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M. K. Chong
University of Durham, United Kingdom

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M.K. Chong
Visiting Fellow, University College, University of Durham
Durham, UK

ABSTRACT

The effects of displacement piling are well documented with many cases of movements caused to adjacent structures and detrimental effects on recently installed piles. The author's experience with dealing with ground displacements of raft piling in deep marine clays in Singapore led to the development of a method for calculating the ground movements to assess the cumulative effects of pile driving. The method is derived from soil mechanics parameters, principle of potential energy, strain energy and work done by the stresses in the soil undergoing a cylindrical cavity expansion process and the stresses in the soil undergoing large strains direct shearing process due to the pile shaft friction. Published case histories of ground displacements have been back-analysed. The calculated movements compared well with these past field tests and laboratory experimental data. In the moderate to far field distances from the pile, the heave to lateral displacements can be expressed as a function of the ratio of lateral forces to soil weight. For near field distances, the calculations show that the heave reaches a maximum, then turns sharply into a downdrag near to the pile shaft. The method is, however, unstable at distances close to the pile shaft.

INTRODUCTION

Problems of Pile Displacement

Displacement piles are common as they are one of the most economical foundations for highrise construction. However, there are many problems associated with their use due to ground heave and lateral displacements causing movements to existing structures. Previous studies on heaving and displacement problems include existing building structures heaved by the nearby pile driving (D'Appolonia, 1971, Healy and Weltman, 1980), uplift of adjacent cylindrical piles that have been already installed. (Cole, 1971, 1972, Hammond et al 1979, Oostveen and Kuppers 1985) as well as steel H-piles driven in soft sediments (Koutsoftas, 1982) and many others. Hagerty and Peck (1971) have shown that the pile uplift is approximately one-half of the soil heave around a single pile. Chow and Teh (1990) theoretical study showed that the pile uplift is approximately one-half to one-third of the soil heave if there were no adjacent piles. Poulos (1994) reported similar results using the deep SPM, Strain Path Method developed at MIT by Baligh, (1986). Sagaseta and Whittle (2001) developed the Shallow SPM to calculate the soil heave without explicitly modeling the pile soil interactions. Such methods have been useful but they are based largely on fluid mechanics.

Displacement piles were used extensively for foundations in deep marine clays in Singapore. There was a need to assess the impact of ground movements due to such massive pile driving on adjacent properties in built-up areas. This paper presents a method for explicitly modeling the pile-soil interaction for calculating such movements.

MODELLING THE DISPLACEMENT PROCESS

Displacement of Soil Around a Closed-Ended Pile

When there are no piles in the vicinity, the soil around a closed-ended pile is assumed to be displaced outwards in an axi-symmetric manner as shown in Figure 1a. For an incompressible soil, there is no change in the volume so that the sum of the integrated heave volume and the volume of the radially displaced soil at a radial distance r from the pile, is equal to the pile volume $\pi r_o^2 H$

$$\int_{r_o}^{r} 2\pi r\, v\, dr \;+\; 2\pi r H w \approx \pi r_o^2\, H \qquad (1)$$

where $w = r - r_i$ is the lateral displacement, v is the vertical displacement of the part of the radially displaced soil from radius r_i to radius r, r_o is the radius of the pile and H is pile length. In the equation the integral term represents the heave volume from r to r_o

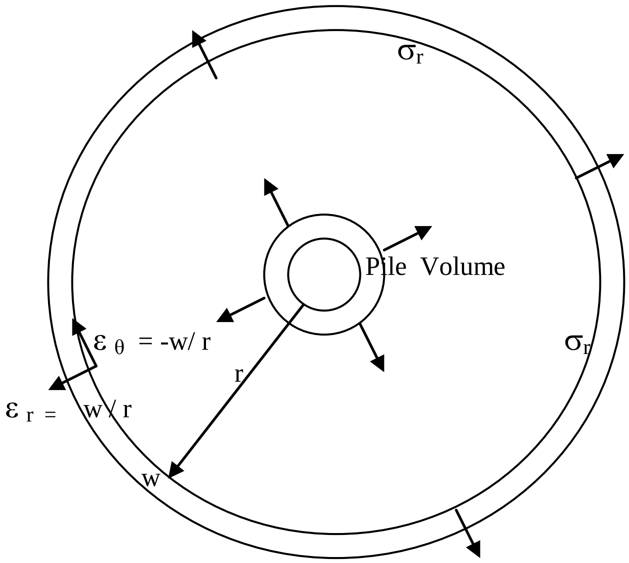

Fig.. 1a Modelling of the displacement process

Then the solution for the displacement w can be expressed as

$$w = \frac{r_o^2}{2r} - \left(\frac{\int_{r_o}^{r} 2\pi r\, v\, dr}{2\pi r H} \right) \qquad (2a)$$

In the simpler case when pile displacement w_c is approximated using that of a purely cylindrical cavity expansion process, then

$$w_c = \frac{r_o^2}{2r} \qquad (2b)$$

In the case of a pile heave and radial displacement taking place simultaneously, the heave volume will then reduce the radial displacement so that the value of the radial displacement will be overestimated by equation (2b). Since $w < w_c$, an expression of w which corrects the over-prediction of equation (2b) may be of the form

$$w = \left(\frac{r_o^2}{2r}\right) e^{-br} < w_c \qquad (2c)$$

where b remains to be determined. Equation (2c) will be applied to correct the over-prediction after the heave function v has been determined. Before considering the form for w given by equation (2c) , it would be useful to first consider the approximate but simpler form for w in equation (2b) which may also be expressed in dimensionless form

$$\frac{w}{r_o} = \frac{r_o}{2r} \qquad (3a)$$

This axisymmetric displacement is inversely proportional to the radial distance from the pile. This expression may be compared to that from cylindrical cavity expansion theory (see for example, Carter et al, 1986)

$$w = \frac{C_u}{G}\frac{R^2}{r} \qquad (3b)$$

where R is the extent of the plastic zone where the soil is sheared to critical state conditions. R is about 7 to 10 times the pile radius (Randolph et al 1979). The shear modulus G is typically about 100 to 200 times the shear strength C_u. This means that the displacement ratio w/r_o values could range from 0.25 r_o / r to 1.0 r_o /r but for corresponding values of $R=7r_o$ and $G=100\,C_u$ and for values of $R=10r_o$ and $G=200C_u$ then equation (3b) would reduce to the expression of equation (3a) with a displacement ratio w/r_o equal to 0.5 r_o/r. Randolph et al 1979 has also derived another expression for the radial displacement

$$\frac{w}{r_o} = \left(1 + \frac{r_i^2}{r_o^2}\right)^{1/2} - \frac{r_i}{r_o} \qquad (3c)$$

The above expressions can be compared with that of SSPM solutions (Sagaseta and Whittle, 2001) for small strain ground movements.

$$\frac{w}{r_o} = \frac{r_o}{2r}\left(1 + \frac{r^2}{H^2}\right)^{-1/2} \qquad (3d)$$

which reduces to equation (3a) since $(1 + r^2/H^2)^{-1/2} \approx 1$. For practical values of r and H , r^2/H^2 is a small number much less than 1. The error of values given by equation (3a) is 20.7% when $r/r_o = 1$, decreases to 5. 9% when $r/r_o = 2$ and are within 1% of those given by equation (3c) for values of $r/r_o > 5$. In the following treatment, equation (3a) is used as it is less unwieldy and is sufficiently accurate for the moderate-to-far field displacement effects.

