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Investigation of the current distribution on an antenna in a waveguide

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INVESTIGATION OF THE CURRENT DISTRIBUTION ON AN ANTENNA IN A WAVEGUIDE

BY
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DAVID J. FREEMAN

A

THESIS

submitted to the faculty of the
SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI
in partial fulfillment of the work required for the
Degree of
MASTER OF SCIENCE, IN ELECTRICAL ENGINEERING

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1956

Approved by Gabriel G. Skitek
Professor of Electrical Engineering

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He wishes also to thank Mr. Robert Stebbins for the expert construction of the special equipment used.

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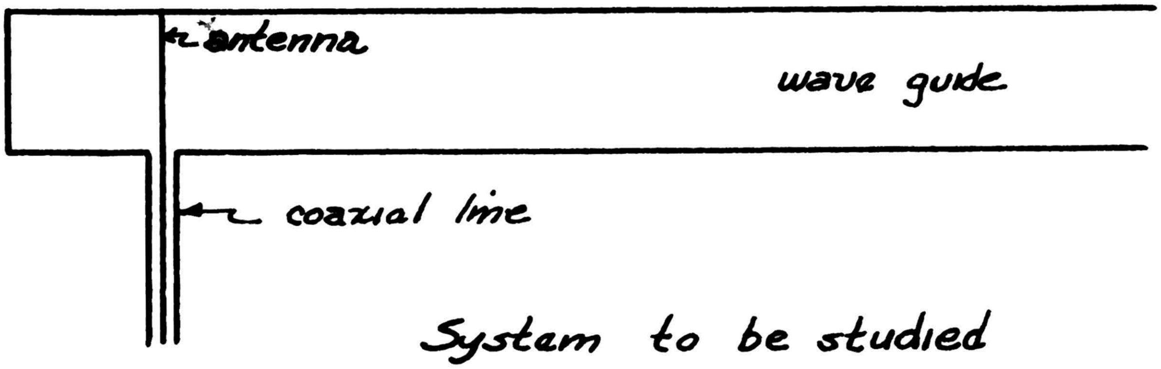
SUMMARY AND INTRODUCTION

In a communication system composed of several networks and transmission systems, it is necessary for all parts of the system to be impedance matched so that maximum energy transfer is possible.

One of the recent transmission systems which has been developed is the wave guide. With the advent of the wave guide the communication engineer is immediately faced with the problem of coupling existing systems and networks to his newly found mode of transmission.

Practically this coupling problem reduces to one of feeding radio frequency energy to the wave guide with some conventional transmission line such as a coaxial line. The modes and transmission characteristics of both the coaxial line and the wave guide have been thoroughly exploited in numerous texts.

Also found in some texts is the actual ultimate link between the two systems; the antenna or excitation source which transfers the signal from one mode of transmission to the other. It is with this link that the communication engineer must ultimately cope in order to properly terminate the coaxial line or wave guide for most efficient transfer of energy from one system to the other. It is only necessary to consider the behavior of such a link with power transfer in one direction since the law of reciprocity holds in this case. That is, a properly terminated coaxial line feeding into a wave guide is also the proper termination for wave guide feeding into the same coaxial line. The system under consideration is shown in Figure 1. To determine if the coaxial line is properly terminated so no reflection occurs, one must find the input impedance of the small antenna projecting into the wave guide. If the impedance seen by the coaxial line is not purely resistive and equal in magnitude to the characteristic impedance of the coaxial line, then there will be reflection.



System to be studied

Figure 1

As will be shown in the review of literature, many theoretical approaches to this input impedance have been made and with success. Each approach is based upon the assumption that the current is evenly distributed along the antenna situated within the wave guide. Proponents of this theory maintain that the assumption is a good one if the length of the antenna with respect to the wave length used is short. Evidently, but not undoubtedly, the theory is reasonably correct since derived formulas for input resistance give fairly accurate results.

It would therefore seem that little is left to explore on this subject. However, there are questions unanswered that are seductively inviting. How long may the antenna become before the uniform current assumption fails? How good is the assumption even if it does give nearly correct results? If the current distribution is not uniform, does it have a pattern and is this pattern practically worth using? In short -- what is the current distribution?

With this question in mind, the project for this thesis was conceived. The project then consists of building a wave guide of considerable size, such that the current distribution on the antenna may be determined.

Such a wave guide was constructed and the current distribution was measured. Figures 8 through 13 show the measured results of this investigation. The current distribution was found in this case to be nonuniform until the antenna's effective electrical length was reduced to the order of 1/8th wave length. In the cases where the current distribution was nonuniform, the current was found to be distributed in such a way as to establish the next possible mode, even though this mode could not be transmitted by the guide. This nonuniform current distribution is assumed to be the sum of a component somewhat cosinusoidally distributed and a component which is uniform, thereby exciting the first two possible modes.

REVIEW OF THE LITERATURE

A search of the available literature disclosed no investigation of this nature. Many investigations have been made to determine the distribution of the current on antennas, but none was found giving the distribution of current on an antenna situated within a wave guide.

Previous work has, however, revealed facts and methods useful in the investigation. Barzilai[1] has made an investigation of current and charge distribution along a cylindrical conductor being used as a transmitting antenna. In his investigation he used a longitudinally slotted hollow conductor for the antenna. A slug containing a magnetic field pickup loop which protruded above the outside surface of the hollow conductor, a capacitor and a crystal rectifier, all connected in series, were placed inside the antenna such that the loop could be moved along the antenna. The voltage induced in the loop was rectified by the crystal rectifier and this rectified voltage would in turn charge the capacitor. The voltage across the capacitor was used as an indication of the current distribution in the region

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(1) Barzilai, G., Experimental Determination of Current and Charge along Cylindrical Antennas, Proc IRE, Vol. 37, July 1949, Pages 825-830

where the loop was located. Mullersman[2] and Sinnamon[3] have subsequently used this method of investigation in the measurement of current distributions. In as much as the necessary measuring equipment was available, this magnetic pickup loop method was used.

Given or assumed the fact that the current actually is uniformly distributed on the antenna, many authors have arrived at the theoretical values for the input impedance to such an antenna in a wave guide.

Moullin[4,5] has derived formulea for the impedance using

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(2) Mullersman, F. H., Design of a Galvanometer to be Placed within a Tubular Antenna for the Purpose of Measuring Antenna Current Distribution, Unpublished Thesis submitted to and accepted by the Missouri School of Mines and Metallurgy, 1952.

(3) Sinnamon, G. F., An Experimental Study of the Effect Upon the Current Distribution and Radiation Pattern of Crossbars on Yagi Receiving Antennas, Unpublished Thesis submitted to and accepted by the Missouri School of Mines and Metallurgy, 1952.

(4) Moullin, E. B., Screening Properties of a Squirrel Cage of Wires, Journal of the Institution of Electrical Engineers, Vol. 91, Part III, March 1944,Pages 14-22

(5) Moullin, E. B., The Propagation of Electric Waves in a Waveguide; Journal of the Institution of Electrical Engineers, Vol. 92, Part III, March 1945, Pages 8-17

the follwoing general idea: An infinite lattice of current filaments equally spaced and alternatly phased as shown in Figure 2 is assumed. The current filaments have a uniform current distribution in the "z" or axial direction. These currents vary with time sinusoidally. It will be noted that half way between any of the current filaments there exists a plane perpendicular to the paper in which the electric field intensity is zero at all times. A conducting sheet such as a wave guide wall may therefore be inserted without disturbing the field distributions. Top and bottom walls may be inserted at will in the plane of the paper since the electric field is at all points perpendicular to the plane of the paper. Hence, if the field around a conductor or current filament "a" can be found then Poynting's vector may be integrated over the surface enclosing the current filament "a", giving the power radiated. The input impedance of the antenna "a" may then be found by properly

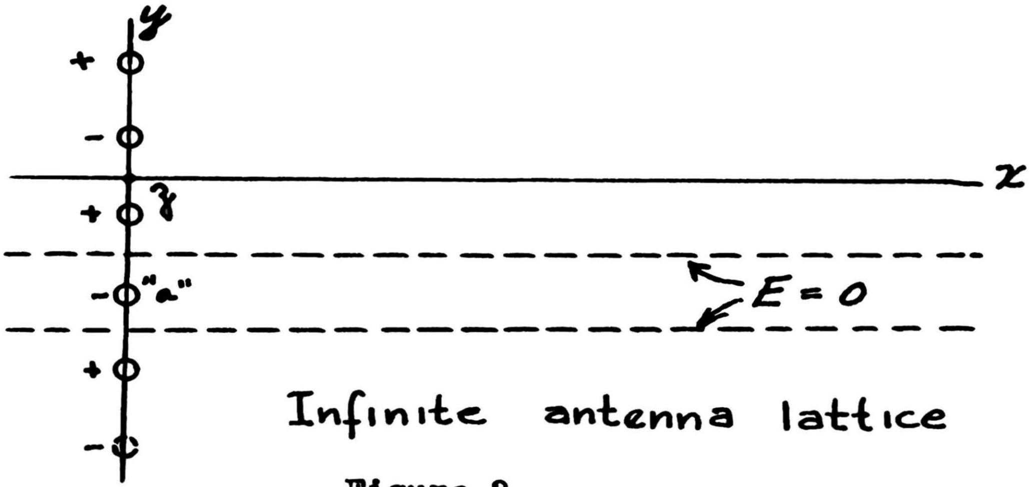

Figure 2

relating the power eminating from the antenna to the current feeding the antenna. This is done with a great flourish of mathematics. All of the results are, nevertheless, contingent upon the assumption that the current filament has uniform current distribution in the "z" direction.

Inasmuch as Moullin's method of the infinite lattice is inherently a method of images, one would expect to find methods which consider first the antenna in the guide and then replace the guide walls by the images caused by the walls. Such a method is used by Schelkunoff[6]. There are numerous other authors who use this basic approach. The current distribution is, however, always assumed to be uniform, and in general all the results involve the evaluation of an infinite series, which is, depending upon certain parameters, often so slow to converge as to be practically impossible to evaluate.

In as much as the apparent reactance of the antenna may be tuned out, it would seem that the resistance of the antenna would be the important thing to study. This is done by Wheller[7]. Wheller points out that the electromagnetic field in front of an infinite flat array of antennas can be sub-

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(6) Schelkunoff, S. A., Impedance of a Traverse Wire in a Wave Guide, Quarterly of Applied Mathematics, Vol. 1, Number 1, April 1943, Pages 78-85

divided into wave channels, each including one of the antennas. Each channel behaves like a hypothetical wave guide similar to a transmission line made of two conductors in the form of parallel strips. A simple derivation then leads to the radiation resistance of each antenna and to some limitations on the antenna spacing. In the usual flat array of half-wave dipoles, each alloted a half wave square area, and backed by a plane reflector at a quarter-wave distance, the radiation resistance of each dipole is 153 ohms. In a finite array, this derivation is a fair approximation for all antennas except those too close to the edge. This derivation also verified the known formula for the directive gain of a large flat array in terms of its area. The same viewpoint leads to the radiation resistance of an antenna in a rectangular wave guide, which has previously been derived by more complicated methods. The details of this method, as usual, disclose that the assumption of uniform current distribution is even made for dipoles.

One common factor in the approaches found was that the investigator used some artificial idea which is then recognized to yield fields which are found in wave guides. An alternate method offered by Shelkunoff[8] is the nearest

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(7) Wheller, H. A., The Radiation Resistance of an Antenna in an Infinite Array or Waveguide, Proc. IRE, Vol. 36, Number 4, April 1948, Pages 478-87.

thing found to a straight forward approach.

Following is another straight forward approach to the problem using the idea of conservation of energy to compute the theoretical input resistance of an antenna in a wave guide.

