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# Effect of the Grain Size on the Energy per Unit Volume at the Onset of Liquefaction

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**SYNOPSIS** Recent exploratory work by the authors indicated the feasibility of relating the development of the pore water pressure leading to liquefaction of soils subjected to earthquake loading to the amount of unit energy imparted to the soil during the dynamic motion. This research also showed that regardless of the mode of stress application, sinusoidal or random, the unit energy needed to initiate liquefaction is nearly constant for a given effective confining stress and a specific relative density, demonstrating that the unit energy is independent of the shear strain amplitude. Data obtained during torsional shear tests on a given soil made possible the development of relationships between the unit energy required for liquefaction (as the dependent variable) and the effective confining pressure and the relative density (as the independent variables). This paper examines the effect of grain size, and in particular that of the amount of silt contained in the liquefiable soil, on the amount of unit energy required for liquefaction. The soils selected for study included soils that liquefied during the recent Northridge Earthquake (Lower San Fernando Valley Dam). Understanding the effect of grain size on the amount of unit energy needed to initiate liquefaction is fundamental if an energy-based method to determine the liquefaction potential of a soil deposit is to be implemented.

## INTRODUCTION

The development of liquefaction in saturated granular soils during earthquakes is of concern to practitioners and researchers alike, because of its damaging effects on civil engineering structures. Research in this field is covering both the prediction of the liquefaction potential of a soil deposit and its post-liquefaction behavior. The first aspect is presently the main concern of designers, whose aim is to prevent or at least minimize damage to new and existing structures. If a soil deposit is found to have a high potential for liquefaction, corrective measures can be undertaken before a seismic event occurs.

Two methods have been advanced and used to determine the liquefaction potential of a soil deposit: The equivalent stress method of Seed and Idriss (1971), Seed et al (1975), and Ishihara and Yasuda (1972, 1975); and the shear strain method of Dobry et al (1982).

With the introduction of the energy concept in the analysis of the densification and liquefaction of cohesionless soils by Nemat-Nasser and Shokooh (1979), a number of experimental studies were conducted by Davis and Berrill (1982), Simcock et al (1983), Berrill and Davis (1985), Law et al (1990), Figueroa (1990), and Figueroa and Dahisaria (1991), Figueroa et al. (1994), and Liang et al (1995) to establish relationships between pore water pressure

development and the dissipated energy during the dynamic motion; and to explore the use of the energy concept, in the evaluation of the liquefaction potential of a soil deposit.

Using hollow cylindrical specimens of Reid Bedford sand subjected to sinusoidal torsional loading, Figueroa et al (1994) demonstrated that there exists a relationship between the dissipated energy per unit volume at the onset of liquefaction and the effective confining pressure, the relative density and the shear strain amplitude. Liang et al. (1995) presented results of liquefaction torsional shear tests conducted under an earthquake-type time series of loading, which demonstrated that the energy per unit volume needed to induce liquefaction was independent of the dynamic loading form. Thus the unit energy can replace both the amplitude of the shear stress and strain and the number of cycles in the evaluation of the liquefaction potential of soils subjected to earthquake loading. The unit energy needed to induce liquefaction can be obtained in the laboratory for predetermined field conditions. In the field, the hysteretic response of a soil deposit subjected to a "design earthquake" that is characterized by its acceleration time history can be obtained through existing numerical procedures (Elgamal, 1991) and used to calculate the field unit energy. This field value is then compared to the one obtained in the laboratory to determine the potential for liquefaction.

Before the energy procedure is implemented the influence

of the grain size distribution must be examined. Relative density has always been considered a key parameter in the study of liquefaction; grain size distribution has not, even though its influence on the relative density is substantial. The percentage of fines in a soil deposit can drastically change its liquefaction characteristics and can become a parameter to reckon with in the process of designing a constitutive model.

This paper studies the effect of the grain size on the amount of unit energy required for the liquefaction of two soils, namely Reid Bedford sand and the silty sands that liquefied during the recent Northridge Earthquake (Lower San Fernando Valley Dam). Of particular interest is the examination of the effect of the amount of silt present in the soil. Recent studies by Singh (1994) point to the importance of fines when looking at the whole liquefaction process.