WORK DONE AND STRAIN ENERGY IN SOIL MASS DUE TO PILE DRIVING

In the following sections, the equations for work done, strain energy and displacements in the loaded soil mass are derived in two stages. Firstly, for the soil undergoing a lateral cavity expansion process but without considering the large shearing process near the pile, the resulting displacement equations are only applicable to moderate-to-far field distances. Secondly, the equations for the work done, strain energy and displacements due to the large shearing process along the pile shaft are then derived for the near field effects. Finally, the complete displacement equation is the obtained by combining both the near field and moderate-to-far field displacement solutions.

Lateral Pressure During Pile Driving

The driving of a pile into the ground causes a build up of lateral pressures in the ground. This lateral pressure in the ground P_i may be obtained from cylindrical cavity expansion theory (see for example Bolton and Whittle, 1999). Under conditions of axial symmetry and undrained expansion, the following relationships apply : axial strain $\varepsilon_a = 0$, circumferential strain $\varepsilon_\theta = -w/r$, the expansion is undrained so $\varepsilon_v = 0$ and the radial strain

$$\varepsilon_r = -\varepsilon_\theta = w/r \qquad (4)$$

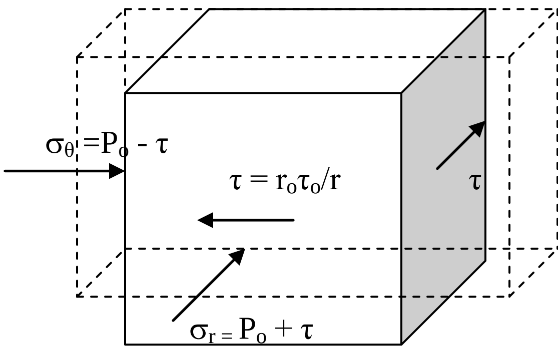

Fig. 1b.Shearing of a soil element in moderate to far distance from pile under cavity expansion.

shear strain is related to the small radial displacement w

$$\gamma = \varepsilon_r - \varepsilon_\theta = 2w/r \qquad (5)$$

The equation for radial equilibrium applies throughout the expansion and within the plastic zone , $\sigma_r - \sigma_\theta = 2C_u$ and for a simple elastic/plastic soil, $\sigma_r = P_o + C_u[1 + \ln(G/C_u)]$, and at the elastic/plastic interface, at some radial distance r_y, $\sigma_r = P_o + C_u$ where P_o is the lateral initial pressure and C_u is the undrained shear strength of the soil. In the elastic loading zone beyond (figure 1b),

$$\sigma_r = P_o + \tau \qquad (6)$$

and

$$\sigma_\theta = P_o - \tau \qquad (7)$$

where τ is given by $G\gamma = G\,2w/r$ or

$$\tau = r_o\tau_o / r \qquad (8)$$

where G is the shear modulus, γ is the shear strain and τ_o is the shear stress at the pile surface. The effective stresses after installation have also been given by Randolph et al 1979 using the modified Cam clay model. For the purpose here, the shear stress τ_o within the plastic zone at the pile surface at r_o is taken as

$$\tau_o = 6C_u \qquad (9)$$

and τ_o reduces to C_u at $r = 7$ to $10r_o$, and thereafter in the elastic zone, it is given by $r_o\tau_o/r$ resulting from the vertical equilibrium of the stresses. (In the rest of the paper, γ is used to denote the unit weight of the soil unless otherwise stated as the shear strain).
At a radial distance r from the pile, the initial lateral effective stress is

$$P_o' = K_o\gamma' H_i \qquad (10)$$

where d is the pile diameter, H_i is the depth of the soil within the impedance zone.

Potential Energy of a Loaded Body of Soil Mass.

In the following a functional expression of the potential energy Ψ in the displaced soil is derived. The potential energy is the strain energy U in the soil less the work done W by the surface and body forces.

$$\Psi = U - W \qquad (11)$$

Based on the principle of minimum potential energy states that, of all possible displacements states of a loaded body, that state of displacement which minimizes the potential energy is the correct one, we obtain a set of equations for the displacements and heave which minimizes the potential energy function :

$$\frac{\partial\Psi}{\partial w} = 0 \qquad (12)$$

$$\frac{\partial\Psi}{\partial v} = 0 \qquad (13)$$

In the following, the appropriate shape of the loaded soil body is identified for analysis and the strains, strain energies, the body and surface forces and the work done by them due to the vertical and lateral displacements are evaluated. Then the functional expression for the potential energy Ψ of the soil body is derived .

Impedance Zone

Figure 2 show a part view of the vertical displacement contours from the Hendon field test of Cooke and Price (Sagaseta and Whittle, 2001).

The soil movements are most pronounced in a conical zone with a lateral extent equal to the length of the pile. For the moderate-to-far distances away from the pile axis, the vertical displacement contours are largely vertical. Thus for soil particles located on any vertical displacement contour, the radial displacement is fairly uniform with depth (but the lateral strains are non zero). Near to the pile shaft, the vertical displacements contours form a complex pattern which are partly heaving in the upper part and downdrag in the lower part. This conical zone of the loaded soil body may be referred to as the impedance zone of the pile.

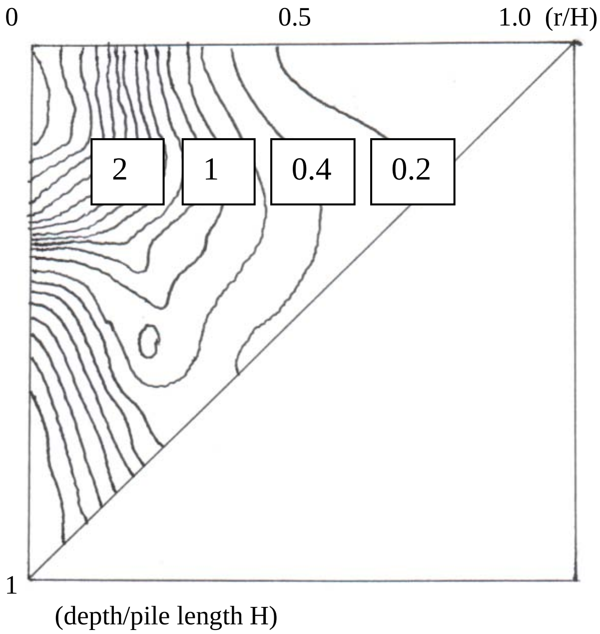

Fig.2 Contours of normalized vertical displacements within soil mass measured at Hendon based on data of Cooke and Price(1973) at all stages of driving of piles (contours adapted from Sagaseta and Whittle, 2001)

The displacements are visualised as being caused by two processes. first there is a lateral cavity expansion process that gives rise to lateral displacement and vertical displacement in the moderate-to-far field distances where the heave is fairly uniform with depth at any radial distance. Second, as the pile plunges into the soil there is simultaneously a large-strain shearing process near to the pile shaft that gives rise to a downdrag vertical movements of the second process and heaving of the first process in the near field with the downdrag being more dominant at close distances while the heaving becomes more dominant with distance from the pile.