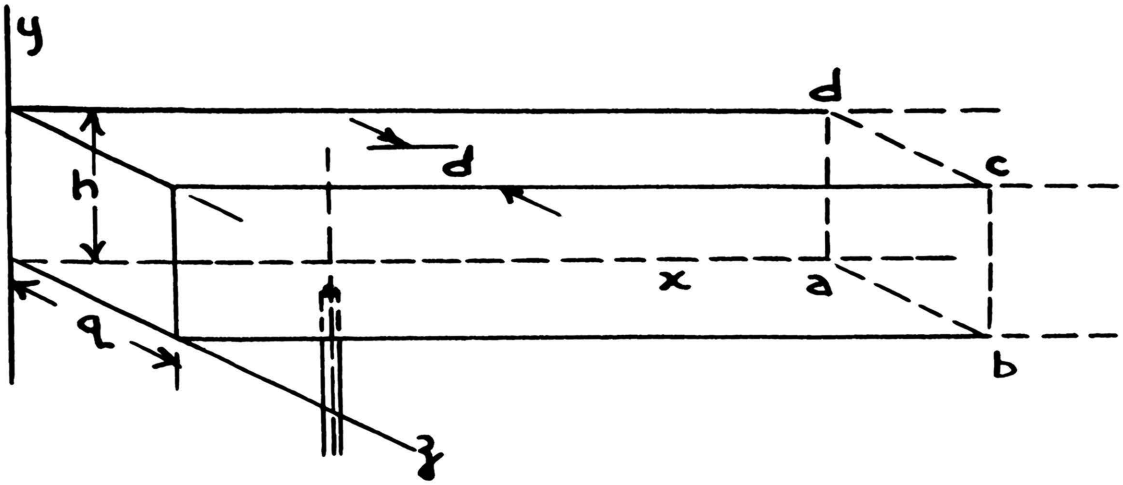

Figure 3

If the guide is assumed lossless as is common, and the reflecting wall at the end is placed 1/4th guide wavelength from the antenna, all of the power radiated from the antenna must pass the cross section abcd located an integral number of wavelengths down the guide, which is shown in Figure 3.

For the pricipal mode of transmission, the fields at abcd are common text book knowledge. Integrating Poynting's

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(8) Schelkunoff, S. A., Impedance of a Traverse wire in a Wave Guide, Quarterly of Applied Mathematics, Vol. 1, Number 1, April 1943, Pages 78-85

vector over the area abcd gives the power flowing at abcd as:

$$P_{avg} = \frac{hq\beta}{2\omega\mu}\hat{E}_y^2$$

where:

h = Refer to Figure 3

q = refer to Figure 3

ω = angular velocity in radians/second

μ = permeability of the medium in the guide space

β = propagation constant for principal mode TE_{01}

$\hat{E}_y$ = Maximum rms electric field intensity in time and space

If $\hat{E}_y$ exists at abcd, it must also exist at some time at the center of the guide where the antenna is located. If the antenna is located d distance from the side of the guide it must be in a place where the maximum rms electric field intensity is:

$$E_y = \hat{E}_y \sin\left[\frac{\pi d}{q}\right]$$

since the electric field is sinusoidally distributed in the z direction. Now the voltage drop on the antenna, assuming <u>uniform current distribution</u> and neglecting the conductivity of the antenna material, will be:

$$V = hE_y = IR_{in}$$

or:

$$I = \frac{hE_y}{R_{in}}$$

but:

$$P_{avg} = I^2 R_{in} = \frac{hq\beta\; E_y^2}{2\omega\mu}$$

substituting:

$$R_{in} = \frac{hq\beta\; E_y^2}{2\,\omega\mu\left[\frac{h E_y}{R_{in}}\right]^2}$$

$$R_{in} = \frac{2\mu\omega\; h^2\, \hat{E}_y^2\, \sin^2\left[\frac{\pi d}{q}\right]}{hq\beta\; \hat{E}_y^2}$$

$$R_{in} = \frac{2\mu\omega h}{\beta q} \sin^2\left[\frac{\pi d}{q}\right]$$

This result agrees with results given by other developements, and seems simple and straight forward.

Disregarding the variations of approach, one thing is common to all: The current distribution is assumed linear. It is therefore, proposed that this assumption be directly verified or disproved by experimental investigation.

DESIGN OF EXPERIMENTAL EQUIPMENT

The controlling factor in the design of this equipment was the physical size of the pickup loop. Since this portion of the equipment was already constructed, the remaining equipment was designed around it. The diameter of the existing pick up loop and slug assembly was approximately 1/2 inch. If the antenna was to be relatively slender, simulating a current filament, then the antenna which contained the slug must be in the order of 12 or more inches long. Twelve inches was chosen.

Having chosen the antenna length, the height of the guide was, therefore, fixed at 12 inches. The principal or TE_{01} mode was chosen by industry in general as the easiest mode which could be used in the wave guide because of the simplicity of the equations describing this mode, and the simplicity of the antenna required to excite this mode. Obviously, the general use of this mode gave rise to the previous theoretical investigations and uniform current distribution assumptions. To make experimental investigation of the assumption, the wave guide was therefore designed for this mode.

With the TE_{01} mode, the electric field is uniformly distributed in the vertical direction. This agrees with the direction of the electric field produced by a vertical antenna. Due to the boundary conditions imposed upon this mode by the wave guide walls, there is a certain frequency below which no energy will be transmitted. This so called cut-off frequency for the TE_{01} mode is a function of the wave guide width only as may be seen from the following equation: (Refer to Figure 3.)

$$F_{cutoff} = \frac{1}{2\sqrt{ue}\ Q} \text{ ------------------------------(1)}$$

For air, $u = 1.257x10^{-6}$, Q = meters, and $e = 8.854x10^{-12}$.

Also, it may be seen from the following equation, that the wave length within the guide approaches infinity as the wave length of the excitation approaches the cutoff wave length, λ_0:

$$\text{Wave length in guide} = \frac{\text{(Free space wave length of source)}}{\left(1-\left[\frac{\text{Free space wave length of source}}{\text{Wave length of cutoff frequency}}\right]^2\right)^{1/2}} \text{ --(2)}$$

From equation (2) it can be seen that it is possible to change the wave length in the wave guide radically by small variation of the excitation wave length. Thus as the excitation wave length is increased toward cutoff, the wave lengths within the wave guide become increasingly longer with respect to the physical length of the antenna. Or it could be said that the antenna seems to become sucessively shorter electrically. Thus by proper choice of excitation wave length and the cutoff wave length, it is possible to simulate shortening and lengthening of the antenna. By employing this procedure, the shortness of the antenna required to produce uniform current distribution may be discovered.

The excitation source available consisted of a UHF transmitter capable of delivering frequencies from approximately 350 megacycles to 600 megacycles.

The wave guide width was therefore chosen to give a cutoff frequency just below the minimum frequency of the excitation source, thereby allowing the maximum possible variation of effective antenna length.

A cutoff frequency of 325 mc was chosen and the width of the wave guide was calculated from equation (1).

The length of the wave guide was chosen to be as long as practical. The longest possible wave guide was desired, but physical housing limitations and available material limitations resulted in choosing a length of approximately three wave lengths at a frequency of 500 mc.

The lack of length was compensated by providing flaps on the open end of the wave guide to match the wave guide to free space, thereby reducing to zero the reflections caused by this discontinuity. This method of matching proved to be insufficient at certain frequencies, and the problem was solved by experimental placement of wooden blocks in the open end of the guide. Depending upon their placement, these blocks offered capacative or inductive reactance or resistance and effectively matched the wave guide to free space.

In order to determine whether or not there was reflection from this discontinuity, a slot was left in the wave guide into which a sliding probe could be inserted to determine the magnitude of the standing waves caused by reflection. This slot was located in a position so as not to interupt current paths in the wave guide wall and thereby cause little or no disturbance of the fields within the guide. Using equation (2) the length of the slot was chosen to make possible the measurement of standing waves for all frequencies to be used. The slot was located as far as possible from the antenna and the open end of the wave guide.

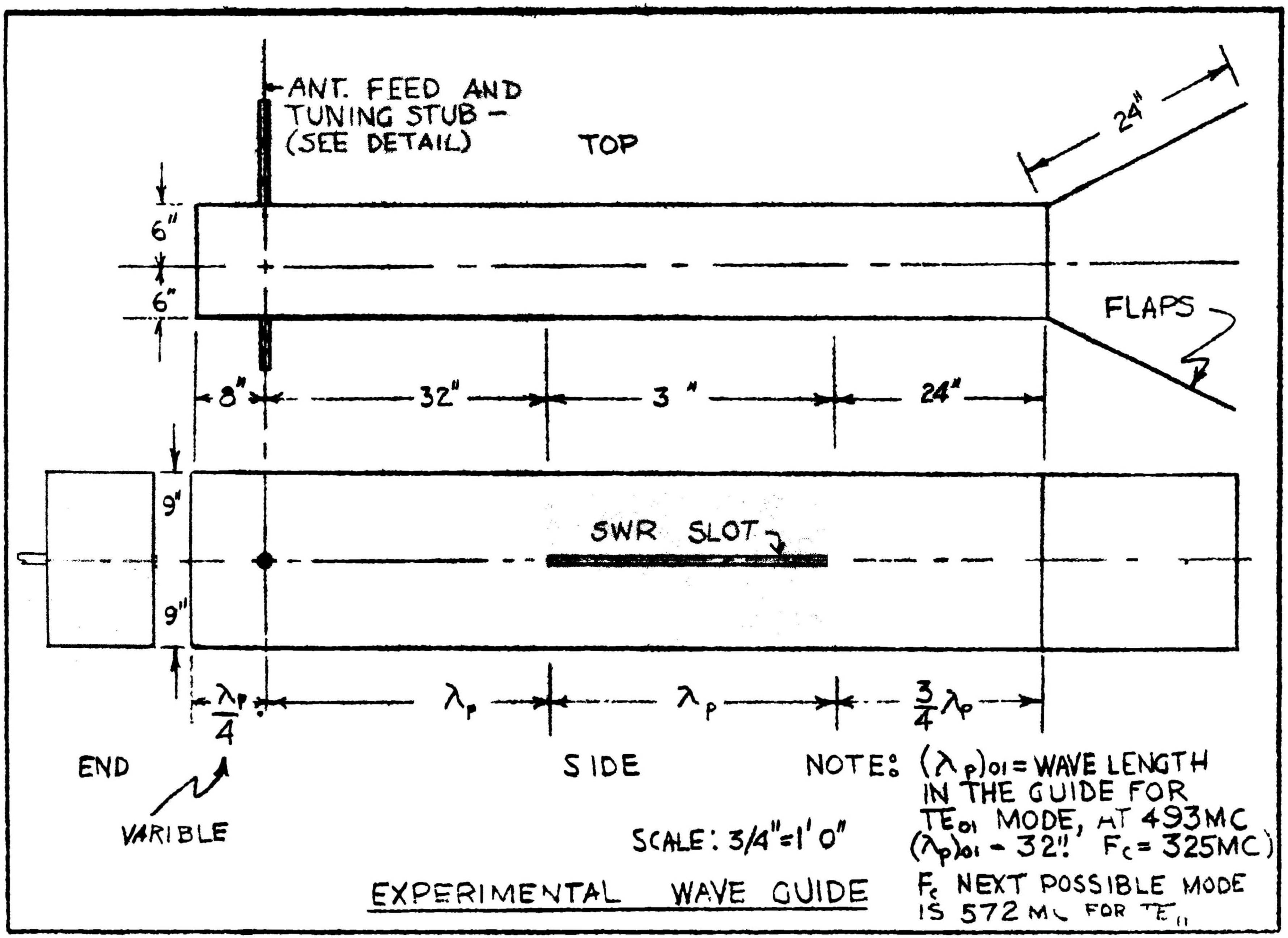
Figures 4 through 6 show descriptive drawings and photographs of the wave guide.

The closed end of the wave guide was constructed as a variable element so that it could always be placed 1/4th guide wave length from the antenna. This variation is necessary because the relative position of the reflecting wall changes the impedance of the antenna, and could possibly affect the antenna current distribution.

The magnetic pickup loop and slotted antenna were already designed. A detailed discussion of their capabilities may be found in the discussion of errors.

