Dynamic Active Earth Pressure Against Retaining Walls

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SYNOPSIS: Equations of equilibrium expressed along the stress characteristics are transformed onto the Zero Extension Line (ZEL) directions. The new dynamic equilibrium equations are then applied to simple ZEL field (composed of Rankine, Goursat, and Coulomb zones) behind retaining walls. Integration of differential equilibrium equations along the assumed field boundary, thus provide the final equations for the active static (K_{ast}) and dynamic (K_{ady}) earth pressure coefficients, which are functions of friction and dilation angles of the soil and friction angle of the wall surface. Numerical evaluation of K_{ast} and K_{ady} indicates that these coefficients are not sensitive to the wall roughness for practical values of angle of friction of backfill material between 35° and 45°. In this range, the coefficients can be approximated by:

$$K_{ast}=\tan^2(\pi/4-\phi/2) \quad \text{and} \quad K_{ady}=\tan(\pi/4-\nu/2)\ .$$

INTRODUCTION

Various approaches for assessment of total active earth pressure on retaining walls, when subjected to horizontal acceleration, have been proposed in the literature. Following is a partial list:

- Mononobe-Okabe approach in which the equilibrium under the concurrent action of horizontal acceleration, a, and the gravity, ρg, is considered.
- Whitman (1991) formula given as $P_{at}=P_{ast}+P_{ady}$ in which $P_{ast}=(\rho gH)\times K_{ast}$ and $P_{ady}=(\rho a_{pst}H)\times 0.75$. Here P_{at}, P_{ast}, and P_{ady} are, respectively the active total, static, and dynamic earth pressures, ρ is the density of backfill material, g is the acceleration of gravity, H is the wall height, K_{ast} is the static active earth pressure coefficient given by $\tan^2(\pi/4-\Phi/2)$ where ϕ is the angle of friction of backfill material, and a_{pst} is the psudo-static acceleration.
- Extension of Sokolovski (1965) stress characteristic method to dynamic equilibrium of backfill proposed by Sabzevari (1974).
- Block and slice equilibrium theory applied by, respectively, Habibagahi (1979) and Ghahramani (1980) on zero extension line networks.
- The approach presented by Behpoor (1993) by considering the equilibrium of elements in different zones of the zero extension line network.

In the present paper a totally different approach is presented in which differential equations of equilibrium expressed along the stress characteristics are transformed onto the zero extension line directions. This makes it possible to evaluate the total active pressure acting on retaining walls when integrated along the boundary of a simple zero extension line field.

It is shown that the newly derived equations for active earth pressure coefficients are comparable with the result obtained by Behpoor (1993). The obtained results are also compared with Whitman recommendation for dynamic earth pressure coefficients and cited experimental results.

GENERAL THEORY

Consider the Mohr-Coulomb granular soil for which the Mohr stress circle is shown in Fig. 1(a). If θ denotes the angle of major compressive stress, σ_{max}, with the x-axis, Fig. 1(b), and s the average compressive stress, then

$$\begin{aligned}\sigma_x&=s\ (1+\sin\phi\ \cos2\theta)\\ \sigma_z&=s\ (1-\sin\phi\ \cos2\theta)\\ \tau_{xz}&=s\ \sin\phi\ \sin2\theta.\end{aligned} \tag{1}$$

By substituting these in the differential equation of equilibrium given below

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xz}}{\partial z}=-\rho a_x=X \tag{2a}$$

$$\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\sigma_z}{\partial z}=\rho(g+a_z)=Z \tag{2b}$$

two stress characteristics (l^+ and l^-) can be obtained; Sabzevari (1974); which make angles $\pm\mu=\pm(\pi/4-\phi/2)$ with σ_{max} direction. Along these two characteristic lines, the differentials of average

