06 Apr 1995, 1:30 pm - 3:00 pm

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Proceedings: Third International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, April 2–7, 1995, Volume I, St. Louis, Missouri

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Paper No. 6.08

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SYNOPSIS: A simplified and efficient numerical model is developed to perform the phenomenological elasto-plastic seismic response analysis of earth dams. The method is based on a two-dimensional Galerkin formulation of the equations of motion for the dam material, and accounts for the presence of water inside the dam. The soil skeleton nonlinear hysteretic response is modelled using an effective stress multi-yield function elasto-plastic constitutive model. The model is validated using actual transient ground motions as input and the computed responses are compared with the motions recorded at the respective sites. The ability of the model to simulate the occurrence of liquefaction in a dam is also demonstrated.

INTRODUCTION

Over the last few years much emphasis has been placed, especially in the U.S.A., on the safety of earth dams subjected to earthquakes (Gazetas 1987). Such an interest is justified by the huge losses of life and property that can result from the catastrophic failure of a dam.

The dynamic analysis of earth dams is a very complicated problem. First, dams are large nonhomogeneous structures and, in general, the material they are composed of is a multi-phase solid-fluid medium. Second, the behavior of the soil skeleton of the dam is highly nonlinear, anisotropic and hysteretic. Finally, the interaction of the dam with its boundaries and the water in the reservoir further complicate the problem. Therefore, the solution of such a problem involves many simplifying assumptions.

However, recent advances in the modelling of fluid-saturated porous media and inelastic soil behavior have enabled researchers to relax some of these assumptions. Because of the complexity of the problem, numerical methods can, in general, account for more factors affecting the response of a dam than analytical solutions. This, coupled by the development of more powerful computers, has led to a growing need for numerical techniques which can model efficiently and accurately the seismic behavior of earth dams.

The objective of this research effort is to develop a model that combines the simplicity and computational efficiency of an elasto-plastic Galerkin-type formulation (Elgamal et al., 1985) and the state-of-the-art features of a finite element dam model considered both a hysteretic and a two-phase system (Lacy and Prevost, 1987).

SIMPLIFYING ASSUMPTIONS

The following assumptions are made hereafter:

1. The dam is an infinitely long symmetrical triangular wedge.
2. Only upstream-downstream shear and vertical normal deformations can take place. Therefore, the only stresses present are the upstream-downstream shear stress τ_{xy} and the vertical normal stress σ_y. The deformations and the stresses are assumed uniformly distributed over every horizontal cross section.
3. The dam is composed of homogeneous material soil layers.
4. The dam rests on a rigid foundation and is subjected to uniform upstream-downstream and vertical ground motions.
5. The phreatic line inside the dam is either parabolic or horizontal and the material below it is saturated and above it dry.

6. The seepage flow in the dam is laminar and the porosity remains constant with time.
7. The static pressure imposed by the reservoir water on the upstream face of the dam is negligible, because its slope is fairly flat.
8. The effect of the truncated top of the dam is insignificant.

Figure 1 shows the proposed two-dimensional model of earth dams of infinite length, which are composed of a saturated and a dry part (Yiagos and Prevost, 1991a).

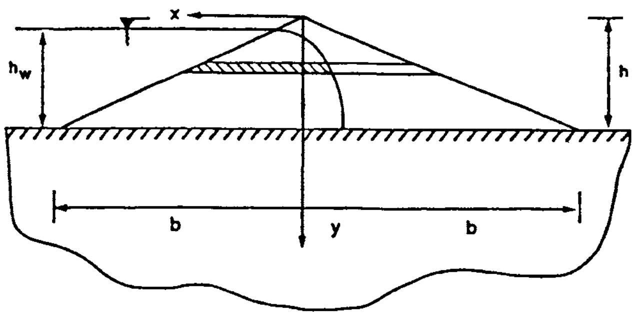

Fig. 1. Two-dimensional model of earth dams of infinite length

MODELLING OF THE POROUS MEDIUM

The two-dimensional consideration of the problem, as opposed to the one-dimensional, is deemed necessary, since for saturated soil deposits of moderate permeability in which drainage can take place, horizontal motions are in general accompanied by vertical motions. Furthermore, the analysis is conducted with fully coupled equations for the soil solid skeleton and the pore water so that their interaction introduces an additional element of damping, apart from the hysteretic damping of the solid skeleton. Finally, in contractive saturated fine sands pore water pressure builds up during the earthquake resulting in a loss of the strength of the soil and thus rendering it vulnerable to liquefaction. Therefore, an effective stress analysis should be conducted, in which the build-up in the pore water pressure and its dissipation with time are computed and their effects on the dynamic response are taken into account.

