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SYNOPSIS: This paper presents a three–dimensional boundary element formulation for the steady–state dynamic analysis of fluid saturated porous media. The coupled differential equations are derived from the field equations by the application of the Fourier Transform. The boundary integral formulation is obtained by the weighted residual method and the associated fundamental solutions are obtained by the method developed by Kupradze. The boundary element model is used to obtain compliance functions of square rigid foundations bonded to the surface of a water–saturated half–space. The effect of the soil permeability on the variation of the compliance functions is also examined.

INTRODUCTION

Dynamic poroelasticity deals with the interactions that take place between the constituents of a fluid–saturated medium under dynamic loads. The three–dimensional theory of this subject was first introduced by Biot (1956,1962). According to Biot's theory, a dynamic disturbance in a saturated medium produces one shear wave as well as two pressure waves. The shear and one of the pressure waves display characteristics which are essentially similar to those of elastodynamics. The second pressure wave is a highly–damped wave which is associated with the out-of-phase motions of the constituents.

Biot's equations use the solid and fluid displacements as the main variables. For the solution of practical boundary value problems, however, these equations are frequently recast in terms of the soil displacement field and the fluid pressure. Such formulations have been utilized by Bonnet (1987), Boutin et al. (1987), and Kaynia (1990) to derive the fundamental solutions of dynamic poroelasticity and by Zienkiewicz and Shiomi (1984), Zienkiewicz et al. (1987), Bougacha and Tassoulas (1991) and Hirai (1992) to solve geomechanics problems by the finite element method. More recently, Cheng et al. (1991) and Kaynia and Khoee (1992) developed boundary integral formulations for the steady–state vibrations of saturated porous media.

Impedances of surface foundations on fluid-saturated soil media have been calculated by Halpern and Christiano (1986), Gazetaz and Petrakis (1987) and Philippacopoulos (1989). These researchers used Biot's $(u-w)$ model in their studied. More recently, Kaynia and Khoee (1992) developed a two–dimensional boundary element $(u-p)$ model to calculate the impedances of strip foundations on a half plane.

The purpose of this paper is to present a three–dimensional boundary element model for poroelastodynamics for the calculation of compliance functions of rigid square foundations on a fluid–saturated half space. This model which is essentially similar to that developed by Suh and Tosaka (1988), is obtained by the application of the weighted residual method to the governing field equations of dynamic poroelasticity.

FIELD EQUATIONS

The basic field equations of poroelastodynamics expressing, respectively, the conservation of total momentum, the flow conservation for the fluid–phase, the constitutive equation and the generalized Darcy's law are :

$$\sigma_{ij,j} + f_i = \rho\ddot{u}_i + \rho_f\ddot{w}_i \qquad (1)$$

$$\dot{w}_{k,k} + \alpha\dot{u}_{k,k} + \frac{1}{Q}\dot{p} = q \qquad (2)$$

$$\sigma_{ij} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} - \alpha p\delta_{ij} \qquad (3)$$

$$p_{,i} = -\frac{\gamma}{k}\dot{w}_i - \rho_f\ddot{u}_i - m\ddot{w}_i \qquad (4)$$

where λ and μ are Lame' constants, p, u and w are the pore pressure, solid displacement and average fluid displacement; ρ_f and ρ are the mass densities of the fluid and solid-fluid mixture, and $m = \rho_f/n$ is another mass parameter; $\alpha = 1 - K_d/K_s$, $Q = [n/K_f + (\alpha - n)/K_s]^{-1}$ are compressibility parameters; K_f, K_s and K_d denote the bulk moduli of the fluid, solid grains and solid skeleton, and n, k, f and Q are the porosity, coefficient of permeability, the body force, and the rate of fluid injection, respectively.

