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SYNOPSIS An effective numerical method for the dynamic analysis of soil pressures on retaining walls has been proposed. In this approach, the retaining wall is considered as a vertical flexible beam and the soil is assumed to be a linear-elastic material with hysteretic damping. The analysis procedure of proposed method, compared with some alternative numerical methods, (such as finite element method), will greatly reduce the computational effort. The properties and the significance of some model parameters, such as hysteretic materials damping and stiffness ratio of flexible beam and soil, are also investigated. As a special case when the retaining wall is considered as a rigid structure, the exact solution of the soil pressure on the walls can be obtained.

INTRODUCTION

The dynamic response of earth retaining structures induced by earthquake or dynamic loading, involves a complex soil-structure interaction problem. In the literature, numerous analytical and experimental investigations have been undertaken for a better understanding of this subject. However, the development of simple and accurate methods for the dynamic interaction analysis of soil pressures and retaining walls is still an active research topic in seismic analysis and design.

This paper deals with dynamic response of a flexible retaining wall and soil systems to seismic loading. With the proposed approach, the retaining wall is treated as a flexible vertical beam and the backfill soil is assumed to be a linear-elastic material with hysteretic damping. A soil layer of finite thickness retained by a rigid bottom is considered. A truncated series of complete functions which satisfy partial boundary conditions of the soil domain are adopted as a weight function. Then, the Galerkin weighted residual method is used to obtain the solution of soil pressures through boundary integration method. Numerical results are presented to illustrate the behavior of the soil pressures on the walls. The computational effort required by the proposed method is small compared with some alternative numerical methods, (such as finite element method). As a special case, when the retaining wall is considered as a rigid structure, an exact solution for the dynamic soil pressure acting on the rigid walls can be obtained by proposed method. This exact solution is also presented in the paper.

Furthermore, a parametric study is presented. The properties and the significance of the primary model parameters, such as stiffness ratio and hysteretic material damping are also investigated for a better understanding of the effects of these parameters.

MODELLING AND METHODS OF ANALYSIS

The soil layer and retaining wall system investigated is shown in Fig.1. The retaining wall is represented by a one dimensional vertical flexible beam with a rigid base. The beam has the flexible stiffness of EI and mass density of $\bar{m}$. The resistance of soil layer against the wall is represented by an interaction force, Q, along the axis of the flexible beam as shown in Fig.2a. The soil layer with depth of H is assumed to be hysteretic and elastic in material properties as shown in Fig.2b. Both the bases of the wall and soil layer are considered to be excited by a space-invariant motion, $\ddot{u}_g(t)$.

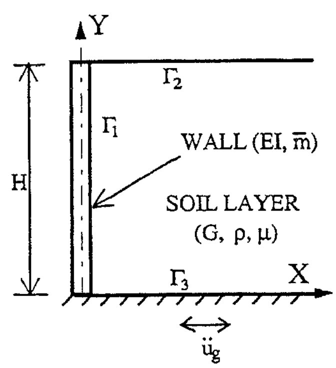

Fig.1. Retaining Wall and Soil System

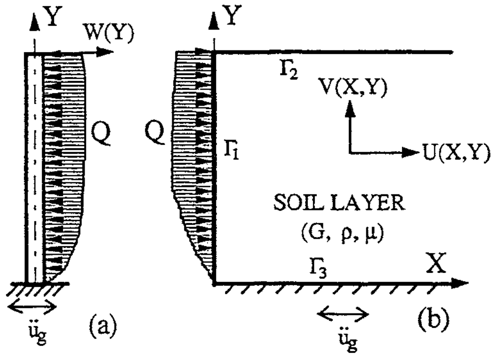

Fig.2. Soil Pressure and Displacement Components

For the soil layer under horizontal base excitation, the vertical displacement, V, as shown in Fig.2b, is considered negligible small compared to the horizontal displacements, U. Under this assumption, the stress-displacement and strain-displacement equations for the soil domain can be expressed as:

