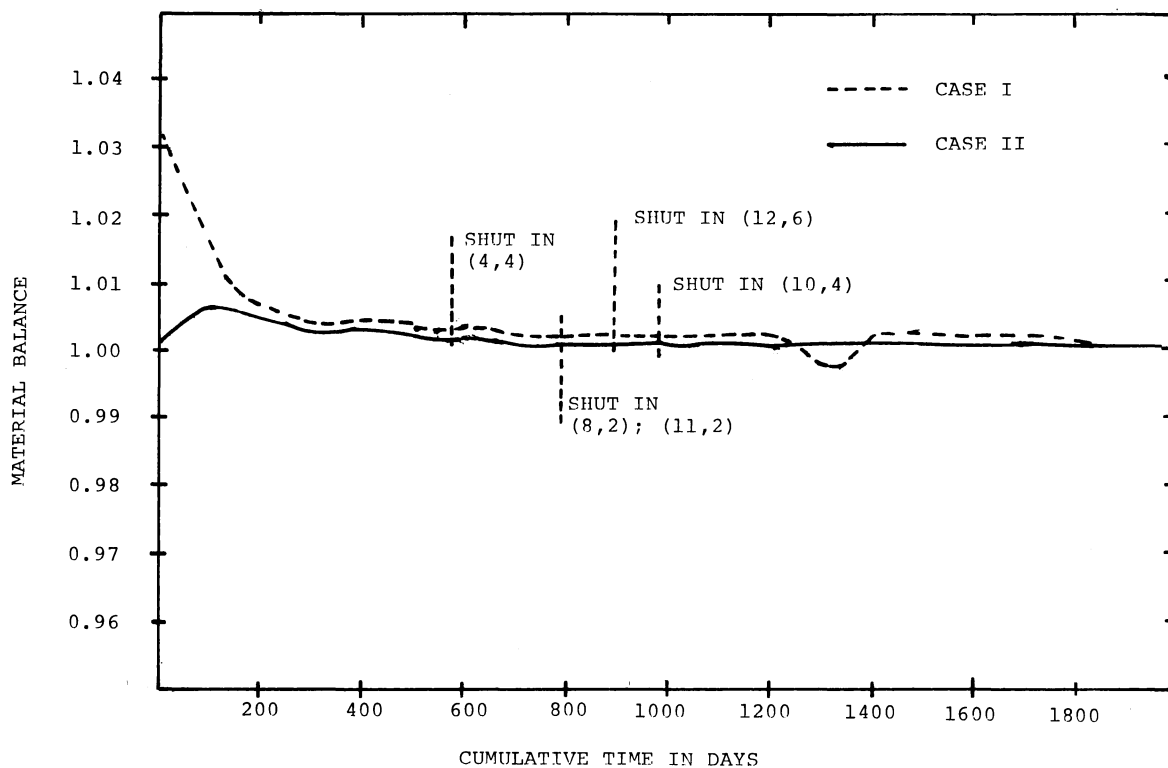


Fig. 3 - Oil material balance as a function of time.