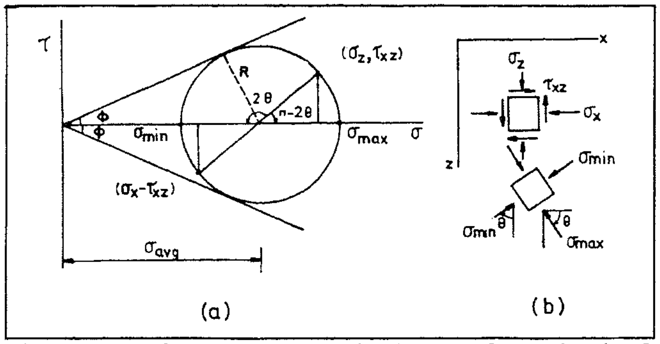

Fig. 1. Mohr Stress Circle and Principal Stresses

stress are expressed by Eq'ns. (7) and (9) of Sabzevari (1974). Changing the derivatives w. r. t. x and z to derivatives w. r. t. l^+ and l^-, the differentials of average stress can be expressed along stress characteristic l^+ as

$$ds+2s\ (\tan\phi)d\theta = s\ [(\tan\phi)d\phi - \frac{1}{\cos\phi}\frac{\partial\phi}{\partial l^-}dl^+] + X\ [dx+(-\tan\phi)dz] + Z\ [(\tan\phi)dx+dz] \quad (3a)$$

and along l^- as

$$ds-2s\ (\tan\phi)d\theta = s\ [(\tan\phi)d\phi - \frac{1}{\cos\phi}\frac{\partial\phi}{\partial l^+}dl^-] + X\ [dx+(\tan\phi)dz] + Z\ [(-\tan\phi)dx+dz]. \quad (3b)$$

Note that in Eq'ns. (2) and (3) X and Z are the body forces in the +ve x- and z-axis directions, respectively, which are assumed to be constant henceforth. The -ve sign in Eq'n. (2a) has been introduced to make the earth pressure coefficients +ve.

Roscoe (1970) has shown that there are two strain characteristic lines (ζ^+ and ζ^-) along which linear strains are zero (ZEL) and make angles $\pm\xi=\pm(\pi/4-\nu/2)$ with the direction of incremental major compressive strain; where ν is the angle of dilation of soil. Assuming the direction of major stress to coincide with the direction of incremental major strain and noting that

$$\begin{Bmatrix} \dfrac{\partial f}{\partial l^-} \\ \dfrac{\partial f}{\partial l^+} \end{Bmatrix} = \frac{1}{\sin 2\xi}\begin{bmatrix} \sin(\xi+\mu) & \sin(\xi-\mu) \\ \sin(\xi-\mu) & \sin(\xi+\mu) \end{bmatrix} \begin{Bmatrix} \dfrac{\partial f}{\partial \zeta^-} \\ \dfrac{\partial f}{\partial \zeta^+} \end{Bmatrix} \quad (4)$$

where f is any function (of s, θ, or ϕ) expressed along stress or strain characteristics, then the equations of equilibrium as represented by Eq'ns. (3) along the stress characteristics, when transformed onto the strain characteristics (ZEL), become

$$ds+2s(\tan\phi)[C_1d\theta+C_2\frac{\partial\theta}{\partial\zeta^-}d\zeta^+] = s\ [(\tan\phi)d\phi-\frac{1}{\cos\phi}\frac{\partial\phi}{\partial\zeta^-}d\zeta^+] + X\ C_3[C_1dx+(-\tan\phi)dz]+Z\ C_3[(\tan\phi)dx+C_1dz] \quad (5a)$$

along ζ^+ and

$$ds-2s(\tan\phi)[C_1d\theta+C_2\frac{\partial\theta}{\partial\zeta^+}d\zeta^-] = s\ [(\tan\phi)d\phi-\frac{1}{\cos\phi}\frac{\partial\phi}{\partial\zeta^+}d\zeta^-] + X\ C_3[C_1dx+(\tan\phi)dz]+Z\ C_3[(-\tan\phi)dx+C_1dz] \quad (5b)$$

along ζ^-. Parameters. C_1 thru C_3 in Eq'ns. (5) are defined as follows:

$$\begin{aligned} C_1 &= (1-\sin\phi\ \sin\nu)/\cos\phi\ \cos\nu \\ C_2 &= (\sin\phi-\sin\nu)/\cos\phi\ \cos\nu \\ C_3 &= \cos\nu/\cos\phi. \end{aligned} \quad (6)$$

