Lateral Stress Ratio on Retaining Structures after Earthquake loading

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SYNOPSIS Equations are derived by analysis giving the earthquake-induced change in the permanent horizontal stress acting on frictionless vertical walls retaining dry sand. These equations exhibit a limit in the coefficient of lateral pressure that depends only on the slope of the critical state line and the Poisson's Ratio of the backfill. The horizontal stress after dynamic shaking increases or decreases towards this limit. Predictions agree qualitatively with results of laboratory tests.

INTRODUCTION

The Mononobe-Ocabe equation that is often used for the seismic design of gravity retaining walls is based upon the assumption that the backfill must deform enough so that full shear resistance is mobilised along the failure plane in the active sense (Whitman, 1990). This assumption implies that the horizontal force (and thus the lateral stress ratio, K) acting on a retaining wall does not change after dynamic loading. Yet, Whitman (1990) states :
"In shaking table tests by Yong in 1985 with unyielding walls, K_o initially was 0.41, but increased to 0.74 after repeated shaking of loose sand and to 0.89 after shaking of dense sand. In centrifuge tests using tilting walls, Andersen et al. in 1987 found residual forces nearly as large as the peak forces during shaking. Steedman in 1984 also observed such increases in his centrifuge model tests upon cantilever walls. Evidence of residual increases has been observed in the field, (for anchored bulkheads) by Iai et al. in 1989."

A general method for predicting earthquake-induced residual displacement and stress redistribution has been developed by Stamatopoulos et al. (1991). In this method, called "Residual Strain Method" (RSM), calculation of transient and residual response is uncoupled over each cycle of loading. This method has been implemented using finite elements in a computer program.

This paper first describes the "Residual Strain Method" (RSM) and gives relevant predictions that illustrate the ability of the method to predict the earthquake-induced permanent change of lateral stress. Then, it uses the RSM to obtain theoretical equations that give the earthquake-induced change in the residual horizontal stress acting on frictionless walls retaining dry sand. These equations exhibit a limiting value of the lateral stress ratio which can lead to improved methods for design of walls with dry backfill.

THE RSM AND ITS PREDICTIONS

The Method

With the "Residual Strain Method" (RSM), attention is focused upon "permanent" (also mentioned in the paper as "residual") strains and stresses in the soil at the end of a load cycle. Under dry conditions, the equations which describe the incremental permanent volumetric and principal deviator strains $d\varepsilon_{vol}$ and de_i are

$$d\varepsilon_{vol} = d\varepsilon^o{}_{vol} + d\sigma_{oct}/K_t \quad (1a)$$

$$de_i = de^o{}_i + dS_i/(2G_t) \qquad (i= 1,2,3) \quad (1b)$$

In the above equations 1,2,3 are principal directions of stress and strain and the strain increments $d\varepsilon^o{}_{vol}$ and $de^o{}_i$ represent permanent strains that would accumulate only in the case of constant average effective stresses, such as in cyclic drained triaxial tests. The stress increments $d\sigma_{oct}$ and dS_i denote changes in the average octahedral and principal deviatoric stress respectively. The symbols K_t and G_t denote the tangent bulk and shear moduli of the soil respectively, used to redistribute $d\varepsilon^o{}_{vol}$ and $de^o{}_i$.

In association with equations (1a) and (1b) Stamatopoulos et al. (1991) give an empirical model that expresses the incremental permanent strains $d\varepsilon^o{}_{vol}$ and $de^o{}_i$ that accumulate in a load cycle of cyclic drained triaxial tests in terms of the cyclic shear strain of the current cycle, the number of cycles of dynamic loading applied so far and the applied average stresses. Stamatopoulos et al. (1991) also give an empirical model that expresses the tangent moduli of the soil in terms of the average state of stress.

A key assumption in the method given above is that residual strains can be separated into two components: strains caused by average stress redistribution and those caused by cyclic loading around constant average stresses. Estimation of permanent strain is simplified in this way, for two main reasons : (a) the actual problem of dynamic equilibrium is reduced into an equivalent static one and (b) the time history of the loading does not need to be simulated explicitly within each load cycle.

