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# Dynamic decision making under uncertainty in renewable energy portfolio management and inventory management

Jianjun Deng



Department: Engineering Management and Systems Engineering

DYNAMIC DECISION MAKING UNDER UNCERTAINTY

IN RENEWABLE ENERGY PORTFOLIO MANAGEMENT

AND INVENTORY MANAGEMENT

by

JIANJUN DENG

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Approved
Zhen Liu, Advisor
Scott E. Grasman
Ruwen Qin
Xiaoping Du
Michael Davis

## ABSTRACT

It is challenging and important for a firm to make effective decisions under uncertainties, such as random fluctuations of products prices or demands, etc. This dissertation formulates mathematic models to help decision makers in energy and retail industries make optimal timing and optimal operational decisions when facing uncertain electricity prices and demands.

As for energy portfolio management, the optimal entry and dispatch strategies are investigated for an electricity generating firm to introduce a renewable power plant as an alternative method for generating electricity, with or without construction delay. In addition, the abandonment strategies of considering shutting down one of the two power plants in the energy portfolio are studied. To develop these strategies, the expected per unit profit is maximized over a finite time horizon by assuming that the price of electricity follows mean reversion stochastic process. This problem is formulated as a mixed optimal stochastic control and optimal stopping problem. The original problem is solved numerically through two auxiliary problems. Numerical experiments are conducted to confirm the results. The sensitivity analysis of the parameters is conducted to reveal how the uncertainty of electricity price, investment, operation cost, and production rate affect the decisions.

A dynamic inventory model is also developed to study optimal control policy in a finite planning horizon with consideration of debt financing and tax. The model assumes that the retailer raises funds from the financial market and replenishes its stock under the constraint of its cash flow facing random demand. The objective is to maximize the expected terminal wealth. The optimal inventory policy and the optimal debt financing decision with the capital constraint and the effect of tax are obtained.

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# NOMENCLATURE

| Symbol | Description |
|---|---|
| $X_t$ | Electricity Price at Time t |
| $\mu$ | Meaning Reverting Coefficient |
| $\lambda$ | Mean Reverting Level |
| $\sigma$ | Volatility |
| $B_t$ | Wiener Process |
| $\rho$ | Risk-adjusted Discount Rate |
| $\alpha$ | Proportion of Total Monetary Input in Alternative Method |
| $\beta$ | Proportion of Total Monetary Input in Traditional Method |
| $\bar{\alpha}$ | Maximum Proportion of Total Monetary Input in Alternative Method |
| $\bar{\beta}$ | Maximum Proportion of Total Monetary Input in Traditional Method |
| $K$ | Capital Investment for New Plant Using Alternative Generating Method |
| $M$ | Liquidation Value for Abandoning Traditional Power Plant |
| $c_1$ | Production Rate of Traditional Generating Method |
| $c_2$ | Production Rate of Alternative Generating Method |
| $D_1$ | Total Cost of Using Traditional Method |
| $D_2$ | Total Cost of Using Alternative Method |
| $T$ | Planning Horizon |
| $x_n$ | The Inventory Levels Before the Replenishment in n Period |
| $y_n$ | The Inventory Levels Before the Replenishment in n Period |
| $D_n$ | Demand in n Period |
| $x_1$ | Initial Inventory Level |
| $S_1$ | Raised Initial Capital |
| $p$ | Unit Sales Price |
| $c$ | Unit Ordering Cost |
| $\gamma$ | Salvage Value |
| $d$ | Interest Rate for Whole Horizon |
| $r_f$ | Risk-free Interest Rate |
| $h$ | Holding Cost |

# 1. INTRODUCTION

## 1.1. ENERGY INDUSTRY AND $CO_2$ CONTROL

The United States is the largest energy consumer in the world in terms of total usage, 40.4% of which is used to generate electricity. Electricity, an extremely flexible form of energy, has been adapted for a great many and growing number of uses. Coal, natural gas, nuclear, hydro, wind, and solar are used primarily to make electricity in the U.S. today (EIA, 2011). Figure 1.1 indicates that coal is still the major source for generating electricity in the U.S. by now, and that fossil sources account for 68.3% of the total electricity. For the short term, coal is so abundant and fits the current grid infrastructure, but the problems are that it will run out some day in the future, and environmental pollution and Greenhouse Gases (GHG) emissions are becoming more and more critical. Natural gas, another major source for generating electricity, is used by most peaking power plants and some off-grid engine generators, since it produces less carbon dioxide during burning and is much cleaner than coal. The data indicate that burning natural gas produces about 45% less carbon dioxide than burning coal (Naturalgas.org, 2012). As the cleanest known source for combined cycle power generation, the natural gas is currently widely used all over the world.

As the third and fourth major sources for generating power, nuclear energy and hydro power plants produce electricity at a lower cost, almost without carbon dioxide emissions, and with high efficiencies and high capacity factors (Ipatov, 2008). The costs associated with nuclear and hydro power are primarily all startup costs, which is similar to most renewable sources. And the total costs to generate electricity from nuclear and hydro power are relatively low. Because of safety concerns, however, no new nuclear plant has been built in the United States since the 1970's.

For solar energy, solar cells conversion efficiencies are relatively low at approximately 14-19% for commercially available multi crystalline Silicon solar cells. The efficiency of a wind power plant depends on two factors, the frequency of wind and its speed. So, wind plant cannot produce electricity 24 hours a day, so does the solar plant. Compared to the environmental impact of traditional energy sources, solar and wind power have relatively minor effects. Solar and wind energy productions consume no fuel, at the same time, they do not produce air pollution or emit carbon dioxide. Unfortunately, the cost of solar or wind power plant is currently high, which can be times that for a coal power plant. In general, both solar and wind energy have extremely potential use in the future because of being renewable. The biggest challenges are the construction and production costs, which hopefully can be reduced by developing new technologies (Pande et al., 2010).

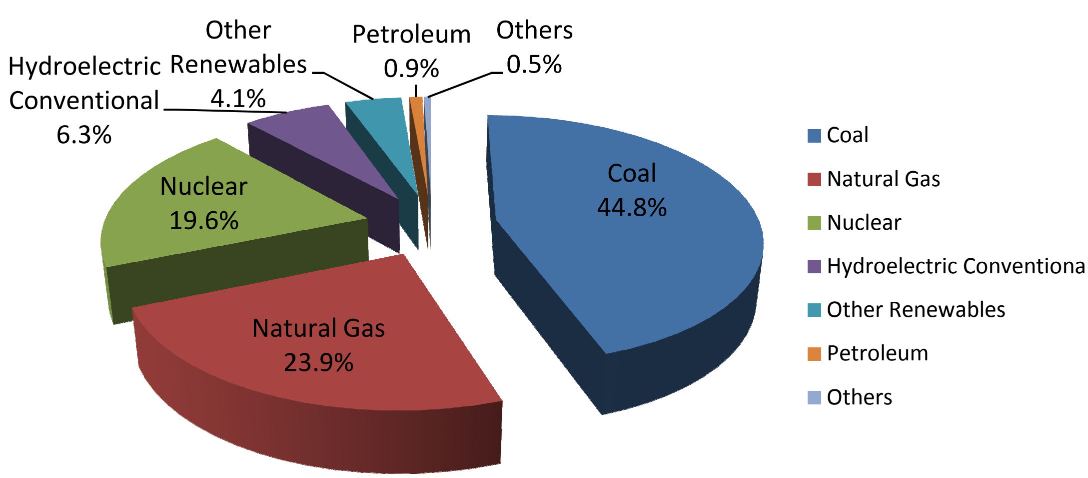


Figure 1.1. U.S Electricity Generation by Source in 2010

Pressures on transferring traditional coal-fired power plants to renewable power plants mainly come from climate change, which is now recognized as the major environmental problem facing our world. Of most concern factors that cause climate change is the increase in carbon dioxide levels due to emissions from fossil fuel combustion. According to a report of the Environmental Protection Agency, power plants are the source of 34% of total greenhouse gas emissions (EPA, 2012), as shows in Figure 1.2. In order to reduce carbon dioxide emissions, the United Nations launched the Kyoto Protocol in 1997 and entered it into force in 2005. Under the Kyoto Protocol, 37 countries have committed to reducing green house gases (GHG) collectively by 5.2% on average for 2008 to 2012 against 1990 levels. In 2007, the European Union committed to cutting its emissions by at least 20% of 1990 levels by 2020 (UN, 1998).

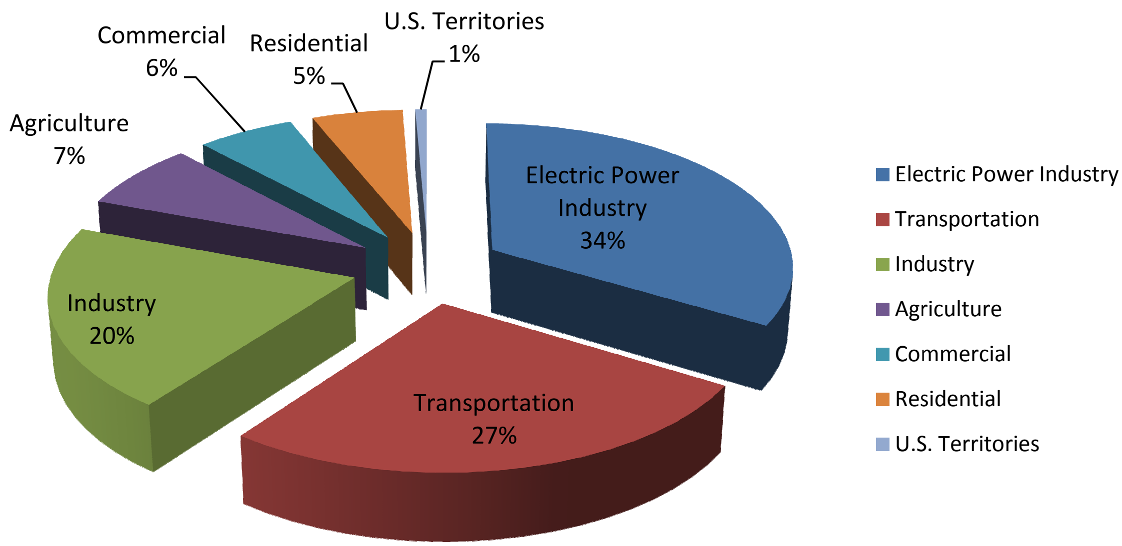


Figure 1.2. 2010 US Greenhouse Gas Emissions by Sectors

While the nations around the world agreed to reduce carbon dioxide emissions, governments tightened regulations on power plants that release carbon dioxide. In 2007, the Supreme Court ruled that greenhouse gases, including carbon dioxide, qualified as air pollutants under the Clean Air Act. In 2008, Congress required that the Environmental Protection Agency (EPA) begin releasing data about carbon dioxide and air quality. On December 23, 2010, EPA issued a proposed schedule for establishing greenhouse gas (GHG) standards under the Clean Air Act for fossil fuel fired power plants and petroleum refineries.

Governments often launch a carbon tax, emission tax, energy tax, and feed-in tariff in order to regulate generators. Carbon tax is an environmental tax levied on the carbon content of fuels; emission tax requires emitters to pay a fee, charge, or tax for every ton of GHG released; energy tax is charged directly to the energy commodities. All of these taxes offer a potential cost-effective means of reducing GHG. Feed-in tariff is a policy mechanism designed to accelerate investment in renewable energy technologies, which offer long-term contracts to renewable energy generators based on the cost of generations. Thus, the tight regulations and incentive policies from governments are the direct motivations that force generators to transfer generation of electricity from traditional methods to renewable methods, since these factors tend to increase the cost of traditional methods and decrease the cost of renewable methods.

## 1.2. MOTIVATIONS AND LITERATURE REVIEWS

Decision making is the essence of management; the quality of managerial decisions has a major influence on whether an organization succeeds or fails (Robbins, 2007). At the same time, uncertainties are always around us, including product prices, production costs, regulations from government, etc. So, the question is how to make effective decisions under uncertainty. Most decision-makers make decisions by intuition or by science (Christensen and Knudsen, 2010). In this dissertation, how to set up mathematical models to help decision-makers to find the optimal solutions for decisions relevant to energy portfolio management and inventory management are studied.