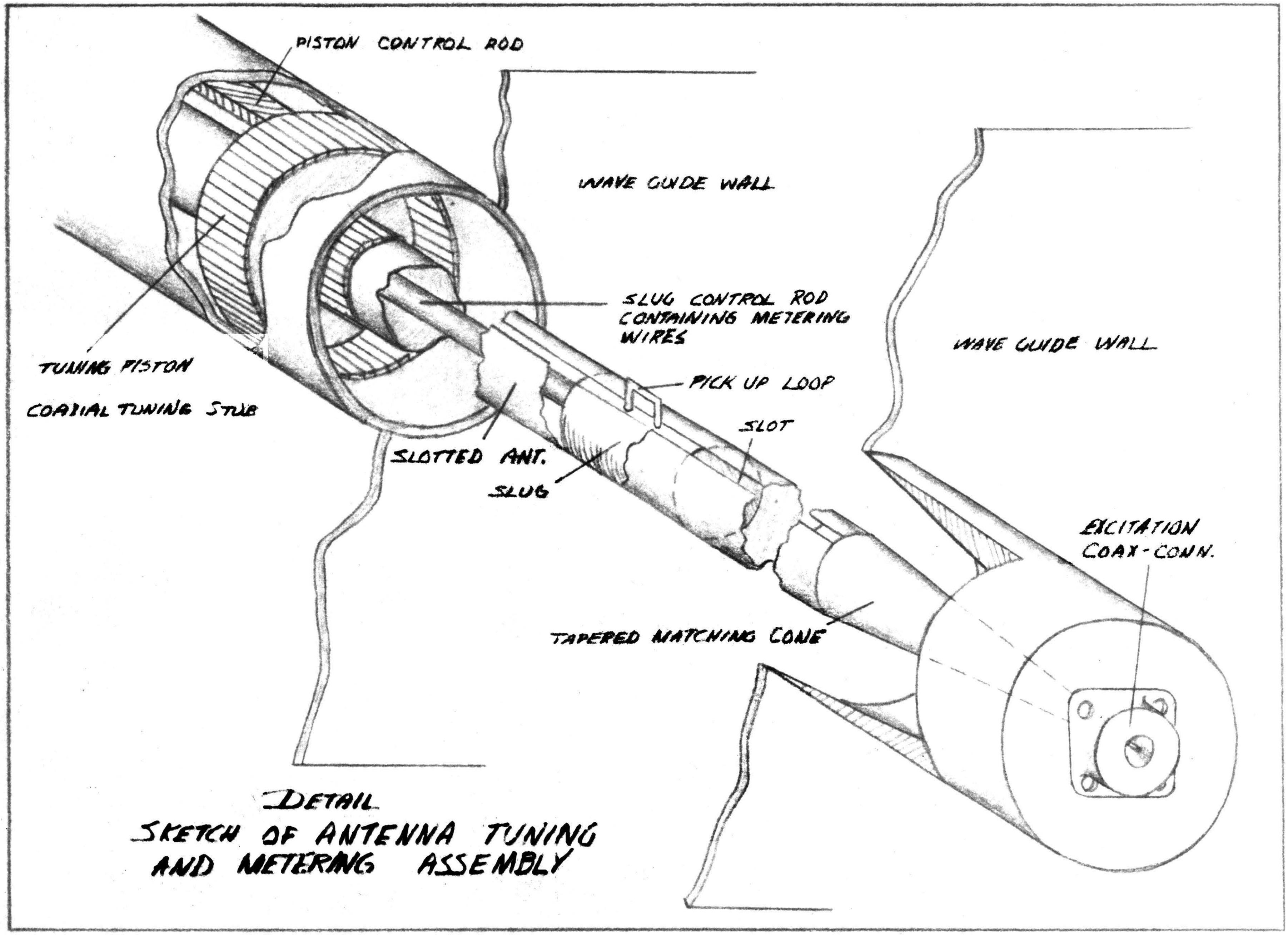
The tapered matching section located at the feed end of the antenna was desirable in order that the maximum energy could be transfered to the antenna. Referring to

