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SYNOPSIS: A unified procedure for calculating active and passive earth pressures on retaining structures for static and seismic (pseudostatic) loading conditions is presented. The procedure is based on the limit equilibrium method, uses the method of slices, and satisfies complete statics. Necessary equations for a typical slice are presented and a solution scheme for solving them is discussed. A sample problem is included to indicate convenience of use of the proposed procedure and accuracy of results obtained. The results are in terms of magnitude, direction, and location of lateral thrust on the wall; however, distribution of earth pressure along the wall height is not obtained and the direction of lateral thrust is user specified.

INTRODUCTION

In geotechnical engineering practice, the commonly used procedure for estimating earth pressure on retaining structures is to use the formula $p = \gamma h K$, where γ is the unit weight of backfill material - its value properly adjusted for the design seismic coefficient in the vertical direction, h is the height of the retaining structure, and K is the earth pressure coefficient. Symbolically, $K = K_0$, K_A, or K_P for static conditions of at-rest, active, and passive earth pressures respectively; and $K = K_{AE}$ or K_{PE} for earthquake conditions of active and passive earth pressures respectively. Appropriate values for K are calculated from published formulae or selected from tables of values or their plotted graphs. Lateral thrust on a retaining structure is given by the area of the triangle representing the linear pressure distribution implied by the above relation. Published resources for obtaining values of earth pressure coefficients incorporate several assumptions such as: the backfill is homogeneous, isotropic, dry, cohesionless, and sloping at a constant angle; the shear surface is a single plane; and the soil mass is on the verge of failure, that is, factor of safety = 1. These assumptions may not necessarily be met by design problems in the field. Also, in the conventional procedures, location of the lateral thrust on the retaining structure is estimated: for static earth pressures it is taken to be at 0.33 h from the base; and for dynamic earth pressures, it is taken to lie between 0.4 h to 0.75 h depending on the manner and extent of the wall movement. Since the physical response of a wall is not known a priori, engineering judgement and past experience are used to select the location of the dynamic thrust. See references by Teng (1962), Seed and Whitman (1970), and Ebeling and Morrison (1992); references included in this paper are representative but not a complete list of works on the subject.

Since the above procedures for static and dynamic earth pressure calculations are based on limit equilibrium theory, it seems reasonable to develop a complete set of equations and their solution shall yield corresponding earth pressure results in terms of magnitude, direction, and location of the thrust. These earth pressure equations are similar to the ones for slope stability equations. Geotechnical engineers routinely analyze stability of slopes with complex geometry, material distributions, material strengths, pore pressure conditions, and shear surfaces of circular, non-circular, or mixed shapes with tension cracks, etc. Most of the slope stability procedures are based on the method of slices and are computerized. Thus, it is advantageous to solve earth pressure problems for the field conditions using a slope stability analysis computer program. However, this procedure cannot give distribution of earth pressure along the height of retaining structure and the direction of earth pressure must be specified by the user engineer.

The objective of this paper is to present an adaptation and use of slope stability analysis procedure to estimate active and passive earth pressures on retaining structures for static and seismic (pseudostatic) conditions. The necessary equations are presented to explain the earth pressure calculations. A sample problem is included to illustrate the application of the ideas presented and to demonstrate the accuracy of results obtained. In classical terminology, the proposed procedure may be viewed as an extension of the trial wedge method via the method of slices.

It is important to mention that there have been several advanced theoretical and experimental studies made to determine the effects of soil-structure interaction on the performance of retaining structures during earthquake conditions. While the end results of these studies remain inconclusive, efforts are always directed at providing the design engineer with a desirable selection of seismic coefficient value for use in the pseudostatic (Mononobe-Okabe) equations and a location for the seismic earth pressure force on the retaining structure. While these are important studies, the material presented in this paper does not deal with any of the soil-structure interaction issues. Casagrande (1973) pointed out that when retaining structures perform satisfactorily despite erroneous earth-pressure assumptions, it is primarily because a cautious and adequate safety factor has been allowed for in the design. Whitman (1990) remarked that the dynamic behavior of gravity retaining walls is much more complicated than envisioned in the simple physical and mathematical model that leads to the Mononobe-Okabe equation. However, this venerable equation, when used with proper choice of input parameters and suitable safety factors, still provides a sound basis for design of many retaining structures.