Relevant Predictions

Finn (1981) presents results from drained cyclic oedometer tests on Ottawa sand to demonstrate the redistribution of average lateral stresses occurring in this type of test. The tests had different initial stress ratios. The results of these tests are shown in fig. 1. Stamatopoulos et al. (1991) present predictions of these tests by a finite-element program that uses the RSM (fig. 1). The model predictions follow the trends of the tests : (a) the lateral stress ratio changes towards a limiting value that is unique and (b) the rate of change increases with the cyclic shear strain and decreases with cycle number. In addition, the model predictions provide a relative accurate estimate for the limit value of the lateral stress ratio.

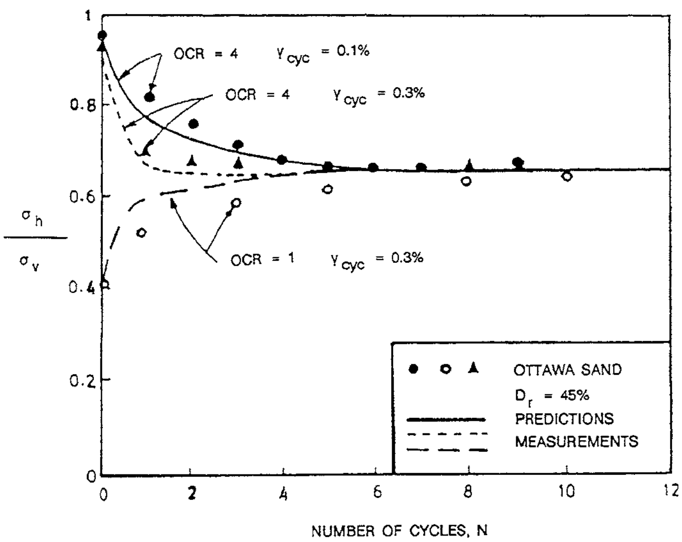

Fig. 1. Predicted and Measured Lateral Stress Redistribution in Cyclic Oedometer Tests (from Stamatopoulos et al., 1991).

Dynamic centrifuge tests simulating the seismic response of tilting walls retaining dry sand were performed by Andersen in 1987. Three different combinations of wall flexibility and seismic intensity were tested, as described by Stamatopoulos and Whitman (1990). The soil behind the wall was initially in an active state of stress, with lateral stress ratio K_O equal to 0.22. After the shakings, K increased to 0.28, 0.31 and 0.32, respectively. Fig. 2 gives the test arrangement, a typical input acceleration record and the corresponding measured change in the spring force. The response of these tests was predicted by a finite-element program that uses the RSM by Stamatopoulos and Whitman (1990). The computed values of the magnitude and location of the horizontal force acting on the wall before and after shaking of the three tests agreed reasonably well with the measured values.

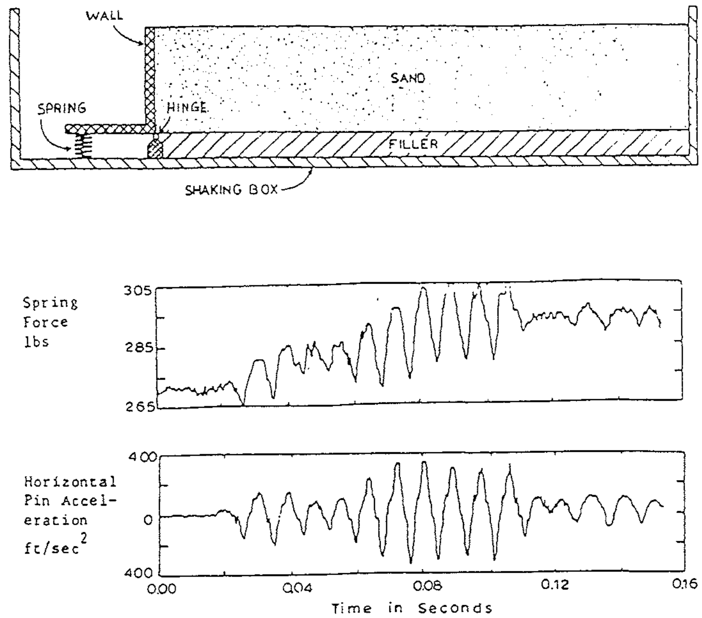

Fig. 2. Centrifuge Test Arrangment, and Change in the Horizontal Force acting on the Wall (from Whitman, 1990).