Facing current pressures to reduce carbon dioxide emissions, the generators are considering the decisions about building a renewable power plant to satisfy the increasing demand for electricity or abandoning a traditional old power plant to get rid of the burden of increasing costs. So, the generators have to find out the optimal time for building a new power plant or the best time for abandoning an old power plant with fluctuating of electricity prices, which are assumed to follow the mean reverting stochastic process. The generators also need to decide the optimal operational dispatch between the two power plants in the energy portfolio. In order to help generators to make these decisions, optimization models are formed to maximize the expected long-term unit profits of the firm, assuming that the price of electricity follows the mean reverting stochastic process. In this model construction delay for the new power plant is considered, which would be significant for the energy industry. The sensitivity analysis is also conducted in the models to reveal how the parameters could affect the decisions.

The optimal investment entry decisions and optimal operations decisions have been widely studied in recent years (Brekke and Øksendal, 1994). Some researchers considered the general investment model based on the productions capacity according to market fluctuations (Dixit and Pindyck, 1994). Pindyck (1988) and Øksendal (2000) studied the capacity decision by modeling a firm with capacity expanding in irreversible investment over an infinite horizon; Chiarolla and Haussmann (2003) modeled irreversible investment in a finite horizon. More recently, Guo and Pham (2005) set up a model to find the optimal entry and production decisions within an infinite time horizon; their model introduced expansion and contraction as partially reversible investment. They also reduced the original control problem into a two-stage procedure: a stochastic control problem (expansion and contraction) corresponding to an immediate entry decision, and a related optimal stopping time problem on the entry decision. They made extensive use of viscosity solutions approach in their paper.

The models on the energy industry can also be found easily (Deng et al., 2010). Tseng and Lin (2007) used a real option framework to value a power plant by generating discrete-time price lattices for two correlated Itô processes for electricity and fuel prices. This model incorporated operational constraints into the decision-making process and the lattice framework can handle general price processes; their method of stochastic dynamic programming, with two-factor price lattices, provides a much more efficient approach to calculating the value of power plant than the Monte Carlo simulation. Tseng and Barz (2002) evaluated a power plant in short-term with unit commitment constraints by using real-options approach, which was tackled using the Monte Carlo simulation.

Chen and Tseng (2011) explored the optimal investment timing for a coal-fired plant generator, and considered introducing a natural gas power plant using the real option approach in the face of tradable permits and carbon taxes, which are two market-based instruments commonly considered by government. Their model considered three stochastic processes: electricity price, natural gas price, and emission permit cost, with the objective function maximizing per MWh profit of the firm. They also found that tradable permits could effectively trigger the adoption of a new technology at a lower level of carbon tax; higher levels of volatility in the permit prices were likely to induce suppliers to take early actions to hedge against carbon risks. Their model employed two-factor price lattices (trees) as the numerical method for the solution procedure.

Bar-Ilan and Strange (1996) focused their research on investment lags, which are significant in the investment process of power generating plants. They concluded that the investment lag would reduce the deterrent effect of uncertainty on investment and tend to lessen inertia. With a short lag, an increase in uncertainty would delay investment, whereas a long lag, and increase in uncertainty may encourage investment. Delay information in the optimal investment problem also was considered by Øksendal (2005), who studied a general optimal problem by considering a time lag between making a decision and the time when the system actually stopped.

## 1.3. STOCHASTIC CONTROL AND FINITE DIFFERENCE METHODS

**1.3.1. Stochastic Process.** A Stochastic Process (SP) is a family of random variables $\{X(t)|t \in T\}$ defined on a given probability space, indexed by the time variable t, where t varies over an index set $T$ (Trivedi, 2002). Stochastic processes can be found anywhere, including stock price, electricity price, coal price, natural gas price, etc. Although there have numerous stochastic processes, here only introduce Markov process, Wiener process and $It\hat{o}$ process (Dynkin, 2006) (Fleming and Soner, 2006).

Any stochastic process, whose present value is only relevant for predicting future value, is called the Markov process. So, the distribution of the Markov process variable in a particular future time is not dependent on the variable changing path in the past. In other words, the future is dependent form the past, given the present. Stock price, electricity price, and many merchandise prices satisfy this Markov property.

The Wiener process is a Markov process with a mean change of zero and a variance rate of 1.0 per year. If $B_t$ follows a Wiener process, so (Hull, 2009):

$$\Delta B = \epsilon\sqrt{\Delta t} \tag{1.1}$$

where $\epsilon$ has a standardized normal distribution $\emptyset(0,1)$ .

And, the values of $\Delta B$ for any two different short intervals of time, $\Delta t$, are independent.

From (1.1):

$$E(\Delta B) = 0 \tag{1.2}$$

$$Var(\Delta B) = \Delta t \tag{1.3}$$

The Wiener process, also called the Brownian motion in physics, has a variety of applications in many areas.

The Wiener process can be expanded as a generalized Wiener process by adding drift rate and variance rate:

$$dx = adt + bd_B \tag{1.4}$$

where constant parameters $a$ and $b$ represent drift rate and variance rate respectively. Thus, the discrete case $\Delta x$ and the expectation and variance of $\Delta x$ can be obtained:

$$\Delta x = a\Delta t + b\epsilon\sqrt{\Delta t} \tag{1.5}$$

$$E(\Delta x) = a \tag{1.6}$$

$$Var(\Delta x) = b^2\Delta t\ . \tag{1.7}$$

Figure 1.3 is a generalized Wiener process with the following parameters: $a = 0.5, b = 1.8, x(0) = 0, T = 10$.

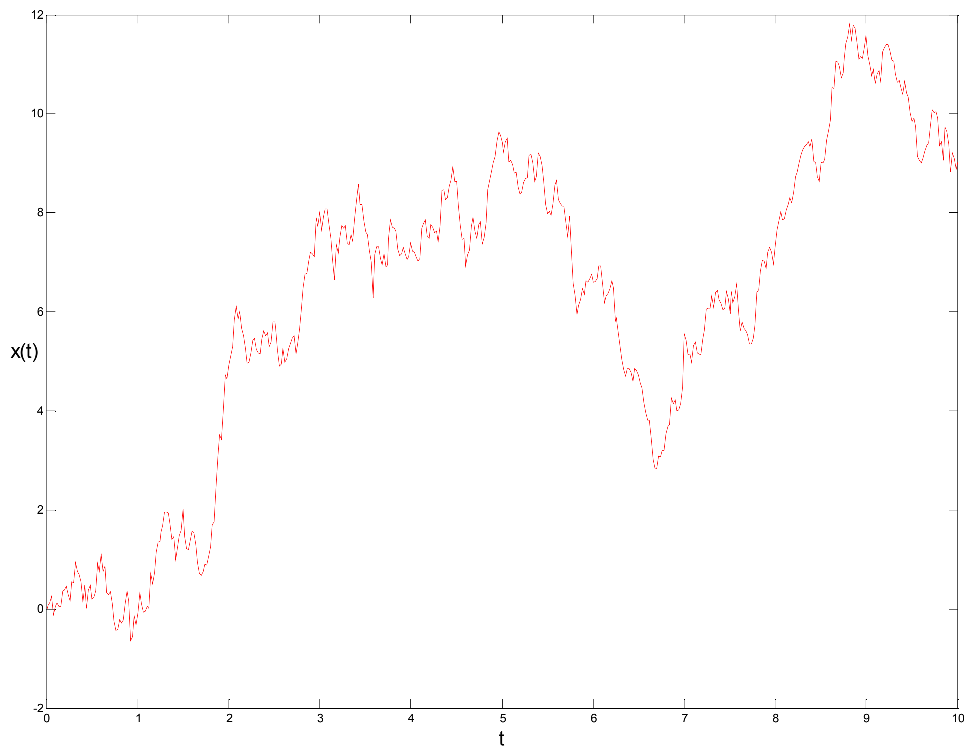


Figure 1.3. A Generalized Wiener Process

If the constant parameters *a* and *b* in the generalized Wiener process in (1.4) are changed to functions of *x* and *t*, the $It\hat{o}$ process can be obtained.

$$dx = a(x,t)dt + b(x,t)d_B. \tag{1.8}$$

The $It\hat{o}$ process is a more general type of stochastic process. Stock prices and electricity prices follow the $It\hat{o}$ process.

For stock price, assume that expected rate of return is $\mu$ and volatility is $\sigma$. Also it's assumed that investors would demand the same rate of return on different stock prices, and would feel the same uncertainty of the same percentage returns on different stock prices. So, the stochastic process model of stock price is:

$$dS = \mu S dt + \sigma S d_B \tag{1.9}$$

or

$$\frac{dS}{S} = \mu dt + \sigma d_B. \tag{1.10}$$

Electricity prices also can be treated as Itô process, but has more characteristics.

First, electricity cannot be easily stored; and the production of power plants is determined by the demand in the market. Accordingly, electricity prices can jump up as much as 1000% of normal prices in a short term, as shown is Figure 1.4 (Kim and Powell, 2011):

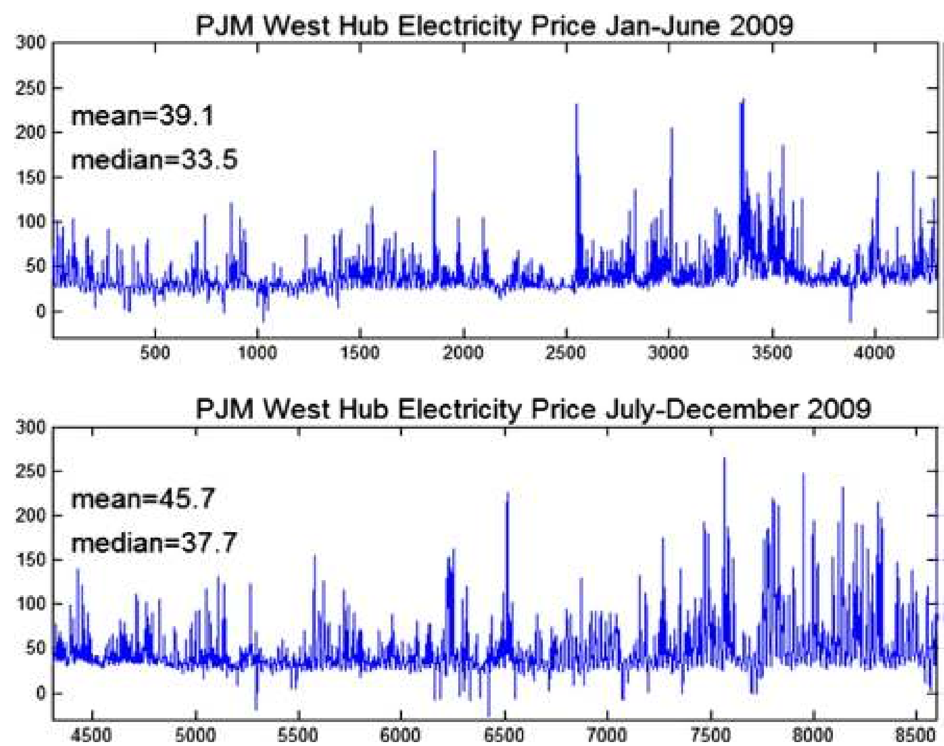


Figure 1.4. PJM West Hub Electricity Price in 2009

Second, seasonal variations occur. Major amount of electricity are used for air conditioning, so, the consumption and price of electricity are much higher in the summer than in the winter. The electricity price can revert to a long-run average level seasonally. The average retail price of electricity is shown in Figure 1.5 (EIA, 2011).