ACTIVE EARTH PRESSURE EQUATIONS

Figure 1 is a general description of an earth pressure problem. For a typical vertical slice, abcd, the forces acting on it are shown in figure 1(b) for active earth pressure condition. $F_e = \lambda W$ is a force, which corresponds to a constant acceleration λ times that of gravity, acting at an inclination γ to the horizontal and through the center of mass of the slice; H_L and H_R are the hydrostatic forces exerted by the subsurface water on the vertical boundaries of the slice; and other forces acting on the slice are self

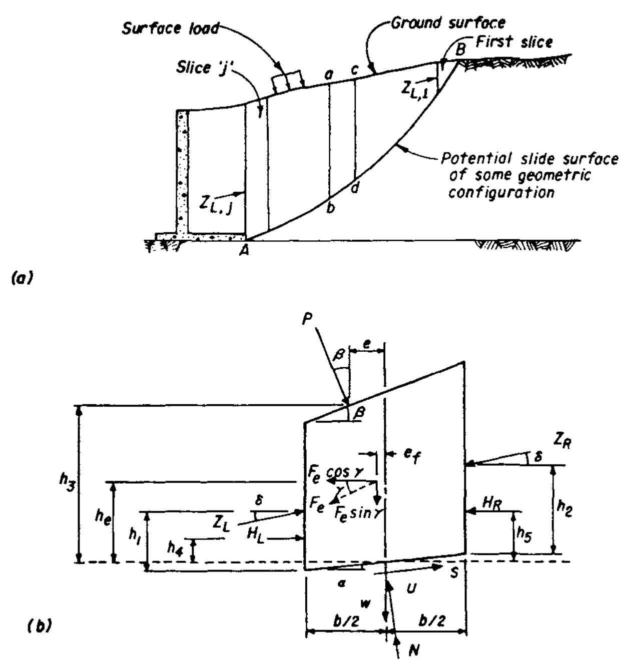

Figure 1-(a) General retaining wall problem description
(b) Forces acting on a typical slice for pseudostatic analysis (active condition)

explanatory. Considering the static equilibrium of forces shown on figure 1(b) and combining them with the Mohr-Coulomb strength equation leads to:

$$Z_L = Z_R - \frac{1}{\cos(\delta-\alpha)\left[1 - \frac{1}{F}\tan(\delta-\alpha)\tan\phi'\right]} * \left[\frac{1}{F}c'b\sec\alpha - W\sin\alpha + \frac{1}{F}(W\cos\alpha - U)\tan\phi' + P\cos(\alpha-\beta)\left\{\frac{1}{F}\tan\phi' - \tan(\alpha-\beta)\right\} + \left(H_L - H_R\right)\cos\alpha\left\{1 + \frac{1}{F}\tan\alpha\tan\phi'\right\} - F_e\cos(\alpha-\gamma)\left\{1 + \frac{1}{F}\tan(\alpha-\gamma)\tan\phi'\right\}\right] \quad (1)$$

Similarly, considering the moment equilibrium of forces leads to the following:

$$h_1 = \frac{Z_R}{Z_L}h_2 - \frac{b}{2}\left(\tan\delta - \tan\alpha\right)\left[\frac{Z_R}{Z_L} + 1\right] - \frac{P\cos\beta}{Z_L\cos\delta}(h_3\tan\beta - e) - \frac{1}{Z_L\cos\delta}(H_L h_4 - H_R h_5) + \frac{F_e\cos\gamma}{Z_L\cos\delta}\left(h_e + e_f\tan\gamma\right) \quad (2)$$

Equations (1) and (2) are for active earth pressure under seismic (pseudostatic) loading. Corresponding expressions for passive earth pressure are given in equations (3) and (4) in the Appendix. Also, passive earth pressure expressions can be achieved by specifying $F = -F$; $\delta = (2\pi - \delta)$; and $\gamma = \pm(\pi - \gamma)$ for downward and upward inertial force, respectively, in equations (1) and (2). For static conditions, set $F_e = 0$.