From the above it can be concluded that the "Residual Strain Method" (RSM) can predict with reasonable accuracy the permanent change of lateral stresses caused by cyclic loading.

ANALYSIS

Boundary Conditions

The response at the end of each cycle of dynamic loading at each infinitesimal element of dry sand located next to a vertical frictionless wall with large width is subjected to the following boundary conditions :

(1) The total vertical stress, given by the condition of vertical equilibrium, does not change. It remains equal to the product of the unit weight of the soil and the depth. The vertical wall affects only the horizontal stress, since it restricts only horizontal movement.

(2) Plane strain conditions exist. The major and minor directions of stress in the plane of strain, denoted as "1" and "3" respectively, correspond to the vertical or horizontal directions, "v" and "h" respectively. The intermediate stress with direction perpendicular to the plane of strain is given by

$$\sigma_2 = \mu (\sigma_v + \sigma_h) = \mu \sigma_v (1 + K) \quad (2)$$

where μ is the Poisson's Ratio, K is the lateral stress ratio and σ_v is the total vertical stress.

(3) The incremental horizontal strain depends on the incremental horizontal stress. Assuming a distribution of incremental stress with depth, the relationship between the incremental horizontal stress and strain is a characteristic of the wall. If the change in horizontal stress is zero, the change in the horizontal strain is also zero. An increase of the horizontal stress in the soil will cause outward movement of the wall and negative (extensive) strain at the soil. Thus, the incremental change in horizontal strain $d\varepsilon_h$ can be expressed as

$$d\varepsilon_h = - d\sigma_h\, f \qquad (3)$$

where $d\sigma_h$ is the incremental change in the horizontal stress and "f" represents a function which is never negative and expresses the flexibility of the wall at the location that may depend on $d\sigma_h$. The function "f" is zero when the wall stiffness is infinite and increases as the wall stiffness decreases. For example, for a tilting wall with linear response, "f" equals the rotational flexibility of the wall, if a linear distribution of residual horizontal stress is assumed with depth.

Theoretical Background

Chang and Whitman (1988) used concepts of soil plasticity to show that the ratio of permanent volumetric to permanent shear strains that accumulate in drained cyclic triaxial compression tests may be expressed in terms of the average applied total stresses as

$$d\varepsilon^o_{vol} / d\gamma^o = 3\, M\, (1 - Q^2) / (4\, Q) \qquad (4a)$$

where γ is the shear strain ($\varepsilon_v - \varepsilon_h$), M is the slope of the critical state line and Q is the ratio of the shear stress by the product of the octahedral stress times the parameter M. The parameter M is largely independent of initial soil density and can be expressed in terms of the friction angle where shearing occurs in triaxial compression at constant volume, φ_{cv} (see fig. 3). Chang and Whitman (1988) estimated M as 1.2, 1.2 and 1.4 for the Oostrechelde, Leighton-Buzzard 120/200 and Connecticut River sands respectively and verified equation (4a) from results of cyclic drained triaxial tests on these sands.

For conditions of plane strain equation (4a) is generalised assuming that the in-plane intermediate stress and strain do not affect the shear stress-strain response. This assumption, which is also made by Bouckovalas and Hoeg (1987), gives that γ and Q in equation (4a) can be expressed as

$$\gamma = \varepsilon_1 - \varepsilon_3 \qquad (4b)$$

$$Q = \frac{\sigma_1 - \sigma_3}{M\, \sigma_{oct}} = \frac{3\,|1 - K|}{(1 + K)\,(1 + \mu)} \qquad (4c)$$

where 1 and 3 are the major and minor directions of stress in the plane of strain and the vertical brackets denote absolute value.