The saturated soil material is viewed herein as a two-phase system consisting of a solid and a fluid phase. When the movement of the solid phase is used as the reference motion, the governing equations of motion for the fluid-saturated porous medium are the following (Prevost, 1985b):

$$y\rho^s \frac{\theta^2 u_x^s}{\theta t^2} + y\zeta\left(\frac{\theta u_x^s}{\theta t} - \frac{\theta u_x^w}{\theta t}\right) - \frac{\theta}{\theta y}(y\tau_{xy}) + n^s \frac{\theta}{\theta x}(y p_w) = 0 \quad (1a)$$

$$y\rho^s \frac{\theta^2 u_y^s}{\theta t^2} + y\zeta\left(\frac{\theta u_y^s}{\theta t} - \frac{\theta u_y^w}{\theta t}\right) - \frac{\theta}{\theta x}(y\tau_{xy}) - \frac{\theta}{\theta y}(y\sigma'_y) + n^s \frac{\theta}{\theta y}(y p_w) - y\rho^s b_y = 0 \quad (1b)$$

$$y\rho^w \frac{\theta^2 u_x^w}{\theta t^2} - y\zeta\left(\frac{\theta u_x^s}{\theta t} - \frac{\theta u_x^w}{\theta t}\right) = 0 \quad (1c)$$

$$y\rho^w \frac{\theta^2 u_y^w}{\theta t^2} - y\zeta\left(\frac{\theta u_y^s}{\theta t} - \frac{\theta u_y^w}{\theta t}\right) + n^w \frac{\theta}{\theta y}(y p_w) - y\rho^w b_y = 0 \quad (1d)$$

$$p_w = p_o - \lambda^w\left[\left(\frac{\theta u_x^w}{\theta x} + \frac{\theta u_y^w}{\theta y}\right) - \frac{n^s}{n^w}\left(\frac{\theta u_x^s}{\theta x} + \frac{\theta u_y^s}{\theta y}\right)\right] \quad (1e)$$

where x and y are the horizontal and the vertical coordinates respectively, u_x^s and u_y^s the unknown horizontal and vertical displacements of the solid phase respectively, u_x^w and u_y^w the unknown horizontal and vertical displacements of the fluid phase respectively, σ'_y the vertical normal effective stress, τ_{xy} the horizontal shear stress, p_w the total pore pressure, p_o the static pore pressure, b_y the gravity force per unit mass, ζ the drag coefficient, λ^w the fluid bulk modulus, ρ^s and ρ^w the macroscopic average densities of the solid and fluid phases respectively, and n^s and n^w the respective fractions of the elemental surface occupied by each phase.

In the dry part of the dam, the above equations simplify as follows:

$$y\rho^s \frac{\theta^2 u_x^s}{\theta t^2} - \frac{\theta}{\theta y}(y\tau_{xy}) = 0 \quad (1f)$$

$$y\rho^s \frac{\theta^2 u_y^s}{\theta t^2} - \frac{\theta}{\theta x}(y\tau_{xy}) - \frac{\theta}{\theta y}(y\sigma'_y) - y\rho^s b_y = 0 \quad (1g)$$

GALERKIN FORMULATION

It is convenient to write the solution to equations (1) as:

$$u_i^a = v_i^a + w_i^a \quad (2)$$

where "i" denotes the direction and "a" the phase, while w is the relative displacement and v the prescribed ground motion. The boundary conditions are:

$$w = 0 \quad (3a)$$

at the base, and

$$\tau_{xy} = \sigma'_y = p_w = 0 \quad (3b)$$

on the inclined surfaces of the dam.

The Galerkin equation is obtained by approximating the displacement w_i^a as follows (Elgamal, Abdel-Ghaffar and Prevost, 1985):

$$w_i^a = \sum_{n=1}^{N} [\phi_n]^a [q_n(t)]_i^a \quad (4)$$

where φ_n are basis functions, which satisfy the homogeneous boundary condition at the base of the dam, $q_n(t)$ the unknown time-dependent generalized modal displacements, and N the number of basis functions used to approximate the solution.

