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Seventh International Conference on

Case Histories in Geotechnical Engineering

and Symposium in Honor of Clyde Baker

LATERAL MOVEMENTS OF A BRIDGE ABUTMENT DUE TO COMPRESSIBLE FOUNDATION SOILS

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ABSTRACT

Highway approach embankments are often constructed over weak and compressible soils. The consolidation settlements induced by the embankment loads are known to result in downdrag forces and lateral forces on the piles supporting the bridge abutments. Several approaches are commonly employed during construction of highway bridges in Ontario in order to reduce these forces. These approaches include: preloading and surcharging to achieve the anticipated settlements prior to the installation of abutment piles; application of lightweight fill materials such as expanded polystyrene and expanded pelletized blast furnace to reduce embankment loads; and measures to expedite pore water pressure dissipation such as wick drains.

This paper presents a case history where the abutments of seven bridges on the Macdonald-Cartier Freeway between Carnwall and the Quebec border experienced excessive movements, which resulted in extensive repair work in three of these structures. A series of numerical analyses are conducted to provide an explanation for abutment movements in Brookdale Avenue underpass structure. Two-dimensional nonlinear finite element analyses were conducted using commercial software package, PLAXIS 2D to evaluate the forces and moments acting on the abutments as a result of embankment loading. The behavior of compressible soils is modeled using Soft Soil Model (SSM). The results indicated that the consolidation settlements and their impact on the abutment piles were estimated with an acceptable accuracy using FE model. The results presented in this study are considered to be of interest to researchers and practitioners.

INTRODUCTION

Deep foundation units are widely used for supporting bridge piers and abutments, where weaker surficial soils preclude shallow foundations. Particularly in Ontario, semi-integral and integral abutment bridges are typically supported on endbearing steel driven piles. The construction of approach embankments over weaker, compressible soils results in significant vertical and lateral soil movements, which induce additional loads on piles supporting the abutments. In order to minimize such effects, preloading and surcharging are commonly employed approaches in Ontario to avoid unanticipated settlements prior to the installation of piles or lightweight fills do avoid the consolidation of approach embankment. Other alternatives such as aggregate pier

supported approach embankment are also being studied as the schedule restrictions imposed by preloading and surcharging becomes cost prohibitive (Turan et al. 2012).

This paper presents a case history, where a number of bridge structures experienced significant abutment movements subsequent to the completion of construction on Macdonald-Cartier Freeway, Highway 401 between Cornwall and the Quebec border. The movements in some structures were such that substantial structural remediation was required. This paper focuses on the Brookdale Avenue Underpass (U/P) Structure, which experienced the most severe deformations. The present study provides details of the Brookdale Avenue

U/P Structure, approach embankments, subsurface conditions and construction history, outlines the methodology adopted for numerical analyses and presents the results. The numerical analyses of soil-embankment-abutment system were performed to model the mechanism by which the abutment piles interacted with the approach embankment and compressible foundation soils and how such interaction led to significant abutment movements. The data presented in this study highlights the unique interaction mechanism between the compressible foundation soil, which experiences vertical and horizontal particle movement, and the piles supporting the abutments.

BACKGROUND

Piles are commonly used to support bridge abutments, where they are typically located within the area of influence of the approach embankments. The vertical and lateral soil movements due to construction of the approach embankment have the potential to impose downdrag and lateral forces on abutment piles. Thus, the construction is typically staged such that the installation of abutment piles follow a preload/surcharge period. The embankment settlements in Ontario Ministry of Transportation (MTO) projects are typically monitored. Subsequent to reaching the stabilized embankment settlements, the bridge construction proceeds with the assumption that the risk of additional forces on the piles is mitigated.

The piles supporting the bridge abutments on compressible soils have to resist the downdrag loads and the pressures exerted by lateral spreading of soil. The lateral response of piles was studied extensively in the past. Two main analysis approaches exist: pressure-based and displacement-based methods (White et al., 2008). The piles subjected to lateral movement of soil can be designed using pressure-based methods (DeBeer and Wallays 1972; Ito and Matsui 1975; Viggiani 1981). In this approach the soil pressures are estimated and applied to the piles as external loads, which are then used to calculate the shear and bending forces in the piles. However, for cases where large pile deformations are expected, the observed values of lateral forces are smaller than the forces predicted by this approach (White et al., 2008). The passive pile response can alternatively be determined using the relative displacement between the soil and the piles, which is a more sophisticated approach due to the pile-soil interaction (Poulos 1973; Byrne et al. 1984). White et al., (2008) performed numerical analyses and physical model tests to study the effect of free field soil movements on a single passive pile. A scaled physical model study on the pile response to lateral soil movement occurring due to excavations is performed by Ong et al. (2006).

