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SYNOPSIS A unified 3-d critical state bounding surface plasticity model (*g*UTS) has been developed which is able to provide realistic simulations of the behaviour of clays, silts and sands both in drained and undrained conditions over a wide range of monotonic and complex cyclic paths. A strong feature of this model is its ability to treat loose and dense states of the same material with a single set of material constants. The link between the two states is made by introducing an apparent normal consolidation line for sands and adopting a volumetric plastic strain hardening/softening model (similar to the critical state models for clays). This and other features enable the model to degenerate to simpler forms including the classic modified Cam-Clay formulation. To date, simulations have concentrated on the medium to high strain range (10^{-3} to 10^{-1}). To address a wider strain range, this paper reports on a new series of simulations for sand in the range 10^{-6} to 10^{-2} which are compared with established dynamic shear modulus and damping ratio data from drained resonant column tests and torsional shear tests.

EXPERIMENTAL OBSERVATIONS OF SOIL RESPONSE UNDER LOW AND HIGH STRAIN CYCLIC LOADING

The reduction in a soil's shear modulus with increasing shear strain under cyclic loading is a well established experimental fact as is the increase in hysteretic damping with increasing shear strain. The data suggest that the void ratio has a minor influence on the normalised shear modulus at a given cyclic shear strain. Similarly, the shear modulus is relatively insensitive to the number of cycles except in the high strain range. However, the mean effective stress level significantly effects the shear modulus degradation; high confinement delays the decay in the shear modulus at a given strain level.

Different experimental methods (for example bender element, resonant column, torsional shear, special triaxial, conventional triaxial) are required to capture the response of soils over the entire range of shear strains. This discontinuous approach is echoed by the distinction between different types of geotechnical problems on the basis of the anticipated strain range (for example, the analysis of machine foundations as compared with the progressive collapse analysis of embankment dams under extreme seismic loading). This division is partly responsible for the use of different models for different classes of problems.

TECHNIQUES FOR MODELLING THE SHEAR RESPONSE OVER A WIDE STRAIN RANGE

Over the years, at least three different classes of models have been proposed to cover the entire strain range (Ishihara 1982). Each of the classes (for example linear elastic, visco-elastic and elasto-plastic) is restricted by its framework to operate over a limited range.

The data from resonant column tests and torsional shear tests as traditionally used in quasi-linear dynamic analyses form the basis of an empirical hyperbolic relationship between the shear modulus and strain. These hyperbolic shear degradation models do not fit neatly into the categories of conventional elasticity or plasticity. This simplified approach lacks a rigorous theoretical basis and is incapable of simulating complex but important deformation phenomena in soils (such as static liquefaction, cyclic mobility under circular stress paths and progressive densification). While secant-based three-dimensional nonlinear elastic models could be used to simulate the drop in shear modulus with increasing cyclic strain, this approach would always predict zero damping contrary to the experimental observations even at low strain levels. A growing body of laboratory data shows that the purely elastic strain range is extremely small (if it exists at all) and so conventional nonlinear elasticity-based models cannot give a good representation of soil behaviour.

Elasto-plastic models based on the effective stress concept have been used with considerable success in soil mechanics to account for the coupling between shear and volumetric effects and provide a general criterion for loading and unloading. Although a great many elasto-plastic models have been proposed, there appears to be very few examples of using these models to predict the small strain shearing response. In fact, it is generally understood that these models are only appropriate for the 'large' to 'failure' strain range.

