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Seventh International Conference on Case Histories in Geotechnical Engineering
April 29 - May 4, 2013 CHICAGO
and Symposium in Honor of Clyde Baker

CALCULATION OF NATURAL FREQUENCY OF EARTH DAMS BY MEANS OF ANALYTICAL SOLUTION

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ABSTRACT

Earth dams are 3-Dimensional, huge and inhomogeneous structures that interact with water and soil. They are the most common type of dams used in the world and developing countries like Iran.
Seismic analysis of earth dam is very important in regions which have been subjected with earthquake impaction several times. Calculating the natural frequency of dams is the essential part for its seismic behavior analysis. So it is very important to present an appropriate solution for calculation of natural frequency.
Natural frequency of earth dams is usually calculated by means of experimental correlations, shear beam analysis (base on the height of dam and shear wave velocity) and time historical dynamic analysis.
The present study concerns a new formulation for natural frequency of earth dams by means of analytical method. In this method, shear wave velocity and height of dam are two parameters which are used for obtaining natural frequency. Geometry of dam body, rigidity of foundation, modulus of elasticity and Poisson's ratio are essential in calculating process. Results from proposed method are compared with case histories, numerical method and other formulations. Comparison shows that there is no significant difference in dams with various heights up to 100m.

INTRODUCTION

Calculating the natural frequency and fundamental period of dams is necessary for evaluating dynamic behavior. So it is very important to present an appropriate solution for calculation of natural frequency. Earthen dams are counted in flexible dams group and can withstand significant deformation during earthquake, so these dams behave relatively well during the past earthquake.

Due to mentioned appropriate parameters, earth dams are preferred to concrete dams in regions which has been subjected with earthquake impaction several times and near the active faults. Evaluating seismic behavior of earth dam is of great importance in design of these structures. Natural frequency of dams is one of the essential parameters that explain the seismic response of dams during earthquake loading. There are several numerical methods for calculating natural frequency but a few analytical solution and experimental correlation are presented in the last years. Parish et al. (2009) determined the natural frequencies of the foundation-dam system by a Fourier analysis of the free vibration response of the dam. Kishi et al (1987) introduced the frequency equation of a dam by a rectangular inhomogeneous truncated wedge according the bending moment as well as shearing force in the vertical direction. Watanabe et al (1996) found natural frequencies and corresponding modes up to fifth modes from the Fourier Spectra of micro tremors and the modal analyses. Zhen et al (1996) evaluate dynamic characteristics, namely natural frequencies and modes of transversal vibration of inhomogeneous earth dams in triangular canyons. Gazetas et al (1991) presented a correlation for natural frequency of earth dam to be resting on a rigid base and composing of cohesive soil by means of shear slice procedure.

This paper develops an approximate method for calculating natural frequency on the base of continuous mass analytical solution. Comparison results with numerical solution and other formulation shows that there is no significant difference between them.

ANALYTICAL PROCEDURE

Calculating natural frequency of dam is carried out by continuous mass solution and under conditions which are listed below:

1. Plane strain behavior is assumed.
2. "Cosine" shape function is used.
3. Homogeneous dam is assumed.
4. Poisson ratio is constant and equal to 0.3.
5. Linear elastic behavior is assumed.
6. Slope of earth dam is 1V:2H
7. Crest of dam width is 0.2 times of height.
8. Dam rests on a rigid foundation.
9. Dams with 20 up to 100m are evaluated.
10. Damping ratio is assumed almost zero.

Natural frequency is one of the significant characteristics of a system and obtained as follows:

$$\omega = \sqrt{\frac{K^*}{M^*}} \tag{1}$$

Where,

$$M^* = \int_0^H \mu(x)\, \varphi^2(x)\, dx \tag{2}$$

$$K^* = \int_0^H k(x)\, (\varphi''(x))^2\, dx \tag{3}$$

In above formulation, $\mu(x)$ and $k(x)$ are in turns mass per unit height and stiffness per unit height, so we can obtain them from the geometry of dam body, that is shown in Fig.1.