FIGURE 4a



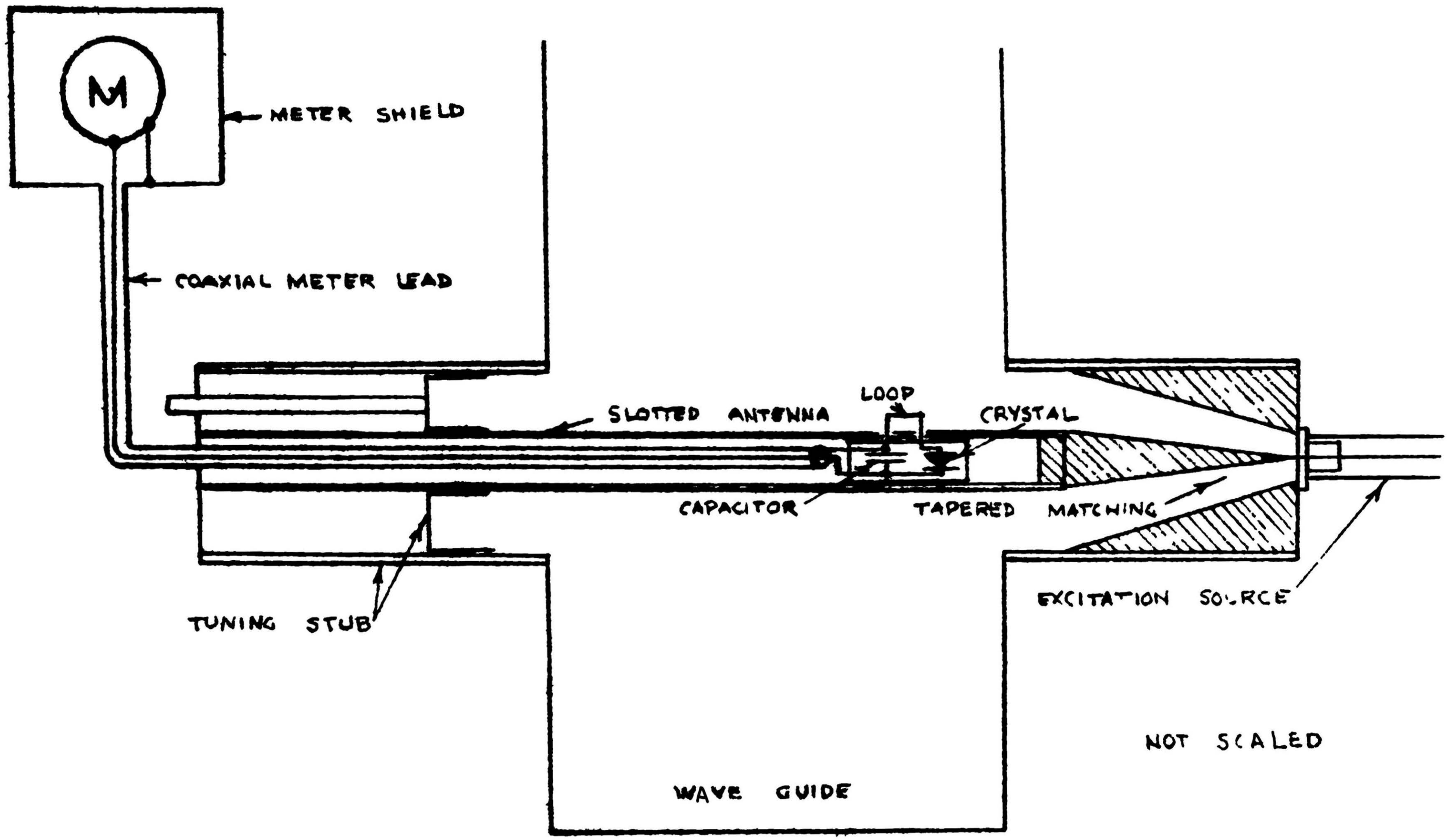
EXPERIMENTAL WAVE GUIDE

Figure 4b



DETAIL
SKETCH OF ANTENNA TUNING
AND METERING ASSEMBLY

Figure 4c



DETAIL OF ANTENNA SHOWING ELECTRICAL CONNECTIONS & SHIELDING

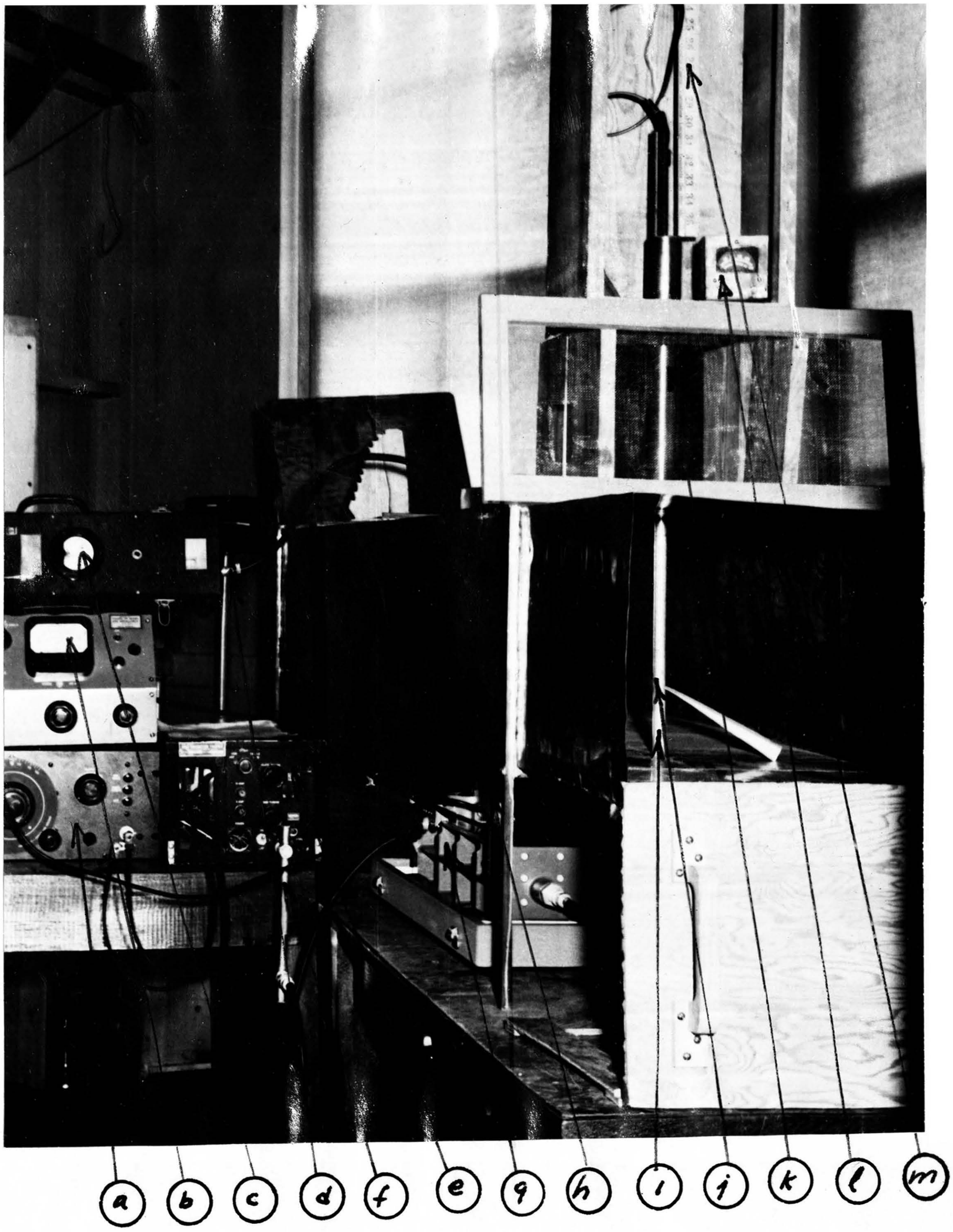


Figure 5

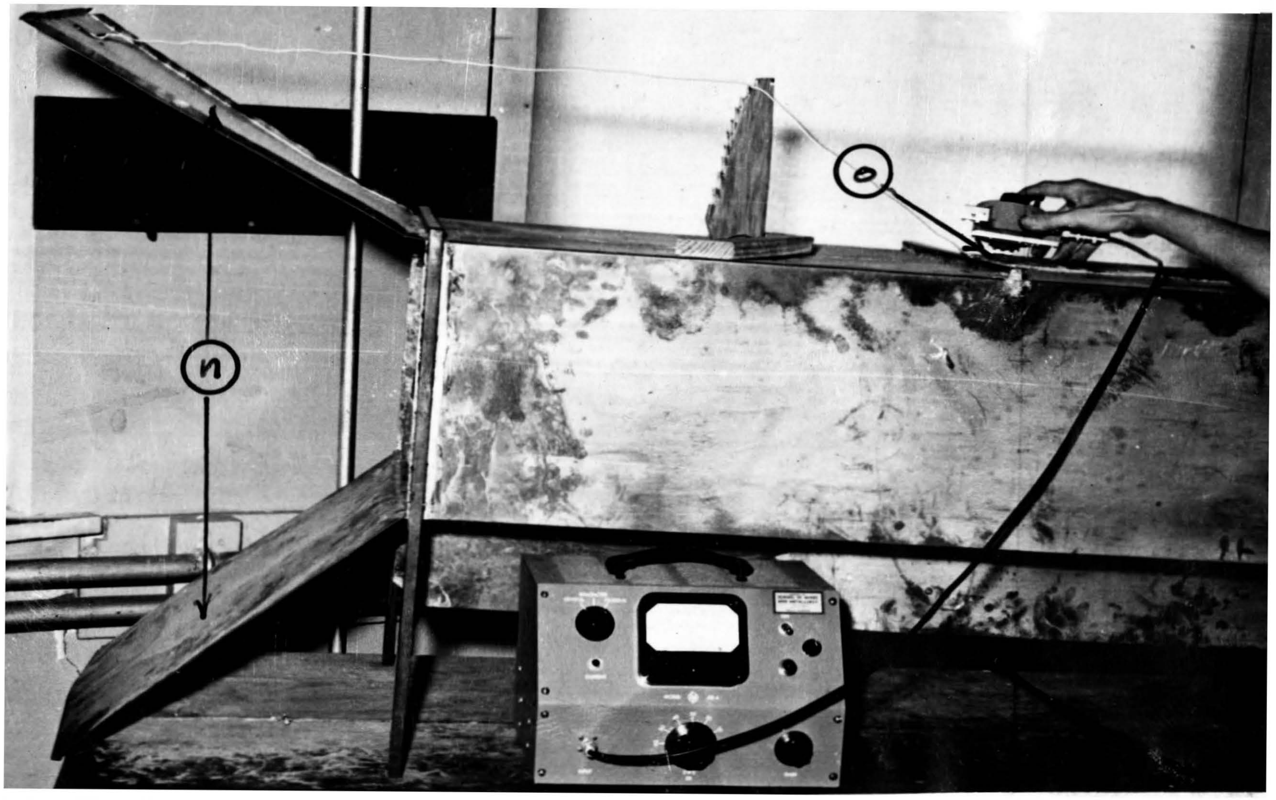


Figure 6

Open End of Wave Guide

A. 1000 cps modulator

B. Standing Wave Meter

C. uhf frequency meter

D. Freq. meter probe

E. U.H.F. Transmitter

F. Low pass filter

G. Slotted line

H. SWR meter probe

I. Slotted antenna

J. Pick up loop

K. Tuning stub

L. Meter for pick up loop

M. Position indicator

N. Matching flap

O. Slot in wave guide

Figure 4c, it will be noted that it was necessary to connect standard RG8U coaxial feed line to the larger coaxial line which feeds the antenna. The design of this matching section was not critical since any reflection at this point could not affect the current distribution on the antenna. Since the design of this taper was not critical, a random taper was chosen. It was found after construction that the connection was virtually reflectionless.

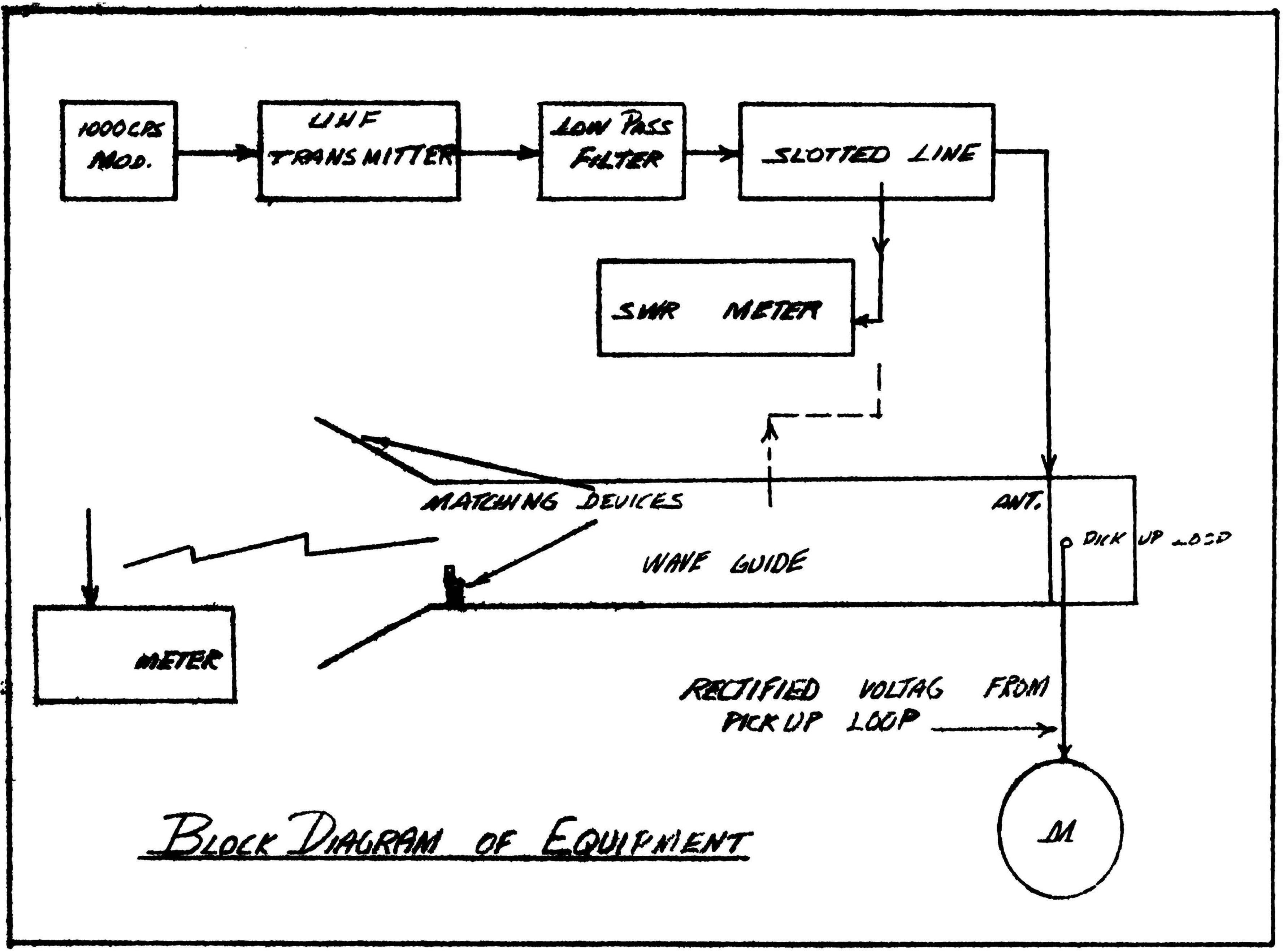
The antenna was terminated in a short circuited stub made of oversize coaxial line. The short was provided by means of a movable piston. Since, depending on the length of this stub, or in other words the distance from the wave guide to the movable piston, the terminating reactance could be made capacitive or inductive, it was thought that this stub could be used to tune out any reactance which the antenna might have.

The shielding design of the metering equipment is shown in Figure 4c. A detailed discussion of its merits may be found in the discussion of errors.

A block diagram of the complete experimental set up is shown in Figure 7.

The equipment functioned as follows: The UHF generator was modulated with a 1000 cps source. The resulting modulated signal was fed through a low pass filter whose cut off frequency was 700 mc. The purpose of this filter may be found in discussion of errors. From the low pass

Figure 7



BLOCK DIAGRAM OF EQUIPMENT

filter the signal progressed through a slotted line of 50 ohms characteristic impedance to the antenna in the wave guide. The probe was moved along the slotted line to determine the standing wave ratio caused by the mismatch between the feed line and the antenna and thereby also determine the input impedance to the antenna. It should be noted that this input impedance, however, was merely a side issue of the investigation. This also served as a rough check on operating frequency. After reachin the antenna the signal was then radiated along the wave guide and out into the atmosphere. Matching of the wave guide to the free space was achieved by use of the adjustable flaps and placement of wooden blocks in the guide. The achievement of matching was determined by the use of the same SWR meter but with the probe in the slot in the side of the wave guide. A cavity resonator type UHF frequency meter was used at the end of the guide to accurately determine the operating frequency.

The pick up loop crystal and meter were calibrated and this calibration data and curves may be found in the appendix "A". A discussion of the calibraion may be found in the discussion of errors.

EXPERIMENTAL PROCEDURE

The equipment was set up as shown in Figure 7 and a number of runs were made at different frequencies as follows:

1. Frequency was calculated which would give the desired wave length in the guide.

2. The reflecting wall of the wave guide was then placed one quarter of this wave length from the antenna.

3. The termination of the wave guide in free space was made reflectionless by use of the matching flaps and wooden blocks to absorb power. The guide SWR was thereby reduced to a minimum. This prevented a mismatch between guide and free space from causing any sort of impedance transformation along the guide.

4. The tuning stub was shortened to a minimum length of one inch.

5. Measurement of the SWR on the slotted line was made and the distance to the first voltage minimum point was measured so the input impedance of the antenna could be calculated.

6. The current distribution was measured.

7. The tuning stub was then placed so as to give a voltage minimum at 1/4 wave length from the antenna and steps 5 and 6 were repeated.

8. The tuning stub was then placed so as to give a voltage minimum at 1/2 wave length from the antenna and steps 5 and 6 were repeated. Steps 7 and 8 were later abandoned.

RESULTS AND CONCLUSIONS

The data recorded may be found in appendix A. Figures 8 through 13 are plots of these data. Since at different frequencies the degree of mismatch between the antenna and feed line causes a different input current, these data are plotted as normalized to the current at position 0, and also position 1, as will be explained.

When the pickup loop is in position "o", its center line coincides with the wave guide wall. This leaves one half of the pick up loop in the feed coaxial line and the other half in the wave guide.

Readings taken in this position are certainly questionable. At this point the fields undergo rapid change with respect to distance, and the pick up loop is no longer of negligible size. The pick up loop in this position therefore not only disturbs the fields, but probably reads incorrectly because field strengths vary rapidly within the pick up loop's area.

There is also the problem of deciding exactly where the junction between feed line and antenna should be considered to be located. It is well known that the junction is almost never located electrically where it is shown physically in a drawing. This difference can be quite large when working with frequencies of 400 megacycles.

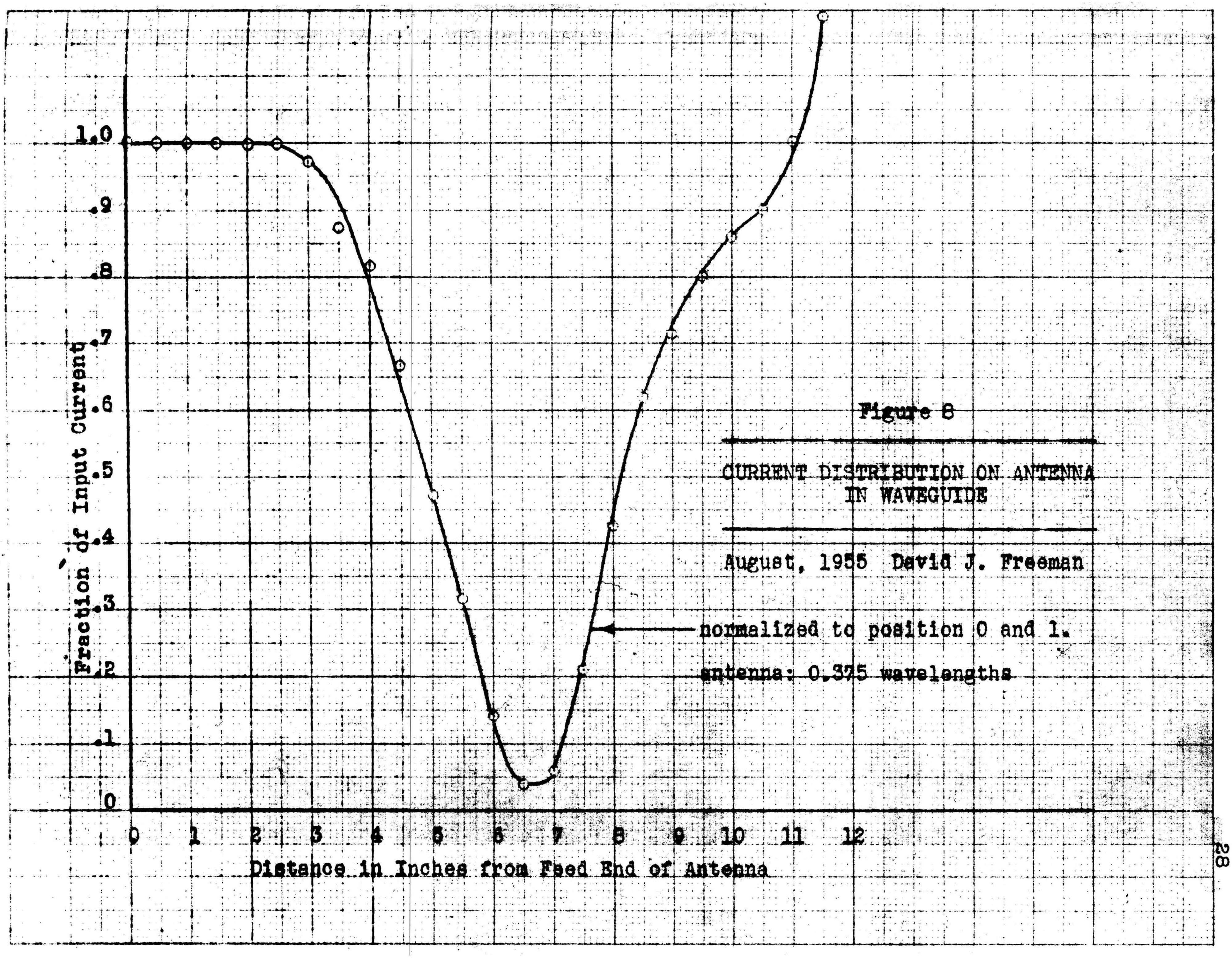


Figure 8

CURRENT DISTRIBUTION ON ANTENNA IN WAVEGUIDE

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normalized to position 0 and 1.