The case history presented here involves large settlements of approach embankments and resultant abutments movements in Brookdale Avenue U/P Bridge. The details of seven structures including Brookdale Avenue U/P and the problems

encountered after construction were presented in Stermac et al., (1968). The present study summarizes the general structural and subsurface information as well as the structural distresses based on Stermac et al., (1968) and MTO archives (MTO, 1965). Subsequently, a series of numerical analyses were performed in order to identify the interaction mechanism between compressible foundation soil, embankment and abutment.

DESCRIPTION OF GEOLOGY AND SUBSURFACE CONDITIONS

The area of interest was covered with a layer of Leda clay underlain by a layer of till which was composed mainly of sand and gravel with boulders. The Leda clay deposits, which comprised the fine sand, silt, and clay, was formed by the Champlain Sea after the glaciers retreated. Subsequent uplift has caused the present non-marine environment (Karrow, 1961; Stermac et al., 1968). Leda clays are highly sensitive, usually normally consolidated and have natural moisture content slightly but sometimes well above the liquid limit (Crawford 1961).

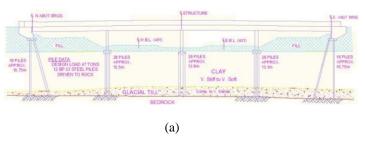
The subsurface conditions in the vicinity of Brookdale Ave. U/P Bridge were determined via a number of field investigations. Initial subsurface investigations were carried out in 1961. Additional investigations were conducted in 1965 after the abutment movements in order to supplement earlier information and allow an in depth evaluation of the causes. The subsurface conditions in the vicinity of Brookdale Avenue U/P structure includes approximately 10.5m thick very stiff to very soft clay layers. In both north and south abutment locations, a 2m thick crust was present in this clay layer. The clay layer is underlain by a layer of glacial till with an approximate thickness of 3m. Immediately below the till layer bedrock was observed. At both abutment locations the height of approach embankment was 6.7m. Figure 1a depicts a crosssection of Brookdale Ave U/P structure and the subsurface model.

DESCRIPTION OF STRUCTURE AND MOVEMENTS

Brookdale Ave. U/P is a four span structure that is supported on north and south abutments and three center piers (see Figure 1).

During the site investigation and subsequent design process, the compressible nature of the foundation soils was identified and it was predicted that the consolidation settlements would be caused by the construction of approach embankments and continue for an extended period of time. However, the subsurface conditions raised concerns that were related to the stability of embankment. The possibility of considerable abutment movements was ruled out based on the fact that the bridge abutments were supported on end bearing H-piles. However, substantial lateral movements in the abutments were

observed after the construction. These movements were such that the bridge abutments displaced away from the bridge deck. The abutment movements that were observed in other ministry bridges due to the settlement of approach embankments were towards the bridge deck. Thus, the movements away from the bridge deck in Brookdale Avenue U/P structure were described as unusual (Stermac et al., 1968).



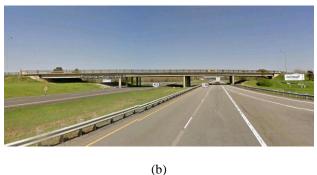


Figure 1. Brookdale Ave. U/P structure (a) subsurface model (b) a view of structure.

Approach Embankment Stability and Settlements

The main concern during the design and construction stages was the prevention of sudden undrained failure under the embankment loads. The original design indicated that the maximum height of the embankment that can be built without a berm was 3.35m with 2H:1V slope. Thus, it was decided that a 6.7m high approach embankment be constructed with 3.35m high and 16m wide berms with 2:1 side slopes. The berms located at the sides of the approach embankment and in front of the abutments are shown in Figures 1 and 2a. Construction of the approach embankment followed pile driving with a consideration that this would allow enough time for the clay remolded due pile driving to recover its original strength.

The settlement points were installed during the construction of approach embankment. Figure 2 shows the plan and cross-section of the settlement points. Figure 3 depicts the measured settlements that lasted over 5 years. The maximum settlement was measured as 1.02m at the highest points of embankment (see Figure 3).

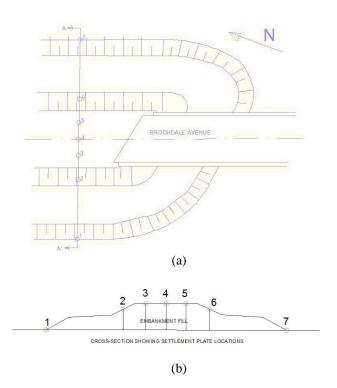


Figure 2. The settlement monitoring for Brookdale Ave. U/P a) plan layout b) A-A' cross-section.