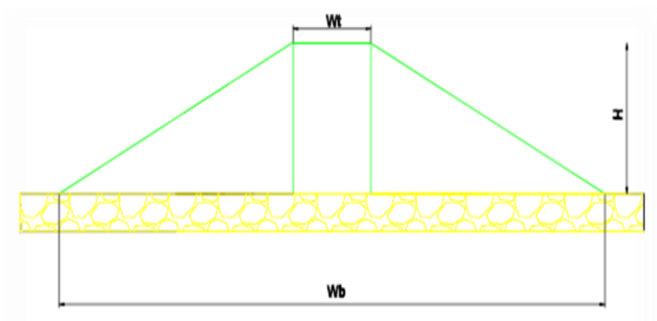

Fig. 1. Geometry of earth dam

Regard to Fig.1. $\mu(x)$ and $k(x)$ are defined as:

$$\mu(x) = \rho \left(Wt + \frac{(H-x)(Wb-Wt)}{H} \right) \tag{4}$$

$$k(x) = \frac{E}{12} \left(Wt + \frac{(H-x)(Wb-Wt)}{H} \right)^3 \tag{5}$$

Where E is Young`s modulus and ρ is density of dam body materials. For calculating natural frequency, it is necessary to assume shape function. The assumed shape function for this paper is 1-cos (πx/2H).

Now the first step is to determine M^* as follows:

$$M^* = \int_0^H \rho \left(Wt + \frac{(H-x)(Wb-Wt)}{H} \right) \varphi(x)^2\, dx \tag{6}$$

$$M^* = 0.23\, \rho\, Wt\, H + 0.04\, \rho\, (Wb - Wt) H \tag{7}$$

The second step is to determine K^* as follows:

$$K^* = \int_0^H \frac{E}{12} \left(Wt + \frac{(H-x)(Wb-Wt)}{H} \right)^3 (\varphi''(x))^2\, dx \tag{8}$$

Using the assumption that mentioned before, mass and stiffness of dam body is determined as:

$$K^* = 0.577E \tag{9}$$

$$M^* = 0.206\, \rho H^2 \tag{10}$$

Then,

$$\omega = \sqrt{\frac{0.577E}{0.206\, \rho H^2}} \tag{11}$$

To make the Eqs. (11) more easier, use Shear modulus instead of Young`s modulus, therefore shear wave velocity is appeared.

$$\omega = 2.69 \frac{Vs}{H} \tag{12}$$

$$T = 2.34 \frac{H}{Vs} \tag{13}$$

For example, fundamental period and natural frequency for dams of height 20 up to 100m and shear wave velocity 500 m/s are listed in Table 1:

Table 1. Natural frequency of earth dam

H (m)	Coefficient	W (rad/s)	T (s)
20	2.69	67.25	0.0933
30	2.69	44.83	0.14
40	2.69	33.63	0.19
50	2.69	26.90	0.233
60	2.69	22.42	0.28
70	2.69	19.21	0.33
80	2.69	16.81	0.373
90	2.69	14.94	0.42
100	2.69	13.45	0.47

Figure 2 shows the natural frequencies of earth dams such as Table 1, but shear wave velocity varies from 500 to 800 m/s and Poisson ratio is 0.3.

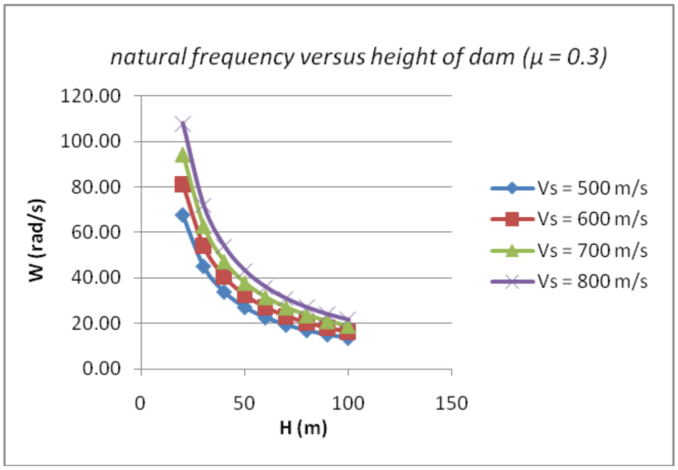

Fig. 2. Natural frequency of earth dam (μ=0.3)

In primary assumptions, Poisson ratio was to be constant and equal to 0.3. For developing proposed formulation, Poisson ratio was changed and varied from 0.2 up to 0.35.

Table 2 shows the natural frequency of dams like Table 1, but here Poisson ratio is equal to for example 0.25.

Table 2. Natural frequency of earth dam (μ=0.25)

H (m)	Coefficient	W (rad/s)	T (s)
20	2.646209966	66.16	0.094928219
30	2.646209966	44.10	0.142392329
40	2.646209966	33.08	0.189856439
50	2.646209966	26.46	0.237320548
60	2.646209966	22.05	0.284784658
70	2.646209966	18.90	0.332248768
80	2.646209966	16.54	0.379712877
90	2.646209966	14.70	0.427176987
100	2.646209966	13.23	0.474641097

Also Fig. 3 illustrates the same parameter as Fig. 1 with a little difference in Poisson ratio.