## LABORATORY TESTING

### Soil Types and Test Specimen Preparation

The soils selected for study included the clean and fairly uniform Reid Bedford sand, obtained from the Reid Bedford Bend, located south of Vicksburg, Mississippi, and the Lower San Fernando Dam (LSFD) silty sand, obtained from the Lower San Fernando Dam, in the Los Angeles, California area. The LSFD silty sand is also fairly uniform but contains up to 28 percent of fines, as depicted by the grain size curves shown in Figure 1. This silty sand was collected from sand boils generated during the Northridge Earthquake of January 17, 1994. Classification tests to determine the properties of these soils included sieve analysis, specific gravity and relative density. These results are shown in Table 1.

Table 1. - Index Properties and Classification

Property	Reid Bedford Sand	LSFD Silty Sand
USCS Group	SP	SM
Specific Gravity	2.65	2.67
Max. Void Ratio	0.85	1.22
Min. Void Ratio	0.58	0.71
D <sub>50</sub>	0.26 mm	0.13 mm

The long, thin, hollow cylindrical specimens prepared for the torsional shear liquefaction tests had the following dimensions:

Outer diameter = 7.1 cm  
 Inner diameter = 5.1 cm

Height = 12.9 cm  
 Volume = 247.2 cm<sup>3</sup>

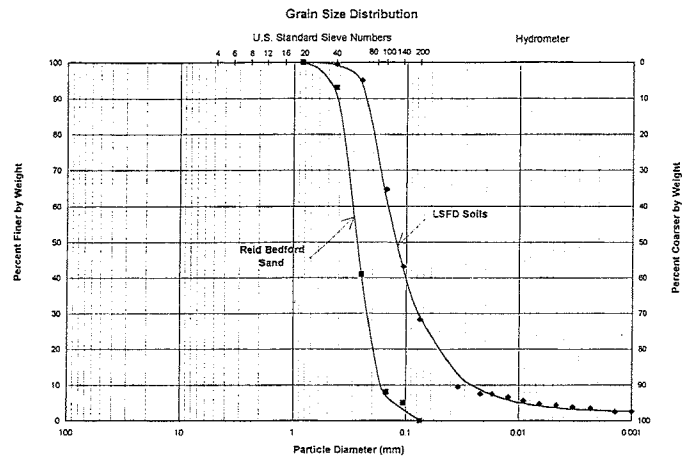


Figure 1. Grain Size Distribution of Tested Soils

These dimensions minimize the radial variation of the shear stresses and the end effects. Sample preparation is described in detail in Figueroa et al. (1994).

### Torsional Shear Liquefaction Testing

Tests were conducted on specimens at nominal relative densities of 50, 60 and 70 percent for the Reid Bedford sand, and at four relative densities ranging between 57 and 92 percent for the LSFD silty sand. No lower relative densities were possible. The three initial effective confining pressures included 41.4 kPa, 82.7 kPa and 124.1 kPa; for each value of the relative density. All specimens were tested with a back pressure of 137.9 kPa (=20 psi).

All the tests were conducted in a controlled-stress-type hollow cylinder torsional shear device. This device is computer-driven. An E/P transducer (Electric-to-Pneumatic Transducer) converts a voltage to a proportional air pressure which is sent to the machine's actuator. The signals from the computer are faithfully translated to positive or negative shearing stresses.

### Random Torsional Loading Control

The torsional load (stress) applied to the hollow cylinder follows a synthetically generated earthquake time series proportional to the time history of the ground acceleration (Seed and Idriss, 1971; Ishihara and Yasuda, 1972 and 1975). The generation of the random synthetic time history of ground acceleration shown in Figure 2 requires the duration of the earthquake, maximum ground acceleration, intensity envelope function, target response spectrum, and damping ratio as input parameters. All specimens were subjected to the same shear stress time series.