For the purpose of the analysis, it is assumed that the loaded body of soil is within a conical surface of radius equal to the length of the pile H (Figure 3). The cone-shaped zone extends from the toe of the pile to the ground surface. This defines a body of soil where the soil heave v for any radial distance r is uniform up to the surface so that the vertical strains are practically zero. In this paper, we consider the case where there are no piles within its impedance zone. (Figure 3).

In the following, the solutions for moderate-to-far field displacement and the near field displacements are separately derived. The total solution is obtained by combining the near field effects of the shearing process close to the pile shaft and moderate-to-far field effects of the cavity expansion process.

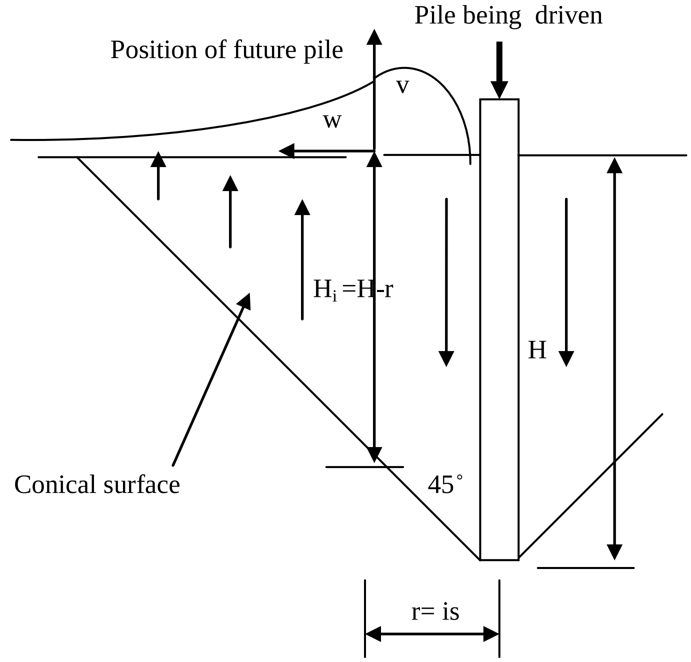

Fig.3. Soil displacements within impedance zone of pile

<u>Installation of a Single Pile (Moderate to Far Field Displacement Functions).</u>

In the following section the moderate-to-far field displacements will be looked at first, ie distances around the pile from 0.1 < r/H < 1.0 Only the effect of a cylindrical cavity expansion and the accompanying radial expansion and the vertical displacements are examined. The effects of the shearing stresses at the pile shaft as the pile penetrates into the soil is addressed in a later section where r/H < 0.1 and the near field displacement functions are introduced. As a group of piles would be considered (elsewhere), it would be convenient to look at the soil around the pile as having 8 segments as most pile configurations are in a rectangular grid at 3 diameter spacing. Consider a segment in the impedance zone. In a segment, the soil pressure P and soil deformations w and v at each grid could be represented as P_1, w_1, v_1 at radial distance r =s, P_2, w_2, v_2, at r=2s away and so on . The depth of the soil within the zone at distance s from the pile being driven is H-s, and at distance ns away is H-ns. The strain energy in an element of the soil undergoing lateral cavity expansion is

$$U_{he} = \frac{1}{2} \left[\sigma_r \varepsilon_r + \sigma_\theta \varepsilon_\theta + \gamma \tau \right] \qquad (14a)$$

$$U_{he} = \frac{1}{2} [(P_o + \tau)w/r + (P_o - \tau)w/r + 2\tau w/r] \quad (14b)$$
$$= (P_o + \tau)w/r = \sigma_r \varepsilon_r$$

In other words the strain energy is equal to the radial pressure times the radial strain. In the following, the total energy is integrated by taking the total radial force. So the strain energy in the soil due to the soil radial pressure P_{io} and radial displacement w is summed as follows :

$$U_h = P_1 s(H-s)w_1 + 2P_2 s(H-2s)w_2 + 3P_3 s(H-3s)w_3 + \ldots$$
$$+ n P_n s(H-ns)w_n \quad (15)$$

In the following treatment, the simpler expression for w given by equation (2b) is first utilized. The correction offered by equation (2c) will then be applied after the derivation of the heave function. Since $w_1 = r_o^2/2r_1$, $w_2 = r_o^2/2(2r_1)$, $w_3 = r_o^2/2(3r_1)$, and $w_n = r_o^2/2(nr_1)$, then $w_2 = w_1/2$, $w_3 = w_1/3$ and $w_n = w_n/n$,

$$U_h \approx [\sum_{i=1}^{n} P_{io}\, s(H-is)]\, w_1 \quad (16)$$

where P_{io} is given by

$$P_{io} = P_o + \tau \quad (17)$$

The work done W_v in the soil due to heave against the gravity forces is as follows

$$W_v = \gamma s^2 (H-s)v_1 + 2\gamma s^2 (H-2s)v_2 + 3\gamma s^2 (H-3s)v_3 + \ldots$$
$$+ n\gamma s^2 (H-is)v_n \quad (18)$$

We assume that the heave v also varies inversely with the radius r, so that $v_2 = v_1/2$, $v_3 = v_1/3$ and $v_n = v_1/n$

$$W_v \approx \gamma s^2 \sum_{i=1}^{n} (H-is)\, v_1 \quad (19)$$

The work done W by the surface forces of the soil segment consist of W_p the work done by the shear stresses at the interface with pile shaft and the base load of the pile in creating the volumetric displacement process

$$W_p = \frac{1}{8} [c_u \pi \frac{d}{2} H^2 + 9 c_u \pi \frac{d^2}{4} H] \quad (20)$$

The factor 1/8 is the fraction of the work done by the pile forces for the segment . W_s, the work done by the frictional sliding forces along the conical surface of the impedance zone from the tip of the pile towards the soil surface at some distance from the pile is

$$W_s = \sqrt{2}\, s^2 c_u w_1 + 2\sqrt{2}\, s^2 c_u w_2 + 3\sqrt{2}\, s^2 c_u w_3 + \ldots$$
$$+ n\sqrt{2}\, s^2 c_u w_n \quad (21)$$

where the $\sqrt{2}\, s^2$ is the area of the surface of the soil in each grid within the segment . Since $w_2 = w_1/2$ $w_3 = w_1/3$ and $w_n = w_1/n$

$$W_s \approx n\sqrt{2}\, s^2 c_u w_1 \quad (22)$$

The potential energy function Ψ_1 of the soil in the impedance zone is

$$\Psi_1 = U - W = U_p - (W_v + W_p + W_s)$$
$$\approx [\sum_{i=1}^{n} P_i\, s(H-is)]\, w_1 - \gamma s^2 [\sum_{i=1}^{n} (H-is)]\, v_1$$
$$- \frac{1}{8} [c_u \pi \frac{d}{2} H^2 + 9 c_u \pi \frac{d^2}{4} H] - n\sqrt{2}\, s^2 c_u w_1 \quad (24)$$

The potential energy Ψ_1 is minimized when

$$\frac{\partial \Psi}{\partial w_1} = \sum P_i\, s(H-is) - n\sqrt{2}s^2 c_u - \gamma s^2 \sum (H-is) \frac{\partial v_1}{\partial w_1} = 0 \quad (25)$$

$$\frac{\partial \Psi}{\partial v_1} = \{\sum P_i s(H-is) - n\sqrt{2}s^2 c_u\} \frac{\partial w_1}{\partial v_1} - \gamma s^2 \sum (H-is) = 0 \quad (26)$$

i.e.

$$\frac{\partial v_1}{\partial w_1} = [\frac{\sum P_i\, s(H-is) - n\sqrt{2}\, s^2 c_u}{\gamma s^2 \sum (H-is)}] \quad (27)$$

$$v_1 = \int \frac{\partial v_1}{\partial w_1} \frac{\partial w_1}{\partial r}\, dr \quad (28)$$

Thus

$$v_1 = [\frac{\sum P_i\, s(H-is) - n\sqrt{2}\, s^2 c_u}{\gamma s^2 \sum (H-is)}] w_1 \quad (29)$$

This equation gives a physical meaning to the relationship between the heave and the lateral displacements, the ratio of which is inversely proportional to ratio of the soil weight to lateral pressure. However, because the radial displacement and the heave functions are both inversely proportional to the radial distance, their influence extends to an infinite distance and decreases at a slower pace than actual measurements of heave and lateral displacements with radial distance. This deficiency is addressed through the imposition of the constraint of conservation of volume in the following section.

