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Effect of Canyon Shape on Longitudi-Vibration of Earth Dam

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SYNOPSIS In this paper, the effects of narrow and sloping canyons on the longitudinal dynamic response of earth dams is investigated by a simplified method for 3-D dynamic analysis. Analytical results for earth dams with several width to height ratios are compared with the results of plane strain solutions for comparable sections. For an embankment in a triangular valley with a width to height ratio of 1∿2 and subjected to earthquake motions, the computed max. accelerations at the midsection were about 30 to 50% higher in a simplified 3-D response analysis than in a plane strain analysis. On the other hand the computed max. displacements, max. velocity, and max. shear stresses for the same section in 3-D analysis were generally about one half the values to 2/3 the values computed from plane strain analysis. It is concluded that the longitudinal dynamic response of an embankment is significantly affected by the width to height ratio of canyon in which it is constructed.

INTRODUCTION

If an earth dam is built in rectangular canyon where the width of canyon is large with respect to the height of dam, then Generally, the assumption of plane strain condition for analyzing the dynamic respone of dam is adopted. However, for cases of narrow rectangular canyons or canyons with sloping walls, the response of a dam is greatly affected by the proximity of the rigid boundaries, and the validity of a plane strain assumption becomes questionable. In such cases, 3-D analysis is desirable to predict accurately the dynamic response of an embankment.

Makdisi at al (1982) studied the effects of canyon shape on the dynamic response of earth dams and the comparisons between 2-D and 3-D analyses by mean of 3-D FEM. A significant difference was shown between the two analyses. Described study was assumed that the earthquake motion is in the upstream-downstream direction(transverse). Although studies of the upstream-downstream of an earth dam are most important, the problem of earthquake-induced longitudinal vibration of earth dam is also significant for the safety of dams. This paper presents a simplified method for 3-D longitudinal dynamic analysis of earth dam in triangular canyons and investigtes the effects of width to height ratio of canyons on the longitudinal dynamic response of dam.

SIMPLIFIED SOLUTION OF LONGITUDINAL VIBRATION FOR EARTH DAM IN 3-D

Author (1992) has presented a simplified solution of longitudinal vibration analysis for earth dam in triangular canyons. It is described briefly as follows.

Differential Equation of Longitudinal Vibration of Earth Dam

Fig.1 shows a max. longitudinal section, a max. transveral section and a longitudinal slice of dam. Assume longitudinal section and transversal section are symmetrical triangle. The assumptions inherent to a shear wedge analysis of dam are as follows: (1) The canyon wall is perfectly rigid; (2) The direction of ground motion is horizontal and parallel to dam axis and there are no displacement in other directions; (3) The dam is homogeneous and the dam materials are linearly elastic; (4) Interaction between water in the reservoir and the dam is negligible, (5) Only shear deformation is taken into account. Forces acting on slice in longitudinal direction, as shown in Fig.1(c), are: shear force on bottom S_{yz}, shear force on top $S_{yz}+dS_{yz}$, axial force on front F_z, axial force on back face F_z+dF_z and inertial force F_i. For the equilibrium of the slice, the equation of motion for undamped free longitudinal vibration of dam is obtained:

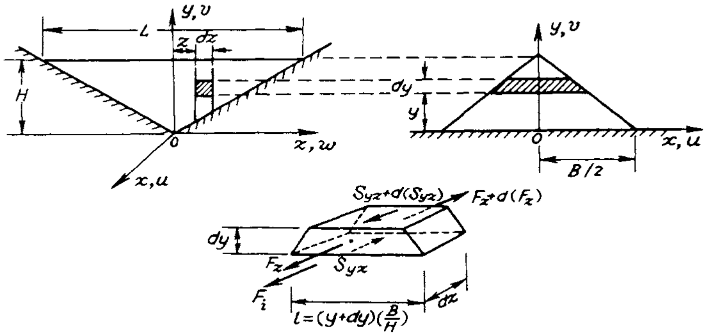

Fig.1 Analytical Model of Dam in Triangular Canyon for Shear Wedge Analysis

$$v_s^2 \frac{\partial^2 w}{\partial y^2} + \zeta v_s^2 \frac{\partial^2 w}{\partial z^2} + \frac{v_s^2}{y-H} \frac{\partial w}{\partial y} - \frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

where w is displacement in z direction, $v_s=\sqrt{G/\rho}$ shear wave velocity, G dynamic shear modulus, ρ density of the material, H the height of the dam and t time. $\zeta=2(1+\mu)$, in which μ is the Poisson's ratio of the material. The following boundary conditions are applicable to the case of a symmetrical dam in triangular canyon:

$$\left.\begin{aligned} \frac{\partial w}{\partial y} &= 0 \qquad \text{at} \quad y = H \\ w &= 0 \qquad \text{at} \quad y = \frac{2H}{L} z \end{aligned}\right\} \tag{2}$$

where L is length of dam crest.