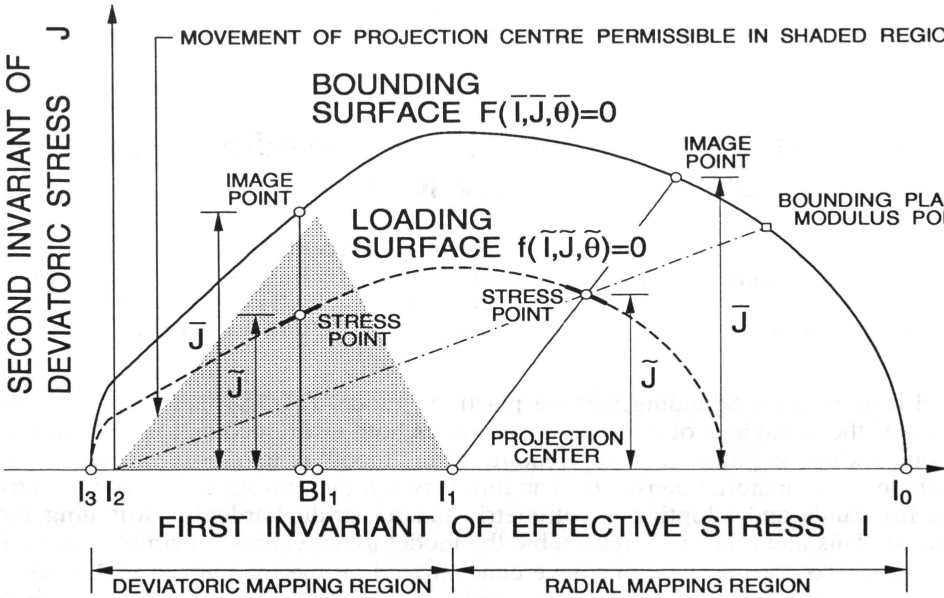

Fig. 1: Meridional section through bounding surface

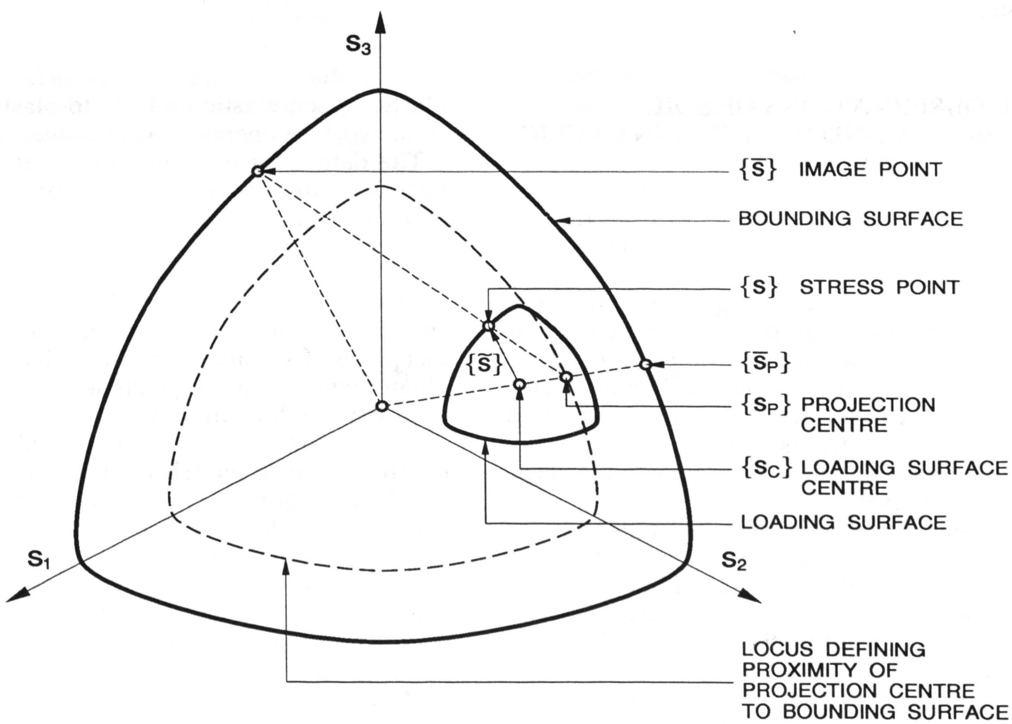

Fig. 2: Deviatoric section through bounding surface

THE CONSTITUTIVE MODEL *g*UTS

From the above discussion it follows that a constitutive framework is required which provides a smooth transition from small-scale irrecoverable straining inside the yield surface to a fully plastic response on the yield surface. A particularly simple and elegant solution to this problem was developed in the late 70s by Dafalias under the name bounding surface plasticity (for a complete description see Dafalias 1986). This framework does indeed allow a plastic response inside the bounding surface (which may be thought of as similar to the conventional yield surface). The magnitude of the plastic response depends on the proximity of the current stress point to the bounding surface. In the original isotropic bounding surface model for cohesive soils (Dafalias and Herrmann 1986), an elastic response is predicted for stress paths directed inside the loading surface (provided the sum of the bounding and additive plastic