antenna: 0.375 wavelengths

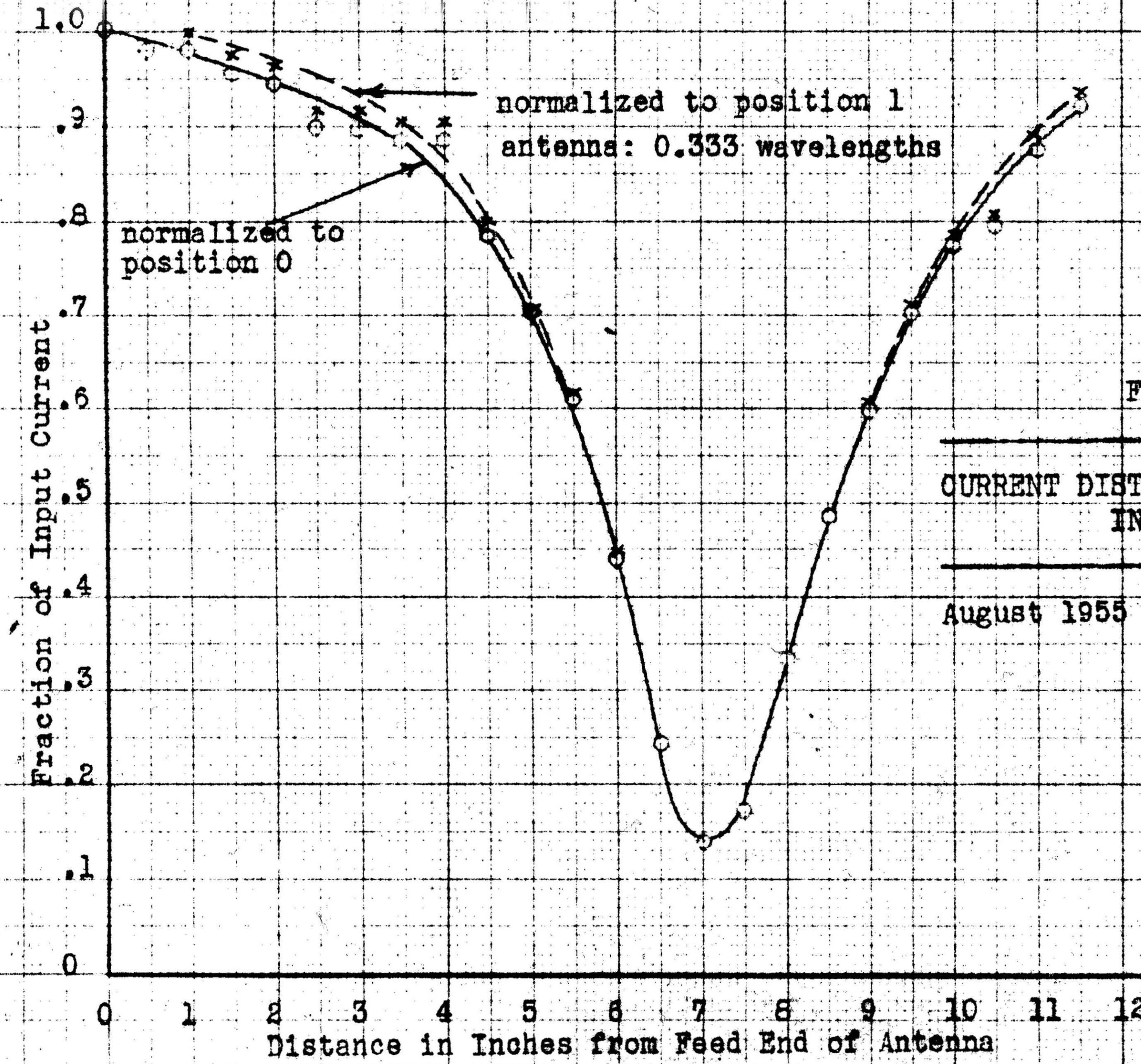

Figure 9

CURRENT DISTRIBUTION ON ANTENNA IN WAVEGUIDE

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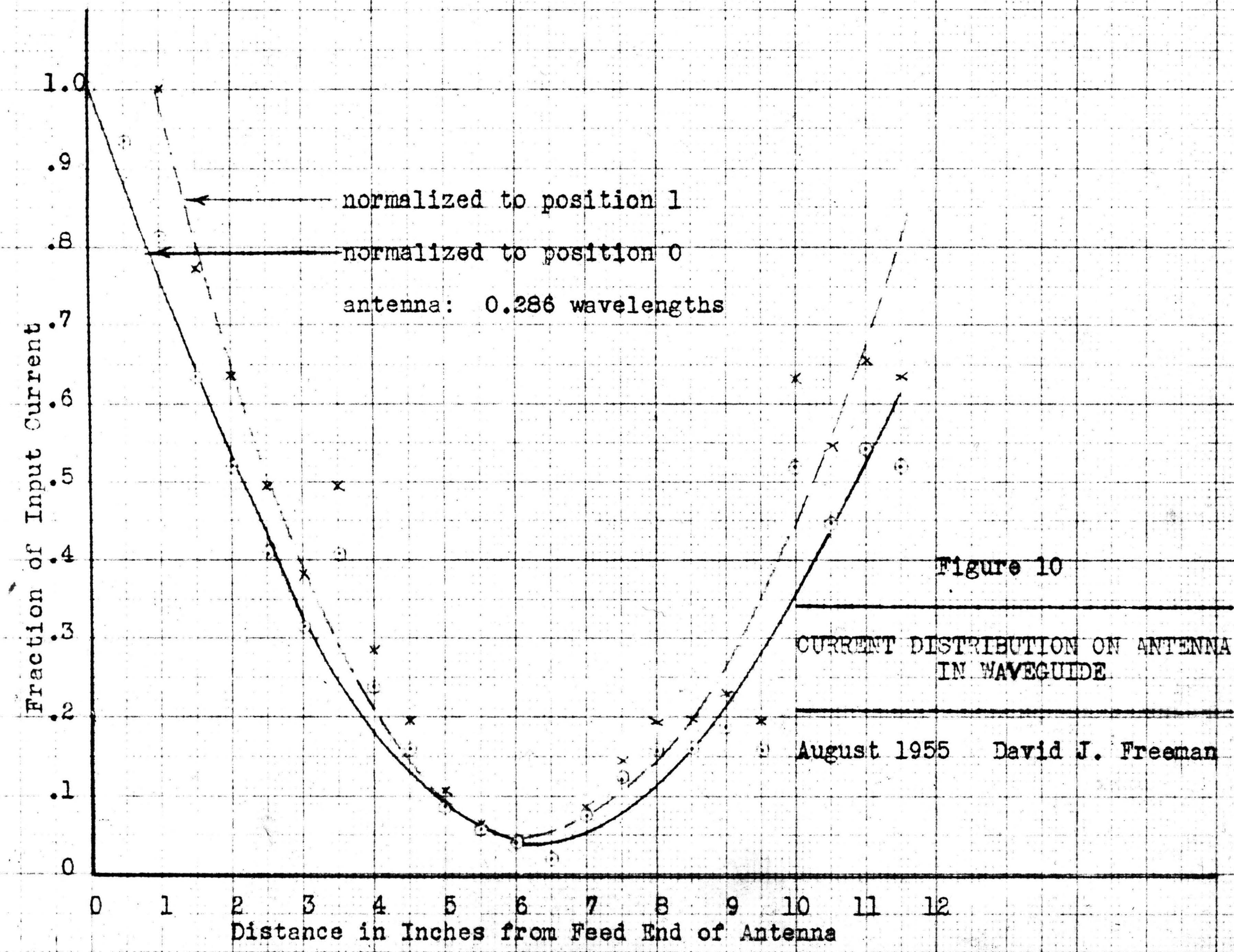

Figure 10

CURRENT DISTRIBUTION ON ANTENNA
IN WAVEGUIDE

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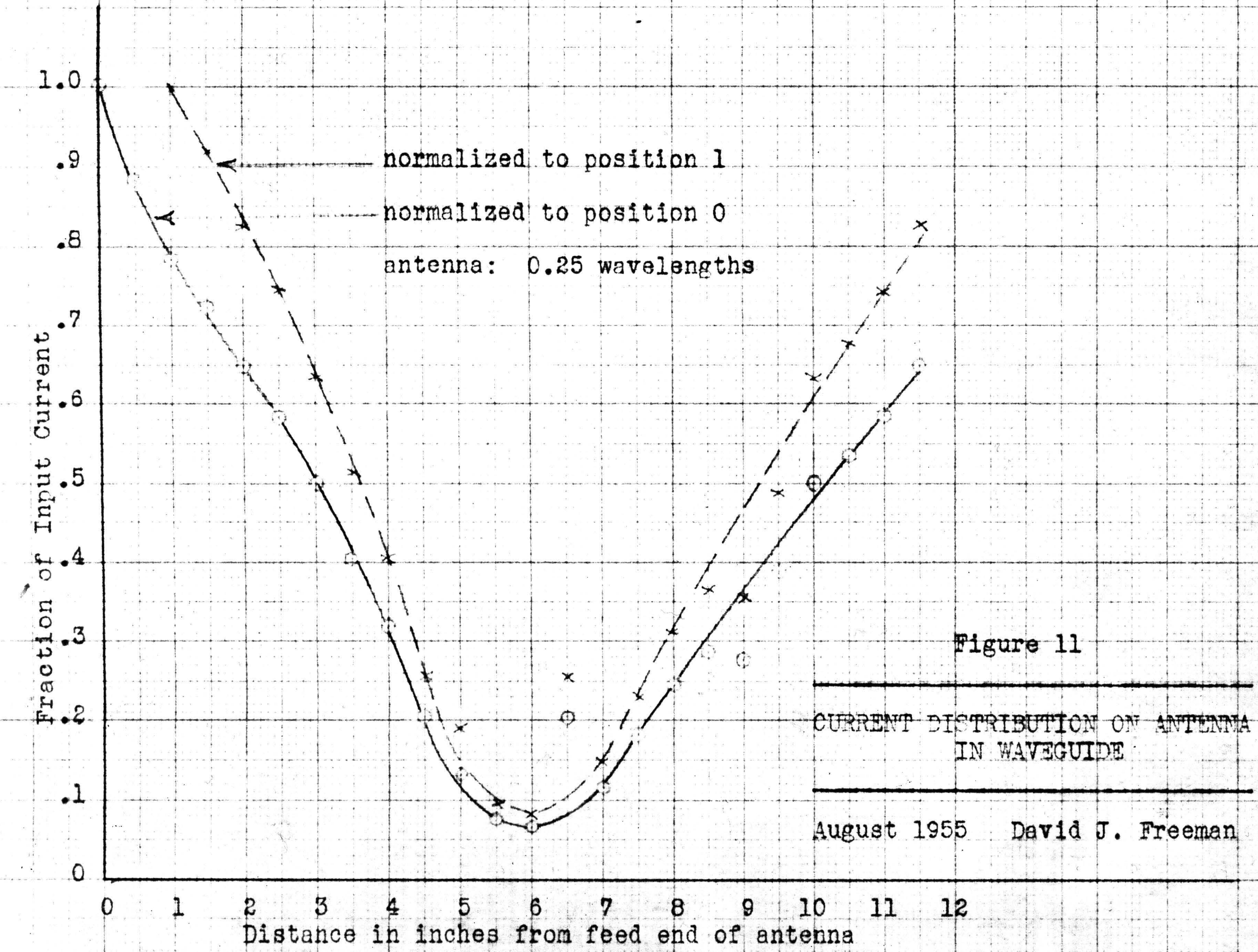