The normal eigenmodes of the vibrating linearized homogeneous triangular wedge are selected as the set of basis functions and are given by:

$$\phi_n = J_o[b_n(\frac{y}{h})] \quad (5a)$$

where J_o is the Bessel function of the first kind and zero order, and h the height of the dam, while b_n is given by the equation:

$$J_o(b_n) = 0 \quad (5b)$$

Integrating the equations (1) along the height of the dam, taking advantage of the boundary conditions described by equations (3) and substituting equations (2), (4) and (5a), the following matrix form of the boundary value problem is obtained:

$$M\ddot{q} + C\dot{q} + s = p + b \quad (6)$$

where M is the mass matrix, C the damping matrix, q the vector of unknown displacements, s the vector of internal forces, p the vector of external forces and b the vector of gravity forces, while the dot denotes differentiation with respect to time.

The spatial integration of the equations is performed by dividing the domain in an adequate number of zones in order to capture the modal wave lengths and using Gaussian quadrature.

Equation (6) describes a system of (4xN)x(4xN) coupled nonlinear equations, which is solved through the use of a finite difference algorithm belonging to the Newmark family of methods (Prevost, 1985b).

In this analysis, the initial linear part of the stiffness is treated implicitly, whereas the subsequent nonlinear elasto-plastic internal restoring force is treated explicitly.

The gravity normal stresses and pore water pressures are computed at the beginning of the calculations by numerically obtaining the purely diffusive part (consolidation part) of the solution.

CONSTITUTIVE ASSUMPTION

The computation of the stresses σ'_y and τ_{xy}, which contribute to the vector s, is performed through a two-dimensional effective stress elasto-plastic constitutive model, belonging to the family of Mohr-Coulomb models and based on the multi-yield function kinematic plasticity theory (Prevost, 1985a, and Yiagos and Prevost, 1991a). The model can simulate observed nonlinear shear hysteretic behavior, as well as shear stress induced anisotropic effects and reflects the dependency of the shear dilatancy on the effective stress ratio. The model is applicable to both cohesive and cohesionless soils. The parameters used in this model can be derived entirely from the results of conventional triaxial soil tests.

The yield function is selected of the form (Prevost, 1985a):

$$f = |\bar{\tau} - \alpha\bar{\sigma}| + M\,\bar{\sigma} = 0 \tag{7}$$

where $\bar{\sigma} = \sigma'_y - a$, a is the attraction ($a = c/\tan\varphi$, c is the cohesion, φ is the friction angle), α the offset of each yield function along the shear stress axis with respect to the origin and M a material parameter associated with the size of each yield function. Figure 2 shows a yield function. A thorough identification of the model parameters is given by Yiagos and Prevost (1991a).

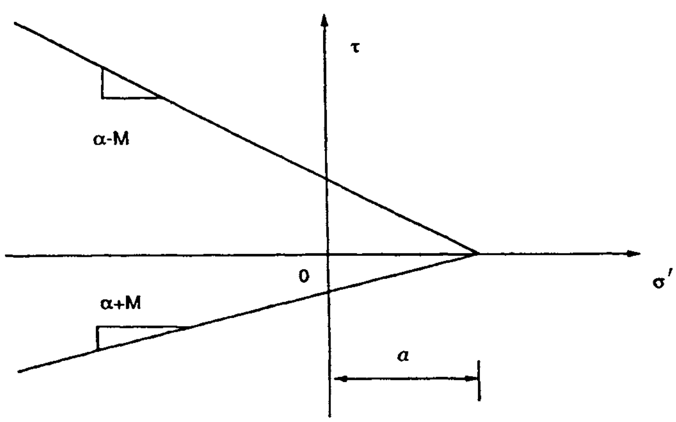

Fig. 2: Yield function

In order to anchor the model to any experimental data, a collection of nested yield functions, having the same origin, is used. Each of them is associated in stress space with a linear segment of the initial smooth loading shear stress-strain or skeleton curve. In the following the skeleton curve is selected to be a modified hyperbolic curve (Prevost and Keane, 1990).

The dependence of the model moduli A (λ_s, G_o or the plastic modulus H') upon the effective normal stress is assumed of the following type:

$$A = A_{(initial)} \left(\frac{\sigma'_y}{\sigma'_{y\,(ref)}} \right)^n \tag{8}$$

where $A_{(initial)}$ the initial value of the respective modulus, $\sigma'_{y(ref)}$ the reference (initial) effective normal stress and n an experimental parameter.