Under steady–state harmonic vibration with frequency ω one can eliminate w from equations (1)-(4) to arrive at the following differential equations :

$$(\lambda+\mu)\bar{u}_{j,ij} + \mu\bar{u}_{i,ji} - \alpha_1\bar{p}_{,i} + \omega^2\rho_1\bar{u}_i + \bar{f}_i = 0 \qquad (5)$$

$$\xi\bar{p}_{,ii} - \frac{i\omega}{Q}\bar{p} - i\omega\alpha_1\bar{u}_{i,i} + \bar{q} = 0 \qquad (6)$$

where $\xi = (\frac{\gamma}{k} + i\omega m)^{-1}$, $\rho_1 = \rho - i\omega\rho_f^2\xi$, $\alpha_1 = \alpha - i\omega\rho_f\xi$ and overbar denotes the amplitude of time–dependent quantities. The transformed displacemants $\bar{u}_i$ and pore pressure $\bar{p}$ are defined by :

$$\bar{u}_i(x,\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t}u_i(x,t)dt$$
$$\bar{p}(x,\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t}p(x,t)dt \qquad (7)$$

It is convenient to rewrite the transformed coupled differential equation system into the following matrix form :

$$L_{ij}\bar{U}_j = \bar{B}_i \qquad (8)$$

where L_{ij} denotes the matrix of the transformed differential operator and the force vector $\bar{B}_i$ and vector $\bar{U}_j$ for three dimensional case are given by :

$$L_{ij} = \begin{pmatrix} \mu\Delta + (\lambda+\mu)D_1^2 + \omega^2\rho_1 & (\lambda+\mu)D_1D_2 & (\lambda+\mu)D_1D_3 & -\alpha_1 D_1 \\ (\lambda+\mu)D_2D_1 & \mu\Delta + (\lambda+\mu)D_2^2 + \omega^2\rho_1 & (\lambda+\mu)D_2D_3 & -\alpha_1 D_2 \\ (\lambda+\mu)D_3D_1 & (\lambda+\mu)D_3d_2 & \mu\Delta + (\lambda+\mu)D_3^2 + \omega^2\rho_1 & -\alpha_1 D_3 \\ -i\omega\alpha_1 D_1 & -i\omega\alpha_1 D_2 & -i\omega\alpha_1 D_3 & \xi\Delta - \frac{i\omega}{Q} \end{pmatrix} \quad (9)$$

Also :

$$\bar{U}_j = \{\bar{u}_1 \ \bar{u}_2 \ \bar{u}_3 \ \bar{p}\}^T \quad (10)$$
$$\bar{B}_i = \{-\bar{f}_1 \ -\bar{f}_2 \ -\bar{f}_3 \ -\bar{q}\}^T \quad (11)$$

and $D_i = \partial/\partial x_i$ and Δ denotes the Laplacian.

BOUNDARY INTEGRAL FORMULATION

One may start with the following weighted residual statement of the differential equation (8) for the weighting tensor G^*_{ik} :

$$\int_v (L_{ij}\bar{U}_j - \bar{B}_i)G^*_{ik}dv = 0 \quad (12)$$

Substituting eqns (9)–(11) into eqn (12) and integrating by parts, one obtains the following integral equation :

$$\int_v (A_{ij}G^*_{jk})\bar{U}_i dv(x) + \int_s (\bar{\sigma}_\alpha G^*_{\alpha j} - \bar{u}_\alpha \tau^*_{\alpha j})ds(x) + \int_s (\xi\bar{p}_{,n}G^*_{4j} - \xi\bar{p}G^*_{4j,n})ds(x) - \int_v \bar{B}_i G^*_{ij}dv(x) = 0 \quad (13)$$
$$(i,j,k = 1,2,3,4 \ \ , \ \ \alpha = 1,2,3)$$

where A_{ij} is the adjoint operator of L (Suh and Tosaka (1988)), $\bar{U}_\alpha = \bar{u}_\alpha$ $(\alpha = 1,2,3)$ and $\bar{U}_4 = \bar{p}$, and v and s denote the domain and its boundary, respectively. The traction vector $\bar{\sigma}_\alpha$ and the corresponding vector $\tau^*_{\alpha j}$, associated with G^*, are given by :

$$\bar{\sigma}_\alpha = \bar{\sigma}_{\alpha\beta}n_\beta = \{(\lambda\bar{u}_{k,k} - \alpha_1\bar{p})\delta_{\alpha\beta} + \mu(\bar{u}_{\alpha,\beta} + \bar{u}_{\beta,\alpha})\}n_\beta \quad (14)$$
$$\tau^*_{\alpha j} = \{(\lambda G^*_{kj,k} + i\omega\alpha_1 G^*_{4j})\delta_{\alpha\beta} + \mu(G^*_{\alpha j,\beta} + G^*_{\beta j,\alpha})\}n_\beta \quad (15)$$