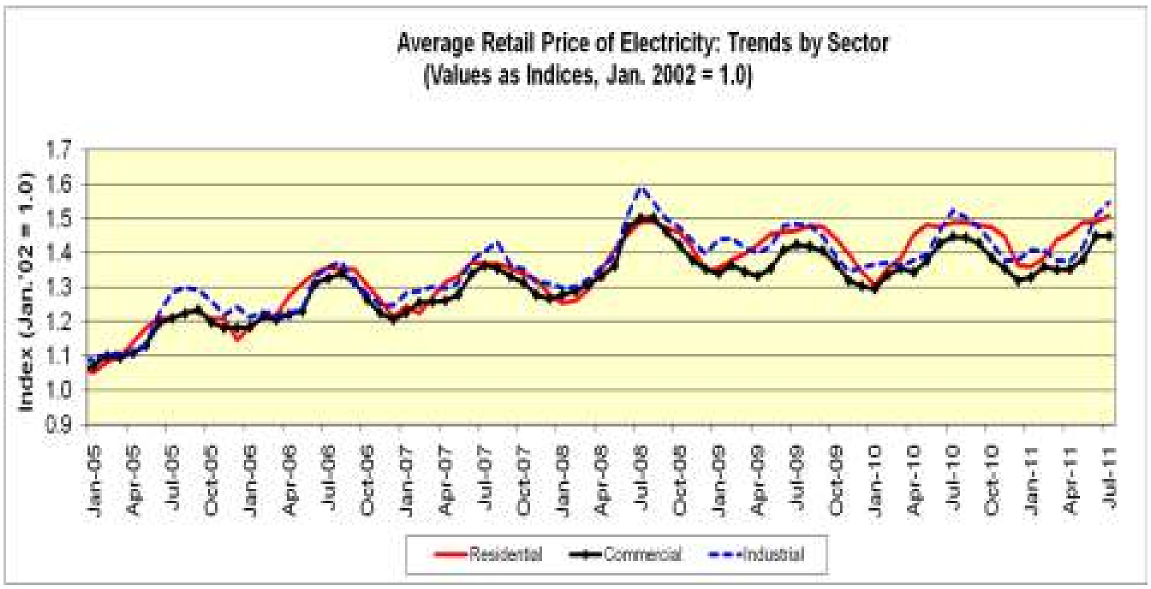


Figure 1.5. Average Retail Price of Electricity in 2005-2011

How to model the price of electricity has long been the focus of research interests, and different researchers have developed different models of electricity prices (Clewlow and Strickland, 2000) (Burger et al., 2003) (Eydeland and Geman, 2003) (Schwartz and Lucia, 2002). Some researchers assumed that the energy prices follow a Geometric Brownian Motion (GBM). Pindyck (1999) studied the long-run evolution of energy prices, which advocated that the prices follow a mean reversion stochastic process, but the rate of mean reversion is slow. Barz (1999) also assume electricity price as a geometric mean reversion (GMR) in order to set up stochastic financial models for electricity derivatives in his Ph.D dissertation. Deng (2000) modeled the electricity spot price as a mean reversion stochastic process with jumps and spikes, which incorporated multiple jumps, regime-switching, and stochastic volatility in his models. He also showed how his model about electricity price determines the value of investment opportunities and the optimal entry decisions. Deng (2005) later formulated a valuation of investment in power generation assets with spikes in electricity prices. This model demonstrated how to determine the value of an opportunity to invest in acquiring the generation capacity and the threshold value above which a firm should invest. He also illustrated the implications of electricity price spikes on the value of electricity generation capacity and the investment timing decisions on when to invest in such capacity. Thompson, Davison, and Rasmussen (2004) presented the valuation and optimal operation of hydroelectric and thermal power plants by considering the electricity price with mean reversion trends and price spikes.

Hull (2009) models the electricity price as:

$$dlnS = [\theta(t) - alnS]dt + \sigma d_B. \tag{1.11}$$

where $S$ is the electricity price, both $a$ and σ are constants. Parameter $a$ measures the speed with which that price reverts to a long-run average level; $\theta(t)$ captures seasonality and trends.

This dissertation will model electricity price like Barz and Hull's, since that is the best way in which to model these optimal investment and operations optimization problems that were described previously.

**1.3.2. Optimal Control and Dynamic Programming.** Optimal control includes objective function, control variables, state variables and constraints. And the decisions need to be made by control variables at each stage or time to maximize or minimize the objective function (profit, cost, time, etc.) under the constraints. Optimal control can be divided into static optimization and dynamic optimization. Dynamic optimization includes discrete-time optimization and continuous-time optimization, while continuous-time optimization consists of deterministic optimal control and stochastic optimal control (Stengel, 1993) (Kloeden, 1992) (Øksendal, 2003) (Øksendal and Sulem, 2005) (Steele, 2001).

A discrete-time optimal control problem can be stated as follows (Sarangapani, 2010):

Objective function:

$$\text{Max or Min} \quad J = \emptyset(x_N) + \sum_{k=0}^{N-1} L^k(x_k, u_k). \tag{1.12}$$

Subject to constraint:

$$x_{k+1} = f^k(x_k, u_k) \tag{1.13}$$

where $\emptyset(x_N)$ represents terminal condition. Expression $L^k(x_k, u_k)$ could be cost function, profit function, energy consumption function, or total time, etc.

The problem is to find optimal control $u_k = u_k{}^*$ and goes through optimal trajectory $x_k = x_k{}^*$, so that $J$ is minimized or maximized. These problems can be solved by introducing Lagrange multipliers to obtain a state equation, co-state equation, and stationary condition. Optimal control problems generally do not have analytic solutions because most of these problems are nonlinear, so, it is necessary to employ numerical methods to solve optimal control problems.

Deterministic continuous-time optimal control problems can be described as:

Objective function:

$$J(x,t) = \min_{u(t_0 \to T)} [\emptyset\big(x(T)\big) + \int_{t_0}^{T} L(x(t), u(t), t)dt] . \quad (1.14)$$

Subject to constraint:

$$\dot{x} = f(x(t), u(t), t). \quad (1.15)$$

For this system, the Hamilton-Jacobi-Bellman equation is:

$$-J_t(x,t) = \min_{u(t_0 \to T)} [L(x,u,t) + J_x(x,t) f(x,u,t)]. \quad (1.16)$$

The Hamilton-Jacobi-Bellman (H-J-B) equation can be solved for all *x* backwards from time *T* to $t_0$. The optimal control decision at *(x,t)* is given by:

$$u(x,t) = arg \min_{u(t_0 \to T)} [L(x,u,t) + J_x(x,t) f(x,u,t)]. \quad (1.17)$$

Stochastic continuous-time optimal control problems would be like this:

Consider the stochastic differential equation:

$$dx = \mu(x(t), u(t), t)dt + \sigma(x(t), u(t), t)d_B \quad (1.18)$$

where *u(t)* is control variable, $B_t$ is a Wiener process.

Objective function:

$$J(x,t) = \min_{u(t_0 \to T)} E\left[\emptyset\big(x(T)\big) + \int_{t_0}^{T} L(x(t), u(t), t)dt\right] . \quad (1.19)$$

By using the Bellman principle of optimality, Hamilton-Jacobi-Bellman equation can be obtained (Kappen, 2005, 2007, 2011, 2012):

$$-\partial_t J(x,t) = \min_{u(t_0 \to T)} [L(x,u,t) + \partial_x J(x,t)\mu(x,u,t) + \tfrac{1}{2}\sigma(x,u,t)\partial_x^2 J(x,t)]. \quad (1.20)$$

Dynamic programming is a solution approach to optimal control problems. It can be introduced to solve discrete-time optimization problems or continuous-time optimization problems by breaking these continuous-time complex problems down into simpler sub-problems in a recursive manner. In dynamic programming, a problem can be divided into stages, with a control (or policy) decision required at each stage; and each stage has states associated with the beginning of that stage. The control decision will transform the current states into new states that are associated with the next stage. The goal is to find the optimal solution at each stage, as well as to determine a solution for the overall problem (Bertsekas, 2011).

The Principle of Optimality is the core of dynamic programming, which can be described as shown below:

An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision (Bellman, 1957).

The principle of optimality means that the optimal decisions in the future are independent of past decisions (actions) which led to the present state. Thus, the optimal decisions for every state can be constructed by starting at the final state and extending backwards. The relationship between the value function in one period and the value function in the next period in recursive form is called the Bellman Equation. The Bellman equation is a very important result of dynamic programming, which has different formats for various dynamic programming problems.

**1.3.3. Finite Difference Methods.** There are a number of numerical methods that can be employed to find the solution for our problem. Finite difference methods are considered to be a good choice for our models. Finite difference methods are numerical methods for approximating the solutions to differential equations by converting differential equations into difference equations and then solving them iteratively. The approximations are based on the Taylor series expansions of functions near the point. As the following Figure 1.6 shows, there are three approximations: forward, backward, and central difference approximations (Wilmott et al., 1995).

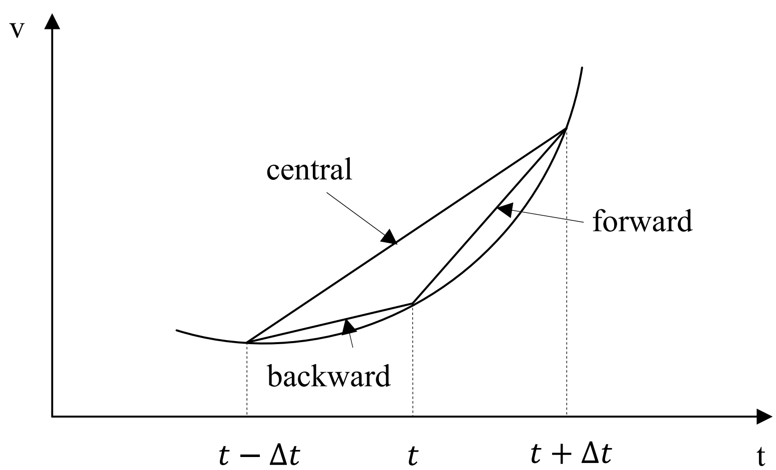


Figure 1.6. Finite Difference Approximations

The partial derivative $\frac{\partial v}{\partial t}$ can be forward approximated as:

$$\frac{\partial v}{\partial t}(x,t) \approx \frac{v(x,t+\Delta t)-v(x,t)}{\Delta t} + O(\Delta t). \tag{1.21}$$

And, the backward difference:

$$\frac{\partial v}{\partial t}(x,t) \approx \frac{v(x,t)-v(x,t-\Delta t)}{\Delta t} + O(\Delta t). \tag{1.22}$$

Central difference:

$$\frac{\partial v}{\partial t}(x,t) \approx \frac{v(x,t+\Delta t)-v(x,t-\Delta t)}{2\Delta t} + O((\Delta t)^2). \tag{1.23}$$

So, the finite difference approximations can be defined for the *x*-partial derivative of *v* in the same way.

Forward difference:

$$\frac{\partial v}{\partial x}(x,t) \approx \frac{v(x+\Delta t,t)-v(x,t)}{\Delta t} + O(\Delta t). \tag{1.24}$$

Backward difference:

$$\frac{\partial v}{\partial x}(x,t) \approx \frac{v(x,t)-v(x-\Delta t,t)}{\Delta t} + O(\Delta t). \tag{1.25}$$

Central difference:

$$\frac{\partial v}{\partial x}(x,t) \approx \frac{v(x+\Delta t,t)-v(x-\Delta t,t)}{2\Delta t} + O((\Delta t)^2)\ . \tag{1.26}$$

For the second partial derivatives, a symmetric finite difference approximation can be defined as the forward difference of backward difference approximation to the first derivative:

$$\frac{\partial^2 v}{\partial x^2}(x,t) \approx \frac{v(x+\Delta t,t)-2v(x,t)+v(x-\Delta t,t)}{(\Delta t)^2} + O((\Delta t)^2)\ . \tag{1.27}$$

The basic premise of the finite difference methods is to divide the x-axis into equally-spaced nodes at a distance of $\Delta x$ apart, and the t-axis into equally-spaced nodes at a distant of $\Delta t$ apart. This will divide the plane (*x,t*) into a mesh, where the mesh points can be presented as $(n\Delta x, m\Delta t)$, let $v_n^m = v(n\Delta x, m\Delta t)$.

Assume that the initial condition (*t=0*) is given as well as boundary conditions, there have explicit and implicit finite difference methods by using the difference forward or backward approximations of derivatives.

An explicit difference method gives the relationship between one value at time $(m+1)\Delta t$ and three different values at time $m\Delta t$. This problem is easy to solve because the initial condition is known. However, the explicit difference method requires the size of time steps as $\alpha = \frac{\Delta t}{(\Delta x)^2} < 0.5$ in order to make the computation stable. The implicit difference method expresses the relationship between one value at time $p\Delta t$ and three different values at time $(p+1)\Delta t$, which are needed to solve numerous simultaneous equations to calculate the value of $v_q^p$ from the values of $v_{q+1}^{p+1}, v_q^{p+1}$, and $v_{q-1}^{p+1}$. The implicity difference method has the advantage of being very robust. The difference between the explicit difference method and the implicit difference method is shown in Figure 1.7 (Hull, 2009).