The so-called "cutting-plane" algorithm, a stress point numerical procedure of explicit nature developed by Ortiz and Simo (1986), is employed to integrate the resulting elasto-plastic constitutive equations.

APPLICATIONS

The Santa Felicia Dam

The Santa Felicia is a modern rolled-fill earth embankment built in 1954-55 and is located in Ventura County, California, about 65 Km northwest of Los Angeles. The dam is 83 m (272.4 ft) high above its lowest foundation and 326 m (1070.0 ft) wide. The dam also has an average height of 72 m (236.3 ft), an average length of 278 m (912.5 ft), and the crest is 9 m (29.5 ft) wide and 388 m (1273.5 ft) long. The upstream and downstream faces slope at 2.25:1 and 2:1, respectively. The dam has an impervious core that rises from the bedrock with slopes of 0.33:1. It is assumed that the height of the water in the reservoir is 60 m (196.9 ft).

The Santa Felicia dam is made of an impervious clay core covered by a sand and gravel shell on a stiff foundation layer of gravel and sand down to the bedrock. The present analysis makes no distinctions between the core and the shell materials, and assumes that the dam is composed be essentially cohesionless materials.

The values of the material properties employed in the calculations are given in Table 1 (Abdel-Ghaffar and Scott, 1981).

Table 1. Soil properties of the Santa Felicia dam

Density	2,177 kg/m^3
Porosity	0.30
Permeability	10^{-6} m/s
Elastic base shear modulus	164,644 kPa
Poisson's ratio	0.40
Fluid bulk modulus	2.14x10^6 kPa
Initial shear stress	0.0
Maximum shear strain	0.015
Cohesion	0.0
Peak friction angle	30°
Dilation angle	25°
Parameter of equation (8)	0.50

The value of the shear modulus G varies along the height of the dam according to the formula:

$$G = G_0 \left(\frac{y}{h}\right)^{0.33} \qquad (9)$$

where G_0 is the elastic base shear modulus.

The linear natural frequencies of the dam in the upstream-downstream direction are computed using a ten basis function expansion and the first three are equal to 1.477, 3.184 and 4.899 Hz, respectively. These values are in good agreement with those reported in the literature. In case of an one basis function expansion, the value of the first linear natural frequency is equal to 1.487 Hz, which is very close to the one calculated through the ten basis function expansion. The slight underestimation of the natural frequencies by the proposed model can be attributed to the presence of relatively rigid abutments, which create a three-dimensional stiffening effect.

In the following one basis function is used to compute the response of the dam. A modified hyperbolic shear-strain curve is generated for each stress point about its reference pressure and 15 yield functions are used to model the skeleton curve at each of these points.

The dam is subjected to the input ground motion recorded at the Santa Felicia dam site, near the outlet works, during the 1971 San Fernando earthquake (M=6.3). The first 16 seconds or 800 time steps of the recorded acceleration are used with data at 0.02 second intervals, and peak accelerations of 0.217 g in the upstream-downstream direction and 0.065 g in the vertical direction. Both components of the ground motion are applied as prescribed uniform input acceleration to both phases at the base of the dam.

Figure 3 shows the horizontal component of the computed and recorded absolute acceleration response at the crest of the dam. The solid line is used for the computed response.

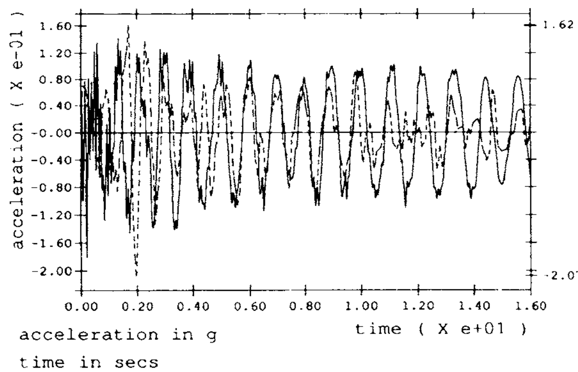

Fig. 3. Computed and recorded absolute acceleration in the upstream-downstream direction at the crest of the Santa Felicia dam (the solid line is used for the computed response)

Despite the simplicity of the model and the minimal computational time required, the comparison of the two responses is very satisfactory, both in the time as well as in the frequency domain. However, it should be noted that the frequency content of the computed acceleration response in the vertical direction does not match the frequency content of the recorded acceleration response, although the magnitude is essentially predicted (Yiagos and Prevost, 1991b).