Derivation and Solution

For the boundary conditions given previously, equation (1a) becomes

$$d\varepsilon_v + d\varepsilon_h = d\varepsilon^o_{vol} + d\sigma_h\,(1+\mu)/(3\, K_t) \qquad (5a)$$

For the vertical and horizontal directions and the boundary conditions given previously equation (1b) can be expressed as

$$d\varepsilon_v - d\varepsilon_h = d\varepsilon^o_v - d\varepsilon^o_h - (d\sigma_h)/(2\, G_t) \qquad (5b)$$

Eliminating $d\varepsilon_v$ from equations (5a) and (5b) gives

$$d\sigma_h = \frac{d\varepsilon^o_v - d\varepsilon^o_h - d\varepsilon^o_{vol} + 2\, d\varepsilon_h}{1/(2\, G_t) + (1+\mu)/(3\, K_t)} \qquad (6)$$

The bulk modulus can be related to the shear modulus and Poisson's Ratio. In addition, it can be assumed the same Poisson's Ratio prior to and during shaking. Then, equation (6) becomes

$$d\sigma_h = G_t\,(d\varepsilon^o_v - d\varepsilon^o_h - d\varepsilon^o_{vol} + 2\, d\varepsilon_h)/(1-\mu) \qquad (7a)$$

Substitution of equation (3) into equation (7a) gives

$$d\sigma_h = \frac{d\varepsilon^o_v - d\varepsilon^o_h - d\varepsilon^o_{vol}}{(1 - \mu)/(G_t) + 2\, f} \qquad (7b)$$

When the vertical stress is greater than the horizontal stress, the directions 1 and 3 correspond to the vertical and horizontal directions, respectively. Combining equation (7b) with equations (4a) and (4b) gives

$$d\sigma_h = d\varepsilon^o_{vol}\, \frac{(4\, Q)/(3\, M\, (1 - Q^2)) - 1}{(1 - \mu)/(G_t) + 2\, f} \qquad (8a)$$

Equation (8a) predicts that $d\sigma_h$ is zero when the stress ratio Q is

$$Q* = -2/(3M) + (\, 4/(9M^2) + 1\,)^{0.5} \qquad (9)$$

This stress ratio Q*, gives a limiting value for the lateral stress ratio, K, after extensive shaking. Fig. 3 gives this limit in terms of M and μ using equations (9) and (4c).

When the vertical stress is less than the horizontal stress, the directions 1 and 3 correspond to the horizontal and vertical directions, respectively. Equations (7b), (4a) and (4b) give

$$d\sigma_h = -d\varepsilon^o_{vol}\, \frac{(4\, Q)/(3\, M\, (1 - Q^2)) + 1}{(1 - \mu)/(G_t) + 2\, f} \qquad (8b)$$

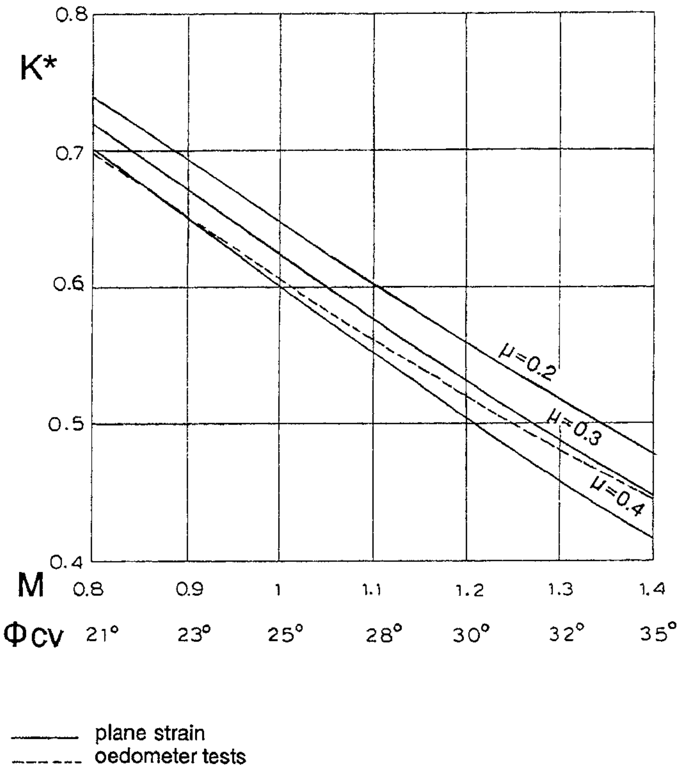

Fig. 3. Limiting Value Towards Which the Coefficient of Lateral Pressure Changes After Dynamic Loading.