Conservation of Volume for Undrained Displacement.

Having found the heave function $v_1 = f\, w_1$ in the previous section , the total volume is obtained by integrating the annular elemental heave volume $2\pi r\, v_I\, dr$ for $r = r_o$ to $r = r_m$. The total heave volume must be equal to the volume of the pile since as r tends to infinity, w tends to zero and equation (1) will then become

$$\int_{r_o}^{\infty} 2\pi\, r\, f w\, dr = \pi\, r_o^2\, H \qquad (30)$$

It is found that the radial displacement function assumed in Eqn (2b) will lead to an infinite volume as r tends to infinity. It is thus necessary that the radial displacement function also satisfy the constraint of volume conservation under the conditions of undrained soil displacement . The form of the radial displacement function satisfying this integral has the form

$$w = \frac{(r_o^2)}{2r}\, e^{-br} \qquad (31)$$

and

$$v = f\,\frac{(r_o^2)}{2r}\, e^{-br} \qquad (32)$$

where b is to be determined. By substituting this into equation (30), the value of b is obtained from the solution of the equation

$$\ln(H/f) + \ln(b) + br_o = 0 \qquad (33)$$

With the new form for the expression for w and v in equation (31) and (32), the expression for the factor f is derived as follows. Since $w_1 = e^{-br} r_o^2/2r_1$, $w_2 = e^{-2br} r_o^2/2(2r_1)$, $w_3 = e^{-3br} r_o^2/2(3r_1)$, and $w_n = e^{-nbr} r_o^2/2(nr_1)$, then $w_2 = w_1 e^{-br}/2$, $w_3 = w_1 e^{-2br}/3$ and $w_n = w_1 e^{-(n-1)br}/n$, the strain energy equation (8) becomes

$$U_h = P_1 s(H-s) w_1 + P_2 s(H-2s)\, e^{-br} w_1 + P_3 s(H-3s)\, e^{-2br} w_1 + \ldots + P_n s(H-ns)\, e^{-(n-1)br} w_1 \qquad (34)$$

or

$$U_h \approx \left[\sum_{i=1}^{n} P_{i\,o}\; s(H-is)\, e^{-bs(i-1)}\right] w_1 \qquad (35)$$

and the work equation (14) becomes

$$W_s = \sqrt{2}\, s^2 c_u w_1 + \sqrt{2}\, s^2 c_u e^{-br} w_1 + \sqrt{2}\, s^2 c_u e^{-2br} w_1 + \ldots + \sqrt{2}\, s^2 c_u e^{-(n-1)br} w_1 \qquad (36)$$

or

$$W_s \approx \sqrt{2}\, s^2 c_u w_1 \frac{(1-e^{-bns})}{(1-e^{-bs})} \qquad (37)$$

and Since $v_1 = fe^{-br} r_o^2/2r_1$, $v_2 = fe^{-2br} r_o^2/2(2r_1)$, $v_3 = fe^{-3br} r_o^2/2(3r_1)$, and $v_n = fe^{-nbr} r_o^2/2(nr_1)$, then $v_2 = v_1 e^{-br}/2$, $v_3 = v_1 e^{-2br}/3$ and $v_n = v_1 e^{-(n-1)br}/n$, the work equation (11) becomes

$$W_v = \gamma s^2 (H-s) v_1 + \gamma s^2 (H-2s)\, e^{-br} v_1 + \gamma s^2 (H-3s)\, e^{-2br} v_1 + \ldots + \gamma s^2 (H-is)\, e^{-(n-1)br} v_1 \qquad (38)$$

or

$$W_v \approx \gamma s^2 \sum_{i=1}^{n} (H-is)\, e^{-bs(i-1)}\, v_1 \qquad (39)$$

and equation (29) for the heave becomes :

$$v_1 = \left[\frac{\sum P_i s(H-is) e^{-bs(i-1)} - \sqrt{2} s^2 c_u (1-e^{-bns})/(1-e^{-bs})}{\gamma s^2 \sum (H-is)\, e^{-bs(i-1)}}\right] w_1 \qquad (40)$$

If we denote the function f_v as follows

$$f_v = \left[\frac{\sum P_i s(H-is) e^{-bs(i-1)} - \sqrt{2} s^2 c_u (1-e^{-bns})/(1-e^{-bs})}{\gamma s^2 \sum (H-is)\, e^{-bs(i-1)}}\right] \qquad (41)$$

Then $v_1 = f_v w_1$, $v_2 = f_v w_2$, and $v_n = f_v w_n$. In the next section the strain in the near field of the pile is addressed.

<u>Effects of Pile Shaft Shearing Action during Installation of a Single Pile (near Field Displacements).</u>

In the case of the shearing near to the pile shaft, Randolph and Wroth (1979) has likened the shear strain to a series of concentric cylinders of soil deforming as the axially loaded pile settled. In a similar way, during the installation as the pile plunges, the continuous shearing process of the shaft causes large shear deformations in the soil near to the shaft.

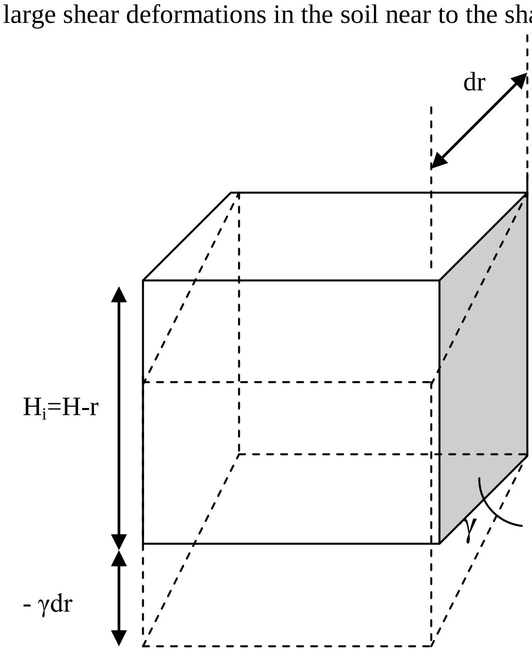

Fig. 4 Shearing of a soil element in the near field of the pile shaft

This will result in large shear strains to the cylindrical layers of soil around the pile. Figure 4 shows the shear deformation of an element in the cylindrical layer. This is in addition to the process of cavity expansion which takes place at the same time. The shearing takes place for the greater part in a thin slip band within an annular region around the pile for the entire length of the pile as the pile penetrates the soil. For the soil extending beyond this band, the shearing is limited by the development of this slip zone. From direct and ring shear tests as well as in the field , the soil reaches its peak strength initially for small movements, then, to a softened critical state after some larger movements (of the order of 5 mm to 10 mm), followed by degradation to the residual strength after much larger movements of about 100 mm to 300 mm and in some clays to larger than 500 mm. (Skempton, 1985, Lupini et al 1981). For pile-soil movements beyond about 300 mm, the pile continues to slip past the soil surface and the shearing strains are confined to an essentially local slip band near to the pile surface. Beyond this local slip band, no further strains deformations takes place in the soil. The mobilized shear stress along the pile surface is close to the residual strength. This shear stress along the pile surface may be represented as follows