The Long Valley Dam

The Long Valley earth dam was constructed during the 30's and is located in a canyon in the Mammoth Lake area in California, approximately 240 miles north of the city of Los Angeles (Griffiths and Prevost, 1988). The dam has a length of 182 m (596 ft) at its crest, a maximum height of 55 m (180 ft), a width at its mid-section of 280 m (920 ft) and upstream and downstream slopes equal to 3:1. We assume that the height of the water in the reservoir is 50 m.

The Long Valley dam is composed of an extensive rolled earth-fill impervious core, that constitutes its major portion, and compacted embankments of a more permeable material. The present analysis makes no distinction between the core and the shell materials and assumes that the dam is composed of essentially cohesive materials.

The values of the material properties employed in the calculations are given in Table 2 (Griffiths and Prevost, 1988).

Table 2. Soil properties of the Long Valley dam

Density	2,110 kg/m^3
Porosity	0.30
Permeability	6.7x10^{-8} m/s
Elastic base shear modulus	224,138 kPa
Poisson's ratio	0.35
Fluid bulk modulus	2.14x10^6 kPa
Initial shear stress	0.0
Maximum shear strain	0.010
Cohesion	45 kPa
Peak friction angle	40°
Dilation angle	35°
Parameter of equation (8)	0.50

The value of the shear modulus G varies along the height of the dam according to the formula:

$$G = G_o \left(\frac{y}{h}\right) \tag{10}$$

where G_o is the elastic base shear modulus.

The linear natural frequencies of the dam in the upstream-downstream direction are computed using a ten basis function expansion and the first two are equal to 1.987 and 3.387 Hz, respectively. These values are in good agreement with those reported in the literature. In case of a two basis function expansion, the value of the first linear natural frequency is equal to 2.012 Hz, which is very close to the one calculated through the ten basis function expansion.

In the following two basis functions are used to compute the response of the dam. A modified hyperbolic shear-strain curve is generated for each stress point about its reference pressure and 15 yield functions are used to model the skeleton curve at each of these points. The three top stress points are assumed to be of a cohesionless nature.

The dam is subjected to the input ground motion recorded downstream at the outlet of the dam during the sixth of the 1980 Mammoth Lake earthquakes. The first 12 seconds or 600 time steps of the recorded acceleration are used with data points at 0.02 second intervals and peak acceleration equal to 0.135 g in the upstream-downstream direction and to 0.084 g in the vertical direction. Both components of the ground motion are applied as prescribed uniform input acceleration to both phases at the base of the dam.

Figure 4 shows the horizontal component of the computed and recorded absolute acceleration response at the crest of the dam. The solid line is used for the computed response.

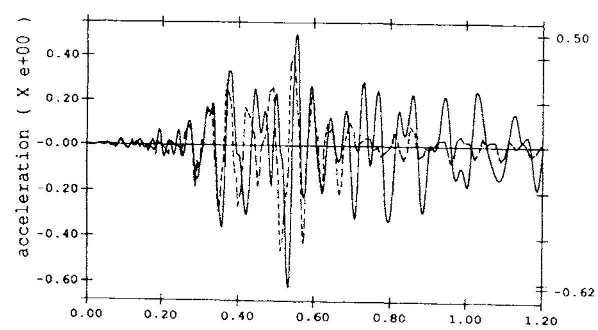

Fig. 4. Computed and recorded absolute acceleration in the upstream-downstream direction at the crest of the Long Valley dam (the solid line is used for the computed response)

The comparison of the computed and the recorded acceleration responses in the upstream-downstream direction at the crest of the dam is very favorable for the proposed model, despite the minimal computational time required. Also the peak value and the frequency content of the acceleration response in the vertical direction are predicted satisfactorily (Yiagos and Prevost, 1991b).

LIQUEFACTION CASE

The proposed model is employed to evaluate the liquefaction potential of a dam with a height of 55 m (180 ft). The height of the water in the reservoir is taken equal to 50 m (164 ft). The dam is assumed to consist of a purely contractive cohesionless material.

In the following two basis functions are used to compute the response of the dam. A hyperbolic shear stress-strain curve with zero slope at maximum shear stress is generated for each of the 40 stress (integration) points about their respective static (reference) pressures, while 15 yield functions are used to model the aforementioned skeleton curve at each of these points. The values of the material properties employed in the calculations are given in Table 3.