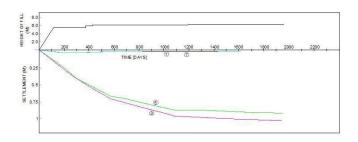


Figure 3. Brookdale Ave. U/P structure settlement monitoring results.

Bridge Distress

Abutments of the structure have continuously moved away from the bridge as the settlement of the approach embankment continued. Such movements have thus caused the tilting of the rocker bearings and severe distortion of the neoprene bearing pads. Figure 4 depicts the rocker bearings on the south abutment of Brookdale Avenue U/P, approximately 900 days after construction (on September 15, 1963). There are no continuous measurements of the abutment movements. However, MTO maintenance records indicate that the lateral movements of the bridge abutments were 19mm as of September 15, 1963 (MTO, 1965). The rocker bearings on the south abutment were repositioned in December 1964. However, due to the ongoing settlements another repositioning

of the bearings was done in autumn 1967.

Stermac et al., (1968) stated that the abutment movements in a direction away from the bridge have not occurred in other MTO structures and described these movements as unusual and area specific. Stermac et al., (1968) provided a qualitative evaluation in order to explain the unusual nature of movements. This evaluation suggests that during the process of elastic-plastic deformations and consolidation, the ground surface under embankment loads takes a dish-like shape. As the consolidation and the accompanying ground surface subsidence progressed, the embankment, resting on the ground, followed this movement by slowly tilting in the direction of the deepest ground. Maintenance work, consisting of additional fill or asphalt placement to bring the embankment to the original grade contributed to this process. Such tilting created a horizontal thrust on the abutment and the supporting piles in a direction away from the bridge.

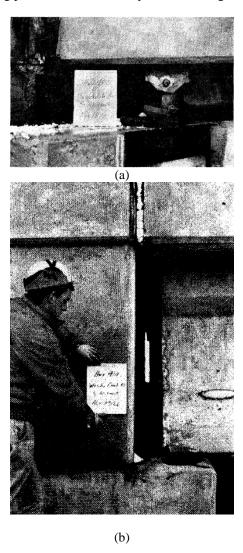


Figure 4. Brookdale Ave. U/P structure rocker bearings distortions (south abutment 900 days after construction).

METHODOLOGY

This section outlines the methodology adopted for the numerical analysis of the interaction between embankment, abutment and foundation piles that took place in Brookdale Avenue U/P structure due to the settlement/yielding of foundation soil.

Plane Strain Simplification

The rows of piles running normal to the section that can be represented using a plane strain model were simplified as an equivalent sheet pile wall in this study. Randolph (1981) proposed that in a plane strain analysis, the structural behavior of a pile row running normal to the section can be represented by an equivalent sheet pile wall. The combined flexural stiffness of piles and soil was assumed to be equal to that of the wall per unit width (Ellis and Springman, 2001). However, since the soil contribution to the total flexural stiffness is very small, it is neglected. It should be noted that the spreading of soft soil between the piles are neglected in this approach.

Problem Geometry

The Brookdale Ave. U/P structure is a four span structure that is supported on two abutments and three centre piers (as shown in Figure 1). Both abutments and piers were supported by 12 BP 53 steel driven piles. A total of 18 piles positioned in 2 rows with 1.5m intervals were used in each abutment. The piles in the front row were inclined towards the highway with a 4V:1H inclination. The piers were also supported on 28 vertical piles positioned in two rows. The height of approach embankment was 6.7 m. The embankment slopes were stabilized using berms that were 3.5m high. The berms were constructed both at the sides and in front of the embankment.

Constitutive Soil Behavior

The constitutive behaviour of the embankment fill, glacial till and bedrock were modeled using an elastic-plastic Mohr-Coulomb model. The behaviour of the thick deposit of soft clay was simulated using the soft soil model (SSM) present in PLAXIS 2D. SSM assumes that there is a logarithmic relationship between the volumetric strain, $\varepsilon_{\rm v}$ and mean effective stress, p'. The relationship between the p' and the volumetric strain is as follows;

$$\varepsilon_{v} - \varepsilon_{v}^{0} = -\lambda^{*} \ln \left(\frac{p'}{p^{0}} \right)$$
 [1]

in order to maintain the validity of Equation [1], the value of p' is set to a unit stress. The λ^* parameter in the model determines the initial compressibility of the material and it is

related to the compression index, λ . Unlike λ , which is related to the void ratio, λ^* is a function of volumetric strain.