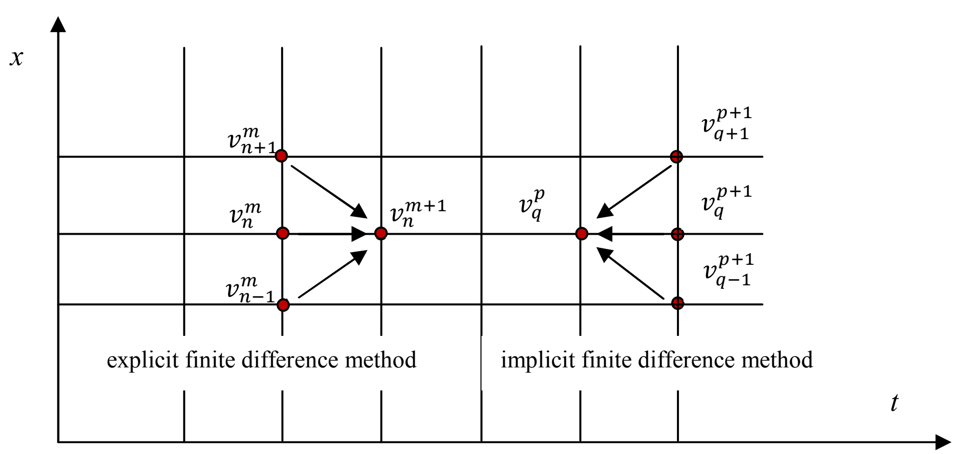


Figure 1.7. Explicit and Implicit Finite Difference Methods

## 1.4. ORGANIZATION OF THE DISSERTATION

The rest of this dissertation is organized as follows. In Section 2, a basic energy portfolio management model introduced is a mixed stochastic optimal control and optimal stopping time problem. An optimization model is set up with the objective function to maximize the long-term unit profit of the generator, with the assumption that the prices of electricity follow the mean reverting stochastic process. The problem will be solved numerically by finite difference methods. As a result, free boundaries will be used to make investment decisions.

In Section 3, a more complex energy portfolio management model with construction delay and relative gain is formed. In this model, sensitivity analysis with different parameters is used to reveal how the parameters affect the optimal decisions and relative gains. In this section, also, the operational cost of new power plant could be decreased over time, different combinations of operation rates and costs are discussed.

In Section 4, the optimal abandonment decision model for the energy portfolio is studied. This problem is formulated by maximizing long-term unit profit and solving it numerically. The free boundary will help the generator making abandonment decisions in a way similar to the energy portfolio management model.

In Section 5, a dynamic inventory optimal control problem, with consideration of debt financing and tax, is set up by maximizing the expected terminal wealth of a retailer facing random demand. The optimal ordering policy and optimal debt financing decision, with capital constraint and the effect of tax, are found at the end of this section.

Section 6 presents the conclusion and possible future work of the models.

# 2. ENTRY DECISION MODEL ON ENERGY PORTFOLIO MANAGEMENT

## 2.1. MOTIVATIONS

This is a basic model about the entry decision on energy portfolio management, which is very useful to the rest models in this dissertation. In this model, given a fixed capital investment, the optimal entry decision for a new plant and the optimal dispatch decision between the existing plant and the new plant are studied with the objective to maximize the long-term profit under the Geometric Mean Reversion (GMR) process for the price of electricity.

In this section, a unique approach is developed to find out the optimal stopping time and the optimal dispatch. At the same time, models are introduced that can help the generators optimize long-term unit profits as well.

This section is organized as follows. In Section 2.2, mathematical models that are developed based on a real problem are described. For simplicity, only the price of the electricity generated by the power plant is considered as a stochastic process; the price of coal and alternative energy, the cost of carbon dioxide emissions, and other costs are all considered constants. The problem is broken down into two separate parts, the optimal stopping time problem (when to build a new plant) and the optimal dispatch problem (how to operate the two power plants). The Hamilton-Jacobi-Bellman (H-J-B) equations are applied separately to each problem. Section 2.3 employs finite difference methods to solve the partial differential equations (PDEs) using forward and backward difference approximations. To ensure a stable and accurate solution, explicit finite difference methods, with a proper step size, are used to calculate the PDEs. A sample case, including data to illustrate the solutions of the PDEs, is also presented in Section 2.3.

## 2.2. MODELS AND FORMULATIONS

Assume that an energy company already owns and operates one existing power plant, which is assumed to be a traditional Coal-fired power plant. The decision makers of the generator are considering building a new renewable power plant to form an energy portfolio. So, the optimal operation policy and optimal time to build the new power plant have to be figured out. This decision-making problem is formulated as mixed optimal stochastic control and optimal stopping time problem to maximize the long-term unit profit of the firm.

There are many random variables that can affect decisions about investment time and operation, including electricity price, carbon dioxide emission cost, and the prices for the energies used by the two power plants to generate electricity. These prices, which follow different stochastic processes, are decided based on their demands and supplies in different markets. To simplify the models, the costs to generate electricity of the two power plants are assumed to be either fixed or locked in through financial contracts. The production rates of the two power plants are also assumed to be constants over a finite time horizon. Electricity price is the only stochastic process in this model.

How to describe the fluctuation in the price of electricity is the base of this model. Various researchers have different models for the electricity price (Barz, 1999) (Clewlow et al., 2001) (Deng, 2001) (Lucia and Schwartz, 2002) (Schwartz, 1998). Despite the presence of spikes in the short term (Deng, 2005), and the low rate of mean reversion in the long term (Pindyck, 1999), this model ignores the electricity jump diffusion and the changing of the mean reversion level, and assumes that electricity prices follow a Geometric Mean Reversion (GMR) process, so that the model can just focus on the optimal stopping time and optimal control. In a mathematical language, the evolution of the electricity price is represented as:

$$dlnX_t = \mu(\lambda - lnX_t)dt + \sigma dB_t. \tag{2.1}$$

That is

$$dX_t = (\mu(\lambda - lnX_t) + \frac{1}{2}\sigma^2)X_t dt + \sigma X_t dB_t. \tag{2.2}$$

Here, the electricity price ($X_t$) follows the Geometric Mean Reversion (GMR) process, where $\mu$, $\sigma$ and $\lambda$ are reverting coefficients, volatility, and the mean-reverting level of $lnX_t$, respectively (Clewlow et al., 2001); $B_t$ is a Wiener Process (Hull, 2009) (Seydel, 2006).

The energy firm is assumed to face a risk-neutral market and is treated as a price taking producer of electricity. Thus, the electricity demand is not considered in this model. Switching costs, which occur when operation switches from one power plant to another, is ignored in this model. The model also assumes that there is no construction delay, which means that the new power plant can be put into operation immediately after the decision is made. Or, it means that the firm purchases a new power plant, which was generating electricity. The model with construction delay will be proposed in Section 3.

In this model, $X_t$ represents electricity price at time *t*; *K* is the capital investment for a new plant using alternative generating method; $c_1, c_2$ represent the production rate of the existing generating method and alternative generating method, respectively; $D_1, D_2$ represent the total cost of generating $c_1$units of electricity, by using exiting method, and the total cost of generating $c_2$units of electricity, using the alternative method, respectively; *T* is the time planning horizon.

Per unit monetary input analysis is used in this model, where it assumes that the existing power plant can generate $c_1$ MWh electricity by inputting one unit of cash to buy the fuel for the existing plant and the total cost to generate $c_1$ MWh electricity is $D_1$, so, the operation profit of the existing power plant, by inputting one unit of cash on fuel, is $c_1 X_s - D_1$. Using the same analysis, the alternative power plant operation profit, by inputting one unit of cash on fuel, can be represented as $c_2 X_s - D_2$.

Parameter $\alpha \in [0, \bar{\alpha}]$ represents the proportion of total monetary input in the alternative method, when $\alpha = 0$, it means that the firm just uses the existing power plant. On the other hand, when $\alpha = \bar{\alpha}$, it means that the firm uses $\bar{\alpha}$ percentage of its cash to buy fuel for the alternative power plant, and input the rest of the cash (1- $\bar{\alpha}$ ) on the existing power plant (Chen and Tseng, 2011).

Assume that the time to make a decision to build a new power plant is $\tau$ ; and the required capital investment for building the new plant is *K* dollars.; the electricity price at time *t* is *x*, which can be expressed as *X(t)=x*.

Thus, per unit profit can be presented by the following functional:

$$J(t, x; \tau, \alpha) \equiv E_x[\int_t^\tau (c_1 X_s - D_1) e^{-\rho(s-t)} ds - K e^{-\rho(\tau-t)} + \int_\tau^T ((1-\alpha)( c_1 X_s - D_1) + \alpha(c_2 X_s - D_2)) e^{-\rho(s-t)} ds]. \tag{2.3}$$

Notes that, before time τ, the profits only come from the old power plant; but after the new power plant is built, the profits come from the energy portfolio formed at time $\tau$. All of the values are discounted to the present value at time *t*. Discount rate $\rho$ is the Risk-adjusted Discount Rate. Parameters $\tau$ and $\alpha$ are control variables: the problem is to determine the optimal time τ to build a new plant, and the optimal dispatch ($\alpha^*$) for the existing power plant and the new plant.

Let function $u_0(t,x)$ represent the best possible value of the objective function with a given time and electricity price *(t,x)*. So, the value function $u_0(t,x)$ is

$$(\mathrm{P}_0) \qquad u_0(t,x) = \sup_{\tau\in\Gamma_{t,T},\alpha\in[0,\bar{\alpha}]} J(t,x;\tau,\alpha) \tag{2.4}$$

where $\Gamma_{t,T,}$ denotes the set of stopping times in $[t,T]$ (Boudarel et al., 1971).

Because of the intractability of this mixed stochastic control and optimal stopping time problem, the above problem (2.4) is decomposed into two sub-problems: optimal dispatch once the new plant is built and the optimal time to build a new plant (Guo and Pham, 2005).

**2.2.1. Optimal Dispatch Once the New Plant is Built.** Suppose a new renewable power plant becomes available to join the operation at time $\tau$. Therefore, an energy portfolio of existing and new generating methods is thus formed, and the optimal proportions of each generating method must be determined. Hence, the problem becomes an optimal dispatch problem (optimal stochastic control problem). The value function *v(t,x)* of the optimal dispatch problem can be defined as:

$$(\mathrm{P1}) \;\; v(t,x) \equiv \max_{\alpha\in[0,\bar{\alpha}]} E_x[\textstyle\int_\tau^T((1-\alpha)(c_1X_s - D_1) + \alpha(c_2X_s - D_2))e^{-\rho(s-\tau)}ds]. \tag{2.5}$$

This stochastic control problem defined above can be transformed into a partial differential equation problem using the principle of dynamic programming (Kirk, 1970). The value function $v$ satisfies the following stochastic Hamilton-Jacobi-Bellman (H-J-B) equation (Kappen, 2007):

$$\frac{\partial}{\partial t}v + \sup_{\alpha\in[0,\bar{\alpha}]}[(1-\alpha_s)(c_1X_s - D_1) + \alpha_s(c_2X_s - D_2) + L_v - \rho v] = 0 \tag{2.6}$$

where

$$L_v \equiv (\mu(\lambda - lnx) + \tfrac{1}{2}\sigma^2)x\frac{\partial v}{\partial x} + \tfrac{1}{2}\sigma^2x^2\frac{\partial^2 v}{\partial x^2}\,. \tag{2.7}$$

From value function $v(t,x)$ (2.5), it is obvious that $v(t,x)$ is a non-decreasing function of α if $x \geq \frac{D_2-D_1}{c_2-c_1}$; $v(t,x)$ is a non-increasing function of α if $x < \frac{D_2-D_1}{c_2-c_1}$. So, the optimal control $\alpha^*$can then be calculated as follows:

$$\alpha^*(t,x) = \begin{cases} \bar{\alpha}, if\ (x \geq \frac{D_2-D_1}{c_2-c_1}) \\ 0\ , if\ (x < \frac{D_2-D_1}{c_2-c_1}) \end{cases}. \tag{2.8}$$

Therefore, the H-J-B equation (2.6) is reduced to

$$\frac{\partial}{\partial t} v + (1-\alpha^*)(c_1 X_s - D_1) + \alpha^*(c_2 X_s - D_2) + Lv - \rho v = 0. \tag{2.9}$$

with the terminal condition:

$$v(x,T) = \begin{cases} c_1 x - D_1 & , if\ (x < \frac{D_2-D_1}{c_2-c_1}) \\ (1-\bar{\alpha})(c_1 x - D_1) + \bar{\alpha}(c_2 X - D_2) & , if\ (x \geq \frac{D_2-D_1}{c_2-c_1}) \end{cases}. \tag{2.10}$$

Finite difference methods can be used to solve equation (2.9) in order to get the evolution of value function $v$, which is the base to solve the second sub problem (optimal stopping problem).