Stamatopoulos et al. (1991) indicate that $d\varepsilon^{o}_{vol}$ is positive for stress ratios Q less than one, zero for Q equal to one and negative for Q greater than one. Thus, the right-hand side of equation (8b) will always be negative. It is concluded that when the initial horizontal stress is greater than the vertical stress, the horizontal stress will potentially decrease until it becomes equal to the vertical stress. Then, the directions 1 and 3 correspond to the vertical and horizontal directions, respectively; the change in the horizontal stress is given by equation (8a) and the limit stress ratio after extensive shaking is given again by equation (9) and fig. 3.

Typical Values of the Limiting Stress Ratio

For sands, typically the Poisson's Ratio μ is taken as 0.3 and the stress ratio at the critical state in triaxial compression M lies between the values of 1 and 1.4. For these values, the limit stress ratio according to fig. 3 lies between 0.65 and 0.45.

EXAMPLES OF USING FIG. 3

Suppose that the soil parameters M and μ of a dry sand layer retained by a frictionless retaining wall are equal to 1.2 and 0.3, respectively. Then, fig. 3 gives that the limiting stress ratio K* equals 0.53. Thus, if the horizontal stress acting on the wall prior to shaking corresponds to the Rankine active state of stress (Terzaghi, 1943), the initial stress ratio K_o is typically about 0.25 and will increase towards the value of 0.53. If, on the other hand, the initial lateral stress ratio is in a Rankine passive state of stress, as a result of shaking, the lateral stress ratio will decrease from a typical value of about 4 towards a value of 0.53.

MAGNITUDE OF CHANGE TOWARDS THE LIMITING CONDITION

The earthquake-induced residual strain $d\varepsilon^{o}_{vol}$ increases when the intensity of shaking increases (Stamatopoulos et al., 1991). Thus, equations (8a) and (8b) predict that as a result of earthquake loading the residual horizontal stress will increase more towards the limit stress ratio of fig. 3 when either one of the intensity of shaking, the tangent shear modulus of the soil, or the stiffness of the wall relative to residual movement increase. Quantitative estimates of the magnitude of the incremental change of the horizontal stress can be obtained upon substitution in equations (8a) and/or (8b) of :

(1) the model given by Stamatopoulos et al. (1991) giving the permanent volumetric strain in triaxial conditions in terms of the cyclic stress history, the average stress ratio and appropriate numerical constants;
(2) the model given by Stamatopoulos et al. (1991) giving the tangent shear modulus in terms of the average stress ratio and appropriate numerical constants; and
(3) the function "f" of equation (3) that expresses the flexibility of the wall under consideration.

The total change in horizontal stress at given depth can be obtained by numerical integration of equations (8a) and/or (8b) after making the substitutions mentioned above.

Alternatively, for a wall that is initially in an active state of stress the above indicate that the increase of the horizontal stress at given depth depends primarily on the intensity of shaking, the stiffness of the soil and the stiffness of the wall to residual movement. This suggests that semi-empirical relationships can express the magnitude of the increase of the horizontal stress acting in the wall at given depth in terms of the factors mentioned above. Such relationships can be obtained from numerical integrations of equation (8a) in connection with appropriate measurements in the field and/or in laboratory and centrifuge tests. Predictions of equation (8a), together with relevant measurements are not given here, as the purpose of this paper is not to predict the magnitude of earthquake-induced changes in the horizontal stress acting on retaining walls

LIMITING STRESS RATIO FOR CYCLIC OEDOMETER TESTS

The derivations given above are for the boundary conditions behind a wall where plane strain conditions exist. For the case of cyclic oedometer tests the intermediate principal stress is not given by equation (2) but is equal to

$$\sigma_2 = \sigma_h \quad (10)$$

In addition, no lateral residual movement can develop and thus the function "f" equals zero and equation (3) becomes:

$$d\varepsilon_h = -d\sigma_h \times 0 = 0 \quad (11)$$

Using equation (10), the stress ratio with respect to the critical state Q can be expressed in terms of the lateral stress ratio K as

$$Q = \frac{\sigma_1 - \sigma_3}{M\,\sigma_{oct}} = \frac{3\,|1-K|}{M\,(1+2\,K)} \quad (12)$$