$$\tau_o = \xi \tau_p \tag{42}$$

where ξ is the strength reduction factor and τ_p the peak strength or intact strength. The residual strength is dependent on the plasticity index and the clay fractions in the soil and also on the rate of shearing. (Skempton, 1985). Randolph and Wroth (1982) reported that, in a pile undergoing static loading, the effect of compressibility of the pile can give rise to $\xi = 0.5$ for movements of 30 to 50 mm. The overall degradation of strength would be 0.50 from a critical state friction angle of 23° and 0.37 from a peak friction angle of 30° to a residual of 12.3°

At high rates of shearing (400 mm/minute), the friction angle of some clays, with an intermediate clay fractions , could drop to one-half the residual value or almost one-third from the maximum friction angle (Skempton, 1985). Changes of residual strength with the rate of displacement have also been studied by Lemos (1991) and Tika et.al. (1996) and this would give rise to different resistances depending on the piling rate. For a constant rate of penetration of 5 mm/min, data from Tika et al (1996) showed that the for London clay, the residual strength ratio is approximately 0.197 (11.1 degrees) at 100 mm/min which is about 2 % to 1% higher than that measured at 11 mm/min. Data reported by Mesri and Cepeda-Diaz, (1986) also showed that the residual strength of clays dropped to below 10° for clays with high liquid limit exceeding 80. The degradation of the shear strength would be in the range of 0.20 to 0.39 for residual shear strength of 4° to 10° and shear strength of 12° to 24° . In the subsequent analysis, it is assumed that this residual strength will be applicable to all large shearing movements beyond 300 mm.

The energy in the soil subject to shear stresses due to the pile shaft is

$$dU_p = \frac{1}{2} \gamma \tau \, dV \tag{43}$$

for a unit volume dV and thus

$$dU_p = \frac{1}{2} \, ro \, \frac{\xi \tau_p}{r} \frac{2\pi r}{8} (H-r)(-\gamma dr) \tag{44}$$

Here the shear strain $\gamma = dv/dr$ is towards the pile while the radial increment is positive away from the pile.

$$\frac{\partial U_p}{\partial r} = ro \, \xi \tau_p \frac{\pi}{8} (H-r) \left(-\frac{dv}{dr}\right) \tag{45}$$

and the differential with respect to the vertical displacement v is

$$\frac{\partial U_p}{\partial v} = - ro \, \xi \tau_p \frac{\pi}{8} (H-r) \tag{46}$$

The next step is to determine the work done by body forces and the work done by the surface forces. The work done W_b (subscript b is used to denote body forces) by the body forces is

$$\partial W_b = \gamma \, 2 \frac{\pi}{8} r (H-r) \, v \, dr \tag{47}$$

where γ which denote unit weight ρg. Equation (47) expressed as differential with respect to v is

$$\frac{\partial W_b}{\partial v} = \gamma \, 2 \frac{\pi r}{8} (H-r) \, v \left(\frac{1}{(dv/dr)}\right) \tag{48}$$

The work done W_p (subscript p is used to denote pile surface forces) by the surface force $\tau_o = \xi \tau_p$ is

$$\partial W_p = \frac{2 \pi r_o}{8} \xi \tau_p (H-r) \, dz \tag{49}$$

The differential $\partial W_p / \partial v$ is zero.

Using the principle of potential energy, by letting the differential of the potential energy function (subscript b denotes with respect to v) equal to zero, we obtain the set of values of v that satisfy the equation as follows

$$\frac{\partial \Psi_b}{\partial v} = \frac{\partial U_p}{\partial v} - \frac{\partial W_b}{\partial v} - \frac{\partial W_p}{\partial v} = 0 \tag{50}$$

ie

$$\frac{\partial \Psi_b}{\partial v} = - ro \, \xi \tau_p \frac{\pi}{8} (H-r) - \gamma 2 \frac{\pi}{8} r (H-r) \, v \left(\frac{1}{(dv/dr)}\right) = 0 \tag{51}$$

so that

$$\frac{1}{v} \frac{dv}{dr} = -2 \left(\frac{\gamma}{(r_o \xi \tau_p)}\right) r \tag{52}$$

from which we obtain

$$v = A\, e^{-f r^2} \qquad (53)$$

where $f = \gamma / r_o \xi \tau_p$ and A is a constant to be determined in the following manner.

To determine A, consider the boundary conditions at the pile soil interface where the total work done (equation (49))could be integrated to give

$$W_{pT} = \frac{\pi r_o}{8} \xi \tau_p H^2 \qquad (54)$$

However only a fraction η of this total work done W_{pT} (subscript pT denotes the total work done due to shearing stress at the pile-soil interface) has a role in the shear deformations in the pre-slip shearing of the soil around the pile.

The displacement at which the residual strength is fully mobilized is assumed to be the maximum shearing displacement associated with the relevant work done ηW_{pT}. This is then regarded as the participating work done prior to the pile soil slippage after which the work done by the pile shearing is dissipated as heat in the slip zone.
After the formation of a slip zone, the deformations of the soil beyond the slip zone is largely unaffected by the remaining shearing work $(1-\eta)W_{pT}$ as the shearing (either turbulent or sliding) becomes confined to a narrow slip band of a few millimeters close to the pile soil interface. So the relevant work done W_p is then given by

$$\eta = \frac{W_p}{W_{pT}} = \frac{\delta}{H} \qquad (55)$$

where η is an efficiency factor and δ is the maximum shearing displacement (about 300 mm) associated with the development of residual strength. The value of δ depends on the clay fraction in the soil.

$$W_p = \eta \frac{\pi r_o}{8} \xi \tau_p H^2 \qquad (56)$$

The total strain energy is as follows :

$$U_p = r_o \xi \tau_p \frac{\pi}{8} \int (H-r) \left(-\frac{dv}{dr}\right) dr \qquad (57)$$

or substituting the expression for v from equation (46),

$$U_p = -\frac{2\pi}{8} \gamma A \int (Hr - r^2)\, e^{-f r^2} dr \qquad (58)$$

The integral of the function $(Hr - r^2)\, e^{-f r^2}$ is denoted by

$$\chi = \int (Hr - r^2)\, e^{-f r^2} dr \qquad (59)$$

The first expression integrate to give $\left[\left\{ -\frac{H}{2f} e^{-f r^2} \right] \right._{r=ro}^{r=H}$