Figure 11

CURRENT DISTRIBUTION ON ANTENNA IN WAVEGUIDE

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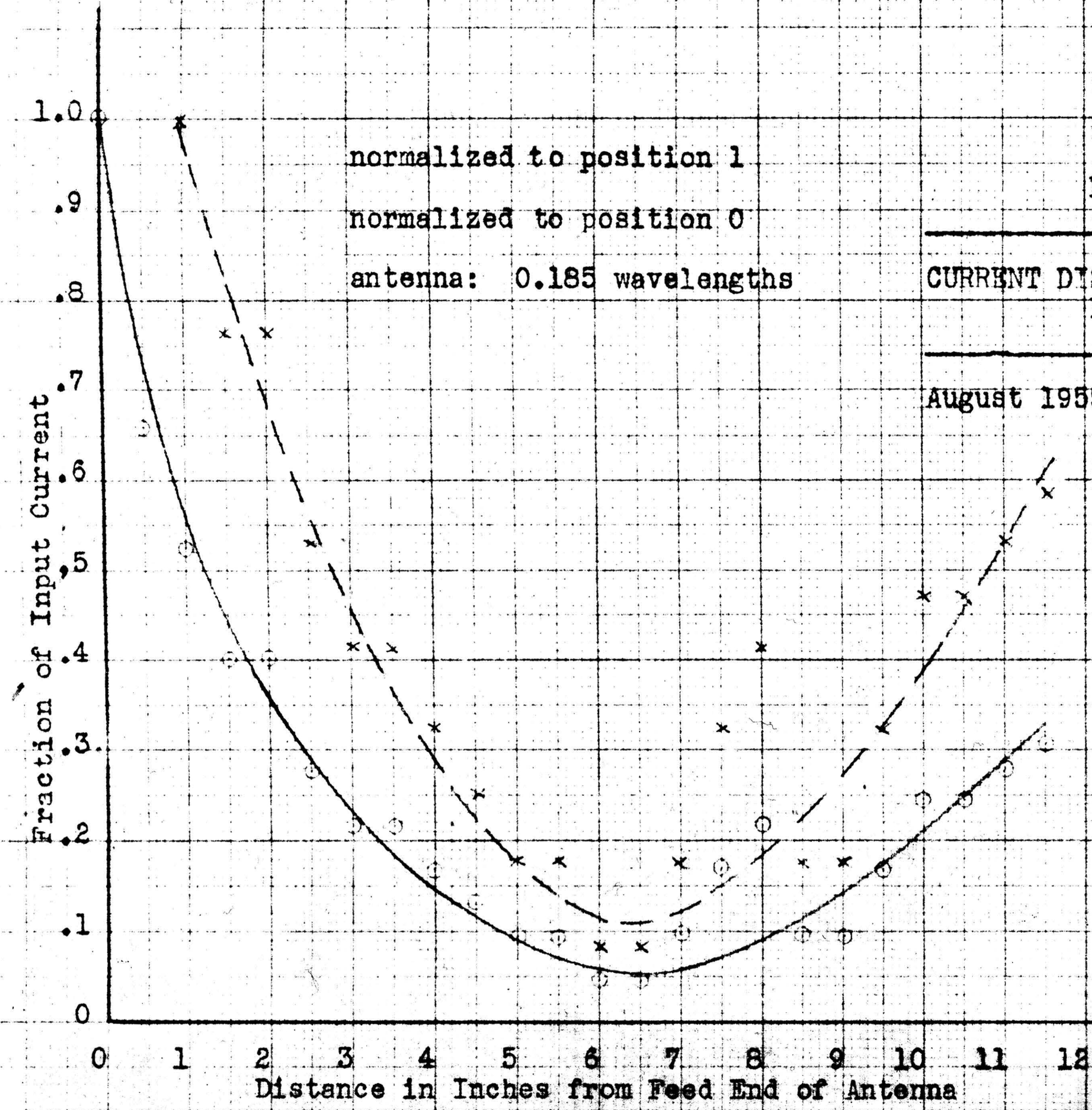

Figure 12

CURRENT DISTRIBUTION ON ANTENNA IN WAVEGUIDE

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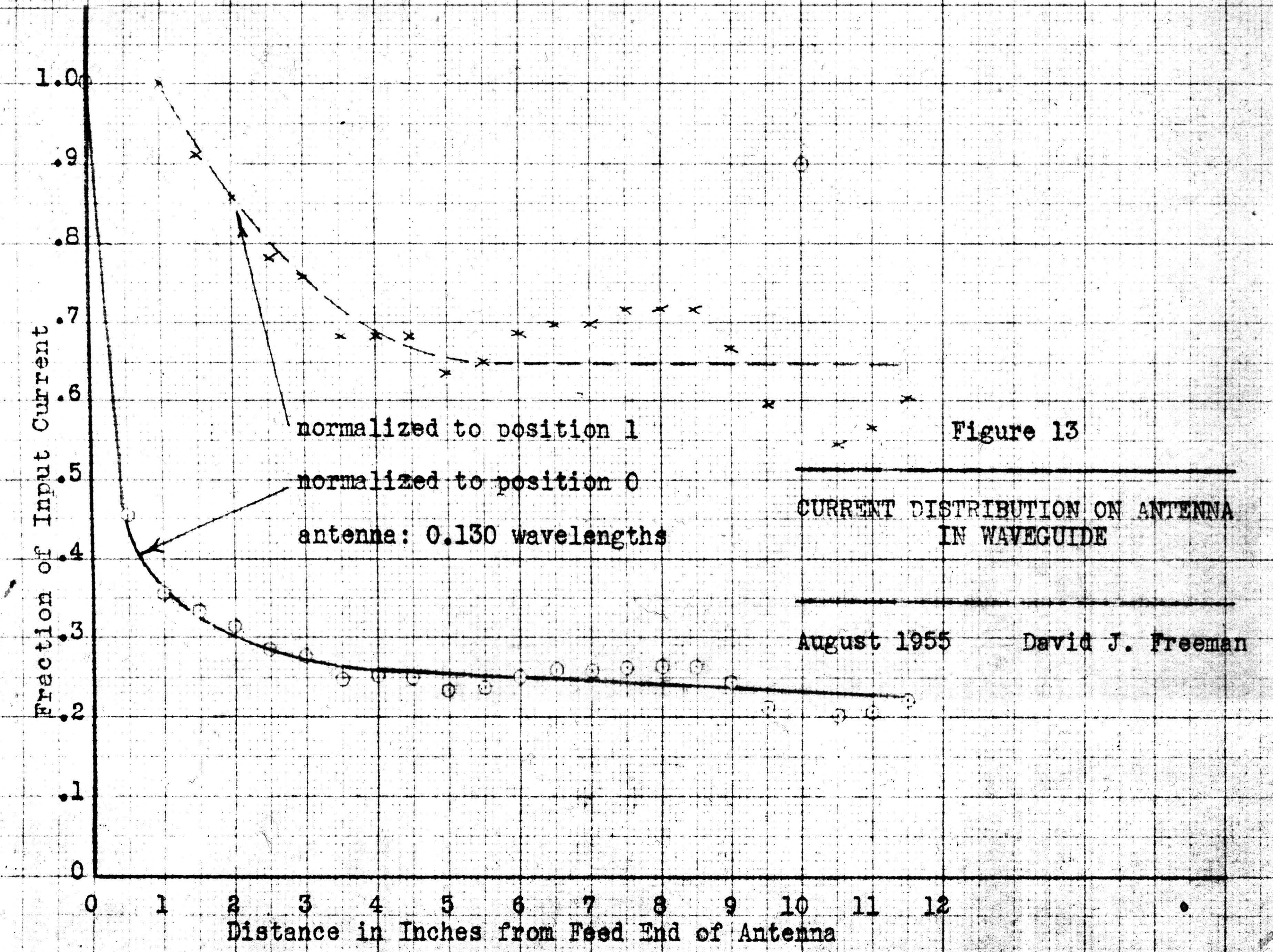

Figure 13

CURRENT DISTRIBUTION ON ANTENNA
IN WAVEGUIDE

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Nevertheless, the curves normalized to position "0" are continuous and seem reliable.

Curves normalized to position "1" are the curves to be examined if the electrical junction of antenna and feed line is actually located 1/2 inch from the wave guide wall in the guide. In position "1", the pick up loop's center line is located 1/2 inch from the wave guide wall.

Quite probably the electrical junction is located at neither position "o" or "1". The two curves are included for convenience only.

In order to justify the curves, refer to Figure 14, which shows the instantaneous field configurations in the wave guide for the first two possible modes of transmission, TE_{01} and TE_{11}. It may be seen that to support both modes the antenna current distribution should be the sum of a uniformly distributed component and a cosinusoidally

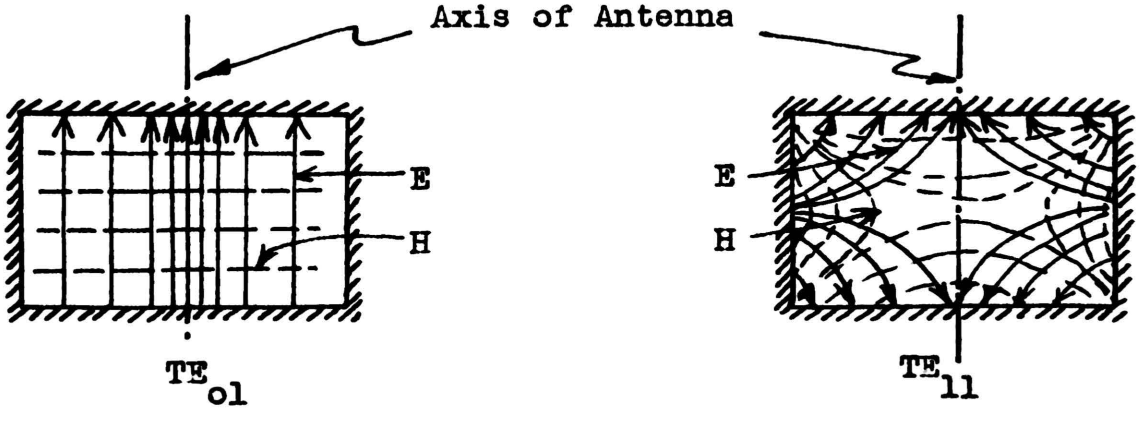

Figure 14

distributed component. The curves, Figures 8 through 12, show a strong resemblance to this type distribution, while the curve, Figure 13, does not.

It is therefore thought that the two modes were being excited Figures 8 through 12, while only one, the TE_{01}, was excited in Figure 13. This theory could be justified by the insertion of properly placed probes in the wave guide, but such a procedure was not followed in this investigation. It was noted, when making standing wave ratio measurements on the guide, that there existed a sizeable difference in the values at voltage minimums. These values decreased with distance away from the antenna. This indicates the presence of the exponentially decaying mode which was not being transmitted. It is true that the mode actually transmitted down the gûide will be attenuated exponentially also, but not at a rate which could be determined by noting the difference in immediately sucessive voltage minimums.

Present theory, being based on assumptions of perfect wave guide conductivity and the like, makes no restrictions on what modes may be excited, but rather only on which modes may be transmitted.

It should be remembered that the modes excited in the wave guide are not so much forced upon the guide by the excitation source as they are allowed to exist by the guide. That is to say, a certain antenna system may be chosen to tend to excite a certain mode, but unless the frequency is above cutoff for that particular mode, it will only exist in the near vicinity of the antenna and will not be transmitted down the guide. There is also the case in which an antenna system is chosen to excite one mode and other modes are nevertheless excited and transmitted as a result of the frequency being above cutoff for several modes.

For the sake of clarity let the modes which are excited but not transmitted be called the "excited modes", and the modes which are both excited and transmitted be called the "transmitted modes".

Using this notation, it seems likely from the results of this investigation that the farther the frequency is below cutoff for "excited " modes, the less likely these "excited modes" are to exist.

Attempts to justify the allowance of the establishment of "excited" modes by the guide, become mathematically involved because the very existance of such modes is based upon Maxwell's equations. If the allowance or disallowance of "excited" modes is to be examined, the assumed perfect boundary conditions must be withdrawn and imperfect boundary conditions substituted. This is because, for example, in the actual wave guide, the electric field does not become zero precisely at the boundary, but rather somewhere within the guide wall. This makes mathematical solution impractical.