As a result of equation (10), equation (1a) does not give as in case of plane strain equation (5a) but gives

$$d\varepsilon_v + d\varepsilon_h = d\varepsilon^o_{vol} + 2\,d\sigma_h/(3\,K_t) \quad (13)$$

Equation (5b) does not change. Equations (5b), (13) and (11) give

$$d\sigma_h = \frac{d\varepsilon^o_v - d\varepsilon^o_h - d\varepsilon^o_{vol}}{1/(2\,G_t) + 2/(3\,K_t)} \quad (14)$$

Equations (14) and (4a) give that $d\sigma_h$ is zero when the stress ratio with respect to the critical state, Q, is given by equation (9). Thus, the limit stress ratio with respect to the critical state, $Q*$, does not change from the case of plane strain that was considered before. Yet, as the relation between the stress ratios Q and K has changed, the limit lateral stress ratio has changed and can be obtained by the solution of equations (9) and (12). This solution is given in fig. 3. It can be observed that it lies between the solutions for plane strain with Poisson's Ratios of 0.3 and 0.4.

THEORY VERSUS EXPERIMENTAL RESULTS

Equation (8a) predicts that the horizontal stress acting on walls retaining dry sand and initially in an active state has the tendency to increase. This agrees with the laboratory measurements and field observations that were given in the introduction.

In shaking table tests by Yong in 1985 with unyielding walls, K_o at rest was 0.41, but increased to 0.74 after repeated shaking of loose sand and to 0.89 after shaking of dense sand (Whitman, 1990). The increase of K towards a limit value agrees qualitatively with predictions of fig. 3. The measured values of the limiting stress ratios do not agree with predictions for typical values of M and μ. Other laboratory or field measurements of a limit stress ratio after repeated shaking on walls retaining dry soil were not found in the literature.

As illustrated by fig. 1, cyclic oedometer tests with various initial stress ratios gave a unique limiting value of K equal to 0.65. The fact that the lateral stress ratio changes towards a limiting value that, additionally, is unique and does not depend on the initial value of the stress ratio, agrees with the theoretical predictions. Furthermore, the measured limiting value of K is in the range of predictions of fig. 3 (between 0.5 and 0.7) for this type of test for typical values of M.

CONCLUSIONS

This paper used the "Residual Strain Method" (RSM) (Stamatopoulos et al., 1991) and a theoretical equation giving the ratio of permanent strains (Chang and Whitman, 1988) to derive equations (8a) and (8b) that give the earthquake-induced residual change in the horizontal stress at given depth acting on frictionless vertical walls retaining dry sand. These equations predict the following :

(1) A limiting value exists for the coefficient of lateral pressure that depends only on the slope of the critical state line and the Poisson's Ratio of the backfill material (fig. 3).
(2) Expected values of this limit lie between 0.5 and 0.7.
(3) Excluding the transient response, as a result of shaking the vertical stress does not change while the horizontal stress increases or decreases towards this limit.
(4) The magnitude of change towards this limit increases when either one of the intensity of shaking, the stiffness of the wall or the stiffness of the soil increase.

Even though theoretical predictions agree qualitatively with test results, more comparisons with relevant laboratory tests or field measurements are needed to certify the above.

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PARTIAL LIST OF NOTATION

d	Earthquake-induced residual differentional change
f	Function expressing the wall stiffness (see equation (2))
G_t, K_t	Tangent shear modulus and tangent bulk modulus
Q	Stress ratio with respect to the critical state
RSM	Residual Strain Method
K	Lateral stress ratio $\{= \sigma_h/\sigma_v\}$
K_o	Lateral stress ratio prior to shaking
M	Slope of the critical state line $\{= 6 \sin \varphi_{cv} / (3 - \sin \varphi_{cv})\}$
μ	Poisson Ratio
ε	Strain
σ	Total stress; average total stress during cyclic loading
φ_{cv}	Friction angle at the critical state in triaxial compression

Subscripts

1, 3	Major and minor directions of stress in the plane of strain
2	Direction perpendcular to the plane of strain
v,h,	Vertical, horizontal
vol, oct	Volumetric, octahedral
*	Limiting value

Superscript

o	Permanent (residual) strain that accumulates during cyclic drained triaxial tests