SOLUTION PROCEDURE

Equations (1) and (2) are in the form of recursive relationships. The solution procedure is initiated by using the known boundary conditions Z_R and h_2 at the far end of the shear surface, and for the assigned values of factor of safety F and interslice force inclination δ. Equations (1) and (2) are used to calculate the Z_L and h_1 for the first slice. Z_R and h_2 for the second slice equal Z_L and h_1 for the first slice. This process is continued until the Z_L and h_1 for the last slice are calculated. Z_L and h_1 for slice j ending at the heel of a retaining wall give, respectively, the magnitude and location of the total thrust per foot length of the retaining structure. Inclination of this force is at the user specified value of δ. The variation of the earth pressure along the height of the wall is not determined by this procedure. The above procedure has been implemented in the computer program SSTAB2 (Chugh, 1992).

GENERAL COMMENTS

1. Equations (1) through (4) are for backfills retained on the right side of the retaining structure shown in figure 1. Similar equations can be derived for backfills retained on the left side.

2. Equations (1) through (4) are for estimating earth pressure forces for a prescribed pore water pressure condition.

3. Selection of seismic coefficients in the horizontal and vertical directions should be made in consultation with experienced engineers and seismologists.

4. Strength values assigned for the backfill soils must be consistent with the displacement of the wall and the backfill soil. Large displacements or accumulation of small displacements may lead to the use of residual shear strengths of the backfill.

5. Use of a factor of safety in earth pressure calculations for static and dynamic loading conditions may include considerations, such as: (a) earth pressure calculations for dynamic loading make sense only if the retaining wall has a reserve of strength after the static needs have been met, that is, retaining structure is designed for static earth pressure using F > 1; (b) appropriate value of F can be used to reflect: (1) the level of uncertainty in shear strength data for the backfill, and (2) the concerns for nondevelopment of fully active or fully passive earth pressure condition in the backfill.

6. For active or at-rest earth pressure, the engineer should investigate shear surfaces which give maximum lateral thrust values. However, for passive earth pressure, the emphasis should be to look for a shear surface which gives minimum lateral thrust.

7. The procedure presented gives results for lateral thrust on a retaining structure for the specified geometry, backfill materials, and loading conditions. For static earth pressures, specify $F_e = 0$.

8. Regular slope stability analysis should be performed to evaluate the stability for a potential shear surface passing through the foundation materials under the retaining structure.

SAMPLE PROBLEM

The problem shown in figure 2 was studied for active and passive earth pressures for static and pseudostatic loading conditions using a modified version of computer program SSTAB2. Table 1 lists the various cases analyzed. Table 2 lists the results. These results compare favorably with the results given in Ebeling and Morrison (1993). Under the

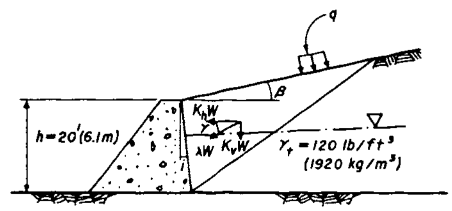

Figure 2 - Sample Problem

Table 1. - Sample problem data.

Analysis No.	Backfill description					Surcharge load	Pressure loading conditions		Seismic coeff. data	
	β°	c'	φ'°	δ°	u	q lbs/sq ft	Active/ Passive	Static/ Dynamic	λ	γ
1	0	0	30	0	0	0	Active	Static	0	0
2	0	0	30	0	0	0	Passive	Static	0	0
3	6	0	30	3	0	0	Active	Static	0	0
4	6	0	30	-3	0	0	Passive	Static	0	0
5	6	0	30	3	0	0	Active	Dynamic	0.12	-33.82
6	6	0	30	3	0	0	Active	Dynamic	0.12	33.82
7	6	0	30	-3	0	0	Passive	Dynamic	0.12	33.82
8	0	0	35	17.5	0	0	Active	Dynamic	0.20	0
9	0	0	35	17.5	*	0	Active	Dynamic	0.20	0
10	0	0	35	17.5	**	0	Active	Dynamic	0.20	0
11	0	0	35	17.5	*	500	Active	Dynamic	0.1	0
12***	0	0	35	0	0	0	Passive	Dynamic	0.32	21.80

* Hydrostatic full depth.
** Hydrostatic 12-foot depth.
*** Batter angle i = 5°

Table 2. - Sample problem results.