The experimental results show that the assumption of uniform current distribution for antennas over 1/8 wave length is definitely incorrect. The current distribution for 1/8 wave length was nearly uniform.

It also seems that the current distribution becomes uniform because of the higher order "excited" modes becoming impossible to establish. It may now be seen that this might be the more proper way of looking at the problem of current distribution. That is, shortening the vertical dimension of the guide might be thought of as merely making it impossible to establish higher order "excited" modes.

There is one further implication in this light which, although not investigated here, seems logical. If an antenna is placed in a guide which is so dimensioned as to allow as nearly as possible one mode only to be transmitted, probably only one mode will be excited, (untransmitted modes will not even be excited, much less transmitted,) and the current distribution in this case will conform only to the one mode. One can not make such a rule on the basis of one experimentation, but this investigation indicates possibilities along this line both in the field of experimentation and theoretical development. If this could be proven or tied down, it might be possible to predict the antenna current distribution in advance.

Two other side issues were investigated. One was the input impedance of the antenna. Unfortunately the antenna impedance was so different from the characteristic impedance of the slotted line which was available, that the standing wave ratio was too high to read.

The other line of investigation was to determine the input impedance and current distribution of the antenna with the terminating reactance, (tuning stub,) located in such a position as to make the antenna impedance purely resistive. Unfortunately the voltage minimum points produced by the standing waves on the slotted line could be adjusted to appear at either the 1/4, 1/2, 3/4, or full wave length points, which means that the tuning stub has the ability to transform the impedance of the antenna into a pure resistance greater or less that the characteristic impedance of the feed line. This indicated that what ever the reactance of the antenna was, it could not be represented by a simple set of series elements, because if it were a simple series element of resistance and reactance, only one value of input resistance could be made to appear. It appeared that the tuning stub was able to both series and parallel resonate the antenna. It was decided that this side issue was a rather profound one which had little bearing on the immediate problem of determining the current distribution on antennas in a wave guide as a function of antenna length. It was therefore abandoned.

DISCUSSION OF ERRORS AND ACCURACY OF RESULTS

In this investigation there were several sources of error. Following will be a discussion of the errors of each component of the measuring system, concluded by an analysis and estimate of the overall error and accuracy of the results.

The Pickup Loop:

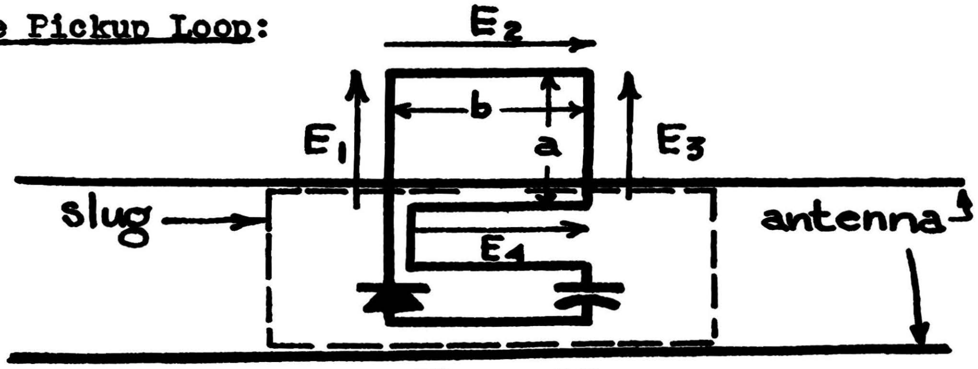

Figure 15

The voltage induced in the loop in Figure 15 may be calculated in two ways. One way is by noting the rate of change of flux linking the loop with respect to time. Another way is to take $(E \cdot dl)$ around the loop. Both should give the same answer. The electric field analysis gives the clearest explanation of the error involved. The induced voltage, V, will be:

$$V = E_1a + E_2b - E_3a - E_4b.$$

In as much as the field E_4 must be inside the antenna, it is zero and:

$$V = (E_1 - E_3)a + E_2 b.$$

The purpose of the pick up loop is to measure the current in the antenna. If the loop is located away from the ends of the antenna,

$$E_1 = E_3, \text{ and } V = a\,E_2,$$

and the voltage induced in the pickup loop is proportional to the proper component of the electric field intensity.

If, however, the loop is located near the junction of the coaxial feed line and the wave guide at a point where the electric field lines are tipping from a position perpendicular to the antenna, as they are in the coaxial line, to a position parallel to the antenna, as they are in the wave guide, it is possible that $E_1 \neq E_3$, and some voltage will be induced which is not proportional to the proper component of field intensity. The magnitude of the error involved here is a function of position and is impossible to evaluate.

The pick up loop and the capacitor form a network which could possibly resonate at frequencies very much higher than the frequencies used in this investigation. In the order of frequencies used in this investigation the pick up loop characteristics should be virtually independent of frequency variation.

This fact, however, presents the problem of the pick up loop's measuring the second, third, etc. harmonic currents which are always present in the output of a radio frequency transmitter such as was used to feed the antenna. There would therefore be no way to tell how much of the antenna current was of the fundamental frequency.

To eliminate this error, the antenna was fed through a low pass filter which attenuated the harmonics before they could reach the antenna to be measured. Before insertion of the harmonic filter, second harmonic standing waves could be detected on the slotted line through which the system was fed. Upon insertion of the low pass filter, the second harmonic standing waves disappeared. Higher harmonics would have been attenuated even more, so it was decided that the harmonic content of the antenna input was sufficiently low.

The rectified voltage taken from the capacitor in the pick up loop may be concluded to be perfect.

Still another error is possible because of the disturbance of the actual field which would exist in the absence of the pickup loop. This error is also impossible to evaluate. Obviously, the smaller the loop, the less the disturbance, and the greater the accuracy. The loop was made as small as possible, consistent with a useable voltage output.

There is a possibility of error as a result of the slot in the antenna through which the pick up loop protruded. A number of texts, however, points out that the slot must be relatively large with respect to the wave length used in order to disturb the pattern established by the antenna. In this case the slot was extremely small compared to a wave length and the resulting error should be nearly zero.

All errors in the pick up loop are, therefore, practically zero except for the effects at the end of the antenna, and the effects of field distrubance caused by the presence of the pick up loop.

The purpose of the investigation was to compare the currents at various points on the antenna, that is to find the current distribution. In other words, a comparison is intended and not an actual measurement of absolute values. It is felt that the field distrubance error would be about the same in all cases and therefore would not enter into the problem of comparison or distribution measurement.

The pick up loop may be assumed to have virtually zero error when used for comparison type investigation except for the times when the readings were made with the pick up loop near the ends of the antenna.

The Transmission Line:

In as much as the energy obtained from the pick up loop was converted to almost pure d.c. energy before being transmitted, there was practically no frequency sensitivity in this component.

The d.c. resistance of the transmission line would affect the scale calibration of the meters used slightly, but would cause no error in a comparison type of investigation. The information presented to the meters is therefore very accurate with the excep ion of instances in which readings are made near the ends of the antenna.

Accuracy of the Meters:

The microammeter-crystal combination was calibrated with the standing wave meter and their accuracies therefore equal. For this reason this discussion will deal only with the accuracy of the standing wave meter.

The meter is known to have an accuracy of 5% of full scale reading. This means that throughout the scale the meter is within plus or minus a certain number of units. Therefore, in as much as the data was normalized with respect to the input current values in a set of data, the error of the plotted point in the graphs is in general reduced by the normalizing process.

Thus it will be noted from the plots that the data normalized with respect to the highest number will show the least apparent variation, while the plots normalized with respect to a lower number show a larger variation, while the accuracy of the measurement is the same throughout.

It is, therefore, no less accurate to draw an average line through the widely scattered points of the graph in Figure 13 than through the apparently unscattered points of the same graph.

Summary of errors:

In as much as the data was measured only once, there can be no statement made as to the precision of the data. The accuracy of results which is dependent upon precision of measurement must therefore be estimated.

Although the overall accuracy may have been only in the order of 5%, it is felt that it is a relatively unimportant matter in this investigation, in as much as the desired result was mainly to discover the general current distribution, and not to measure with great precision and accuracy the absolute values of current.

APPENDIX A

Symbols used in following Data

f = Frequency

GSWR = Standing wave ratio of waves in the guide

λ_o = Wave length at critical or cutoff frequency

λ_1 = Wave length in free space

λ_p = Wave length in the guide

S' = Distance in cm to first voltage minimum on slotted line

LSWR = SWR on the slotted line

Zero = Distance of zero position on slotted line from antenna - coax junction

f - 493 m.c. λ_1 = 24" LSWR = ∞
GSWR 1.03 λ_p = 32" Zero 424
λ_o = 36" S' 16.3 cm Run 1

Position	With Shunt	Without Shunt	Corrected	Normalized to Pos. 0	Pos. 1
0	22		70	1	
1/2	22		70	1	
1	22		70	1	1
1½	22		70	1	1
2	22		70	1	1
2½	22		70	1	1
3	20		68	.97	.97
3½	15	132	61	.87	.87
4	12.7	95	57	.815	.815
4½	8.5	63	47	.67	.67
5	4.8	36	33	.471	.471
5½	2.5	19	22	.314	.314
6	.93	7	10	.143	.143
6½	.26	2	3	.043	.043
7	.4	3	4	.057	.057
7½	1.6	12	15	.214	.214
8	4	30	30	.428	.428
8½	7.33	55	43	.615	.615
9	10	75	50	.715	.715
9½	12.4	93	56	.8	.8
10	14.1	106	60	.858	.858

computed

Position	With Shunt	Without Shunt	Corrected	Normalized to Pos. 0	Normalized to Pos. 1
10½	16.7	125	63	.9	.9
11	22		70	1	1
11½	35		83	1.19	1.19

f = 465 m.c.

λ_1 = 25.5"

LSWR = ∞

GSWR = 1.03

λ_p = 36"

Zero 42.4

λ_o = 36"

S' = 28.6 cm

Run 2

Position	With Shunt	Without Shunt	Corrected	Normalized to Pos. 0	Normalized to Pos. 1
0	40		87	1	
½	38		85	.98	
1	38		85	.98	1
1½	35		83	.955	.975
2	34		82	.944	.965
2½	30		78	.898	.918
3	30		78	.898	.918
3½	28		77	.886	.906
4	28		77	.886	.906
4½	20		68	.783	.8
5	15	128	61	.703	.718
5½	10.7	80	53	.61	.624
6	6	45	38	.437	.448
6½	2.3	17	21	.242	.247
7	1.1	8	12	.137	.141

damped

Position	With Shunt	Without Shunt	Corrected	Normalized to	
				Pos. 0	Pos. 1
7½	1.5	11	15	.173	.177
8	3.9	29	29	.334	.341
8½	7.0	53	42	.483	.494
9	10.7	80	52	.598	.612
9½	15	112	61	.702	.718
10	19	145	67	.772	.788
10½	21		69	.793	.812
11	27		76	.875	.895
11½	32		80	.92	.941

f = 436

λ_1 = 27.2"

LSWR = ∞

GSWR = 1.04

λ_p = 42"

Zero = 42.4

λ_0 = 36"

S' = 5.3 cm

Run 3

Position	With Shunt	Without Shunt	Corrected	Normalized to	
				Pos. 1	Pos. 2
0		150	150	1	
½		123	140	.934	
1		93	122	.814	1
1½		60	95	.633	.772
2		45	78	.52	.634
2½		32	61	.407	.496
3		23	47	.314	.382
3½		32	61	.407	.496
4		16	35	.233	.285

Position	With Shunt	Without Shunt	Corrected	Normalized to Pos. 1	Normalized to Pos. 2
4½		10	24	.160	.195
5		5	13	.087	.106
5½		3	8.5	.057	.069
6		2	6	.04	.049
6½		1	3	.02	.024
7		4	11	.073	.089
7½		7	18	.12	.146
8		10	24	.16	.195
8½		10	24	.16	.195
9		12	28	.187	.228
9½		10	24	.16	.195
10		45	78	.52	.634
10½		37	67	.446	.545
11		47	81	.54	.658
11½		45	78	.52	.635

f = 411 m.c.