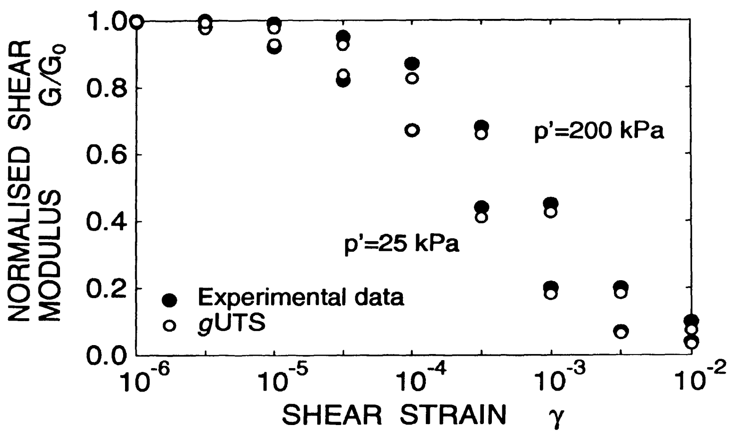

Fig. 3: Normalised shear modulus versus shear strain

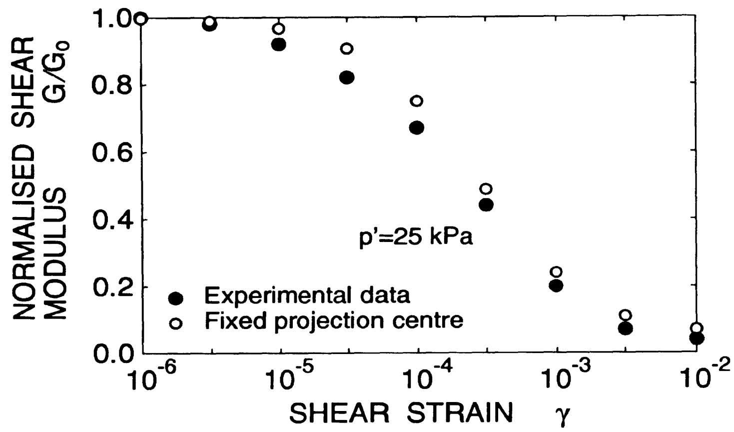

Fig. 4: Normalised shear modulus versus shear strain for case when projection centre is fixed

moduli is greater than zero at that point). This feature reproduces the behaviour of cohesive soils quite well but not of cohesionless soils where significant plastic straining has been observed upon stress reversal. Crouch and Wolf (1994) extended the original model by Dafalias and Herrmann (1986) by introducing a number of new features including the use of a radial mapping rule in the lightly over-consolidated region and a deviatoric mapping rule in the heavily over-consolidated region (Figure 1). The latter enables a plastic response to be predicted upon stress reversal by means of movement of the projection centre. Figure 2 shows this feature in a deviatoric plane. The complete model is named *g*UTS; an acronym for grand Unified Theory for Soil (note the lower case italic g, it would be foolish to imagine it is truly Grand).

It is important to emphasise that the intention is not to determine shear modulus degradation and damping ratio curves using *g*UTS for application in quasi-linear dynamic analyses. The small to moderate strain data from resonant column and torsional shear devices are treated as just another data set. The principal objective of this paper is to show that a single consistent approach (involving the use of a comprehensive consititutive model for soil, like *g*UTS) may be adopted for a wide range of engineering problems.

SHEAR MODULUS DEGRADATION AND EFFECT OF MEAN EFFECTIVE STRESS FOR SAND

The secant shear modulus is equal to the slope of the line joining the extremes of shear stress and shear strain for a given cycle after a predefined number of repeat paths. The material damping ratio D is defined as the area of the hysteresis loop normalised with respect to 2π times the product of the maximum shear stress and shear strain values.

Comparisons between the model simulations and the experimental data of Iwasaki et al (1978) are shown in Figure 3. The tests on Toyoura sand (with a mean void ratio of approximately 0.68) were conducted in both a hollow cylinder resonant column device and a hollow cylinder torsional shear apparatus. The former was used for the strain range from 10^{-6} to $5\text{x}10^{-4}$, and the latter for the range from $5\text{x}10^{-5}$ to 10^{-2}. The two devices showed good quantitative agreement in the overlap range. 4 drained tests at constant mean effective stresses p' of 25kPa, 50kPa, 100kPa and 200kPa were undertaken This mean stress results from equal internal and external fluid pressures and a balancing axial load acting on the ends of the specimen. Figure 3 shows the simulations for mean effective stresses of 25kPa and 200kPa. The constitutive model *g*UTS is normally calibrated on the basis of laboratory data from drained and undrained triaxial compression and extension tests. As such data for Toyoura sand are not reported by Iwasaki et al (1978), typical material constants determined from Sacramento River sand have been used and modified where appropriate. In particular, the slope of the hydrostatic rebound curve (in the void ratio e versus log p' plot) κ and Poissons ratio υ have been reduced to match the initial values of the shear moduli given by Iwasaki et al (1978) The material constants used for the simulations are specified in Crouch and Wolf (1994) with the following modifications: $\kappa = 0.002$, $\upsilon = 0.2$, $n_{g_c} = 1.5$, $h_{l_c} = 0.185$ and $n_H = 0.75$.