The second expression is a Guassian integral

$$-\chi = \int_{r=ro}^{r=H} r^2 e^{-f r^2} dr = \int_{r=o}^{r=H} r^2 e^{-f r^2} dr - \int_{r=o}^{r=ro} r^2 e^{-f r^2} dr \qquad (60)$$

where

$$\int_{r=o}^{r=x} r^2 e^{-f r^2} dr = \frac{\sqrt{\pi}}{4f^{3/2}} \operatorname{erf}(x\sqrt{f}) - \frac{x}{2f} e^{-f x^2}$$

and

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{r=o}^{r=x} e^{-u^2} du$$

The integral form may be retained for approximate numerical computation. Now W_b may be evaluated from equation (47) since v has been determined from equation (53). By integrating the work done by the body forces in equation (47) we obtain the expression for W_b

$$W_b = \frac{2\pi}{8} \gamma A \chi \qquad (61)$$

From equation (56), (58) and (61), the total potential energy equation can then be expressed as

$$\Psi = U_p - W_p - W_b$$

$$= -\frac{4\pi}{8} \gamma A \chi - \eta \frac{\pi r_o}{8} \xi \tau_p H^2 \qquad (62)$$

From this equation and letting $\Psi = 0$ (conservation of energy) we obtain an expression for A as follows

$$A = -\eta \frac{r_o \xi \tau_p}{4 \gamma \chi} H^2 \qquad (63)$$

By combining the solutions for the near-field effects due to the shaft shearing action from equation (53) and for the moderate-to-far field effects from equation (40) we obtain the complete solution for the heave around a pile as follows

$$v = \left[\frac{\sum P_i\, s\,(H - is)e^{-bs(i-1)} - \sqrt{2}\, s^2 c_u\,(1-e^{-bns})/(1-e^{-bs})}{\gamma s^2 \sum (H - is)\, e^{-bs(i-1)}}\right] w$$

$$- \eta \frac{r_o\, \xi\, \tau_p}{4\,\gamma\,\chi} H^2 e^{-(\gamma / r_o \xi \tau_p) r^2} \qquad (64)$$

The value of b determined in equation (33) ought to be checked with this final expression for v. Equation (64) may also be expressed in dimensionless form as

$$\frac{vH\gamma}{r_o\tau_p} = \left[\frac{\sum P_i s(H-is)e^{-bs(i-1)} - \sqrt{2}s^2 c_u(1-e^{-bns})/(1-e^{-bs})}{\gamma s^2 \sum (H-is)\, e^{-bs(i-1)}}\right] \frac{wH\gamma}{r_o\tau_p}$$

$$- \eta \frac{\xi H^3}{4\,\chi} e^{-(\gamma / r_o \xi \tau p) r^2} \qquad (65)$$

COMPARISON WITH FIELD AND EXPERIMENTAL DATA FOR A SINGLE PILE

In the following, the field tests measurements in Figure 5, 6, 8, 10, 11 and 12 and the experimental data from the calibration chamber tests in Figures 7 and 9 are each compared with calculations of the analytical model given above by equations (31) for radial displacements, equation (41) for moderate-to-far field heave and equation (64) which incorporates the near field pile-soil shearing

Figure 5 plots the calculated movements from equation (64) compared with the measurements from the field pile test at Hendon in London clay by Cooke et al . The predicted maximum heave was about 10.9 mm at a distance of 0.6 m away from the pile axis compared with the measured maximum heave of 10.2 mm at 0.35 m away from the pile axis. The calculated heave becomes negative at distances less than 0.4 m from the pile axis. There is a spike in the calculated value when r is near to the pile shaft. This appears to be an anomaly of the functions at small r values. In this field test, the pile penetration was H=3.5 m and the clay shear strength increases uniformly from 35 kPa to 65 kPa at the depth of 3.5 m. The analysis uses the following data for London clay : average undrained shear strength C_u = 50kPa, peak friction angle = 30 degrees, the residual friction angle 12 degrees (Skempton ,1985) and unit weight 17 kN/m^3. For calculating the moderate-to-far field values, n was taken to be 26 divisions for the summation of the factors in equation (41). From equation (33) the value of b = 1.446 was determined and from equation (41), f_v = 5.71. For calculating the near-field values, ratio of residual shear strength to peak shear strength is (tan 12/tan 23°) ξ = 0.37. The value of the fraction of relevant work done $\eta = (300/3500) = 0.0857$, from equation (61), the value of $\chi = 0.185$ is determined. From equation (60) A = 0.1643 and from equation (52), f =12.25.

It is interesting to look at the predictions of equation (41) without yet considering the influence of the near-field effects. Equation (41) does not predict well in the near field where r/H < 0.1. It is also unable to account for the reversal of direction of the displacement near to the pile shaft. This is because the strain within the near field zone is large and is influenced primarily by the shearing action of the pile shaft. The downward movement predicted by equation (53) is much more than the heave component from equation (41). That is why equation (64) which takes into account the near field shearing process, is able to predict the maximum displacements in the near field distance and a reversal of direction of the displacement when the values of equation (53) become dominant..

Figure 6 shows the predictions from Equation (64) and (41)

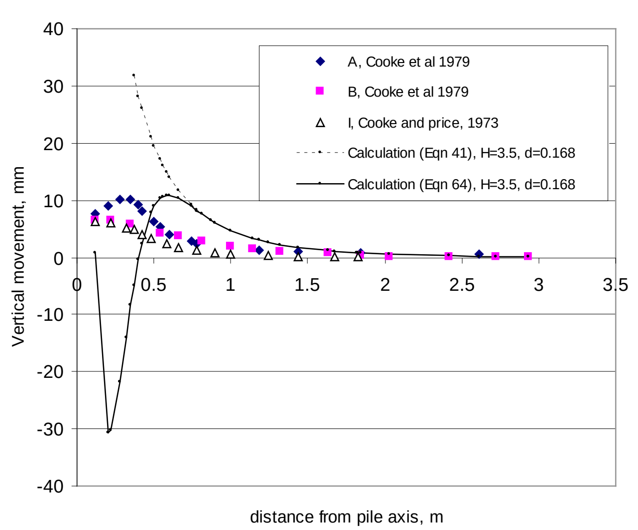

Fig. 5 Comparison of calculated and measured soil movements at Hendon Test pile installation

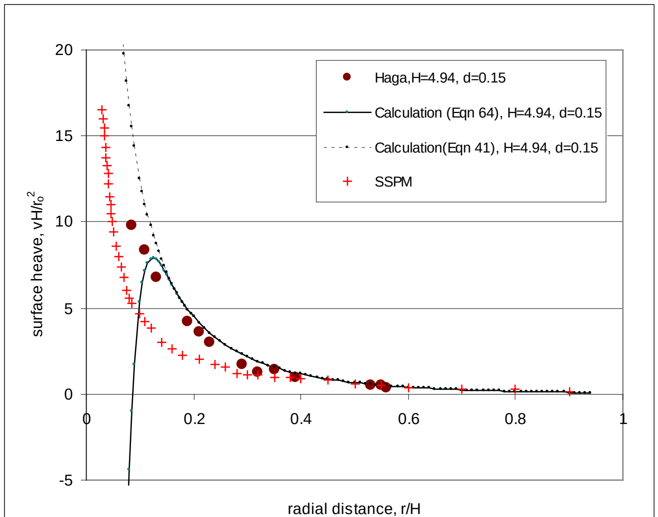

Fig. 6 Comparison of surface heave calculations for pile instalation at Haga site.

are in close agreement with the data at the Haga site in Norway (Karlsrud and Haugen(1983) . The data are plotted in non-dimensional heave (vH/r_o^2) and radial distances (r/H).
Also plotted in these figures are the predictions by the Shallow Strain Path Method (SSPM) developed at MIT by Baligh (1985) and subsequently developed further to include the effects of stress-free ground surface by Sagaseta (1987). The SSPM underpredicts the data by a factor of 2 . The Haga test pile were jacked into a sensitive, relatively uniform overconsolidated clay underlain by free draining sand. For the analysis of the Haga test pile, the following data were assumed : Cu =50 kPa, critical φ = 30°, unit weight =17 kN/m3 and ξ = 0.37 for the overconsolidated clay

Figure 7 plots the data for the calibration chamber in normalized vertical displacements (v/r_o) versus initial radial position (r/r_o) together with the model predictions of equation (64) which is in close agreement with the experimental data.