**2.2.2. Optimal Time to Build a New Plant.** Based on the solution of the optimal dispatch problem, the optimal stopping problem can be found. Noting that the value of the energy portfolio (when the new renewable power plant is built) is given by $v(\tau,X_\tau)$,which can be used in the optimal stopping problem. Therefore, the value function of the optimal stopping problem $w(t,x)$ is defined as follows:

(P2)

$$w(t,x) \equiv \sup_{\tau \in \mathrm{T}_{t,T}} E_x\left[\int_t^\tau (c_1 X_s - D_1) e^{-\rho(s-t)} ds + (v(\tau,X_\tau) - K) e^{-\rho(\tau-t)}\right],\ \ t \in [0,T) \tag{2.11}$$

The solution to this optimal stopping time problem $w$ satisfies the following linear complementarity problem:

$$\begin{cases} \frac{\partial}{\partial t} w + L_w + (c_1 X_s - D_1) - \rho w \leq 0 \\ w \geq v - K \\ (\frac{\partial}{\partial t} w + L_w + (c_1 X_s - D_1) - \rho w)\,(w - v + K) = 0 \end{cases} . \tag{2.12}$$

That is,

$$Min\left(-\frac{\partial}{\partial t} w - L_w - (c_1 X_s - D_1) + \rho w, w - (v - K)\right) = 0 \tag{2.13}$$

with the terminal condition:

$$w(x, T) = 0. \tag{2.14}$$

According to Guo and Pham, Problems (P2) and (P0) are equivalent. Hence, the original problem (P0) can be decomposed into an optimal stochastic control problem (P1), which can be solved by the PDE equation (2.9), and an optimal stopping time problem (P2), which can be solved by the PDE equation (2.13). An example is given on how to solve this problem in the case study section.

## 2.3. NUMERICAL SOLUTION AND RESULTS

**2.3.1. Numerical Solution.** Finite difference methods can be used to solve the PDE equations. According to the Taylor series expansion of functions near a point, forward and backward difference approximations are applied (Wilmott et al., 1995):

$$\frac{\partial v}{\partial t} \approx \frac{v(x,t+\Delta t) - v(x,t)}{\Delta \mathrm{t}} + O(\Delta t) \tag{2.15}$$

$$\frac{\partial v}{\partial x} \approx \frac{v(x,t) - v(x-\Delta x,t)}{\Delta x} + O(\Delta x) \tag{2.16}$$

or $$\frac{\partial v}{\partial x} \approx \frac{v(x+\Delta x,t) - v(x,t)}{\Delta x} + O(\Delta x) \tag{2.17}$$

$$\frac{\partial^2 v}{\partial x^2} \approx \frac{v(x+\Delta x,t) - 2v(x,t) + v(x-\Delta x,t)}{(\Delta x)^2} + O((\Delta x)^2). \tag{2.18}$$

The x-axis is then divided into *n* equally spaced nodes with an interval of $\Delta x$, and the t-axis is divided into *m* equally spaced nodes with an interval of $\Delta t$. Thus, the *x* - *t* plane is divided into a mesh with the cross point ($n\Delta x$, $m\Delta t$). The value of $u(x,t)$ at the mesh point $(n\Delta x, m\Delta t)$ can be expressed as $v_n^m = v(n\Delta x, m\Delta t)$.

First, considering equation (2.9)

$$\frac{\partial}{\partial t}v + (1-\alpha^*)(c_1x - D_1) + \alpha^*(c_2x - D_2) + L_v - \rho v = 0$$

$$L_v = \mu' \frac{\partial v}{\partial x} + \frac{1}{2}\sigma^2 \frac{\partial^2 v}{\partial_{x^2}} \tag{2.19}$$

$$\mu' = \mu(\lambda - lnx) + \frac{1}{2}\sigma^2 . \tag{2.20}$$

To calculate the equation, the terminal condition needs to be changed to initial condition by considering $t' = T - t$. However, in this dissertation *t* is still employed to represent $t'$ for convenient. Thus, the following equation can be achieved:

$$-\frac{\partial}{\partial t}v + (1-\alpha^*)(c_1x - D_1) + \alpha^*(c_2x - D_2) + L_v - \rho v = 0 \tag{2.21}$$

with the boundary conditions:

$$v(X_{min}, t) = -\frac{D_1}{\rho}(1 - e^{-\rho t}) \qquad (X_{min} = 0) \tag{2.22}$$

$$v(X_{max}, t) = -\frac{(c_1 + \alpha^*(c_2 - c_1)) * X_{max} - (1-\alpha^*)D_1 - \alpha^* D_2}{\rho}(e^{-\rho t} - 1)$$

$$(\, X_{max} \text{ is a big number}) \tag{2.23}$$

with the initial condition:

$$v(x,0) = \begin{cases} c_1x - D_1 & , if \ (x < \frac{D_2 - D_1}{c_2 - c_1}) \\ (1-\bar{\alpha})(c_1x - D_1) + \bar{\alpha}(c_2x - D_2) & , if \ (x \geq \frac{D_2 - D_1}{c_2 - c_1}) \end{cases} . \tag{2.24}$$

In order to improve computationally efficiency, a change of variable is used in the calculation as $Y = lnX$ (Hull, 2009). To overcome the artificial oscillations, an upwind scheme is used to represent $\frac{\partial v}{\partial x}$ (Seydel, 2006). Thus, equation (2.16) is used when ${\mu_x}' > 0$ ; and equation (2.17) is used when ${\mu_x}' < 0$.

Using equations (2.15),(2.16),(2.17) and (2.18), ignoring terms of $O(\Delta t)$ and $O(\Delta x)$, and plugging in equation (2.21), the formula for $v_n^{m+1}$ is obtained:

If ${\mu_x}' < 0$

$$v_n^{m+1} = \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}v_{n+1}^m - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right)v_n^m + \left(\frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu'\frac{\Delta t}{\Delta y}\right)v_{n-1}^m +$$

$$\Delta t\big((c_1 + \alpha^*(c_2 - c_1))e^y - (1 - \alpha^*)D_1 - \alpha^* D_2\big). \tag{2.25}$$

If ${\mu_x}' > 0$

$$v_n^{m+1} = \left(\frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right)v_{n+1}^m - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right)v_n^m + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}v_{n-1}^m +$$

$$\Delta t\big((c_1 + \alpha^*(c_2 - c_1))e^y - (1 - \alpha^*)D_1 - \alpha^* D_2\big). \tag{2.26}$$

As the equations determining $v_n^{m+1}$ in terms of $v_n^m$ are explicit, this process can be solved by Matlab. Thus, the numerical solution of *v* is found.

Second, consider H-J-B equation (2.13) after getting the value of *v*,

$$Min(-\frac{\partial}{\partial t}w - L_w - (c_1 X_s - D_1) + \rho w, w - v + K) = 0.$$

Consider first part of equation (2.13):

$$\frac{\partial}{\partial t}w' + L_{w'} + (c_1 X_s - D_1) - \rho w' = 0 \tag{2.27}$$

$$L_{w'} = \mu'\frac{\partial w'}{\partial x} + \frac{1}{2}\sigma^2\frac{\partial^2{}_{w'}}{\partial_{x^2}}. \tag{2.28}$$

By using the same strategy as before to change *t*:

$$-\frac{\partial}{\partial t}w' + L_{w'} + (c_1 X_s - D_1) - \rho w' = 0 \tag{2.29}$$

with the boundary conditions as follows:

$$w'(X_{min}, t) = -\frac{D_1}{\rho}(1 - e^{-\rho t}) \qquad (\tau = T, X_{min} = \frac{D_1}{c_1}) \qquad (2.30)$$

$$w'(X_{max}, t) = \frac{c_1 X_{max} - D_1}{\rho}(1 - e^{-\rho t}) \qquad (\tau = t, X_{max} \text{ is a big number}). \quad (2.31)$$

with the initial condition as follows:

$$w'(x, 0) = 0. \qquad (2.32)$$

Also the formula for $w'^{m+1}_{n}$ is obtained:

If $\mu_x' < 0$

$$w'^{m+1}_{n} = \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} w'^{m}_{n+1} - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right) w'^{m}_{n} + \left(\frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu'\frac{\Delta t}{\Delta y}\right) w'^{m}_{n-1} +$$

$$\Delta t(c_1 e^y - D_1). \qquad (2.33)$$

If $\mu_x' > 0$

$$w'^{m+1}_{n} = \left(\frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right) w'^{m}_{n+1} - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right) w'^{m}_{n} + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} w'^{m}_{n-1} +$$

$$\Delta t(c_1 e^y - D_1). \qquad (2.34)$$

Once the evolution of $w'$ is obtained, $w'^{m}_{n}$ in this solution can be compared with $v_n^m - K$; if the electricity price $X_s$ makes $v_n^m - K > w'^{m}_{n}$, which means the portfolio value is greater than the value without portfolio, value function *w* should be equal to $v_n^m - K$, because value function *w* satisfy the equation $w_n^m = max\,\{w'^{m}_{n}, v_n^m - K\}$. In other words, the new green plant should be built. The free boundary curve is formed by the electricity price at different times that satisfy equation $w'^{m}_{n} = v_n^m - K$.

**2.3.2. Case Study and Results.** To illustrate this mathematic model and solution technique, the parameters are established in Table 2.1. Values are obtained from literature reviews and experience on energy industry.

Table 2.1. The List of Parameters for Entry Decision Model

| Notation | Value | Unit |
|---|---|---|
| $\mu$ | 0.053 | |
| $\sigma$ | 0.13 | |
| $\lambda$ | 4.788 | |
| $\rho$ | 6% | |
| $\bar{\alpha}$ | 80% | |
| $c_1$ | 2 | MWh/unit monetary input |
| $c_2$ | 3 | MWh/ unit monetary input |
| $D_1$ | 100 | $\$/c_1$MWh |
| $D_2$ | 130 | $\$/c_2$MWh |
| $K$ | 500 | \$/unit monetary input |
| $T$ | 10 | Year |

First, the mean price of electricity $120/MWh is derived from the parameters. Second, form free boundary of $w$ by using the equation $w_n^m = v_n^m - K$, which shows when the new plant should be built at a given time with respect to the electricity prices. The case study finds that $v_n^m - K > w_n^m$ in the area above free boundary, which means the value with portfolio is greater than the value without portfolio in that area. So, the generator should invest the new plant when the electricity price goes up above the free boundary. The free boundary of $w$ is shown in Figure 2.1.

Figure 2.2 shows that the decision maker should excise the investment option at time $\tau$ when the electricity price is about to across over the free boundary.