Analysis No.	Computed results			Converted results	
	Total thrust (lbs per foot of wall length)	Location h_1 (ft)	α	Equivalent pressure coeff.	Fraction h_1/H
1	8,000	6.74	60°	0.333	0.337
2	72,000	6.934	29.7°	3.0	0.347
3	8,312	6.970	56.9°	0.346	0.349
4	96,480	8.936	32.3°	4.020	0.447
5	9,554	8.420	52°	0.427	0.421
6	10,640	8.285	52°	0.415	0.414
7	90,140	9.01	30.7°	4.026	0.451
8	9,113	8.020	49.6°	0.380	0.401
9	20,560	5.819	29.1°	0.857	0.291
10	12,650	6.699	45°	0.527	0.335
11	15,440	5.25	59°	0.643	0.263
12	72,210	5.51	23°	3.419	0.276

present practice, lateral pressure calculations are made individually for the various loadings of interest, each analysis giving the magnitude and location of the lateral thrust for the corresponding condition. However, in the procedure presented, the problem is considered as one whole and the analysis yields the magnitude, direction, and location of lateral thrust on the wall.

SUMMARY: The proposed procedure provides an effective and efficient means for using the limit equilibrium method to determine the magnitude, direction, and location of lateral thrust on a retaining structure for static and seismic (pseudostatic) conditions without resorting to charts and in a manner analogous to slope stability analysis procedure. The geometry of the retaining wall and backfill materials, pore water pressure conditions, surface loads, geometry of shear surface, soil strengths, reinforcement, seismic coefficients in horizontal and vertical directions, and desired factor of safety and angle of inclination of lateral thrust on the wall can all be specified by the engineer to reflect the field conditions and design needs. However, the limit equilibrium method has its limits of applicability, and the decision to use this method for determining earth pressures for a particular job should be made by the engineer in charge of the project.

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APPENDIX: PASSIVE EARTH PRESSURE EQUATIONS

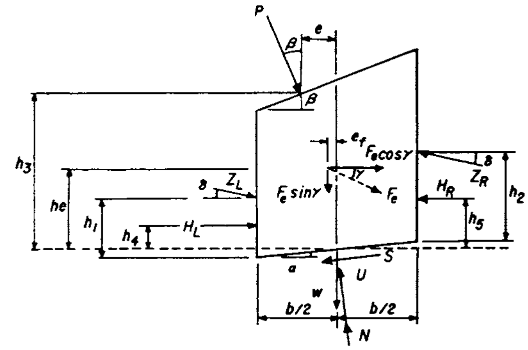

Figure 3 - Forces acting on a typical slice for pseudostatic analysis (passive condition)

Figure 3 is the free body diagram of a typical slice for passive earth pressure condition. Considering the static equilibrium of forces and combining them with the Mohr-Coulomb strength equation leads to:

$$Z_L = Z_R + \frac{1}{\cos(\delta + \alpha)\left[1 - \frac{1}{F}\tan(\delta + \alpha)\tan\phi'\right]} *$$

$$\left[\frac{1}{F} c'b \sec\alpha + W \sin\alpha + \frac{1}{F}(W\cos\alpha - U)\tan\phi'\right.$$

$$+ P\cos(\alpha - \beta)\left\{\frac{1}{F}\tan\phi' - \tan(\beta - \alpha)\right\}$$

$$- \left(H_L - H_R\right)\cos\alpha\left\{1 - \frac{1}{F}\tan\alpha\tan\phi'\right\}$$

$$\left.- F_e\cos(\alpha + \gamma)\left\{1 - \frac{1}{F}\tan(\alpha + \gamma)\tan\phi'\right\}\right] \quad (3)$$

Similarly, the moment equilibrium of the forces acting on the slice, figure 3, gives:

$$h_1 = \frac{Z_R}{Z_L} h_2 + \frac{b}{2}(\tan\delta + \tan\alpha)\left[\frac{Z_R}{Z_L} + 1\right]$$

$$- \frac{P\cos\beta}{Z_L\cos\delta}(h_3\tan\beta - e)$$

$$- \frac{1}{Z_L\cos\delta}(H_L h_4 - H_R h_5)$$

$$- \frac{F_e\cos\gamma}{Z_L\cos\delta}(h_e - e_f\tan\gamma) \quad (4)$$