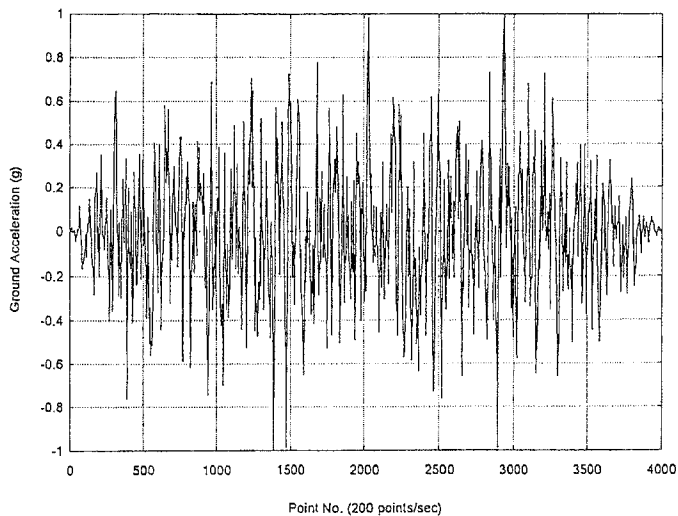


Figure 2. Simulated Ground Acceleration (Liang et al., 1995)

A similar package could be used in the actual application of the unit energy method to determine the liquefaction potential of a soil deposit.

## RESULTS AND ANALYSIS

Liquefaction during torsional shear testing is considered to have occurred when the pore pressure in the undrained specimen equals the confining pressure. The accumulated energy per unit volume ( $\delta W$ ) absorbed by the specimen, can be calculated from the area of the hysteresis loops showing the variation of the shear stress with the shear strain. It is given by (Figuroa et al, 1994):

$$\delta W = \sum_{i=1}^{n-1} \frac{1}{2} (\tau_i + \tau_{i+1}) (\gamma_{i+1} - \gamma_i) \quad (1)$$

where:

- $\tau$  = shear stress;
- $\gamma$  = shear strain;
- $n$  = number of points recorded to liquefaction.

A typical trace of the hysteresis loops is shown in Figure 3 which displays the characteristic decay in the shearing resistance of an LSFDF silty sand specimen before the development of liquefaction; and the progressive flattening of the loops, indicating the softening of the soil as the pore water pressure increases. This specimen was prepared at a relative density of 60% and was subjected to an initial effective hydrostatic pressure of 82.7 kPa. The area of the loops is initially small because of the relatively high stiffness of the soil during the initial stages of shearing. However as the pore pressure increases, the soil gradually loses its stiffness allowing for larger strains and a rapid buildup of the accumulated energy per unit volume. When the specimen is near liquefaction it has suffered significant

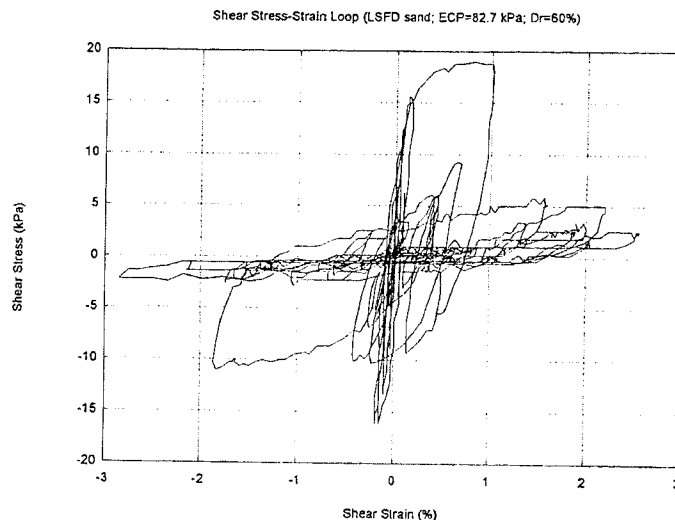


Figure 3. Shear Stress-Strain Relationships During Random (Stress Controlled) Torsional Shear Testing - LSFDF Silty Sand

softening and a loss of shearing resistance. In such case the area of the elongated hysteresis loops becomes very small.