TABLE 3. Soil properties of the model dam

Density	2,110 kg/m^3
Porosity	0.30
Permeability	6.7x10^{-8} m/s
Elastic base shear modulus	224,138 kPa
Poisson's ratio	0.35
Fluid bulk modulus	2.14x10^6 kPa
Initial shear stress	0.0
Maximum shear strain	0.010
Cohesion	0.0
Peak friction angle	40^o
Dilation angle	40.5^o
Parameter of equation (8)	0.50

The value of the shear modulus varies along the height of the dam according to equation (10).

The dam is subjected to the input ground motion recorded at the outlet of the Long Valley dam during the sixth of the 1980 Mammoth Lake earthquakes. The peak acceleration is equal to 0.135 g in the upstream-downstream direction and to 0.084 g in the vertical direction. Both components of the ground motion are applied as prescribed uniform input acceleration to both phases at the base of the dam.

Figure 5 shows the shear stress-strain history close to the mid-height of the dam. It is evident that the area of the hysteresis loops diminishes gradually and that the shear stress finally becomes equal to zero. Figure 6 shows the vertical effective normal stress - shear strain history close to the mid-height of the dam. The vertical effective normal stress attains its maximum possible value by becoming equal to zero, while the pore pressure builds up without any subsequent dissipation.

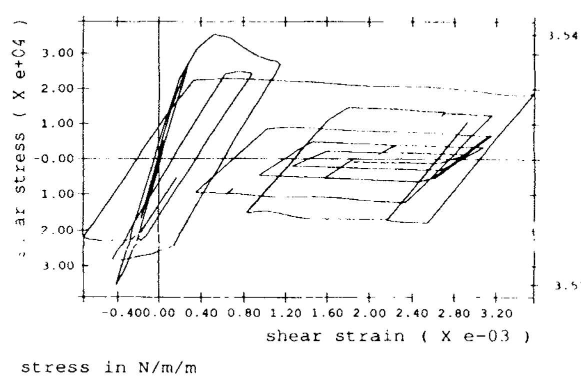

Fig. 5. Shear stress-strain history close to the mid-height of the dam

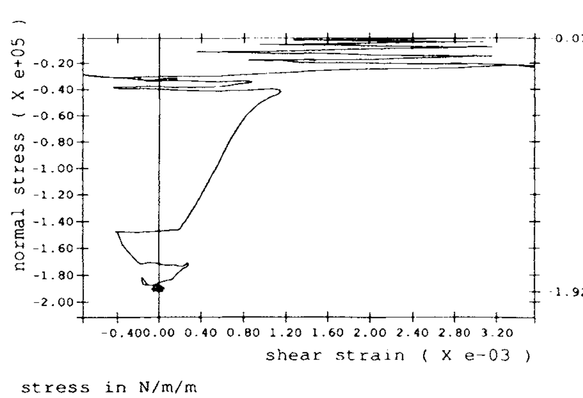

Fig. 6. Vertical effective normal stress - shear strain history close to the mid-height of the dam

Figure 7 shows the distribution of the shear stress along the height of the dam following 12 seconds of shaking. The shear stress has become equal to zero close to the mid-height of the dam and therefore the soil material in this area has liquefied.

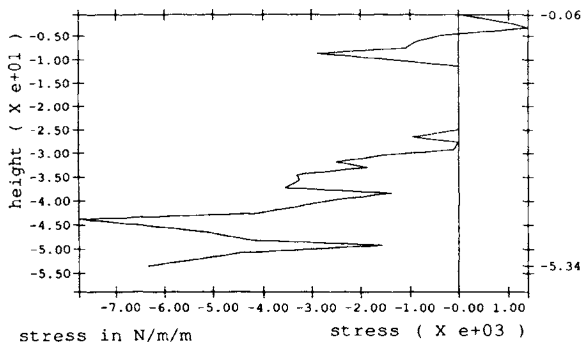

Fig. 7. Distribution of the shear stress along the height of the dam following 12 seconds of shaking

CONCLUSIONS

The proposed model is an easy-to-use tool for the realistic evaluation of the seismic response of earth dams as well as their liquefaction potential. In addition, its computational efficiency renders it attractive for the performance of parametric studies and stochastic analyses. A procedure like this can be also easily extended to provide a reliable tool for analyzing the dynamic response of general geotechnical structures that are both nonlinear and two-phase systems.

ACKNOWLEDGEMENTS

The author would like to express his sincere appreciation to Prof. J.-H. Prevost of Princeton University for his guidance during the course of this work.

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