$$\sigma_X=(\lambda^*+2G^*)\frac{\partial U}{\partial X} \qquad \sigma_Y=\lambda^*\frac{\partial U}{\partial Y} \qquad \tau_{XY}=G^*\frac{\partial U}{\partial Y} \tag{1}$$

$$\varepsilon_X=\frac{\partial U}{\partial X} \qquad \varepsilon_Y=0 \qquad \gamma_{XY}=\frac{\partial U}{\partial Y} \tag{2}$$

where σ, τ are the normal stress and shear stress of soil domain, respectively, ε, γ are the strain components, and G^* and λ^* are complex Lame's parameters of the soil, i.e.,

$$G^*=G(1+i\delta) \qquad \lambda^*=2\mu G^*/(1-2\mu) \tag{3}$$

in which $i=\sqrt{-1}$, μ is Poisson's ratio, and δ represents the hysteretic material damping parameter of the soil which is considered to be frequency independent.

The governing equation of the soil displacement under harmonic input acceleration, $\ddot{u}_g(t)=\ddot{u}_g\exp(i\omega t)$, can be expressed, by assuming $x=X/H$, $y=Y/H$, $u=U/H$, $w=W/H$, and $q=Q/G$, as:

$$\varepsilon^2\frac{\partial^2 u}{\partial x^2}+(1+i\delta)\frac{\partial^2 u}{\partial y^2}+\left(\frac{H\omega}{v_s}\right)^2 u-\frac{H\ddot{u}_g}{v_s^2}=0 \tag{4}$$

with the following boundary conditions:

$$q-\varepsilon^2\frac{\partial u}{\partial x}=0 \qquad (x=0) \tag{5a}$$

$$u=0 \qquad (x\to\infty) \tag{5b}$$

$$u=0 \qquad (y=0) \tag{5c}$$

$$\frac{\partial u}{\partial y}=0 \qquad (y=1) \tag{5d}$$

where $v_s^2=G/\rho$ is shear velocity of the soil medium, ρ is mass density of soil, and $\varepsilon^2=2(1-\mu)(1+i\delta)/(1-2\mu)$.

The equation of motion of the flexible beam under the base excitation may be expressed as:

$$\frac{d^4 w}{dy^4}-\frac{H^4\overline{m}\omega^2}{EI}w=\frac{H^3 G}{EI}q-\frac{H^3\overline{m}}{EI}\ddot{u}_g \tag{6}$$

with the boundary conditions:

$$w=0 \qquad (y=0) \tag{7a}$$

$$\frac{dw}{dy}=0 \qquad (y=0) \tag{7b}$$

$$\frac{d^2 w}{dy^2}=0 \qquad (y=1) \tag{7c}$$

$$\frac{d^3 w}{dy^3}=0 \qquad (y=1) \tag{7d}$$

where w is the dimensionless relative displacement of the beam, EI and $\overline{m}$ are the flexible stiffens and mass density of the beam, respectively.

Furthermore, the dynamic pressure of soil layer to the flexible retaining wall can be achieved by solving the differential equations described above, i.e., Eqs.4-7, using a boundary integration method (Sun and Pires, 1994). These solutions are discussed in the following.

BOUNDARY INTEGRAL SOLUTION

A weight function u^* which satisfies the differential equation:

$$\varepsilon^2\frac{\partial^2 u^*}{\partial x^2}+(1+i\delta)\frac{\partial^2 u^*}{\partial y^2}+\left(\frac{H\omega}{v_s}\right)^2 u^*=0 \tag{8}$$

and the boundary condition of the soil boundary Γ_2 and Γ_3 (see Eqs.5b-5d), is introduced. The solution of this weight function can be expressed in a truncated series as:

$$u^*=\sum_{j=1}^{M}A_j^*\phi_j(x,y)=\sum_{j=1}^{M}A_j^*\exp(-\beta_j x)\sin(\alpha_j y) \tag{9}$$

where M is the number of selected function terms, A_j^* $(j=1,2,...M)$ are arbitrary constants, and the parameters α_j and β_j are:

$$\alpha_j=(2j-1)\pi/2 \qquad \beta_j=\frac{\alpha_j}{\varepsilon}\sqrt{(1+i\delta)-a_0^2/(2j-1)^2}$$

in which $a_0^2 = \frac{\omega^2}{\varpi^2}$ and $\varpi = \frac{\pi v_s}{2H}$.