In *g*UTS the elastic shear modulus G_e is equal to

$$G_e = \frac{3p'(1+e_{in})(1-2\upsilon)}{2\kappa(1+\upsilon)} \tag{1}$$

where e_{in} is the initial void ratio.

This simple relationship is the same as that used in the modified Cam-Clay model (assuming a constant Poisson's ratio). It seems reasonably satisfactory although a number of researchers have found a better agreement with experimental data when the mean effective stress is raised to the power m where $0<m<0.5$ (for example see Ishibashi and Zhang 1993).

The simulation starting at a hydrostatic state of stress consists of a series of cyclic drained paths which are obtained by applying a sequence of shear stress increments up to predefined shear strain levels. Since p' does not vary

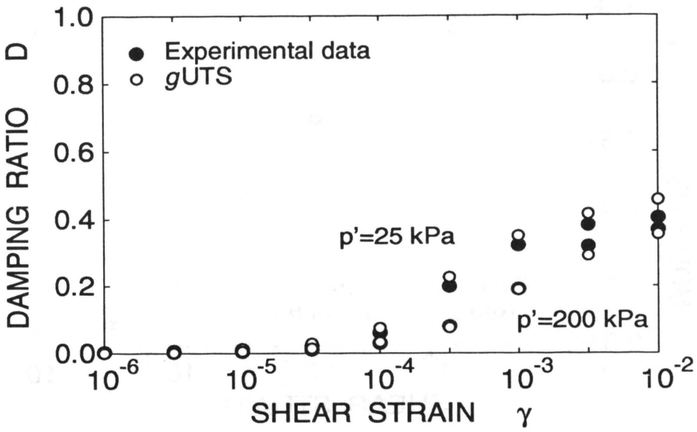

Fig. 5: Damping ratio versus shear strain

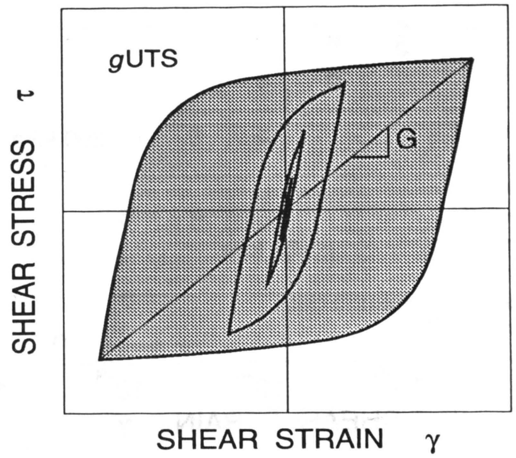

Fig. 6: Typical shear stress versus shear strain output of final cycles for different shear strain limits

during the tests, there is no change in the elastic shear modulus G_e and so the reduction in G in the model results solely from the accumulation of plastic strains. It is clear that the general behaviour is accurately reproduced over the entire strain range and the model is able to account for the different responses under varying mean effective stresses using a single set of material constants. Small increases in the initial void ratio (i.e. loosening) produce a slightly more rapid decay of the shear modulus (not shown). Similarly small decreases in the initial void ratio (i.e. densification) produce a slower decay in the shear modulus.

The effect of preventing movement of the projection centre is illustrated in Figure 4. By not allowing the projection centre to move away from the hydrostatic axis, plastic straining during stress reversals can only occur once the stress point has crossed the hydrostatic axis. Thus a greater shear stress is required to reach the predefined strain limit and the equivalent shear modulus is larger than for the case where the projection centre can move upon load reversal.

HYSTERETIC DAMPING

Details of the experimentally determined damping ratios for Toyoura sand are given by Tasuoka et al (1978). Note that the damping ratios obtained from the resonant column device and the torsional shear apparatus in the overlap strain range deviate more than the shear moduli do (the torsional shear apparatus consistently gave higher damping ratios than the resonant column device).

In the high strain range the simulations overpredict and underpredict the damping ratios at low and high mean effective stresses, respectively (Figure 5). Larger damping ratios are predicted if the movement of the projection centre is suppressed (not shown).

The growth in the size of the hysteresis loops and the reduction in the shear modulus as the shear strain range increases determined from *g*UTS are clearly illustrated in Figure 6.

CONCLUSIONS

Just as the limits of the operational ranges of different experimental devices are expanding as very high-precision displacement-measuring devices are developed, the range of applicability of refined constitutive models is also increasing. The success of the constructive model *g*UTS may be attributed to its high degree of flexibility yet unified approach based on established concepts.

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