APPLICATION ON RETAINING WALLS

Consider the vertical retaining wall shown in Fig. 2 with the simple zero extension line net behind it for the active case, which is composed of three zones:

1) Rankine zone ABO making an angle of $\zeta=(\pi/4+\nu/2)$ with the surface,
2) Coulomb zone CDO making an angle β with the wall,
3) Goursat zone, a logarithmic spiral zone in between.

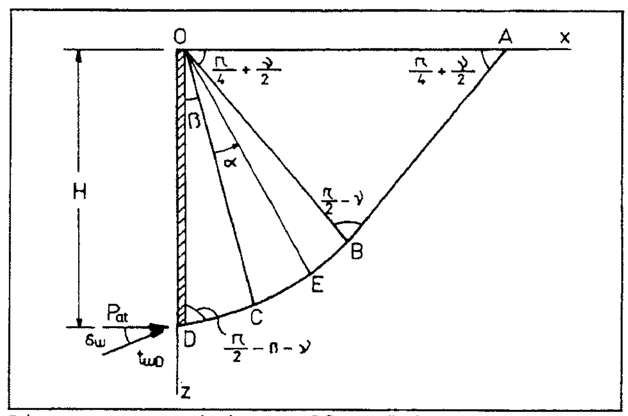

Fig. 2. Retaining Wall and its ZEL Net for Active Case

Line AB coincides with ζ^+ direction along which $\partial\theta/\partial\zeta^-=0$ and $d\theta=0$. Then for a constant ϕ granular soil (normally used backfill material behind retaining walls,) differential of average stress s along line AB can be deduced from Eq'n. (5a) to be

$$ds = C_3\ \{C_1\ [X\ dx+Z\ dz] + (\tan\phi)\ [Z\ dx-X\ dz]\}. \quad (7)$$

Integrating Eq'n. (7), and noting that $s_A=0$, results in

$$s_B = \left(-X\frac{\tan\xi}{1-\sin\phi} + Z\frac{1}{1+\sin\phi}\right) H \,\frac{\cos(\beta+\nu)\cos\xi}{\cos\nu}\, e^{-(\xi-\beta)\tan\nu} \qquad (8)$$

where s_B is the average stress at point B on the ZEL.

In Goursat zone, consider the radial line OE which makes an angle α with OC, Fig. 2. Then α can be expressed as

$$\alpha = (\pi/2) - \beta + \xi - \theta \qquad (9)$$

and the differential of average stress s on line BC, deduced from Eq'n. (5a), with some manipulations, as

$$d[s\, e^{-(2C_1\tan\phi)\,\alpha}] = \frac{\cos(\beta+\nu)}{\cos\nu\cos\phi}\, H\, e^{-C_4\alpha}\Big\{ \frac{1}{\cos\phi}[X\cos(\alpha+\beta) - Z\sin(\alpha+\beta)] + C_2\,[X\sin(\alpha+\beta) + Z\cos(\alpha+\beta)]\Big\} \qquad (10)$$

where

$$C_4 = \tan\lambda = 2C_1\tan\phi + \tan\nu. \qquad (11)$$

Integrating Eq'n. (10) from B to C yields

$$s_C = s_B\, e^{-2C_1(\xi-\beta)\tan\phi} + H\,\frac{\cos(\beta+\nu)\cos\lambda}{\cos\nu\cos\phi}\Big\{\Big(\frac{X}{\cos\phi} + C_2 Z\Big)[\sin(\beta-\lambda) - \sin(\xi-\lambda)\, e^{-(\xi-\beta)C_4}] + \Big(C_2 X - \frac{Z}{\cos\phi}\Big)[-\cos(\beta-\lambda) + \cos(\xi-\lambda)\, e^{-(\xi-\beta)C_4}]\Big\} \qquad (12)$$

where s_C is the average stress at point C on the ZEL.