Applying Galerkin's procedure in Eq.4 by taking u^* as a weight function and using Green's second theorem, the soil displacement, u, can be expressed by the following boundary integral equation:

$$\int_0^1 \left(\frac{\partial u}{\partial x} u^* - \frac{\partial u^*}{\partial x} u \right) dy = \int_0^\infty \int_0^1 \frac{H \ddot{u}_g}{v_s^2} u^* dx dy \quad (10)$$

Substituting boundary condition given in Eq.5a into Eq.10, yields:

$$\int_0^1 q u^* dy - \varepsilon^2 \int_0^1 \frac{\partial u^*}{\partial x} u dy = \varepsilon^2 \int_0^\infty \int_0^1 \frac{H \ddot{u}_g}{v_s^2} u^* dx dy \quad (11)$$

Furthermore, if the interaction force, q, is expressed as a truncated series in $\phi_j(0,y) (j=1,2,...M)$, that is:

$$q = \sum_{j=1}^{M} q_j \phi_j(0,y) = \sum_{j=1}^{M} q_j \sin(\alpha_j y) \quad (12)$$

a group of integration equations will be obtained by substituting Eqs.9 and 12 into Eq.11. That is:

$$\sum_{j=1}^{M} q_j \int_0^1 \phi_i(0,y) \sin(\alpha_j y) dy - \varepsilon^2 \int_0^1 \frac{\partial \phi_i(0,y)}{\partial x} u dy = \varepsilon^2 \frac{H \ddot{u}_g}{v_s^2} \int_0^\infty \int_0^1 \phi_i(x,y) dx dy$$
$$(i=1,2,...M) \quad (13)$$

This equation will be coupled with the beam equation given in Eq.6 to solve the unknowns, $q_j (j=1,2,...M)$, and therefore the soil pressure on the retaining wall.

SOIL PRESSURE ON RETAINING WALL

On the basis of Eq.6, the relative displacement of the flexible beam, w, may be represented by the sum of the displacement due to the base movement, $\hat{w}$ and the displacement induced by the soil pressure, $\overline{w}$. By introducing the interaction force given in the truncated series functions of Eq.12 and expanding the base excitation, $\ddot{u}_g$, in the form:

$$\ddot{u}_g = \frac{4}{\pi} \ddot{u}_g \sum_{j=1}^{M} \frac{\sin(\alpha_j y)}{2j-1} \quad (14)$$

the relative displacement of the flexible beam can be expressed as:

$$w = \overline{w} + \hat{w} = \frac{H^3 G}{EI} \sum_{j=1}^{M} q_j w_j(y) - \frac{4H^3 \overline{m} \ddot{u}_g}{\pi EI} \sum_{j=1}^{M} b_j w_j(y) \quad (15)$$

in which the constant b_j can be expressed as $b_j = 1/(2j-1)$ and the functions $w_j (j=1,2,...M)$ are the solution of the equation:

$$\frac{d^4 w_j}{dy^4} - \lambda^4 w_j = \sin(\alpha_j y) \quad (16)$$

with the boundary condition given in Eq.7. The solution of w_j can be expressed as:

$$w_j = [sh(\lambda y) \quad ch(\lambda y) \quad \sin(\lambda y) \quad \cos(\lambda y)] [c_{1j} \quad c_{2j} \quad c_{3j} \quad c_{4j}]^T \quad (17)$$

where $\lambda^4 = H^4 \overline{m} \omega^2 / EI$ and the constant factors, c_{ij} $(j=1,2,3,4; i=1,2,...M)$, can be fund by solving Eq.16 and the boundary conditions in Eq.7.