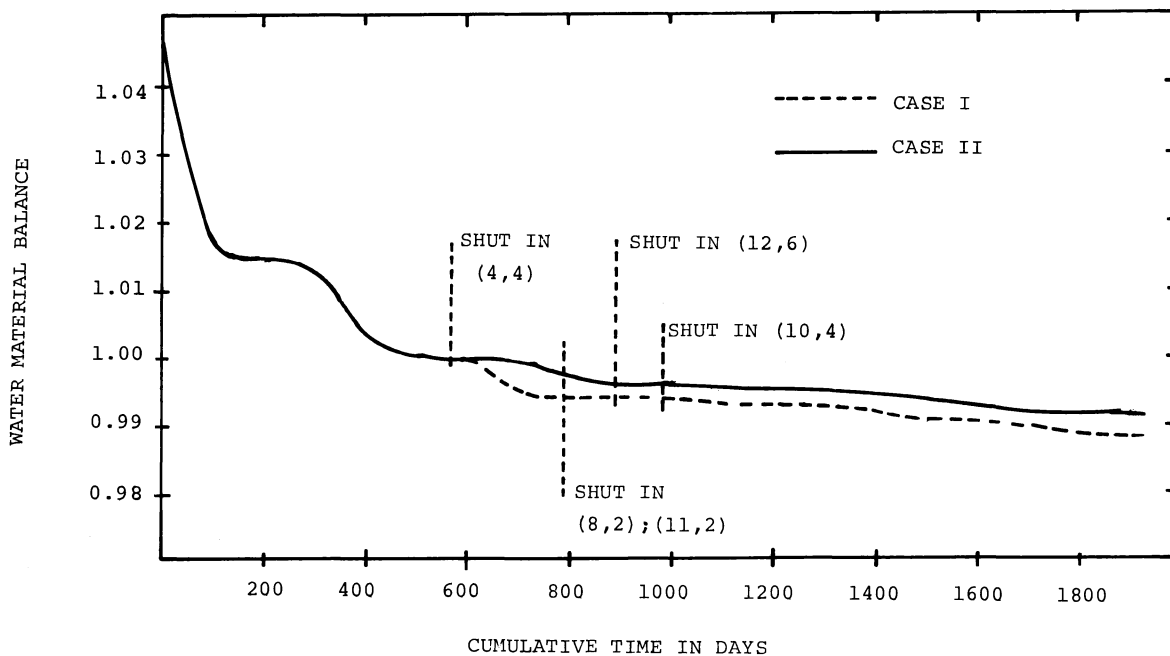


Fig. 4 - Water material balance as a function of time.

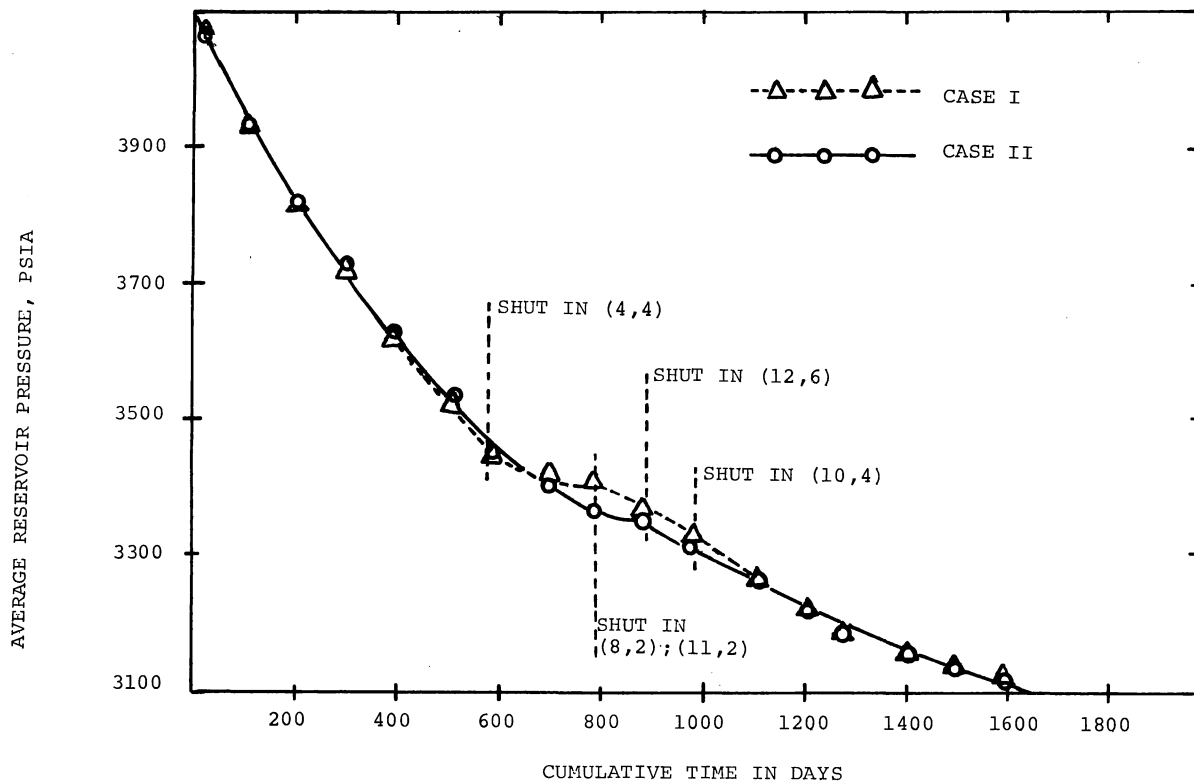


Fig. 5 - Average reservoir pressure as a function of time.