The energy per unit volume ( $\delta W$ ) at the onset of liquefaction calculated from the hysteresis loops was related to the relative density ( $D_r$ ) and the initial effective confining pressure ( $\sigma'_c$ ) by multivariable regression analyses. Linear, second order polynomials and logarithmic relationships were examined. Equation 2 provided the best fit to the data obtained from nine non-uniform torsional loading tests, conducted at combinations of three relative densities and three effective confining pressures on Reid Bedford Sand:

$$\text{Log}_{10}(\delta W) = 2.062 + 0.0039 \sigma'_c + 0.0124 D_r \quad (2)$$

$R^2 = 0.925$

This equation indicates that:

1. A higher amount of energy is required to liquefy a clean sand with a higher relative density, considering a constant effective confining pressure.
2. More energy per unit volume is required to liquefy a clean sand with higher effective confining pressure, considering a constant relative density.

Liang et al. (Liang et al., 1995) stated that the response time history of the resisting shear stress and the corresponding power spectrum show a frequency band width similar to that of the excitation. However, specimens prepared at a higher relative density and subjected to a higher initial effective confining pressure develop a shear stress power spectrum closer to the excitation spectrum than the spectrum of specimens prepared at a lower relative density and subjected to a lower initial effective confining pressure.

To examine the influence of grain size on the amount of

energy per volume at the onset of liquefaction random-

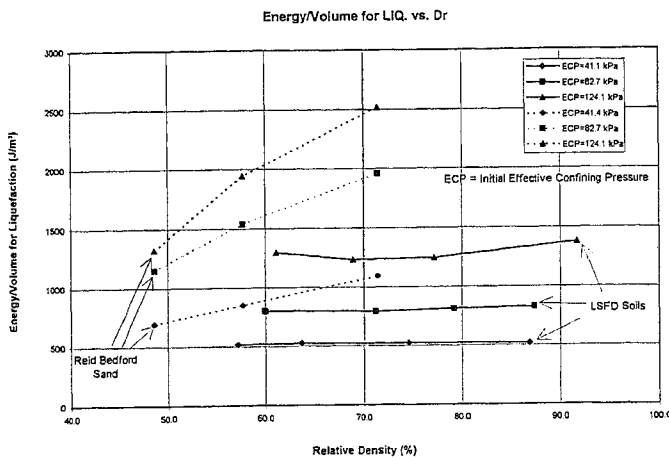


Figure 4. Effect of Grain Size on the Energy/Volume for Liquefaction

loading torsional shear tests were also conducted on silty sands that liquefied during the recent Northridge Earthquake (Lower San Fernando Valley Dam). Four nominal relative densities and three effective confining pressure were used for a total of twelve tests. These soils contain up to 28% of silt.

Here too, regression analyses were conducted between the energy per unit volume (dependent variable) and the relative density and the effective confining pressure (independent variables). Equation 3 provided the best fit to the data obtained from the non-uniform torsional loading tests conducted on the LSFDF silty sand:

$$\text{Log}_{10}(\delta W) = 2.484 + 0.00471 \sigma_c' + 0.00052 \text{Dr} \quad (3)$$

$$R^2 = 0.995$$

The minor effect of the relative density in determining the amount of unit energy at the onset of liquefaction in the silty sand is noted by the very low coefficient corresponding to Dr. This is also observed after eliminating the relative density from the regression analysis to obtain the equation:

$$\text{Log}_{10}(\delta W) = 2.529 + 0.00474 \sigma_c' \quad (4)$$

$$R^2 = 0.994$$

where the constant, the coefficient for  $\sigma_c'$  and the coefficient of determination, are barely affected. The presence of a high percentage of silt seems to have changed the kinematics of the soil mass. Simple models, based on positive or negative dilations caused by sand particles rolling on each other, cease to be satisfactory. It appears that the relative density alone is an unsatisfactory parameter in representing the degree of densification of a granular soil containing an appreciable amount of fines (Head, 1992).

Figure 4 shows the amount of energy per unit volume at the onset of liquefaction as a function of the relative density for the Reid Bedford Sand and the LSFDF silty sand. The finer LSFDF soil requires lower unit energy for liquefaction than the coarser Reid Bedford Sand at the same effective confining pressure. This figure also shows that the influence of the relative density on the energy per volume is practically eliminated with increased silt content, (LSFDF silty sand) regardless of the value of the effective confining pressure. One also notices that LSFDF silty sand specimens liquefy at relative densities above 80 and 90%, contrary to the belief that liquefaction only occurs at lower Dr.