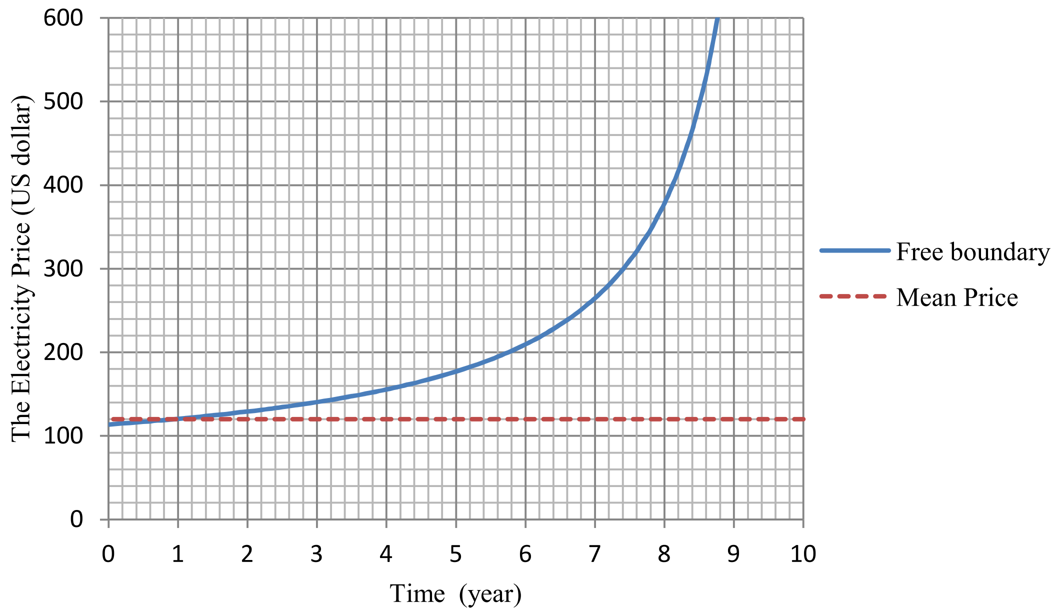


Figure 2.1. Free Boundary of *w*

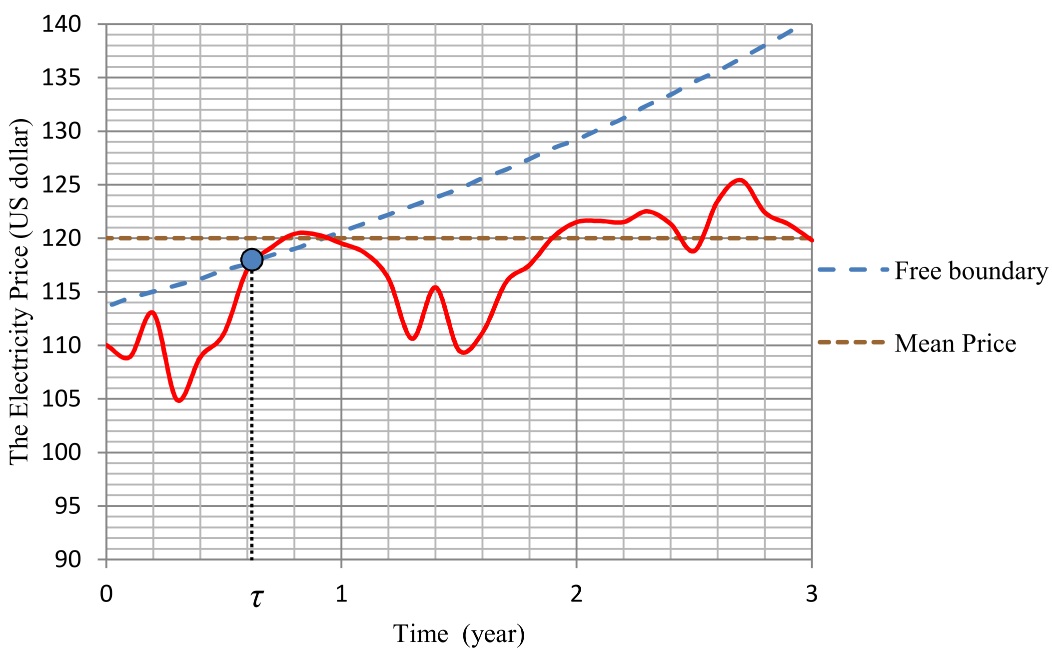


Figure 2.2. Entry Decision by Using Free Boundary

## 2.4. CONCLUSIONS

This section models the optimal entry problem for a new renewable power plant, with the given fixed capital investment, and the optimal dispatch between the existing power plant and the new power plant in order to maximize the long-term profit under the Geometric Mean Reversion (GMR) process for the price of electricity. Optimal control and finite difference methods are used to solve the entry decision and optimal dispatch problems in the energy portfolio investment.

The results provide some valuable data for the generators to make viable decisions. The free boundary can be employed to decide when the generator should build the new renewable power plant. That is, the new plant is to be built when the price of electricity jumps up above the free boundary at a particular time. After the new plant is built, the new plant should be set on the maximum proportion ($\bar{\alpha}$) of monetary input when the electricity price is higher than $\frac{D_2-D_1}{c_2-c_1}$.

The contributions of this basic model are two-fold: First, the model of optimal entry decisions of a firm to form an energy portfolio by the stochastic control approach is formulated. Second, the intractable problem is decomposed into two sub-problems, and then solves them numerically.

# 3. ENTRY DECISION MODEL WITH DELAY

## 3.1. MOTIVATIONS

The previous section introduces a basic model of the entry decision on energy portfolio management. However, the reality is much more complex than the assumption of that model. For example, the new power plant cannot be put into operation immediately after the investment decision is made, since it needs several years to finish the construction. Also, the operation cost could change over the finite horizon time, especially for the new technology. Accordingly, in this section, the construction delay and changing operation cost are considered, and, the optimal entry decision for the new plant and the optimal dispatch decision are investigated further. The model maximizes the long-term expected profit under the geometric mean reversion process for the electricity prices.

It is assumed that a firm owns a plant, and considers adding a new plant while maximizing the expected long-term profit. Since the firm can generate electricity by a portfolio of two plants, the optimal dispatch of these two plants needs to be determined after the new plant is constructed. Under the geometric mean reversion process for electricity prices, this decision problem is formulated as a mixed stochastic optimal control problem and optimal stopping problem. Due to the intractability of the mixed problem, it is decomposed into two auxiliary problems: one is a regular stochastic control problem, and the other one is an optimal stopping problem with a delay. The solutions to the auxiliary problems are equivalent to the original control problem. As an extension of Section 2, the optimal stopping problem with a delay can be transferred into an optimal stopping problem without delay by the Markov property of a Markov process.

The rest of this section is organized as follows. Section 3.2 describes a mathematical model of the problem. The intractable problem is decomposed into two sub-problems, the optimal stopping time problem and the optimal dispatch problem. The Hamilton-Jacobi-Bellman (HJB) equation or Variational Inequality (VI) of the value functions for the sub-problems are obtained, respectively. Section 3.3 employs finite difference methods to solve the partial differential equations (PDEs) and get the gain percentage by using forward and backward difference approximations. To ensure a stable and accurate solution, explicit finite difference methods with a proper step size are used to calculate the PDEs. Numerical experiments are presented in the Section 3.4.

## 3.2. MODELS AND FORMULATIONS

This problem is formulated as a mixed stochastic control and optimal stopping time problem, to maximize the long-term profits of the company, using the same assumption that is presented in Section 2. The difference is that a construction time to build the new power plant (delay) is considered in this model. The same standard notations are used in this section as are used in Section 2, with $\delta$ representing the construction time for building the alternative power plant.

The price of electricity is also assumed to follow the stochastic process. In addition, the cost to generate electricity by the existing power plant is assumed to be either fixed or locked in through financial contracts, as well as by the constant production rate of this power plant. The cost for the new power plant to generate electricity is assumed to have decreased over time because of the new plant's improved technology. It should be noted that constant costs are used to set up the model. The decreasing costs of the new power plant are discussed in the sensitivity section.

The electricity price is assumed to follow the Geometric Mean reversion (GMR) process just as the basic model describes in Section 2. The assumption is made that construction of the new power plant would take $\delta$ years to be complete. Here, per unit monetary input analysis used is the same as described in Section 2. Thus, the long-term profit functional is

$$J(t,x;\tau,\alpha) \equiv E_x[\int_t^{\tau+\delta}(c_1X_s - D_1)e^{-\rho(s-t)}ds - Ke^{-\rho(\tau+\delta-t)} + \int_{\tau+\delta}^{T}((1-\alpha)(c_1X_s - D_1) + \alpha(c_2X_s - D_2))e^{-\rho(s-t)}ds]. \quad (3.1)$$

So, the value function $u_0$ *(t,x)* is defined as

$$(\mathrm{P}_0) \qquad u_0(t,x) = \sup_{\tau\in\Gamma_{t,T-\delta,}\alpha\in[0,\bar{\alpha}]} J(t,x;\tau,\alpha). \quad (3.2)$$

That is

$$u_0(t,x) = \sup_{\tau\in\Gamma_{t,T-\delta,}\alpha\in[0,\bar{\alpha}]} E_x[\int_t^{\tau+\delta}(c_1X_s - D_1)e^{-\rho(s-t)}ds - Ke^{-\rho(\tau+\delta-t)} + \int_{\tau+\delta}^{T}((1-\alpha)(c_1X_s - D_1 + \alpha(c_2X_s - D_2))e^{-\rho(s-t)}ds]. \quad (3.3)$$

where $\Gamma_{t,T-\delta,}$ denotes the set of stopping times in $[t, T-\delta]$ (Bar-Ilan and Strange, 1996).

Due to the intractability of this mixed stochastic control and optimal stopping time problem, the above problem is decomposed into an optimal control (dispatch) problem and an optimal stopping time problem (Guo and Pham, 2005).

**3.2.1. Optimal Dispatch Once the New Plant is Built.** It is assumed that construction of the new power plant has been completed and becomes available to join the operation at time *τ*. The value function *v* is defined as:

$$v(t,x) \equiv \max_{\alpha\in[0,\bar{\alpha}]} E_x[\int_\tau^T((1-\alpha)(c_1X_s - D_1) + \alpha(c_2X_s - D_2))e^{-\rho(s-t)}ds]. \quad (3.4)$$

By using the principle of dynamic programming, the value function $v$ satisfies the following H-J-B equation:

$$\frac{\partial}{\partial t}v + \sup_{\alpha\in[0,\bar{\alpha}]}[(1-\alpha_s)(c_1x - D_1) + \alpha_s(c_2x - D_2) + L_v - \rho v] = 0 \tag{3.5}$$

where

$$L_v \equiv (\mu(\lambda - lnx) + \frac{1}{2}\sigma^2)x\frac{\partial v}{\partial x} + \frac{1}{2}\sigma^2x^2\frac{\partial^2 v}{\partial_{x^2}}, \tag{3.6}$$

with the terminal condition:

$$v(x,T) = \begin{cases} c_1x - D_1 & ,if\ (x < \frac{D_2-D_1}{c_2-c_1}) \\ (1-\bar{\alpha})(c_1x - D_1) + \bar{\alpha}(c_2x - D_2) & ,if\ (x \geq \frac{D_2-D_1}{c_2-c_1}) \end{cases}. \tag{3.7}$$

It is not difficult to solve equation (3.4); the optimal control variable $\alpha^*$can then be calculated as follows:

$$\alpha^*(t,x) = \begin{cases} 0 & ,if\ (x < \frac{D_2-D_1}{c_2-c_1}) \\ \bar{\alpha} & ,if\ (x \geq \frac{D_2-D_1}{c_2-c_1}) \end{cases}. \tag{3.8}$$

Therefore, the H-J-B equation (3.5) is reduced to

$$\frac{\partial}{\partial t}v + (1-\alpha^*)(c_1x - D_1) + \alpha^*(c_2x - D_2) + L_v - \rho v = 0. \tag{3.9}$$

By solving equation (3.9), the maximal discounted portfolio values for different $t$ and $x$ after building the new power plant can be used in the next step to solve the optimal stopping time problem.

**3.2.2. Optimal Time to Build a New Plant with Construction Delay.** Based on the previous calculations, the value of the portfolio is given by $v(\tau+\delta, x_{\tau+\delta})$ when the new plant is built; therefore, the value function $w(t, x)$ , with consideration of the construction time (delay), is defined as follows:

$$w(t,x) \equiv \sup_{\tau \in \Gamma_{t,T}} E_x \left[ \int_t^{\tau+\delta} (c_1 X_s - D_1) e^{-\rho(s-t)} ds + (v(\tau+\delta, X_{\tau+\delta}) - K) e^{-\rho(\tau+\delta-t)} \right]$$

$$t \in [0, T-\delta) \tag{3.10}$$

Although it is difficult to solve *w*(*t*,*x*) directly, some researchers have identified a method for solving the delay problem (that is similar to the problem in this model) by transforming it into an easily solvable problem without the time delay. By introducing the function $g_0(t,x)$ (Øksendal, 2005), the delayed optimal stopping problem is transformed into a non-delayed optimal stopping problem as follow:

$$w(t,x) \equiv \sup_{\tau \in \Gamma_{t,T-\delta}} E_x \left[ \int_t^{\beta} (c_1 X_s - D_1) e^{-\rho(s-t)} ds + g_0(\beta, x_\beta) e^{-\rho(\beta-t)} \right], t \in [0, T-\delta) \tag{3.11}$$

where

$$g_0(t,x) \equiv E_{t,x} \left[ \int_t^{t+\delta} (c_1 X_s - D_1) e^{-\rho(s-t)} ds + (v(t+\delta, X_{t+\delta}) - K) e^{-\rho\delta} \right], t \in [0, T-\delta) \tag{3.12}$$

Note that $\beta = \tau + \delta$ is the optimal stopping time for a non-delayed problem. The $g_0$ can be solved by the following problem by using the Feynman-Kac theorem:

$$\begin{cases} \frac{\partial}{\partial t} g_0 + L_{g_0} + (c_1 X_s - D_1) - \rho g_0 = 0 \\ g_0(\tilde{T}, x) = v(\tilde{T}, x) - K \end{cases} \tag{3.13}$$

where $\tilde{T} = t + \delta$ is the terminal time.