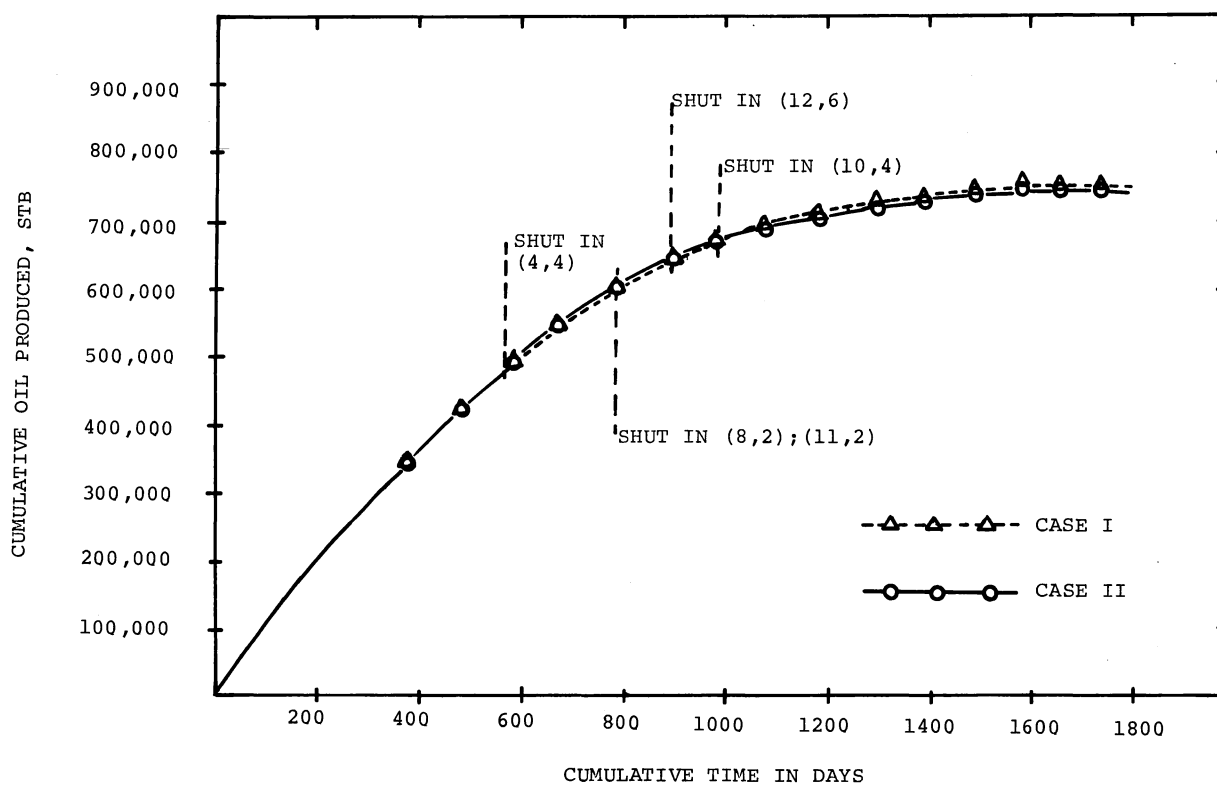


Fig. 6 - Cumulative oil produced as a function of time.