GSWR = 1.04

λ_0 = 36"

λ_1 = 288"

λ_p = 48"

S' = 11.35

LSWR = ∞

Zero = 42.4

Run 4

Position	With Shunt	Without Shunt	Corrected	Normalized to Pos. 0	Normalized to Pos. 1
0	50		94	1	
½	35		83	.882	
1	25		74	.787	1

Position	With Shunt	Without Shunt	Corrected	Normalized to Pos. 0	Normalized to Pos. 1
1½	20		68	.723	.92
2	15	115	61	.65	.825
2½	12	90	55	.585	.743
3	8.7	65	47	.5	.635
3½	60	45	38	.405	.514
4	4.0	30	30	.319	.406
4½	2.0	15	19	.202	.257
5	1.33	10	14	.131	.19
5½	.66	5	7	.075	.095
6	.534	4	6	.064	.081
6½	2.0	15	19	.202	.257
7	1.07	8	11	.117	.149
7½	1.87	14	17	.181	.23
8	2.66	20	23	.245	.311
8½	3.46	26	27	.287	.365
9	3.2	24	26	.277	.352
9½	5.34	40	36	.383	.487
10	8.7	65	47	.5	.635
10½	10	75	50	.532	.676
11	11.7	88	55	.585	.743
11½	14.7	110	61	.65	.825

Computed

f = 377 mc.

$\lambda_1 = 31.4''$

LSwR = ∞

GSWR = 1.03

$\lambda_p = 65''$

Zero = 42.4 cm

$\lambda_0 = 36''$

S' = 20.8

Run 5

Position	Without Shunt	With Shunt	Corrected	Normalized to Pos. 0	Pos. 1
0	35		65	1	
½	20		43	.66	
1	15		34	.523	1.0
1½	11		26	.40	.765
2	11		26	.40	.765
2½	7		18	.277	.53
3	5		14	.215	.412
3½	5		14	.215	.412
4	4		11	.169	.324
4½	3		8.5	.131	.25
5	2		6	.092	.177
5½	2		6	.092	.177
6	1		3	.046	.088
6½	1		3	.046	.088
7	2		6	.092	.177
7½	4		11	.169	.324
8	5		14	.215	.412
8½	2		6	.092	.177
9	2		6	.092	.177
9½	4		11	.169	.324

Position	Without Shunt	With Shunt	Corrected	Normalized to Pos. 0	Normalized to Pos. 1
10	6		16	.246	.471
10½	6		16	.246	.471
11	7		18	.277	.53
11½	8		20	.308	.588

f = 354 mc

λ_1 = 33.5"

LSWR = ∞

GSWR = 1.03

λ_p = 92"

Zero = 42.4

λ_0 = 36"

S' = 29.0 cm

Run 6

Position	With SWR Meter	Recipricol	Normalized to Pos. 0	Normalized to Pos. 1
0	1	1	1.	
½	2.2	.455	.455	
1	2.8	.357	.357	1
1½	3	.333	.333	.91
2	3.2	.312	.312	.852
2½	3.5	.286	.286	.78
3	3.6	.277	.277	.754
3½	4	.25	.25	.682
4	4	.25	.25	.682
4½	4	.25	.25	.682
5	4.3	.232	.232	.634
5½	4.2	.238	.238	.65
6	4	.25	.25	.682
6½	3.9	.256	.256	.699

Position	With SWR Meter	Recipricol	Normalized to	
			Pos. 0	Pos. 1
7	3.9	.256	.256	.699
7½	3.8	.263	.263	.718
8	3.8	.263	.263	.718
8½	3.8	.263	.263	.718
9	4.1	.244	.244	.666
9½	4.6	.217	.217	.592
10	1.1	.9	.9	2.46
10½	5	.2	.2	.546
11	4.8	.208	.208	.568
11½	4.5	.222	.222	.606

Meter Calibration Data

Run 7

Ammeter Reading With Shunt	Corrected Value With Shunt	SWR Meter Reading	Ammeter Reading Without Shunt	Corrected Value Without Shunt	SWR Meter Reading
150	150	1	* 150	150	1.0
140	143	1.05	140	146	1.03
130	139	1.08	130	140	1.07
120	134	1.12	120	136	1.1
110	128	1.17	110	131	1.14
100	123	1.22	100	125	1.2
90	118	1.27	90	121	1.24
80	113	1.33	80	116	1.29

Ammeter Reading With Shunt	Corrected Value With Shunt	SWR Meter Reading
70	106	1.41
60	101	1.49
50	93	1.60
40	87	1.72
30	79	1.9
*20	68	2.2
13.4	56.7	
10.0	53.5	2.8
8.0	42.5	
4.0	26.0	
1.3	16.0	

Plot Figure 16

Ammeter Reading Without Shunt	Corrected Value Without Shunt	SWR Meter Reading
70	103	1.45
60	94	1.60
50	86	1.75
40	71	2.1
30	58	2.6
20	43	3.5
10	35	4.2

Plot Figure 17

* Note: Shunt multiplier is 7.5;

Meter is measuring same current in this case.

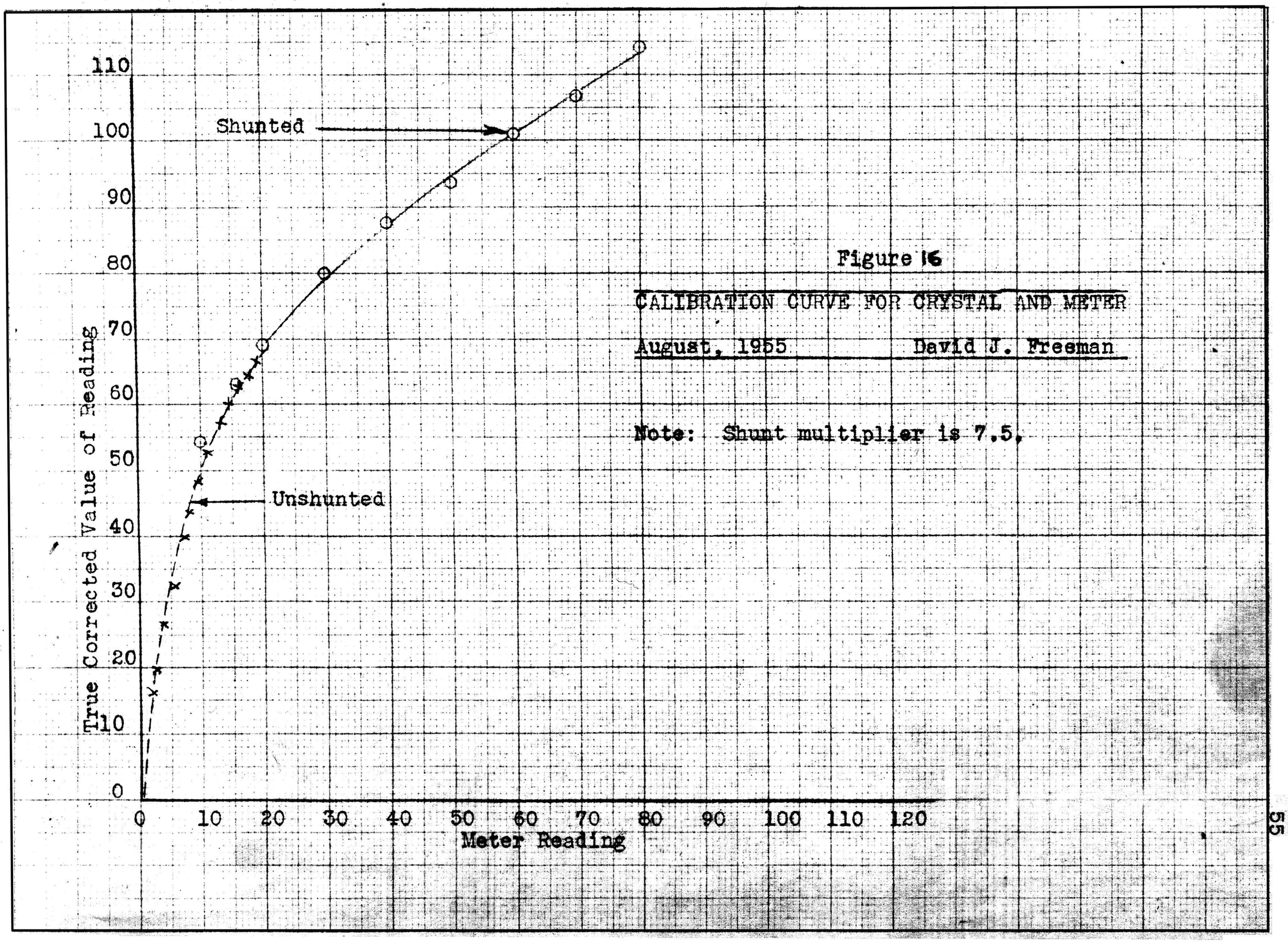

Figure 16

CALIBRATION CURVE FOR CRYSTAL AND METER

August, 1955 David J. Freeman

Note: Shunt multiplier is 7.5.

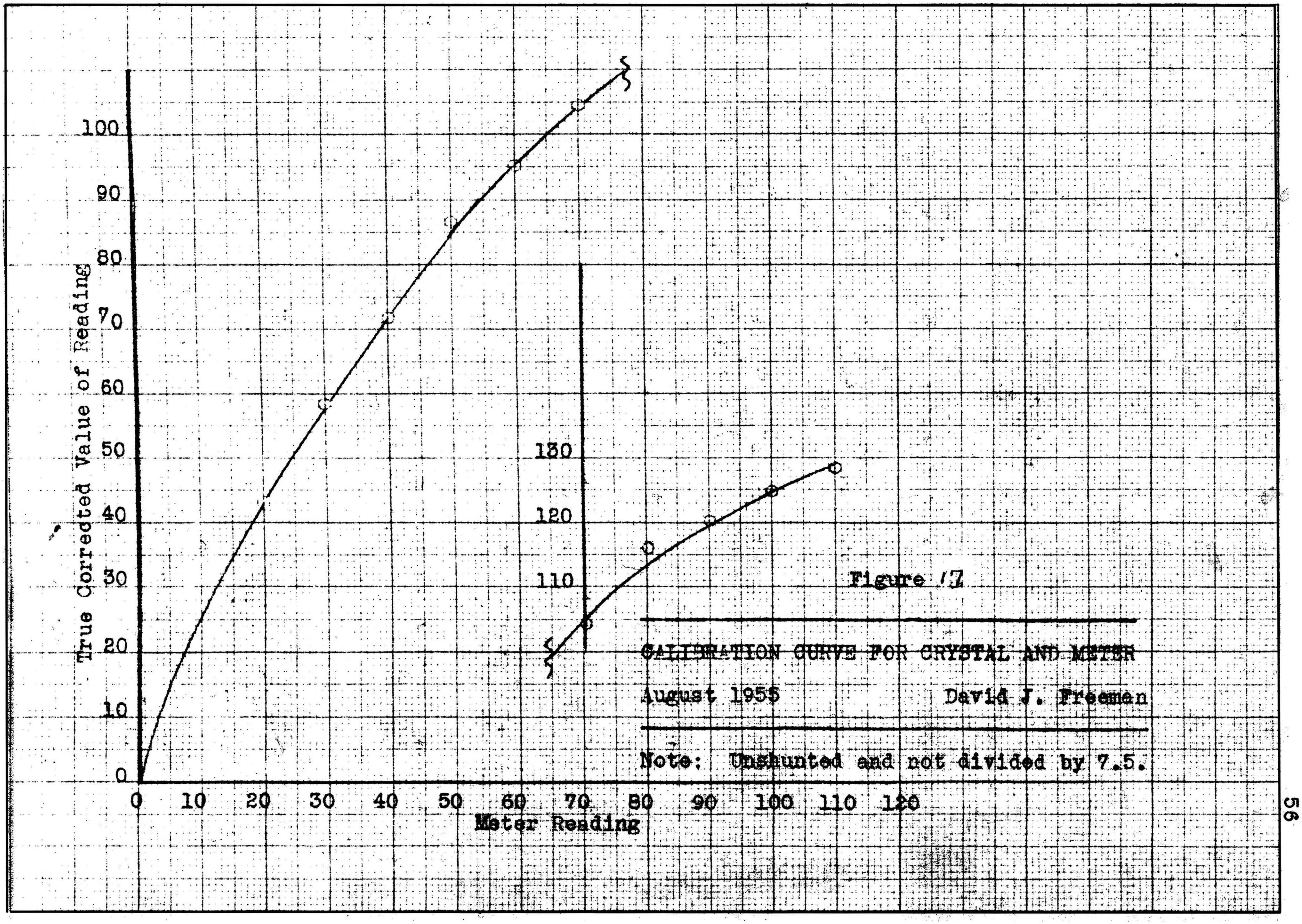

Figure 17

CALIBRATION CURVE FOR CRYSTAL AND METER

August 1955 David J. Freeman

Note: Unshunted and not divided by 7.5.

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VITA

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