The weighting tensor G^*_{jk} must be chosen as the solution which statisfies the differential equation :

$$A_{ij}G^*_{jk} + \delta_{ik}\delta(x-y) = 0 \quad (16)$$

Then the first ferm in eqn (13) can be replaced by $-C_{kj}\bar{U}_k(y)$ where $C_{kj} = \frac{1}{2}\delta_{kj}$ for a smooth boundary and x and y represent field point and singularity point, respectively. Incorporating this result in eqn (13) and assuming zero body force, $\bar{B}$, one obtains the following integral equation in matrix form :

$$\mathbf{C}\bar{\mathbf{U}} + \int_s \mathbf{P}^*\bar{\mathbf{U}}ds = \int_s \mathbf{U}^*\bar{\mathbf{P}}ds \quad (17)$$

where $\mathbf{C}$ is a diagonal matrix and :

$$\mathbf{U}^* = \begin{pmatrix} G^*_{11} & G^*_{21} & G^*_{31} & \xi G^*_{41} \\ G^*_{12} & G^*_{22} & G^*_{32} & \xi G^*_{42} \\ G^*_{13} & G^*_{23} & G^*_{33} & \xi G^*_{43} \\ G^*_{14} & G^*_{24} & G^*_{34} & \xi G^*_{44} \end{pmatrix}, \quad \bar{\mathbf{U}} = \begin{Bmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \\ \bar{p} \end{Bmatrix} \quad (18)$$

$$\mathbf{P}^* = \begin{pmatrix} \tau^*_{11} & \tau^*_{21} & \tau^*_{31} & \xi G^*_{41,n} \\ \tau^*_{12} & \tau^*_{22} & \tau^*_{32} & \xi G^*_{42,n} \\ \tau^*_{13} & \tau^*_{23} & \tau^*_{33} & \xi G^*_{43,n} \\ \tau^*_{14} & \tau^*_{24} & \tau^*_{34} & \xi G^*_{44,n} \end{pmatrix}, \quad \bar{\mathbf{P}} = \begin{Bmatrix} \bar{\sigma}_1 \\ \bar{\sigma}_2 \\ \bar{\sigma}_3 \\ \bar{p}_{,n} \end{Bmatrix} \quad (19)$$

To develop a boundary element formulation one needs to solve eqn (17) numerically. This can be achieved by discretizing the boundary into N segments (elements) and using interpolation functions to define the unknowns in terms of the corresponding nodal values. In the present study triangular constant element, has been utilized. In this case one needs to write eqn (17) for the N nodes to arrive at a system of algebraic equations of order $4N$ in the form :

$$\mathbf{H}_1\bar{\mathbf{u}} + \mathbf{H}_2\bar{\mathbf{p}} = \mathbf{G}_1\bar{\sigma} + \mathbf{G}_2\bar{\mathbf{p}}_{,n} \quad (20)$$

or :

$$\mathbf{H}\bar{\mathbf{U}} = \mathbf{G}\bar{\mathbf{P}} \quad (21)$$

Reorganizing the system of equations [eqn (21)] with respect to the unknown, the final system of equations can be solved with conventional solution techniques.

FUNDAMENTAL SOLUTION

The fundamental solution pertaining to the present boundary integral formulation is the solution of eqn (16) and can be derived by Kupradze (1979) Method (The reader is referred to Kaynia (1992) for details of the derivation). The fundamental solutions for this problem define the solid displacement and fluid pressure due to unit concentrated load and unit rate of fluid injection in the pores.