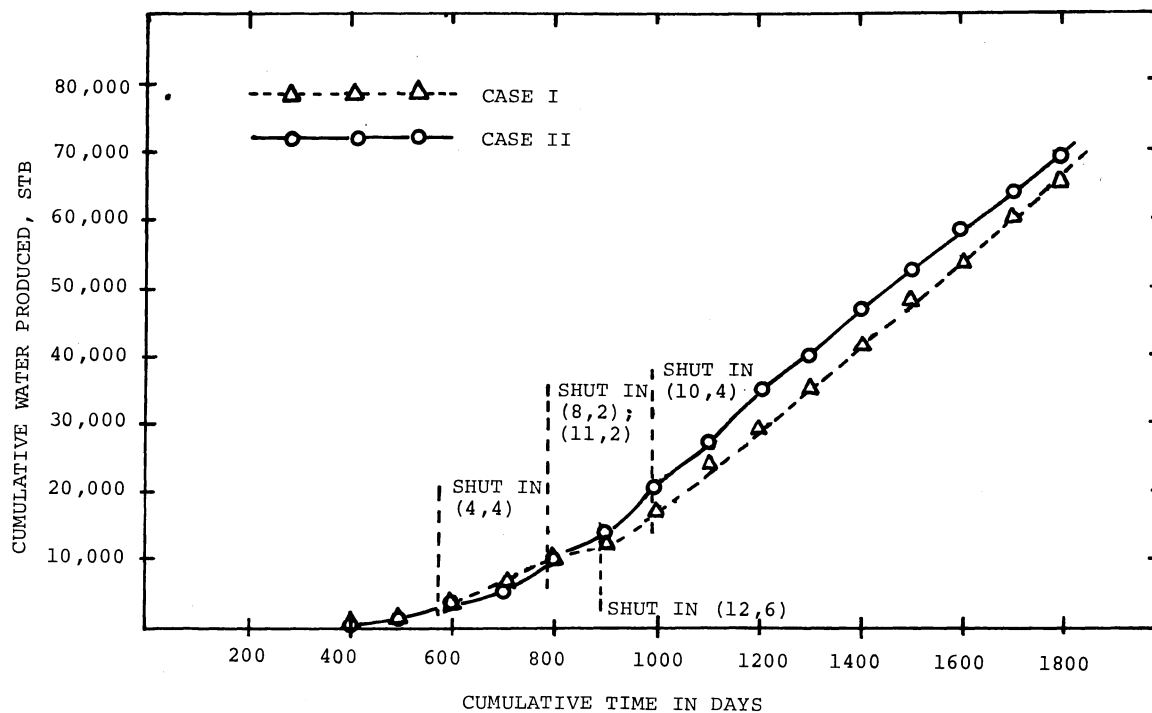


Fig. 7 - Cumulative water produced as a function of time.