Let $w(y) = u(0,y)$ and substitute Eq.15 into Eq.13, yields:

$$\sum_{j=1}^{M} q_j \int_0^1 \sin(\alpha_i y) \sin(\alpha_j y) dy + \varepsilon^2 \xi \sum_{j=1}^{M} q_j \int_0^1 \beta_i \sin(\alpha_i y) w_j(y) dy$$

$$- \frac{4\varepsilon^2 \xi}{\pi} \frac{\overline{m} \ddot{u}_g}{G} \sum_{j=1}^{M} b_j \int_0^1 \beta_i \sin(\alpha_i y) w_j(y) dy$$

$$= \varepsilon^2 \gamma \frac{\overline{m} \ddot{u}_g}{G} \int_0^\infty \int_0^1 \exp(-\beta_i x) \sin(\alpha_i y) dx dy \quad (18)$$

Which can be written in a matrix form as:

$$([G] + \xi \varepsilon^2 [S][C]) \{q\} = \gamma \varepsilon^2 \frac{\overline{m} \ddot{u}_g}{G} \{P\} + \frac{4 \xi \varepsilon^2}{\pi} \frac{\overline{m} \ddot{u}_g}{G} [S][C] \{b\} \quad (19)$$

where $\xi = H^3 G / EI$ is a stiffness ratio, and $\gamma = \rho H / \overline{m}$ is a mass density ratio of the soil and the beam, respectively. By integration of Eq.18, the matrices in Eq.19 can be written as:

$$G_{ii} = 1/2 \; ; \quad G_{ij} = 0 \, (i \neq j) \quad (20a)$$

$$S_{j1} = \beta_j \lambda ch(\lambda) \sin(\alpha_j) / (\alpha_j^2 + \lambda^2) \quad (20b)$$

$$S_{j2} = \beta_j [\lambda sh(\lambda) \sin(\alpha_j) + \alpha_j] / (\alpha_j^2 + \lambda^2) \quad (20c)$$

$$S_{j3} = \beta_j \lambda \cos(\lambda) \sin(\alpha_j) / (\alpha_j^2 - \lambda^2) \quad (20d)$$

$$S_{j4} = \beta_j [\alpha_j - \lambda \sin(\lambda) \sin(\alpha_j)] / (\alpha_j^2 - \lambda^2) \quad (20e)$$

$$P_j = 1/\beta_j \alpha_j \quad (20f)$$

$$b_j = 1/(2j-1) \quad (20g)$$

where the parameter, λ, can be expressed further by the dimensionless factors as: $\lambda^4 = \xi \pi^2 a_0^2 / 4\gamma$. After obtaining the vector $\{q\}$ by solving the complex matrix equation (Eq.19), the dynamic soil pressure acting on the axis of the retaining wall can be calculated by Eq.12.

NUMERICAL RESULTS

As first example, we consider a rigid wall with hysteretic and elastic soil system. In this case, the stiffness ratio $\xi \to 0$ which means the relative displacement of the wall is $w=0$. From Eqs.19 and 20, we have:

$$q_j = \frac{8H\varepsilon^3 \rho \ddot{u}_g}{(2j-1)^2 \pi^2 G \sqrt{(1+i\delta) - a_0^2/(2j-1)^2}} \quad (j=1,2,...) \tag{21}$$

Therefore, the exact solution for the dynamic soil pressure on the rigid retaining wall, induced by the base excitation, can be obtained as:

$$Q(y) = \frac{8H\varepsilon^3 \rho \ddot{u}_g}{\pi^2} \sum_{j=1}^{\infty} \frac{\sin\left[\frac{(2j-1)y}{2}\right]}{(2j-1)^2 \sqrt{(1+i\delta) - a_0^2/(2j-1)^2}} \tag{22}$$

The real part, imaginary part and the amplitude of the harmonic soil pressure acting on the top of rigid retaining wall are plotted in Fig.3 as a function of the frequency ratio a_0. The Poisson's ratio $\mu=0.3$ and the mass density ratio $\gamma=1.0$ are adopted, and the material damping factor is taken as $\delta=0.05$. In the figures, the vertical axis represents $\Psi = Q/\rho H \ddot{u}_g$.