In Coulomb zone, the differential of average stress s along line CD is given by Eq'n. (7). Integrating this equation from C to D, yields the average stress at point D.

$$s_D = s_C + H\Big\{-\frac{\cos(\beta+\nu)\sin\beta}{\cos\phi}[X\, C_1 + Z\,(\tan\phi)] - \frac{1}{\cos\phi}[\cos\nu - \cos(\beta+\nu)\cos\beta][X\,(\tan\phi) - Z\, C_1]\Big\} \qquad (13)$$

Now consider the Mohr's circle of stress at point D; Fig. 3. The following equations can be deduced by using sines law.

$$\beta = \pi/4 - \nu/2 + \delta_w/2 - \psi/2 \qquad (14)$$

and

$$t_{wD} = [\sin(\psi - \delta_w) / \sin\psi]\; s_D \qquad (15)$$

where ψ is shown in Fig. 3 and can be obtained from the following equation

$$\psi = \sin^{-1}[\sin\delta_w / \sin\phi]. \qquad (16)$$

The actual total pressure; P_{at}; on the wall can be obtained (see Fig. 2) from $P_{at} = t_{wD}\cos\delta_w$. By progressive substitution of Eq'ns. (8), (12), (13), and (15) into the equation for P_{at}, and

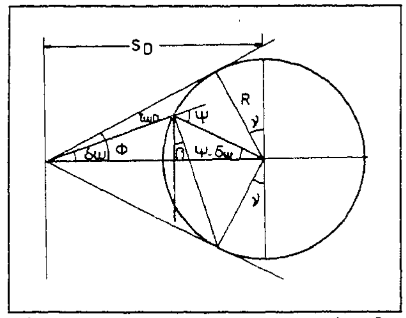

Fig. 3. Mohr Stress Circle of Point D

taking the body forces Z and X to be equal to ρg and $-\rho a$, respectively, then

$$P_{at} = P_{ast} + P_{ady} = (\rho g H)\, K_{ast} + (\rho a H)\, K_{ady} \qquad (17)$$

where

$$K_{ast} = \frac{\sin(\psi-\delta_w)\cos\delta_w}{\sin\psi\cos^2\nu\cos^2\phi}\Big\{\sin\beta\cos\nu\,[\sin(\beta+\nu) - \sin\phi\cos\beta] + \cos(\beta+\nu)\,\{\cos\lambda\,[\sin\phi\sin(\beta-\lambda) + \cos(\beta-\lambda+\nu)] + e^{-(\xi-\beta)C_4}[\cos\xi\cos\nu(1-\sin\phi) - \cos\lambda\,[\sin\phi\sin(\xi-\lambda) + \cos(\xi-\lambda+\nu)]]\}\Big\} \qquad (18)$$

and

$$K_{ady} = \frac{\sin(\psi-\delta_w)\cos\delta_w}{\sin\psi\cos^2\nu\cos^2\phi}\Big\{\sin\beta\cos\nu\,[\cos(\beta+\nu) + \sin\phi\sin\beta] + \cos(\beta+\nu)\,\{\cos\lambda\,[\sin\phi\cos(\beta-\lambda) - \sin(\beta-\lambda+\nu)] + e^{-(\xi-\beta)C_4}[\sin\xi\cos\nu(1+\sin\phi) - \cos\lambda\,[\sin\phi\cos(\xi-\lambda) - \sin(\xi-\lambda+\nu)]]\}\Big\}. \qquad (19)$$

K_{ast} and K_{ady} as given by Eq'ns. (18) and (19), respectively, were evaluated for ϕ between 15° and 45° and δ_w between 0°