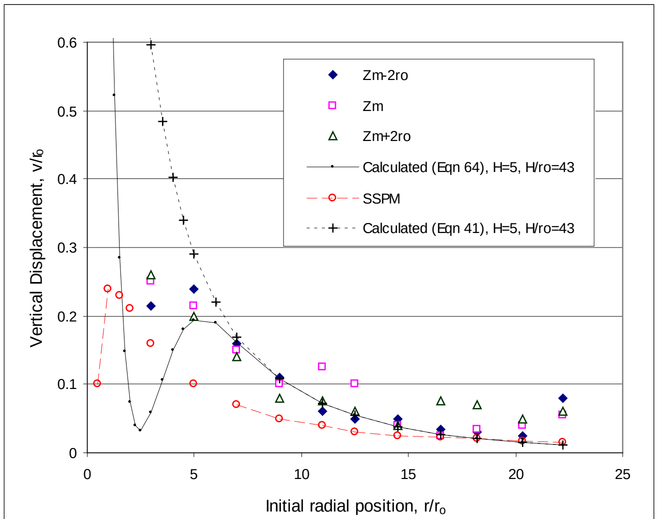

Fig. 7 Comparison of Calculations with measured Heave during pile installation in Calibration Chamber tests

The SSPM under-predicts by a factor of 2 at moderate distances r/H =0.1 to 0.3. Equation(64) predicts a maximum heave v/r_o = 0.19 at r/r_o = 5 while the observed experimental maximum was 0.26 at radial distance of 3. At moderate-to-far distances r/ro =5 onwards, agreement between the predictions of Equation (64) and the experimental data is good. Equation (64) is able to predict the maximum heave but at a slightly offset location and the heave in the near to far field distances. There is again a spike in the predicted values as r approaches the pile shaft. This is due to the errors in the functions at small r leading to an overprediction of the lateral displacement in equation (3a) and (31). For the Chamber tests by Gue, the pile diameter was 16 mm and penetrate 344 mm into a chamber of 450 deep of Speswhite Kaolin which was consolidated under pressures from 200 to 600 kPa and overconsolidation ratios of 1 to 10. The following data were assumed : Cu =50 kPa, peak φ = 30, unit weight =17 kN/m3 and ξ = 0.37 for the overconsolidated clay. The length of the pile was taken to be 5 m and the diameter =0.232 m such that the pile depth to diameter ratio H/d is 344 as in the test chamber.

Figure 8 plots the data of Karlsrud and Haugen (1983) and those of Oostveen and Kuppers (1985) on non-dimensional heave (vH/r_o^2) versus radial distance (r/H). Notice that the heave for that of a larger diameter in the Baghdad case of Oostveen and Kuppers is slightly higher than when the diameter is smaller. This trend is also correctly reflected in the predictions by equation (64) for Oostveen and Kuppers's data. The SSPM again under-predicts by a factor of 2. Sagaseta and Whittle (2001) have attributed the under-prediction of the SSPM to the presence of horizontal tensile zones or cracks in the surface. This may have accounted for the heave in some cases. In the present analysis this factor has not been taken into account. The proposed equation (64) is able to predict the heave in the moderate-to-far field distances well.

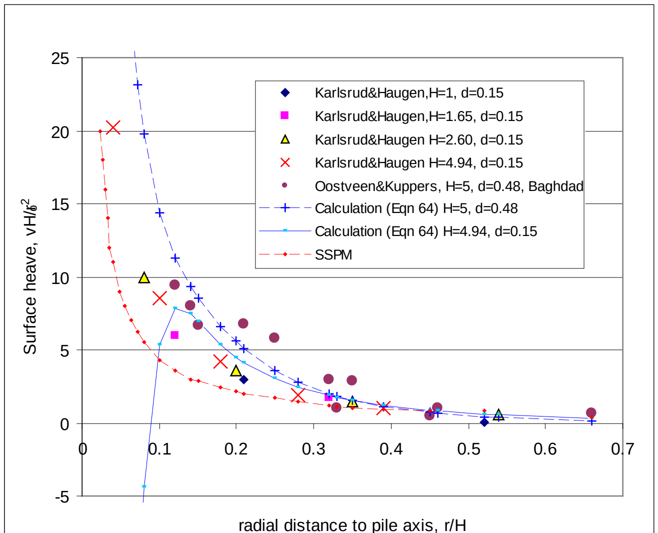

Fig. 8 Comaprison of Calculations with measured heave due to driving of single pile in field tests.

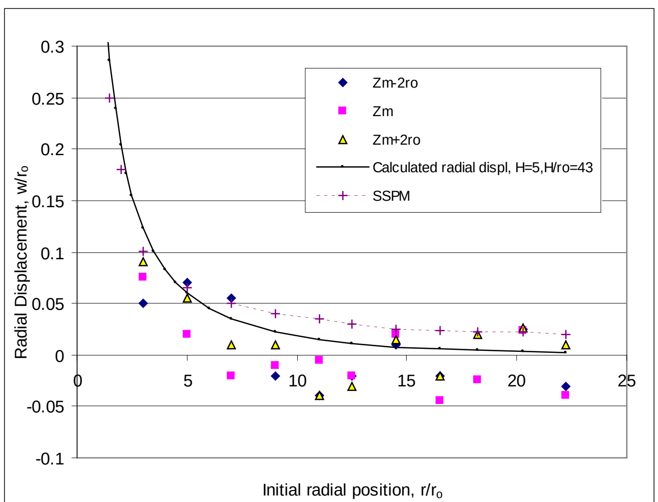

Fig. 9 Comparison of Calculations with measured radial displacements from Calibration chamber tests

Figure 9 plots the predictions of equation (31) and that of SSPM with measured radial displacements (w/r_o) versus the initial radial position (r/r_o) from the calibration chamber tests of Gue. Both Equation (31) and the SSPM are in reasonable agreement with the measured radial displacements for the moderate distances except that the SSPM tends to predict higher values at far distances.

Figure 10 and figure 11 plots the predictions for the surface heave and the radial displacements with the field tests measurements of Hwang et al (2001).