BOUNDARY ELEMENT IMPLEMENTATION

The standard boundary element procedure can be applied to solve numerically the system of boundary integral equations [eqn (17)]. The integration in eqn (17) can be carried out numerically over each element s_e by using a standard numerical integration scheme such as that developed by Hammer et al. (1956). However, when the source point y coincides the element under consideration, the integral should be treated in a different manner because the fundamental solution contain singularities. In three–dimensional problems, each triangular element is divided into three sub-elements. In each sub–element, integrals can be calculated analytically in a local polar coordinate system by using the following expression :

$$[H_{\alpha\beta}]_{ii} = \sum_{e=1}^{3}\int_{s_e}\tau^*_{\beta\alpha}\,ds = \sum_{e=1}^{3}\int_{-\theta_2}^{\theta_1}\int_0^{R(\theta)}\tau^*_{\beta\alpha}(r)\,r\,dr\,d\theta \quad (22)$$

Consequently, the integration (22) can be performed with using the standard Gaussion quadrature scheme. The results of singular integrals $[H_{\alpha\beta}]_{ii}$ and $[G_{\alpha\beta}]_{ii}$ for the three dimensional case contain integrals with integrands of the form $exp(-hr)/r^a$ $(\alpha = 0,1,2,3)$. These integrals can then be evaluated numerically.

NUMERICAL RESULTS

The preceding boundary element formulation has been used to obtain compliance functions for rigid pervious square foundations bonded to the surface of a saturated half space. The material properties used for the half space are as follows :

$$\lambda = 1.154\times10^8 N/m^2, \ \ \mu = 0.769\times10^8 N/m^2$$
$$Q = 7.5\times10^9 \ N/m^2, \ \ \alpha = 1.0, \ \ n = 0.3$$
$$\rho_f = 1000, \quad \rho = 2190 \ \ \& \ \ m = 3333Kg/m^3$$

The drained elastic properties used for the soil (λ and μ) correspond to a medium with $E = 2.0\times10^8$ N/m^2 and $\nu = 0.3$.

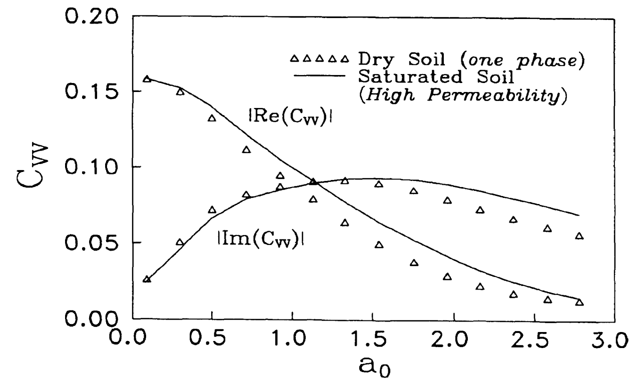
The quantities to be presented are the conpliance functions (or flexibilities) of rigid square foundations for the three modes of horizontal, rocking and vertical motions. Under steady-state conditions each of these functions are frequency-dependent complex quantities which will be presented by their real and imaginary parts. The presented results display the variations of these functions with the nondimensional frequancy $a_0 = \omega b / V_s$, where b is the half-width of the foundation and V_s is the drained shear wave velocity (i.e. $\sqrt{\mu/\rho_s}$)of the medium. The vertical and horizontal compliances are normalized by μb and the rocking compliance is normalized by μb^3 .

Figures 1(a) and 1(b) portray the variations of the vertical compliance of a square foundation for three values of the coefficient of permeability $k = 200, 0.2, 0.02\, cm/sec$. Also plotted in these figures are the results corresponding to an equivalent one phase undrained medium (the solid curve with dots). According to Kaynia and Banerjee (1993) the dynamic behavior of a low-permeability medium can be simulated by an equivalent one-phase medium with the following equivalent elasticity parameters :

$$\lambda_u = \lambda + \alpha^2 Q \quad (23)$$

$$\nu_u = \frac{1}{2}\frac{\lambda + \alpha^2 Q}{\lambda + \mu + \alpha^2 Q} \quad (24)$$