Solution for First Natural Frequency

By the method of separation of variable, letting $w = \phi(y,z)T(t)$ and substituting it into equation (1) and (2), the following equations are obtained:

$$\frac{\partial^2 T}{\partial t^2} + \omega^2 T = 0 \tag{3}$$

$$\frac{\partial^2 \phi}{\partial y^2} + \xi \frac{\partial^2 \phi}{\partial z^2} + \frac{1}{y-H} \frac{\partial \phi}{\partial y} + \frac{\omega^2}{v_s^2} \phi = 0 \tag{4}$$

The boundary conditions can be imposed on the function ϕ:

$$\left.\begin{aligned} \frac{\partial \phi}{\partial y} &= 0 \qquad \text{at} \quad y = H \\ \phi &= 0 \qquad \text{at} \quad y = \frac{2H}{L} z = Kz \end{aligned}\right\} \tag{5}$$

Solution in closed form of Eq.(4) is difficult to obtain. However, an approximate eigenvalue solution of Eq.(4) can easily be used to obtain a rather accurate value for the first natural frequency of vibration of the system. According to the Bubnov-Galerkin method (Zienkiewicz 1971), if a function ϕ which satisfies the boundary conditions given by Eq.(5) can be found, the following integral:

$$\int_0^H \int_0^{y/k} \left(\frac{\partial^2 \phi}{\partial y^2} + \xi \frac{\partial^2 \phi}{\partial z^2} + \frac{1}{y-H} \frac{\partial \phi}{\partial y} + \frac{\omega^2}{v_s^2} \phi \right) \phi \, dz\, dy = 0 \tag{6}$$

yields an algebraic equation from which the frequency of the system can be determined. It can easily be shown that the the function:

$$\phi = \frac{1}{H^4} (y+Kz)(y-Kz)(y-2H+Kz)(y-2H-Kz) \tag{7}$$

satisfies the Eq.(5). After substituting Eq.(7) into Eq.(6) and performing the integration the following algebraic equation is obtained:

$$\frac{32}{225} \frac{\omega^2}{K} \frac{H^2}{v_s^2} - \frac{8}{5K} - \frac{32}{45} \xi K = 0 \tag{8}$$

Solving Eq.(8) for ω, we get:

$$\omega = \frac{v_s}{H} \sqrt{\frac{45}{4} + 40(1+\mu)\frac{H^2}{L^2}} \tag{9}$$

This expression gives the first natural frequency, i.e. ω_1 of a symmetrical dam in a triangular canyon under longitudinal vibration, the function ϕ in Eq.(7) is corresponded to the first mode shape of vibration, i.e. ϕ_1.

Longitudinal Earthquake Response of Dam

It is easily proved that the equation governing longitudinal vibration of the dam with damping under earthquake can be written as:

$$\frac{\partial^2 w}{\partial t^2} + \frac{c}{\rho} \frac{\partial w}{\partial t} + \frac{G}{\rho} \left(\frac{\partial^2 w}{\partial y^2} + \xi \frac{\partial^2 w}{\partial z^2} + \frac{1}{y-H} \frac{\partial w}{\partial y} \right) = - \ddot{w}_g(t) \tag{10}$$

where $\ddot{w}_g(t)$ is acceleration of rigid canyon in z direction and c is coefficient of damping. By the method of separation of variables [letting $w = \sum_1^\infty \phi_n(y,z)T_n(t)$] and based upon the orthogonality of mode shape, the following two equations for the first mode shape are obtained:

$$\frac{\partial^2 \phi_1}{\partial y^2} + \xi \frac{\partial^2 \phi_1}{\partial z^2} + \frac{1}{y-H} \frac{\partial \phi_1}{\partial y} + \frac{\omega_1^2}{v_s^2} \phi_1 = 0 \tag{11}$$

$$\frac{\partial^2 T_1}{\partial t^2} + 2\lambda_1 \omega_1 \frac{\partial T_1}{\partial t} + \omega_1^2 T_1 = -\eta_1 \ddot{w}_g(t) \tag{12}$$

where ω_1 is first natural frequency given by Eq.(9), λ_1 damping ratio of first mode, it is equal to $c/2\rho\omega_1$, η_1 is mode participate coefficient

$$\eta_1 = \frac{\int_0^H \int_0^{y/k} \phi_1 (H-y) dz dy}{\int_0^H \int_0^{y/k} \phi_1^2 (H-y) dz dy} \tag{13}$$