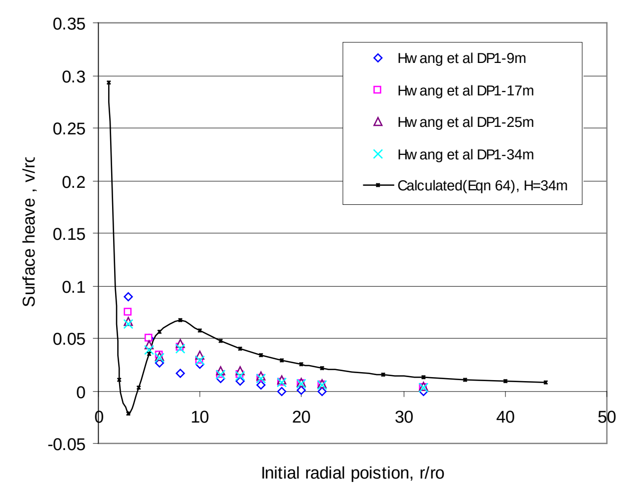

Fig. 10 Comparison of calculations with heave measurements in installation of pile in field tests

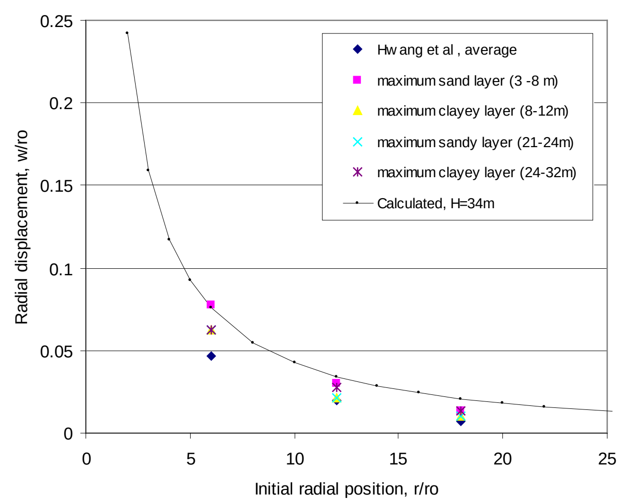

Fig. 11 Comparison of radial displacement calculations with field tests measurements

The tests involved driving precast RC piles 800 mm in diameter to depths of 34 m through layers of silty clay, (3-8m), soft clays(8-12m), medium sands(12-21m), a clay layer interbedded with thin layers of sand(21-32m) and medium dense sands(32-40m). It is instructive to look at the ability of the analytical model to predict the movements in a layered clay and sandy soil. The near field surface heave prediction rises sharply rise at the shaft. This is an anomaly of the calculations due to the errors of functions at small values of r. The predicted maximum of v/ro =0.067 ar r/ro =8 as compared to the measured data of v/ro = 0.064 at r/ro =3. The values are over-predicted at the moderate field distances. The predicted radial displacements are closer to the measured maximum values but over-predicts the average values.

The reason for the over-prediction could be that the sandy soils tend to be densified during the driving leading to smaller heave and radial displacements. For the input data, the average shear strength was 135 kPa (increasing with depth from about 50 kPa at 10 m to about 250 kPa at 32 m). The average friction angle was about 31 degrees and the unit weight 19.1 kN/m3. As the soil is largely sandy, it was assumed that ξ = 0.95 since the ratio of the residual strength to peak strength would be nearer to 1 for soils with a low clay fraction (Skempton,1985).

Figure 12 replots the predictions of equation (64) in non-dimensional heave (vH/r_o^2) and radial distances (r/H) for the chamber test of Gue (1984) for different pile penetration depths having the same H/ro ratio of 43. For the chamber tests, varying the depth of the pile H but maintaining the H/ro ratio of 43 has produced different displacement curves that are influenced primarily by the near field displacement component of equation (53). Radial displacement equation (31) has an error up to 20 % near to the shaft and has amplified the heave predictions at near field distances when r/ro =1 to 2.

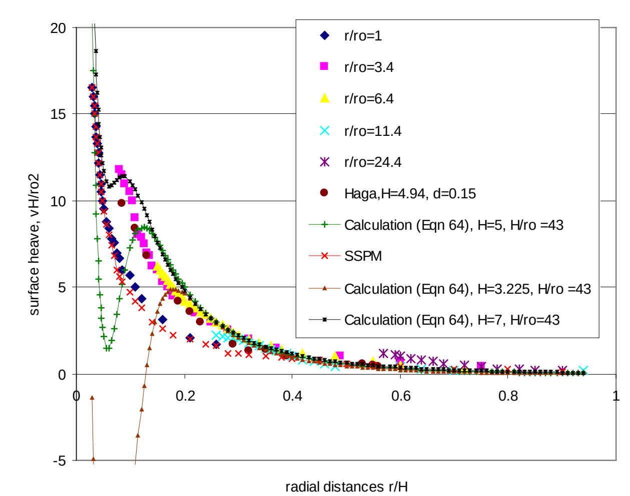

Fig. 12 Comparison of heave calculations for different penetration depth H but with same H/ro ratio=43 for Gue Chamber tests. Haga test site data are also shown.

The spikes in predicted values near to the pile shaft are likely due to computation errors. Equation (64) does not predict well near to the pile shaft where r/H < 0.1. The predictions are in good agreement with the data for r/H from 0.1to 0.2 onwards. Also plotted in these figures are the predictions by the Shallow Strain Path Method (SSPM) (Sagaseta and Whittle, 2001) which was derived using fluid mechanics and therefore does not depend on any soil parameters. The SSPM under-predicts the data by a factor of 2 . While the SSPM is in close agreement with the case of shallow penetration when r/r_o =1, the rest of the data for all the other stages of pile penetration are consistently away from this trend.

The proposed method contains certain shortcomings which are not fully explored. It makes an assumption that the heave is some function of the inverse of the radius. This may not be entirely correct but it makes a tractable analysis possible. The imposition of the conservation of volume of the displaced soil helps to place a sensible constraint on the suitable form of the displacement function. The kinematics of the soil rate of movements had not been considered. In energy formulation, the displacement process is viewed more as pseudo-static with dissipation of the momentum and velocities being contained in the efficiency factor in the energy transfer. The contribution of soil movements below the pile outside of the loaded soil zone has been neglected in the simplified analysis. There are also errors in the approximation and computation. These and the particular manner of formulation may have been responsible for instability at small radial distances from the pile and thus further developmental work remains.

CONCLUSIONS

The model for calculating the surface heave and lateral displacements presented in this paper offers a soil-mechanical basis for its analysis and interpretation. The analysis has been developed for a single cylindrical pile and is based on conservation of volume, work done and energy principles in cylindrical cavity expansion and pile shaft-soil shearing process and is applicable to large strains close to the pile shaft.

At distances away from the pile, in the moderate-to-far field distances away from the pile, ie where radial distance to pile length r/H > 0.1, the ratio of the heave to the lateral displacements is largely a function of the ratio of the lateral forces to the soil weight. The form of the function illustrates an intuitive physical basis to the mechanics of the heaving phenomenon

The model predicts a local maximum heave in the near field distance and a directional reversal of the displacement close to the pile shaft. The peaking of heave and reversal of direction in the near field distances of r/H 0.1 to 0.2 shows a dominance of downdrag shearing forces at close distances to the pile shaft.

The model's calculations are in close agreement with the measured heave movements in the back-analysed case histories of field tests and laboratory chamber tests for moderate-to-far field distances from the pile

The spikes in values calculated for distances near to the pile shaft are distortions due to the errors of approximation in the displacement functions and formulation when r is small.

For practicing engineers making a choice of displacement piles in built-up areas, the impact on adjacent structures often need to be assessed and the model can be applied to the evaluation of soil movements due to the installation process.

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