From these figures and tables, it can be said that, height of dam, shear wave velocity of dam body soil and Poisson ratio are affective parameters in obtaining natural frequency. In other words, with increasing height of dam, natural frequency reduces and increases when Poisson ratio and shear wave velocity increase. Approximate and fast estimation of natural frequency of dams without using complex relations and spending time, is the benefit of the proposed formulation.

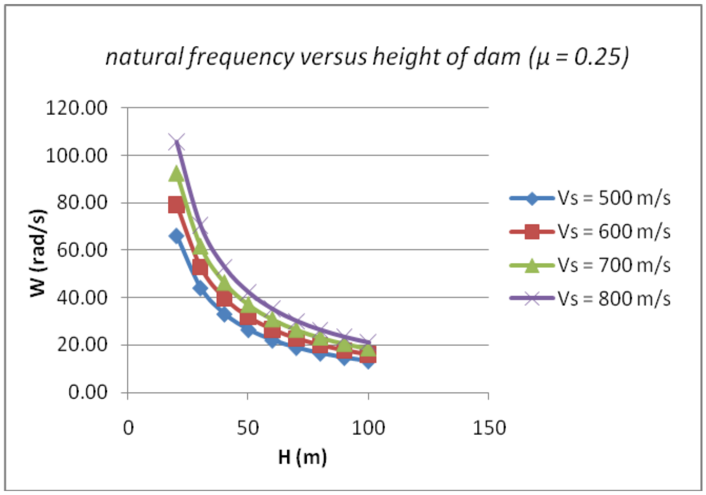

Fig. 3. Natural frequency of earth dam (μ=0.25)

DYNAMIC ANALYSIS

The QUAKE/W finite element software has been used to model and carried out the dynamic analyses (QUAKE/W user's guide, 1991). To verify the analysis results, the dam has been analyzed first under static conditions. Then, taking into account the seismicity studies of the region, an appropriate acceleration has been selected and applied to the model to do the dynamic analyses and to estimate seismic behavior of the dam during the earthquake loading. To prepare the initial requirements for doing the dynamic analyses, the dam was modeled from the beginning of the construction. Two dimensional finite element model of dam with rigid foundation is presented in Fig. 4.

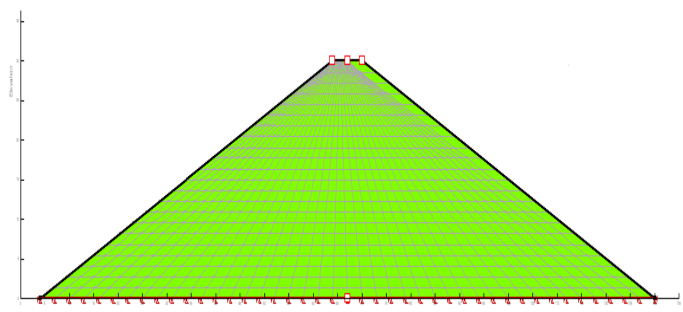

Fig. 4. Model of dam with rigid foundation

Fixed support in x & y direction is used for modeling rigid foundation as shown in Fig. 4. The material properties of dam that used in this study are presented in Table 3.

Table 3. Material properties of dam

Material Model	Unit Weight	Poisson ratio	Damping ratio	Shear Modulus
Linear elastic	20 kN/m^3	0.3	1e-008	980 Mpa

As seen, the model was performed and is analyzed in full reservoir condition. The QUAKE/W is two-dimensional, dynamic finite element software that uses equivalent linear strain-dependent modulus and damping properties. It is a time-step analysis that uses Rayleigh damping and allows variable damping for different elements

The analyses are performed under sample earthquake loading with 0.1g peak acceleration and result in dynamic responses of modeled dam. Y- Total Stress and deformation of dam are shown in Fig. 5.

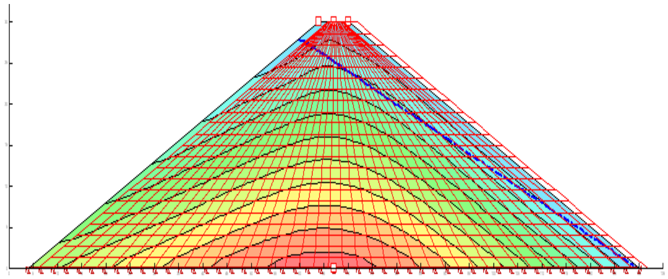

Fig. 5. Dynamic responses of dam

The period corresponding to the maximum point of x-spectral velocity versus period graph is approximate the fundamental period of dam and use for obtaining natural frequency. Fig. 6 presents x-spectral velocity graph.