Thus, the solution to this optimal stopping time problem *w,* with time delay, satisfies the following linear complementarity problem:

$$\begin{cases} \frac{\partial}{\partial t} w + L_w + (c_1 x - D_1) - \rho w \le 0 \\ w \ge g_0 \\ (\frac{\partial}{\partial t} w + L_w + (c_1 x - D_1) - \rho w)\,(w - g_0) = 0 \end{cases} . \tag{3.14}$$

That is,

$$Min\left( -\frac{\partial}{\partial t} w - L_w - (c_1 x - D_1) + \rho w, w - g_0 \right) = 0 \tag{3.15}$$

with the terminal condition:

$$w(x, T-\delta) = 0. \tag{3.16}$$

The solution of $w$ in (3.15) is found after solving $g_0(t,x)$ in (3.13).

Hence, the original problem is decomposed into a stochastic control problem, which can be solved by the PDE equation (3.9), and an optimal stopping time problem with delay, which can be solved by the PDE equation (3.15). Examples of how to solve these problems numerically is presented in the case study section.

**3.2.3. No Investment Option.** In order to determine the effect of the energy portfolio, this section introduces relative gain, which measures the profit increment comparing the case with that where no portfolio is involved. So, by considering a case where there is no investment option throughout the finite planning horizon, and only the existing power plant is operated.

The value function $u$ for the case where is no investment option is defined as

$$u(t,x) \equiv E_x[\int_t^{\tau}(c_1 X_s - D_1)e^{-\rho(s-t)}ds]. \tag{3.17}$$

Using the same strategy, the H-J-B equation is found:

$$\frac{\partial}{\partial t}u + (c_1 x - D_1) + L_u - \rho u = 0 \tag{3.18}$$

$$L_u \equiv (\mu(\lambda - lnx) + \frac{1}{2}\sigma^2)x\frac{\partial u}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 u}{\partial_{x^2}} \tag{3.19}$$

with the terminal condition:

$$u(x,T)=0. \tag{3.20}$$

After PDE (3.18) is obtained, it can be solved numerically. Thus, the numerical solution of $u$ can be used to calculate the relative gain of portfolio investment, which is defined as $p = (w-u)/u$.

## 3.3. NUMERICAL SOLUTION AND RESULTS

**3.3.1. Numerical Solution.** Finite difference methods are also employed to solve these PDEs, with the solution result of the problems as follows. Detailed solution procedures are shown in Appendix Section.

In order to improve calculation efficiency, $Y = lnX$ is set when solving equation (3.9),(3.13),(3.15),(3.18). The formulas for $v_n^{m+1}$ , ${g_0}_n^{m+1}$, ${w'}_n^{m+1}$ and $u_n^{m+1}$ are obtained by using an upwind scheme (Seydel, 2006), note that $\mu' = \mu(\lambda - lnX_t) + \frac{1}{2}\sigma^2$.

If $\mu' < 0$

$$v_n^{m+1} = \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}v_{n+1}^m - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right)v_n^m + \left(\frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu'\frac{\Delta t}{\Delta y}\right)v_{n-1}^m$$
$$+\Delta t\big((c_1 + \alpha^*(c_2 - c_1))e^y - (1-\alpha^*)D_1 - \alpha^* D_2\big). \tag{3.21}$$

$${g_0}_n^{m+1} = \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}{g_0}_{n+1}^m - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right){g_0}_n^m + \left(\frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu'\frac{\Delta t}{\Delta y}\right){g_0}_{n-1}^m +$$
$$\Delta t(c_1 e^y - D_1). \tag{3.22}$$

$${w'}_n^{m+1} = \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}{w'}_{n+1}^m - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right){w'}_n^m + \left(\frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu'\frac{\Delta t}{\Delta y}\right){w'}_{n-1}^m +$$
$$\Delta t(c_1 e^y - D_1). \tag{3.23}$$

$$u_n^{m+1} = \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}u_{n+1}^m - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right)u_n^m + \left(\frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu'\frac{\Delta t}{\Delta y}\right)u_{n-1}^m +$$
$$\Delta t(c_1 e^y - D_1). \tag{3.24}$$

If $\mu' > 0$

$$v_n^{m+1} = \left(\frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right)v_{n+1}^m - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right)v_n^m + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}v_{n-1}^m$$
$$+\Delta t\big((c_1 + \alpha^*(c_2 - c_1))e^y - (1-\alpha^*)D_1 - \alpha^* D_2\big). \tag{3.25}$$

$$g_{0}{}_{n}^{m+1} = \left(\frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right) g_{0}{}_{n+1}^{m} - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right) g_{0}{}_{n}^{m} + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} g_{0}{}_{n-1}^{m} + \Delta t(c_1 e^{y} - D_1). \quad (3.26)$$

$$w'^{m+1}_{n} = \left(\frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right) w'^{m}_{n+1} - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right) w'^{m}_{n} + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} w'^{m}_{n-1} + \Delta t(c_1 e^{y} - D_1). \quad (3.27)$$

$$u_{n}^{m+1} = \left(\frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right) u_{n+1}^{m} - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right) u_{n}^{m} + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} u_{n-1}^{m} + \Delta t(c_1 e^{y} - D_1). \quad (3.28)$$

As the equations of $v_n^{m+1}$,$g_{0}{}_{n}^{m+1}$,$w'^{m+1}_{n}$,$u_n^{m+1}$, in terms of $v_n^m$, $g_{0}{}_{n}^{m}$, $w'^{m}_{n}$, $u_n^m$, respectively, are explicit, these processes can be solved by MATLAB. Thus, the numerical solution of *v*, $g_0$, $w'$, $u$ are found. According to equation (3.14), once $w'^{m}_{n}$, $g_{0}{}_{n}^{m}$ are obtained, the evolution $w_n^m = max\,\{w'^{m}_{n},\ g_{0}{}_{n}^{m}\}$ is found. Also, the free boundary presents $w'^{m}_{n} = g_{0}{}_{n}^{m}$ at a given point (*n,m*). Now, the optimal investment problem becomes clear: if the electricity price $X_s$ makes $g_{0}{}_{n}^{m} > w'^{m}_{n}$, $w_n^m$should be equal to $g_{0}{}_{n}^{m}$. In another words, the new green plant should be built. Both free boundary and relative gain ($p = (w_n^m - u_n^m)/u_n^m$) can be used to help the generators to make investment decisions.

**3.3.2. Case Study and Results.** To illustrate this technique, the parameters established in Table 3.1 are used in the case study. Some data about the electricity price come from the research of Tseng and Lin (2007), other values of the parameters come from literature reviews and author's experience. Most values in Table 3.1 are the same as values in previous model. The construction delay is assumed to be 1 year, which indicates the new power plant is assumed to be a small or medium size power plant.

Table 3.1. The List of Parameters for Delay Model

| Notation | Value | Unit |
|---|---|---|
| $\mu$ | 0.053 | |
| $\sigma$ | 0.13 | |
| $\lambda$ | 4.788 | |
| $\rho$ | 6% | |
| $\bar{\alpha}$ | 80% | |
| $c_1$ | 2 | MWh/unit monetary input |
| $c_2$ | 3 | MWh/ unit monetary input |
| $D_1$ | 100 | $\$/c_1$MWh |
| $D_2$ | 130 | $\$/c_2$MWh |
| $K$ | 500 | $/unit monetary input |
| $T$ | 10 | Year |
| $\delta$ | 1 | Year |

First of all, the mean price of electricity ($120/MWh) is derived from the parameters. Second, use the equation $w'^m_n = {g_0}^m_n$ to form the free boundary of $w$, which shows when the new plant should be built at a series of times in the planning horizon with respect to the electricity prices. Last, the relative gain line of portfolio investment at the beginning of the time horizon is formed, which is defined as: $p = (w_n^0 - u_n^0)/u_n^0$. The free boundaries of $w$, with a 1 year delay, and relative gain of the portfolio investment are shown in Figure 3.1 and Figure 3.2, respectively.

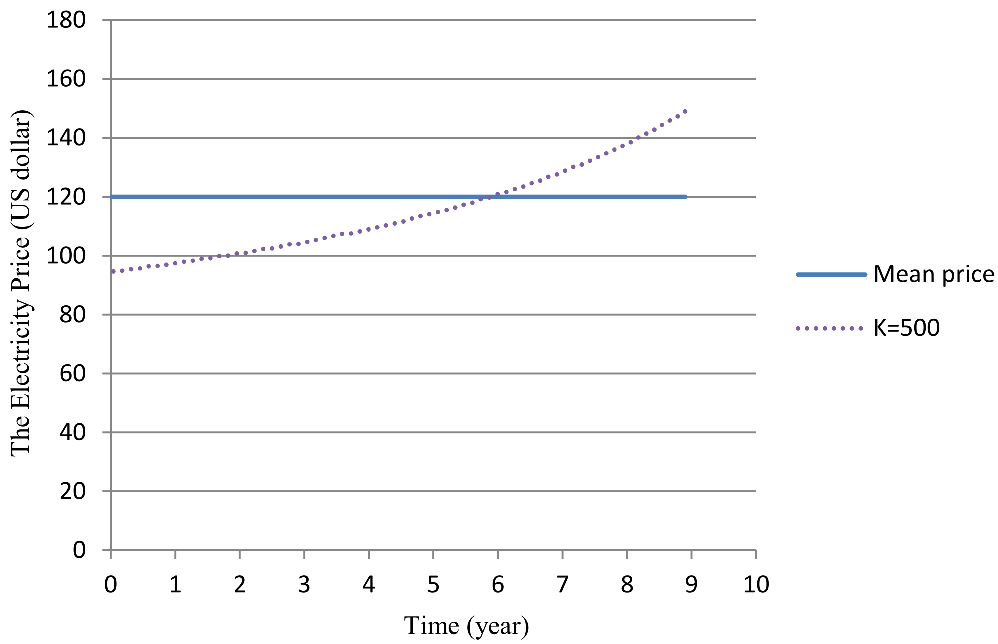


Figure 3.1. Free Boundary of *w* with Delay

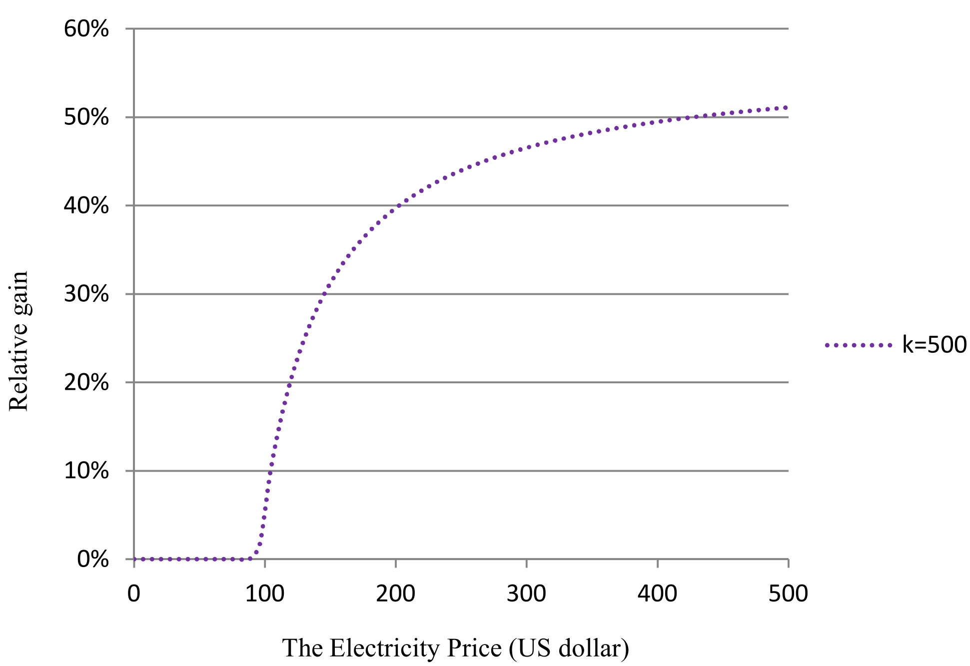


Figure 3.2. Relative Gain of Portfolio Investment with Delay

According to the calculations in case study, the free boundary is formed by the points (*x,t*), which satisfy the equation ${w'}_n^m = {g_0}_n^m$ ; and it's also found that $w_n^m = {g_0}_n^m$ when electricity prices are above the free boundary at a given time in Figure 3.1, and $w_n^m = {w'}_n^m$ when electricity prices are below the free boundary at a given time. Thus, the entry decisions with a construction delay become very clear according to the free boundary of $w$, which is similar to the previous model. The decision maker should invest the new power plant at a given year if the electricity price is jumping above the free boundary at that given time, since the company's profit with the energy portfolio is more that its profit without the energy portfolio. Otherwise, the firm should not invest the new power plant but just keep operating the old power plant.