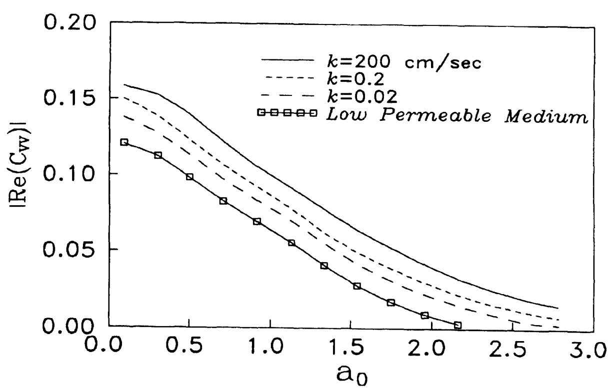
The results in Figures 1(a) and 1(b) suggest that as the soil premeability decrasees, the vertical compliances decrease as well. It is interesting to note that the compliance functions of the one-phase medium constitue lower bound curves for the results associated with different permeabilities. Figure 1(c) compares the real and imaginary pents of the vertical compliances for a highly permeable medium. The results in this figure suggest that the existence of pore water does not play any role on the compliances of a highly permeable medium.



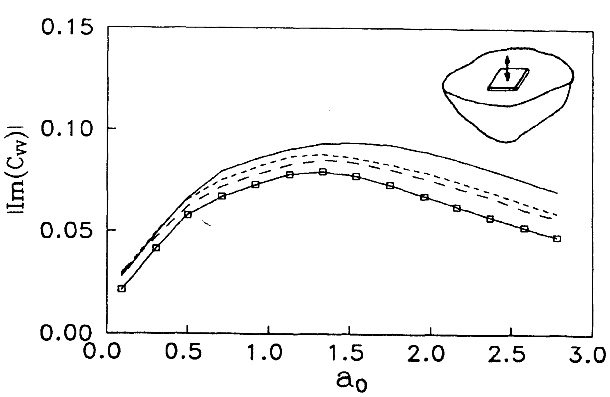
(c)

Fig. 1. Vertical compliance functions; a) Real part, b) Imaginary part, c) Highly-permeable vs. dry soil

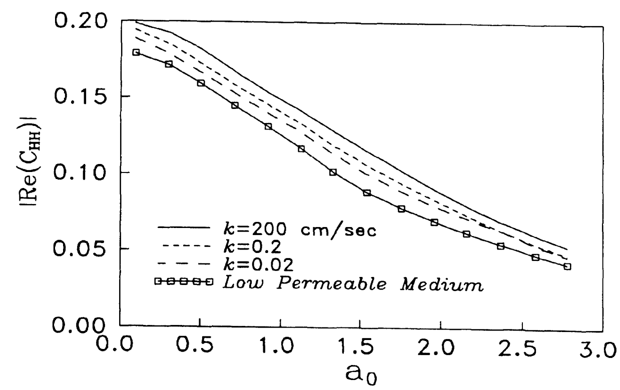
Similar results are presented for the horizontal and rocking compliances in figures 2 and 3. These figures portray the variations with a_0 of the real and imaginary parts of the horizontal and rocking compliances. The results show trends similar to those observed in the vertical case. It is also instructive to note that the compliances of saturated media display characteristics which are essentially simillar to those of dry (one-phase) media. This observation is helpful as it allows one to infer the general features of the compliances from those of dry media.