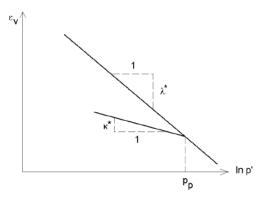


Figure 5. Logarithmic relationship between the volumetric strain and mean stress.

The κ^* is modified swelling index, and determines the compressibility of the material during unloading and reloading. Unlike κ , which is related to the void ratio, κ^* is a function of volumetric strain (see Figure 5). The parameters λ^* and κ^* are related to λ and κ parameters given in Burland (1965). The relationships are described in equations 2 and 3. The e value used in equations 2 and 3 is the average void ratio measured during the odometer test. Figure 6 depicts the yield surface assumed in SSM model.

$$\lambda^* = \frac{\lambda}{1+e} \tag{2}$$

Table 1. Soil Parameters.

Soil Type	Friction Angle, ϕ	Cohesion,	Unit Weight,	Modified compression	Modified swelling	Young Modulus	Poisson's Ratio	Permeabilit y (m/day)*
	(°)	(kPa)	$\rho (kN/m^3)$	index, λ^*	index, κ*	(kPa)		
Embankment Fill	32	0	22	-	-	3E4	0.3	1E-3
Clay - Crust	4	38	18	0.036	0.01	-	-	1E-4
Clay - Soft	4	19.5	17.5	0.042	0.011	-	-	1E-4
Till	35	5	18	-	-	1E5	0.3	1E-4
Concrete	50	0	25	-	-	3E7	0.25	-

^{*} The horizontal and vertical permeability were assumed equal.

Numerical Model and Boundary Conditions

Since the objective of the numerical analyses were to simulate the consolidation of the approach embankment and determine the effect of soil deformations on the pile foundations, a series of finite element analyses based on the solution of coupled stress—diffusion equations were performed. In such analyses, each point of the saturated soil mass is subjected to a total

 $\kappa^* = \frac{\kappa}{1+e} \tag{3}$

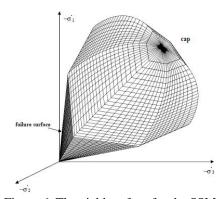


Figure 6. The yield surface for the SSM.

Soil Parameters

The Mohr-Coulomb and SSM parameters used in analyses were summarized in Table 1. The Mohr-Coulomb parameters were interpreted from geotechnical design report. These parameters were based on shear vane test, unconfined compression test and triaxial compression test. A number of odometer tests were also performed to determine the consolidation characteristics of the soft clay. SSM parameters for the clay layers were calculated based on the odometer test and triaxial compression test results.

stress, σ , which is the sum of the effective stress, σ' carried by the soil skeleton, and the pore pressure, u. With the application of external loads, a hydraulic gradient of pore pressure develops between two points within the soil mass. This hydraulic gradient will cause the water to flow. Gradually, the additional stresses are transferred to soil matrix, pore pressures decrease and thus the soil consolidates.

The coupled finite element analyses presented in this study were carried out using commercial finite element program PLAXIS 2D. Figure 7 shows the FE model used in the analyses. The approach embankment, bridge abutment and the foundation soil were modeled using 15-node plain strain elements. The piles were modeled using linear elastic beam elements fully bonded with the soil (e.g. separation and slippage are not allowed). Fixed head conditions were considered for the pile-cap connection. The lateral boundaries were extended 50 m from the approach embankment at each direction to minimize boundary effects.

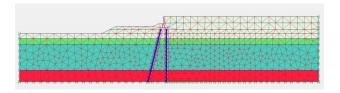


Figure 7. The Finite Element Model.

Initially, the native ground stratigraphy was constructed and the effective self-weight was applied to all layers. The top surface of the clay layer was assumed to be permeable, whereas the glacial till layer underlying clay layer was considered to be impermeable. The initial step was a geostatic step that established the equilibrium within the clay layer. During the subsequent steps, the installation of piles and staged construction of the approach embankment were simulated. The model allowed sliding between the piles and surrounding soft soils.

The embankment soil was assumed to have a larger permeability compared to the clay layer. After the completion of the staged construction, the model was analyzed for a total analysis time of 2000 days.

RESULTS AND DISCUSSION

The results presented in this study show the interaction mechanism between the approach embankments, compressible foundation soil and the bridge supported on piles. Finite element analyses were performed to simulate the soil-structure-interaction phenomenon that led to the movement of abutments. The results obtained from the FE analyses were compared with field monitoring results.