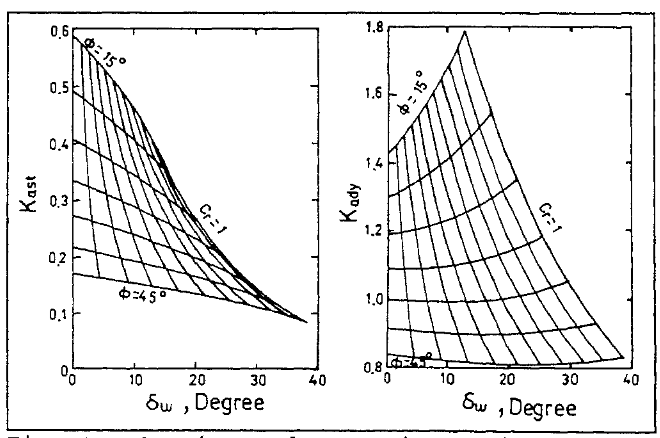

Fig. 4. Static and Dynamic Active Earth Pressure Coefficients

and 40° with an increment of 5° and the results are shown in Fig. 4. Note that contours of constant roughness coefficients, C_r, are also drawn in Fig. 4 for ease of application. For numerical evaluation of static and dynamic active earth pressure coefficients, the angle of dilation of backfill material has been related to the angle of internal friction, based on the authors' judgement, by $\nu = \phi - 35^\circ$ (in the computer code PLAXIS developed by Verneer (1993), ν has been taken as $\phi - 30^\circ$).

COMPARISON AND DISCUSSION OF THE RESULTS

The values obtained for K_{ast} and K_{ady} and presented in Fig. 4 when compared with the results obtained by slice theory of Behpoor (1993) were identical, as expected. But, the new approach is fundamentally different and more versatile than the slice or block theories, and can accommodate variable ϕ .

The values of K_{ast} and K_{ady} were specifically determined for $\phi = 39^\circ$ and $\delta_w = 18^\circ$ and the result as given by

$$K = (P_{at} / \rho g H) = 0.176 + 0.917 \ (a / g) \qquad (20)$$

is compared, in Fig. 5, with the experimental results of Sherif (1982) and Sherif (1984); taken from Bakeer (1990); and Mononobe-Okabe equation. It is evident from this comparison that the results obtained by this new approach can quite successfully predict the experimental results.

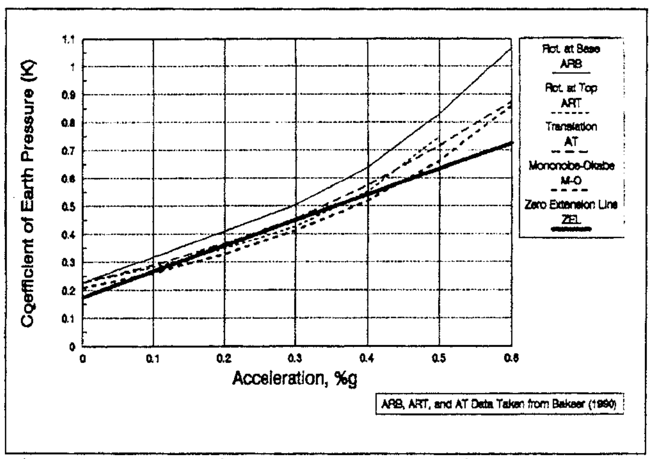

Fig. 5. Comparison of Experimental and Theoretical Results

Fig. 4 indicates that in the practical range of angle of friction for backfill materials; taken between 35° and 45°; K_{ast} and K_{ady} are not sensitive to wall roughnesses and the coefficients can be best approximated by the following formulas:

$$K_{ast} = \tan^2(\pi/4 - \phi/2)$$
$$K_{ady} = \tan(\pi/4 - \nu/2) \qquad (21)$$

where ϕ and ν are the angles of friction and dilation of backfill material, respectively.

Last but not least, the new results indicate that, in the same line as proposed by Whitman (1991), P_{ady} can be obtained by

$$P_{ady} = (\rho \ a \ H) \times 0.90 \ . \qquad (22)$$

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