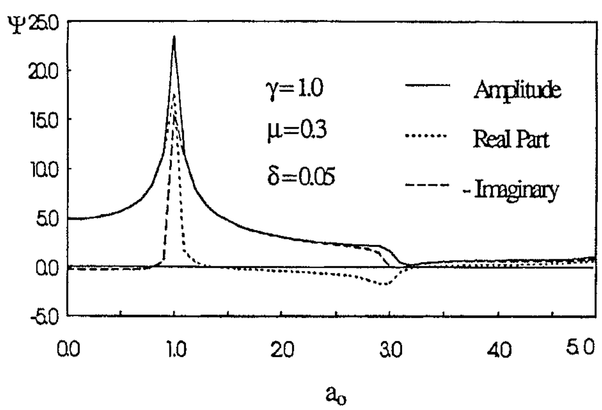

Fig.3. Frequency Response of Soil Pressure on the Top of a Rigid Retaining Wall

To investigate the effect of the stiffness ratio of soil and flexible wall on the dynamic response of soil pressure, some numerical results are produced for difference values of the factor ξ. The complex frequency response functions for the dynamic soil pressure acting on the top of flexible retaining wall to harmonic base excitations are shown in Fig.4 for $\delta=0.05$, $\gamma=1.0$, and $\mu=0.3$. The vertical axis is also represents $\Psi = Q/\rho H \ddot{u}_g$.

The behavior of the complex frequency response of soil pressure on the top of beam for various values of the material damping ratio, δ, is shown in Fig.5. As expected, the method predicts a decrease in dynamic amplification near the fundamental frequency of the retaining wall and soil system. The amplitude of the frequency response will be reduced significantly as increasing the material damping of soil.

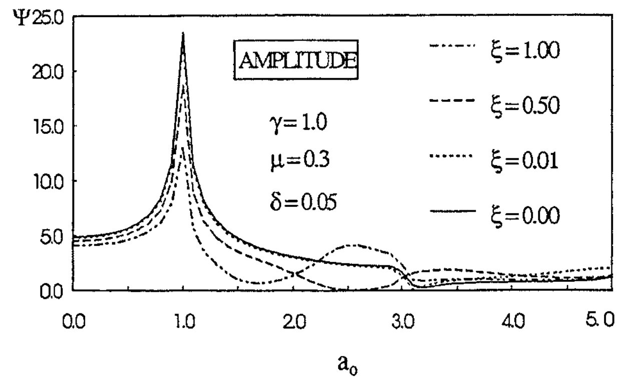

Fig.4. Effect of Stiffness Ratio on Frequency Response of Soil Pressure

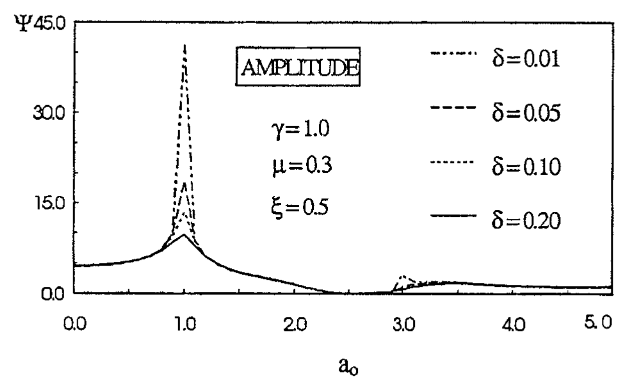

Fig.5. Effect of Material Damping on Frequency Response of Soil Pressure

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