Foundation Soil Settlements

The field measurement performed using settlement plates were depicted in Figure 3. The results shown in Figure 3 indicate that a maximum total settlement of 1.02m and 0.92m took place within the duration of 1900 days at plates 3 and 5, respectively. Figure 8 shows the variation of vertical settlement simulated using the numerical model at the location

of maximum depression. The results indicate that the maximum settlement calculated using numerical model was 0.93m at the surface of clay layer. The results show that a good agreement was obtained between the measured and simulated magnitude and time rate of settlements. It should be noted that the ultimate consolidation settlement calculated using Terzaghi's 1D consolidation theory was 0.74m. This indicates that embankment loads also resulted in some yielding.

Significant lateral movements in soft clay layer were also observed. Figure 9 depicts the distribution of the lateral soil movements. The results depicted in Figure 9 show that a maximum lateral displacement of 0.5m was created by the embankment loads. There are no field measurements of the lateral movements that took place. However, larger values should be anticipated due to creep effects.

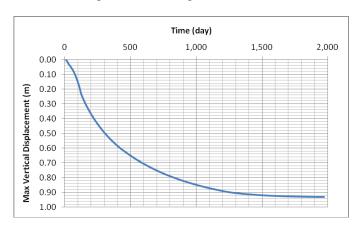


Figure 8. Simulated Maximum Settlement.

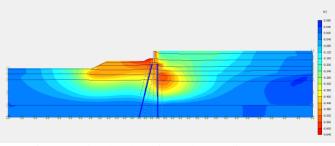


Figure 9. Simulated Horizontal Spreading Contours.

Embankment Deformations

Figure 10 shows the total deformations contours and deformed shape of the embankment surface took place in compressible foundation. The deformations on the embankment surface resulted in a concave shape. The largest vertical deformation of this depression was 1.1 m as shown in Figure 10. This depression is approximately 10 m from the abutment. Thus, the resultant force on the abutment is in the direction away from the bridge. These observations are in line with the study by Stermac et al. (1968).

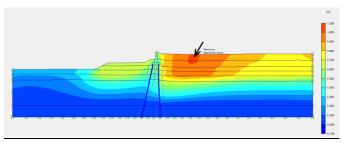


Figure 10. Simulated Total Displacement Contours.

Abutment Movements

The results indicate that the abutment movements were caused by the deformation of foundation soil and embankment fill. Figure 10 depicts the deformed shape of the abutment and piles. Since the complete field notes taken during remedial work were not available, the only field measurement regarding the abutment movements remain to be the 19mm abutment movements away from the bridge, which was noted during 1964 remediation. Thus, it was not possible to make a direct comparison between the simulated values and actual movements. However, the results obtained from FE analyses indicate that the abutment movements away from the bridge also took place in the numerical analyses (See Figure 10). Simulating the mode of deformations away from the bridge is considered to be a sufficient outcome given the simplified plain strain modeling approach of the piles, which neglects the spreading of soft soil between the piles, absence of the bridge deck and bridge loads in the model and missing monitoring history of abutments.

SUMMARY AND CONCLUSIONS

The static behavior of embankments constructed on soft compressible foundation at Brookdale Avenue U/P was investigated with a particular focus on the abutment movements observed after the completion of construction. The following are the conclusions arising from the results obtained in this study:

- The lessons learned from Brookdale Avenue U/P
 case led to stringent requirements for the construction
 of approach embankments which may include
 solutions such as preload/surcharge, light weight fills
 and wick drains. These practices prevented
 reoccurrence of similar problems.
- The Brookdale Avenue U/P case showed that the embankments constructed on highly compressible foundations may result on large vertical and lateral forces on the piles which may lead to movements of abutments and thus leading to costly remedial measures.
- The construction of a 6.7m high embankment fill on a 10.5m thick soft clay layer resulted in consolidation settlements ranging from 0.92 to 1.02m. The simple

SSM model was used in a coupled analysis to estimate the time dependent consolidation behavior of the embankment using the parameters derived from laboratory tests. A maximum settlement of 0.93m was estimated using FE model, which simulated the construction stages including the grade raise of embankment at day 350. The results showed that large settlements at the approach embankments could be simulated with good accuracy using simple SSM model.

- The deformations on the embankment surface caused by consolidation of compressible foundation soil resulted in a concave shape. The largest vertical deformation of this depression was 1.1 m at the embankment surface, which was 10 m away from the abutment. This caused the resultant force on the abutment to be in the direction away from the bridge. These observations are in line with those presented in Stermac et al. (1968).
- The abutment deformations estimated from the FE analysis showed similar trend that was observed in the field. Estimating the mode of deformations is considered to be a sufficient outcome given the simplified plain strain modeling approach of the piles, which neglects the spreading of soft soil between the piles, absence of the bridge deck and the bridge loads in the model.

ACKNOWLEDGEMENTS

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