After substituting the ϕ_1 from Eq.(7) into Eq.(13) and performing the integration, then obtains $\eta_1 = 1.839$. The solution of Eq.(12) is:

$$T_1 = \frac{-1.839}{\omega_1} \int_0^t \ddot{w}_g(\tau) e^{-\lambda_1 \omega_1 (t-\tau)} \sin \omega_1' (t-\tau) d\tau \tag{14}$$

where $\omega_1' = \omega_1 \sqrt{1-\lambda_1^2}$, the Duhamal integral may be calculated by numerical integration method. Because the higher modes little effect on earthquake response of dam, only a few lower modes ($1\sim3$ order) are adopted for practical requirement. Then the longitudinal earthquake response of dam in triangular canyons can be approximately written as follows: $w \doteq \phi_1 T_1$; $\dot{w} \doteq \phi_1 \dot{T}_1$; $\ddot{w} \doteq \phi_1 \ddot{T}_1$; $\tau_{yz} \doteq G \phi_{1y}' T_1$; $\sigma_z \doteq E \phi_{1z}' T_1$, where ϕ_1, ϕ_{1y}', and ϕ_{1z}' can be determined by Eq.(7) and its derivative, T_1, $\dot{T}_1$ and $\ddot{T}$ can be obtained by Eq.(14) and its first derivative and second derivative for t, E Young's modulus and σ_z longitudinal normal stress. In engineering it is most interesting in the max. response of dam and so the following formulas of max. response are useful for earthquake-resistant design of dam

$$w_{max} \approx |\eta_1 \phi_1| S_d \doteq |1.839 \phi_1| S_d \tag{15}$$

$$\dot{w}_{max} \approx |\eta_1 \phi_1| S_v \doteq |1.839 \phi_1| S_v \tag{16}$$

$$\ddot{w}_{max} \approx |\eta_1 \phi_1| S_a \doteq |1.839 \phi_1| S_a \tag{17}$$

$$\tau_{yz,max} \approx |\eta_1 \phi_{1y}| S_d \doteq |1.839 \phi_{1y}| S_d \tag{18}$$

$$\sigma_{z,max} \approx |\eta_1 \phi_{1z}| S_d \doteq |1.839 \phi_{1z}| S_d \tag{19}$$

where S_d, S_v and S_a are displacement response spectrum, velocity response spectrum and acceleration response spectrum respectively.

EFFECT OF WIDTH TO HEIGHT RATIO OF CANYONS ON LONGITUDINAL VIBRATION OF DAM

Effect on First Natural Frequency

By using of Eq.(9) and assuming $\mu = 0.3$, the first natural frequencies for width to height ratio

L/H=1,2,...6 of earth dam are computed. Fig.2 shows the variation of the first natural frequency of longitudinal vibration of the earth dam with the width to height ratio, L/H, for

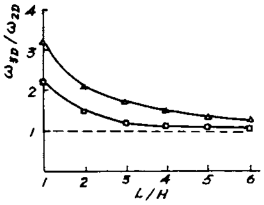

Fig.2 Variation of the ω_{3D}/ω_{2D} of dam with L/H.

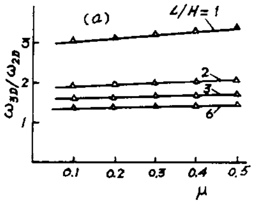

Fig.3 The effect of Poisson's ratio on the ω_{3D}/ω_{2D}.

triangular shaped canyons. The three-dimensional frequency ω_{3D} is presented as a ratio of plane strain natural frequency, ω_{2D} , which is calculated by formular $\omega_{2D}=2.41 v_s/H$. It may be seen from Fig.2 that the effect of width to height of triangular shaped canyons on the first natural frequency is significant. While L/H=1∿2, the three-dimensional natural frequencies for triangular shaped canyons ω_{3D} are about 3∿2 times of plane strain natural frequency ω_{2D}. This shows that for earth dams in narrow V-shaped canyons, the plane strain analysis may lead a large errors in the computed response.

Values of the ratio of the first natural frequency computed from Eq.(9) to that of the plane strain analysis for different values of Poisson's ratio μ of dam meterial and various values of L/H are presented in Fig.3. It may be seen from Fig.3 that the ω_{3D}/ω_{2D} is slightly increased with the increase of μ . While the μ increase from value of 0.1 to value of 0.5, the ω_{3D}/ω_{2D} increases about 15%. This shows, the effect of the value of μ on the first natural frequency of longitudinal vibration of earth dam is not large.

Effect on Various Dynamic Responses

As an example for investigation of L/H of canyons on various dynamic responses, suppose the symmertrical earth dam in triangular canyon is subjected a longitudinal earthquake (EL Centro in 1940), the max. height of dam H=50m, the dynamic properties of dam meterial are G=80MPa, v_s=200m/s, damping ratio λ_1=0.1, μ=0.3. Determine the various max. responses in the central and L/4 sections of dam, for the cases of L=50m, 100m, 200m, and 300m(i.e.L/H=1,2,4,6) respectively.