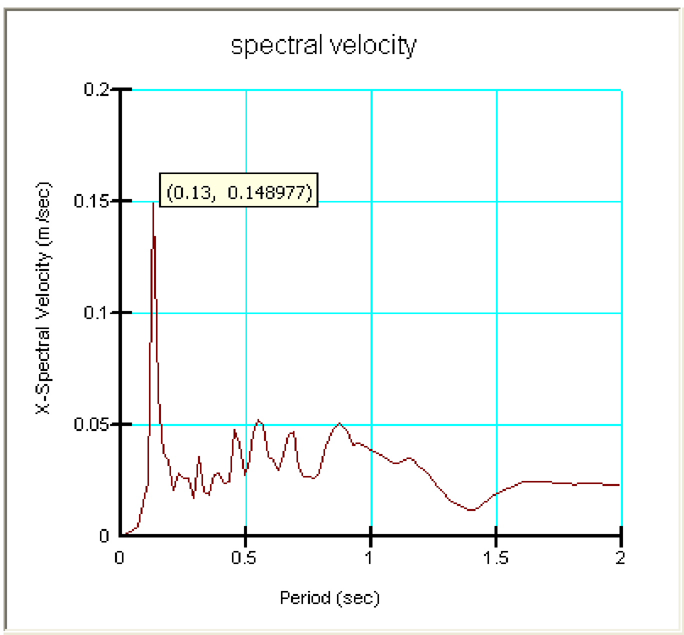

Fig. 6. X-spectral velocity vs. period

It is so obvious that natural frequency is calculated by dividing 2π over period of earth dam. Results of these dynamic analyses are shown in Table 4 only for shear wave velocity 500 m/s.

Table 4. Results of dynamic analyses (Vs = 500 m/s)

H (m)	W (rad/s)	T (s)
20	48.31	0.13
30	33.05	0.19
40	25.12	0.25
50	20.26	0.31
60	16.97	0.37
70	14.6	0.43
80	12.82	0.49
90	11.42	0.55
100	10.30	0.61

Dynamic analyses are performed for four cases which shear modulus varies from 500 up to 1280 Mpa. This is indicated in Fig. 7.

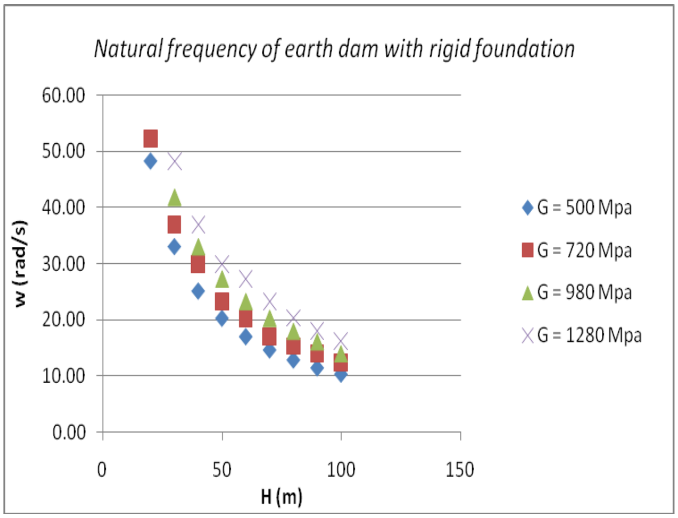

Fig. 7. Natural frequency of dam with rigid foundation

Like previous section, natural frequency is reduced with increasing of height of dam and also increases when shear modulus increases.

DISCUSSION ABOUT RESULTS

It is always necessary to compare every new proposed formulation with previous relations and numerical method so as to evaluate correctness and accuration of new correlation. As a result, in this section, the paper attempts to compare this analytical solution with other relations and results of dynamic analysis.

Gazetas et al (1991) presented one of the most useful correlations for calculating fundamental period of earth dam as follows:

$$T = 2.6\frac{H}{Vs} \tag{14}$$

Where H is height of dam (m) and Vs is shear wave velocity (m/s). For example, fundamental period and natural frequency for dams of height 20 up to 100m and shear wave velocity 500 m/s are listed in Table 5:

Table 5. Natural frequency of earth dam

H (m)	Coefficient	W (rad/s)	T (s)
20	2.6	60.15	0.10
30	2.6	40.10	0.16
40	2.6	30.08	0.21
50	2.6	24.06	0.26
60	2.6	20.05	0.31
70	2.6	17.19	0.37
80	2.6	15.04	0.42
90	2.6	13.37	0.47
100	2.6	12.03	0.52

Figure 8 indicates comparison between proposed formulation and Eqs. (14), results of presented formulation are almost agreeable in front of Gazetas et al (1991).