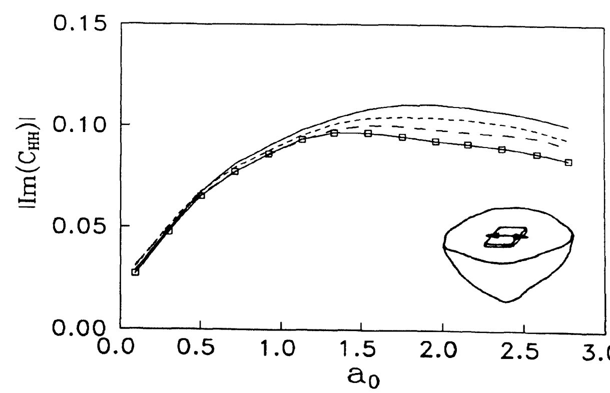
(a)



(b)



(a)



(b)

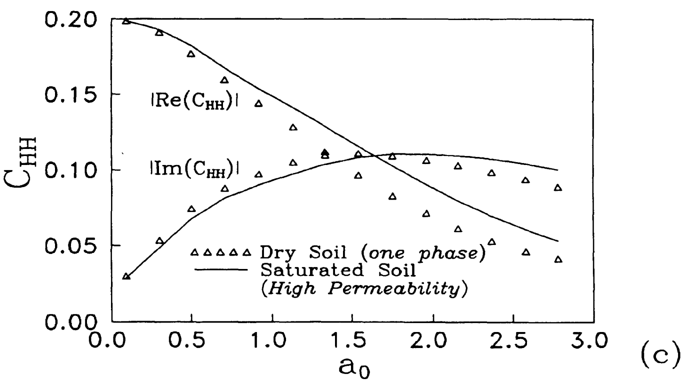


Fig. 2. Horizontal compliance functions; a) Real part, b) Imaginary part, c) Highly-permeable vs. dry soil

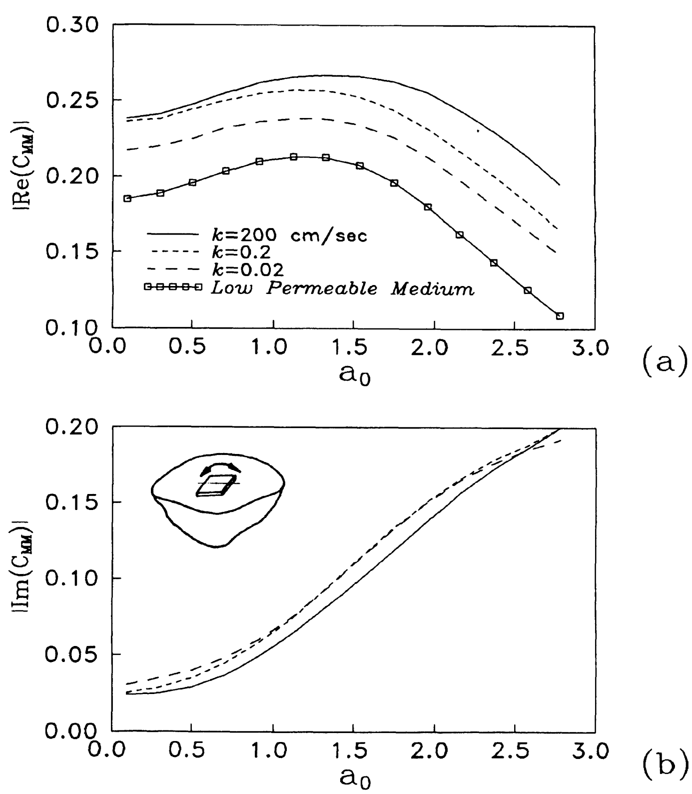


Fig. 3. Rocking compliance functions; a) Real part, b) Imaginary part

CONCLUSIONS

In this paper, a boundary element formulation is presented for obtaining approximate solutions to boundary vlaue problems in the three dimensional theory of dynamic poroelasticity. The integral equation was obtained by applying the weighted residual method to the equations of dynamic poroelasticity and the required fundamental solution was derived by the Kupradze method. As a numerical example, the boundary element model is used to obtain compliance functions of rectangular rigid foundations bonded to the surface of a water-saturated half-space under steady-state vibrations. The limited presented results suggested that an increase in the soil permeability increases the compliances of the foundation. Also, the characteristics of the compliances of a highly permeable saturated medium are essentially similar to those of the corresponding dry medium, In addition, a relatively impermeable medium behaves similarly to a dry medium with equivalent elasticity properties as proposed by Kaynia and Banerjee (1993).

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