The influence of the relative density in the development of pore pressure during liquefaction is also seen in Figures 5 and 6, where the normalized pore water pressure (pore water pressure/initial confining pressure) is shown as a function of the energy per volume. for specimens prepared at several relative densities. The same constant confining pressure of 82.7 kPa is used in both figures. While the curves corresponding to the different relative densities are widely separate in Figure 5 they nearly coincide in Figure 6; thus demonstrating the near lack of influence of this parameter on the energy per unit volume. Also, the build up of the pore water pressure in the clean granular loose soil is faster for a smaller relative density. This is hardly noticeable in the case of the silty sand.

The relation between the normalized pore pressure build up and the unit energy can be expressed by an equation of the form:

$$\frac{u}{\sigma_c'} = a + \frac{b\delta W}{c + \delta W} \quad (5)$$

where:

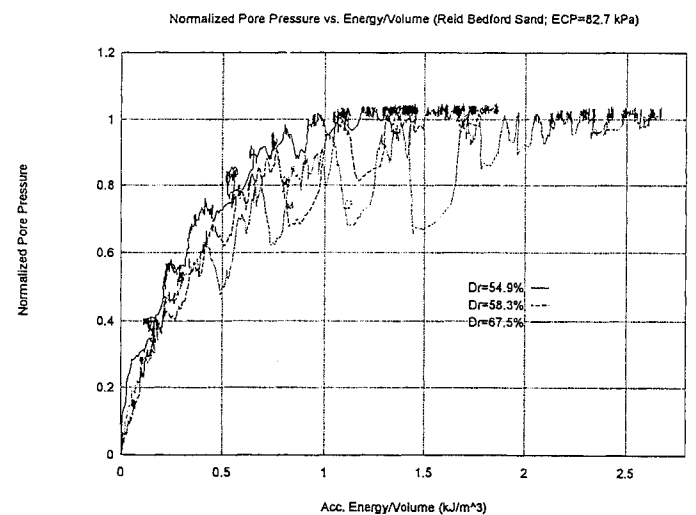


Figure 5. Variation of Normalized Pore Pressure Build-up with Energy/Volume (Reid Bedford Sand)

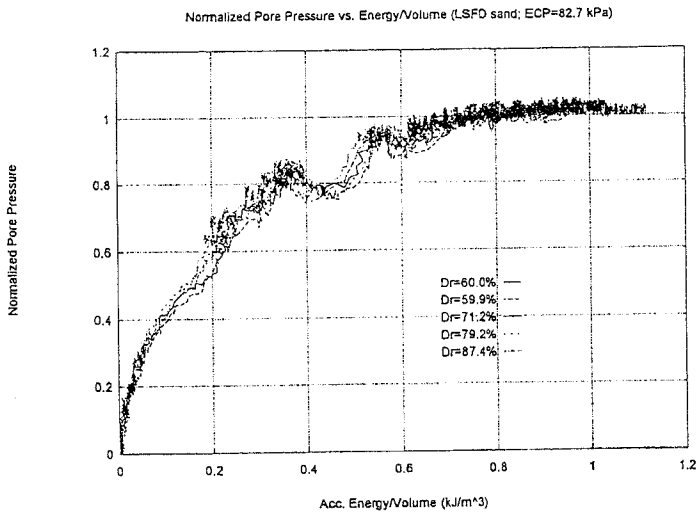


Figure 6. Variation of Normalized Pore Pressure Build-up with Energy/Volume (LSFD Silty Sand)

$u$  = pore water pressure  
 $\delta W$  = energy/volume  
 $\sigma_c'$  = initial effective confining pressure  
 $a, b, c$  = regression constants

The general form of this equation partially follows the hyperbolic relationship commonly used in soil dynamics.