Solution: For the case of L=50m, K=2H/L=2×50/50 =2. Substituting the values of v_s , H and K into Eq.(9), we get the first natural frequency $\omega_1$and first natural period T_{D1} for longitudinal vribration of dam as follows:

$$\omega_1 = \frac{200}{50}\sqrt{\frac{45}{4} + 40(1+0.3)\frac{50^2}{50^2}} = 31.8 \text{ rad/s}$$

$$T_{D1} = \frac{2\pi}{\omega_1} = \frac{2\times 3.14}{31.8} = 0.07 \text{ s}$$

According to T_{D1} and from the charts of response spectrum of EL Centro earthquake in 1940 (Wiegel (1970)) we get:

S_v=12cm/s; S_d=0.8cm; S_a=300cm/s²

Substituting the values of S_d , S_v , S_a and G into Eqs.(15)-(19), the various max. response distributions with depth for the case of L=50m are obtained. For the cases of L=100m, 200m and 300m, an analogous computation has been performed. The computed results are summarized as in Fig.4.

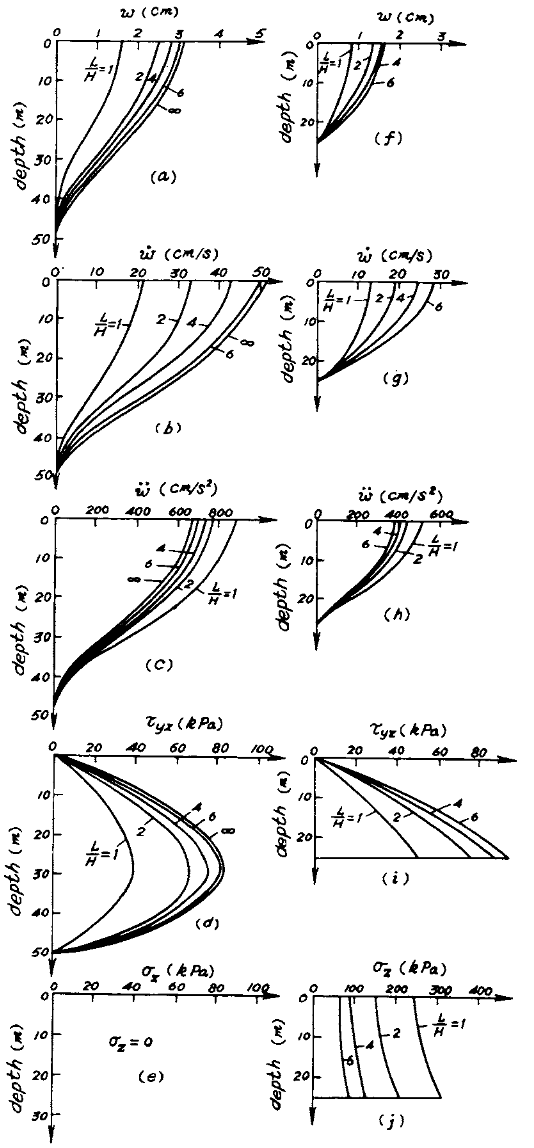

Fig.4 Distribution of Various Max. Responses (a)∿(e)—central section; (f)∿(j)—L/4 section.

In order to comparison, the results computed with plane strain assumption (L=∞) for midsection of embankment are also given in Fig.4. It may be seen from Fig.4 that the results computed with 3-D model presented in this paper are considerablly different from that computed with a plane strain model, especially as L/H small.

The max. displacements w_{max} , max. velocities $\dot{w}_{max}$ and max. shear stresses $\tau_{yz,max}$ are decreased with decrease of the value of the width to height ratio L/H, while the max. accelerations and max. longitudinal normal stresses $\sigma_{z,max}$ are increased with decrease of those value. For an embankment in a triangular canyon with L/H=1∿2 and subjected to earthquake motions, the computed max. accelerations at the midsection were about 30 to 50% higher in a 3-D response analysis than in a plane strain analysis, the computed max. displacements, max. velocities and max. shear stresses for the same section in the 3-D analysis were generally about one half the values to 2/3 the values computed from strain analysis.

CONCLUSION

The effect of width to height ratio of triangular canyons on the longitudinal vibration of earth dam is significant. For an embankment in a triangular canyon with L/H=1∿2, the 3-D natural frequency ω_{3D} is 3-2 times of plane strain natural frequency ω_{2D}, the computed max. accelerations at the midsection were about 30 to 50% heigher in a 3-D response analysis than in a plane strain analysis, the max. displacement, max. velocities and max. shear stresses for the same section in 3-D analysis were generally about one half the values to 2/3 the values computed from strain analysis. Thus, for earth dams in narrow V-shaped canyons, the plane strain analysis may lead a large errors in the computed response.

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