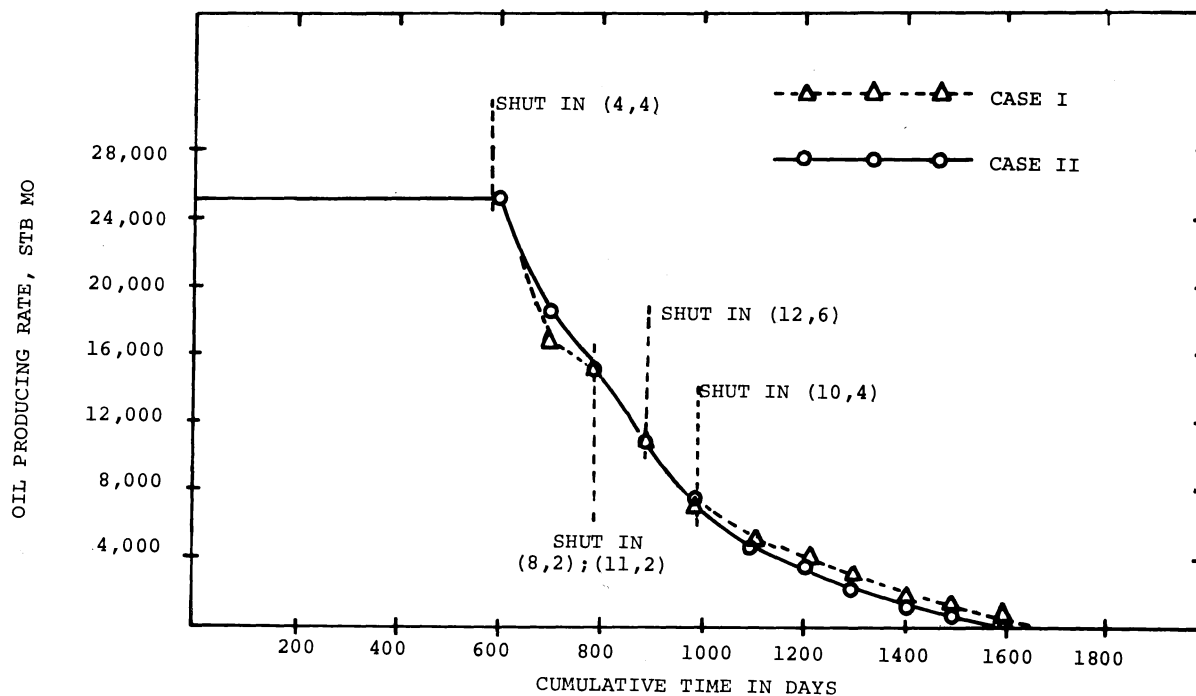


Fig. 8 - Oil producing rate as a function of time.

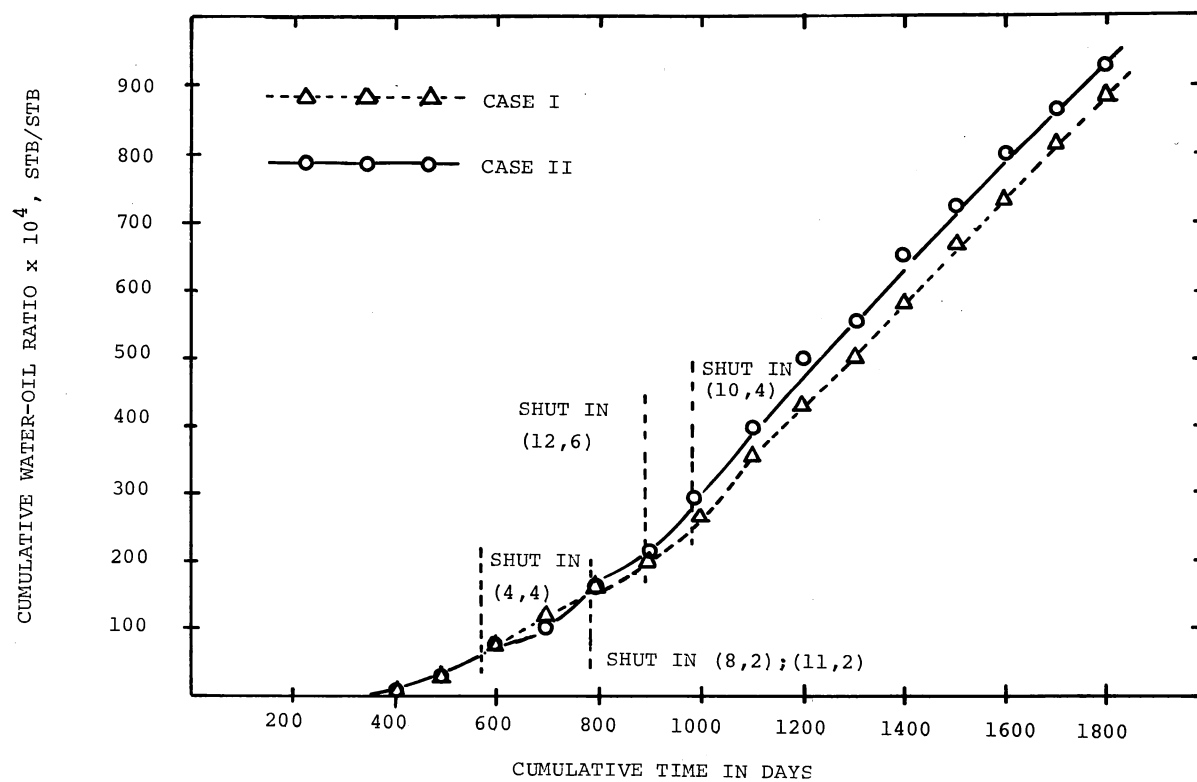


Fig. 9 - Cumulative water-oil ratio as a function of time.