The equations and curves of best fit are shown in Figures 7 and 8 for three of the curves shown in Figures 5 and 6 respectively. All correlations through the hyperbolic relationship described by Equation 5 yielded a coefficient of determination greater than 0.97. The correlation equations shown in Figure 7 for the Reid Bedford sand clearly depict the influence of the relative density in the pore pressure build up as a function of the energy per volume. Similarly, the very minor influence of  $D_r$  in the pore pressure build up as a function of the energy per volume is evident in Figure 8 where all regression curves essentially coincide.

Energy-based degradation models used in the prediction of soil response when subjected to random type earthquake loading could follow the general form of Equation 5 which can be readily obtained from torsional shear liquefaction tests.

## SUMMARY AND CONCLUSIONS

The use of the energy concept to define the liquefaction potential of a soil has been examined through a series of torsional shear liquefaction tests conducted on Reid Bedford sand and Lower San Fernando Dam silty sand. The influence of the grain size distribution on the amount of energy per volume at the onset of liquefaction was examined by comparing the results of liquefaction tests on specimens of these two soils. The finer LSFD silty sand requires lower energy per volume to reach liquefaction. In

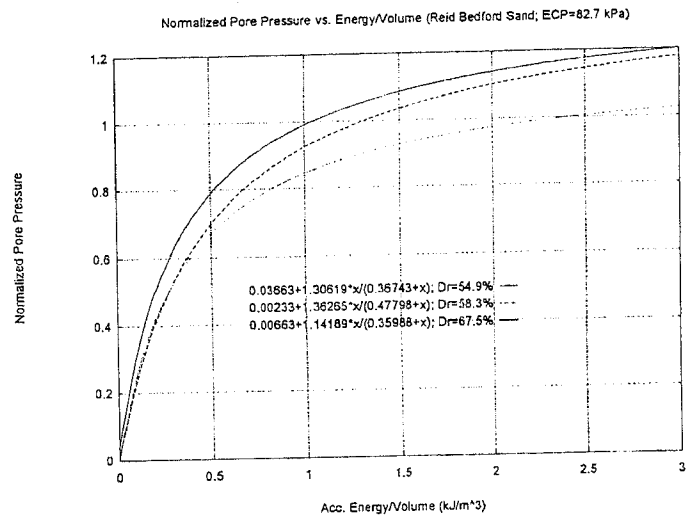


Figure 7. Hyperbolic Relationships for Reid Bedford Sand

addition the influence of the relative density of the soil as a determining factor in the level of energy required for liquefaction is practically eliminated with increased silt content; regardless of the value of the effective confining pressure. The kinematics of the granular soil mass seems to have been modified by the significant presence of silt in the inter granular spaces.

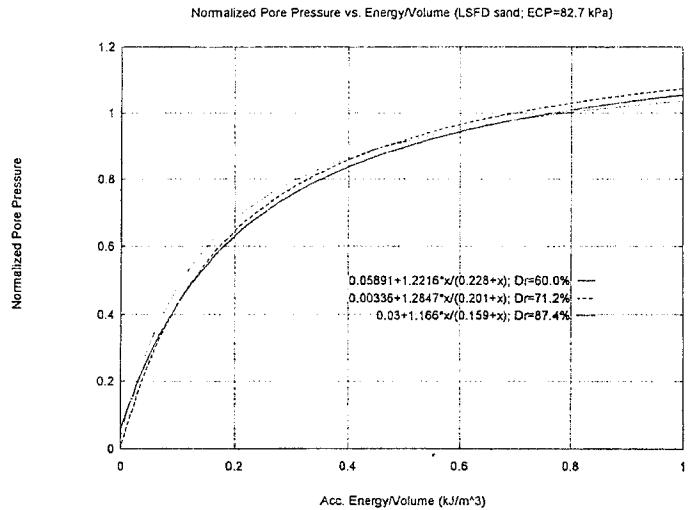


Figure 8. Hyperbolic Relationships for LSFD Silty Sand

The pore water pressure build up during liquefaction tests was found to relate reasonably well to the energy per volume through a hyperbolic relation. This type of equation could very well serve as the basis for an energy-based degradation model which could be used in the prediction of soil response when subjected to random type earthquake loading. The predicted energy per unit volume can then be compared to the energy needed for liquefaction. The latter being obtained from liquefaction tests in the laboratory.

## ACKNOWLEDGMENTS

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