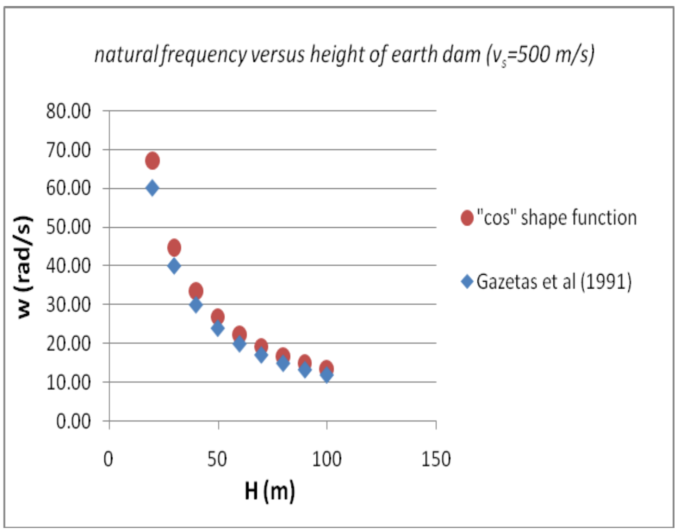

Fig. 8. Natural frequency of dam (comparison of results)

The same discussion is also performed for dynamic analysis and proposed formulation results; the comparison is shown in Fig. 9. It seems that there is also no significant difference between them, but is not as accurate as the previous one. Although we know that dynamic analysis is the most accurate solution for such problems.

An accurate observation on Fig. 9 and Table 6 shows that it can be a relationship between dynamic analysis and analytical solution results. We can claim that natural frequency of earth dam by dynamic analysis is obtained by dividing natural frequency of earth dam calculated by proposed analytical method over 1.33. This number is the average of the 10 numbers corresponding to the various heights of dams. This work is performed for shear wave velocity 500 m/s.

Table 6. Comparison of frequency (Vs = 500 m/s)

H (m)	W (rad/s) Dynamic analysis	W (rad/s) Analytical solution	Proposed coefficient
20	48.31	67.25	1.39
30	33.05	44.83	1.36
40	25.12	33.63	1.34
50	20.26	26.90	1.33
60	16.97	22.42	1.32
70	14.6	19.21	1.32
80	12.82	16.81	1.31
90	11.42	14.94	1.31
100	10.30	13.45	1.31

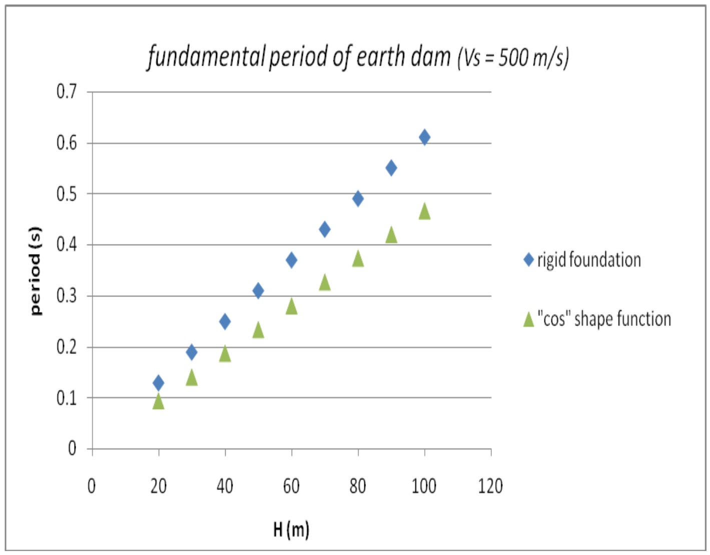

Fig. 9. Fundamental period of dam (comparison of results)

CONCLUSION

In this paper, based on continuous mass solution, presented a new formulation for computing natural frequency of earth dam. This solution is founded on some assumption such as shape function, rigid foundation and so on. It is clear that natural frequency corresponding to dam on flexible foundation is more than that one. Geometry, shear wave velocity and Poisson ratio of dam body, are affective parameters in calculating process. Results of new formulation show that for usual dams with 20 up to 100m height, fundamental periods vary from 0.03s up to 0.48s. As we said, natural frequency of earth dam by this method has reverse relation with height of dam and direct relation with shear wave velocity and Poisson ratio.

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