Figure 3.1 also reveals that the free boundary function increases before the $9_{th}$ year, and then suddenly stops at the end of $9_{th}$ year because of the construction delay. So, the decision makers for the energy company need a higher price of electricity in order to make a decision about an investment in a new power plant as time goes by.

Figure 3.2 shows the relative gains of the portfolio at the beginning of the time. It is obvious that the relative gain increases with respect to the electricity price increase after \$94.5/MWh, which is the exercise price at time zero in Figure 3.1. The relative gains should be zero when the electricity price is below \$94.5/MWh, since no new power plant investment decision is made at that time. Figure 3.2 also reveals that the increase rate of relative gain line decrease.

**3.3.3. Sensitivity Analysis.** In order to reveal how the parameters affect the decisions, the following sensitivity analyses are conducted. Different combinations for production rates and operational costs are also studied in this section.

**3.3.3.1 Sensitivity analysis for investment amount.** Different investments $K$ derive different free boundaries, just as shown in Figure 3.3, and, the free boundaries move up when the investment requirement capital $K$ increases. This means that high electricity prices are required to trigger the new power plant investment decision if investment $K$ increases. In other words, the possibility to invest the new power plant becomes smaller based on the stochastic process of electricity price. On the other hand, the less a new power plant costs, the more possible it is that a new plant should be built.

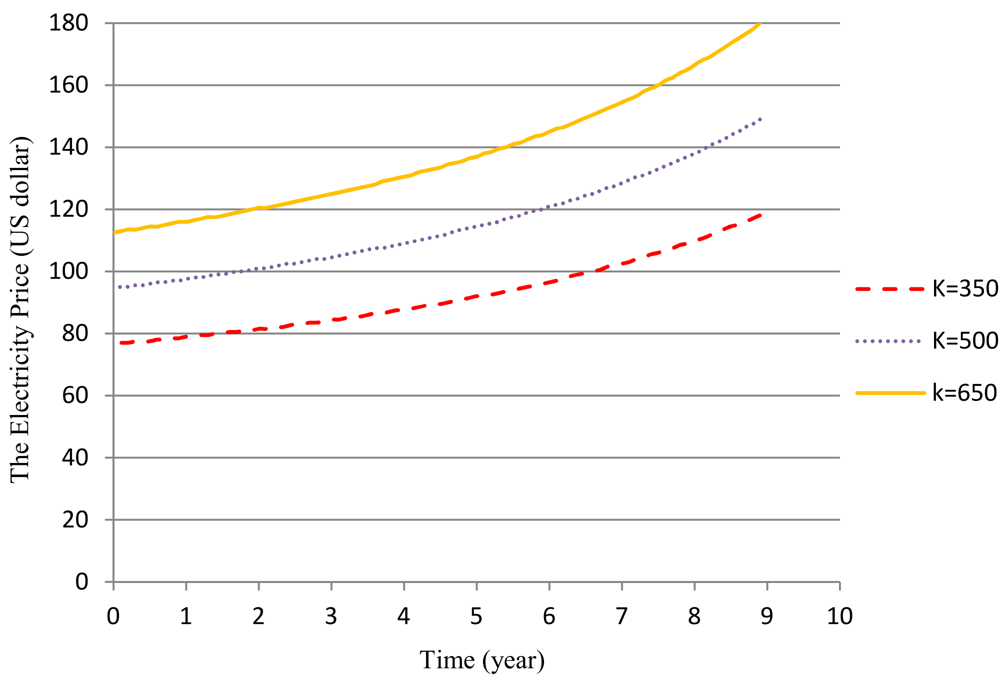


Figure 3.3. Free Boundary for Investment with Different $K$

This relative gain sensitivity analysis for $K$ in Figure 3.4 reveals that the smaller the investment amount is, then, the larger the relative gain will be.

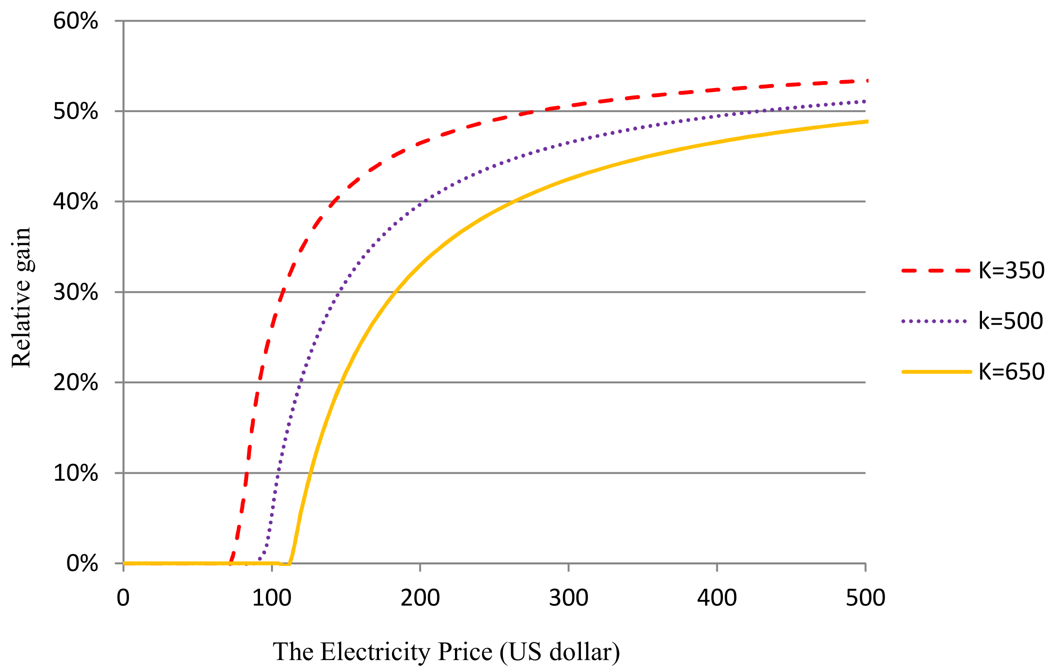


Figure 3.4. Relative Gain with Different $K$

**3.3.3.2 Sensitivity analysis for delay.** The sensitivity analysis for delay is shown in Figure 3.5 and Figure 3.6. If the construction delay time is changed from 1 year to 2 years, the free boundary of $w$ becomes lower and flatter, which means that a lower electricity price can trigger the new power plant investment decision. The reason for this is that this model assumes that the investment $K$ is paid at the end of the construction, which means more delay and less Net Present Value (NPV) of the investment. From Figure 3.3, it is clear that less investment means a lower free boundary.

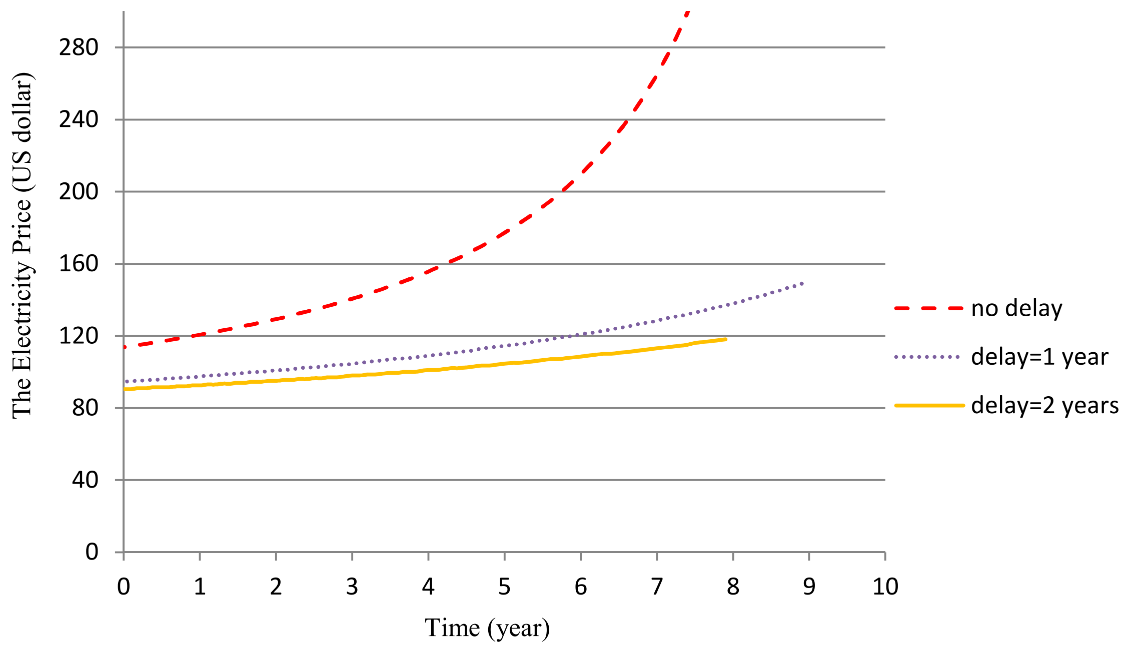


Figure 3.5. Free Boundary for Investment with Different Delays

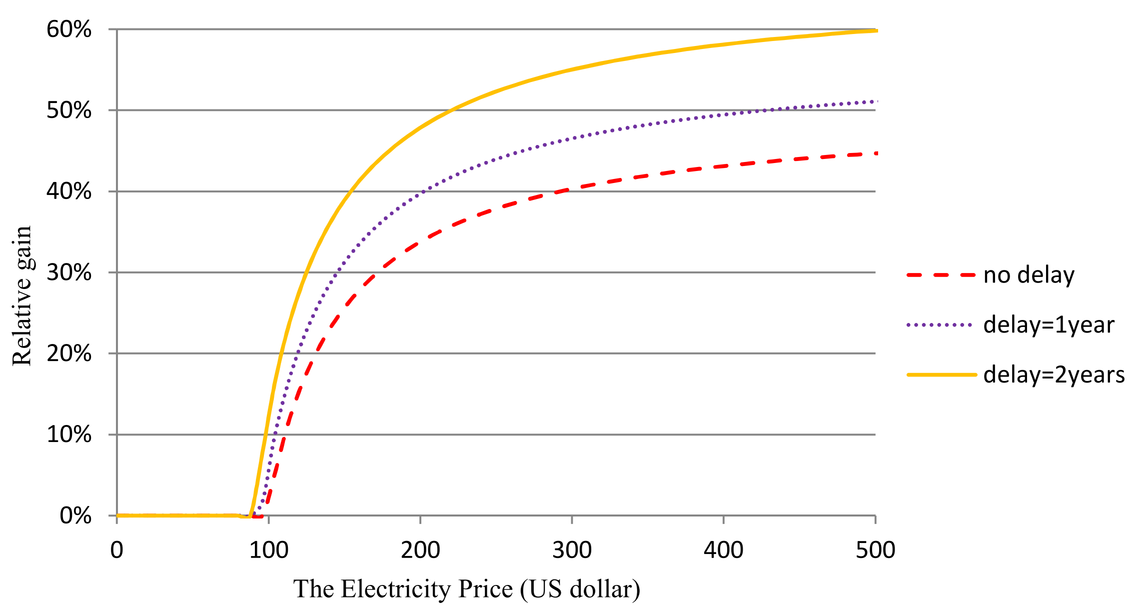


Figure 3.6. Relative Gain with Different Delays

Figure 3.6 shows that the relative gain line moves up when the construction delay increase from 1 year to 2 years. This result is consistent with sensitivity analysis of free boundary: more delay could increase the possibility to make the investment decision to build a new power plant.

**3.3.3.3 Sensitivity analysis for volatility.** From the following electricity price volatility analysis (Figure 3.7 and Figure 3.8), it can be seen that the uncertainty of electricity prices would also affect the free boundary and relative gain. More uncertainty would lower the free boundary and raise the relative gain curve. These results are also consistent with Bar-Ilan and Strange (1996). That is, the increase in uncertainty with investment construction delay would decrease the investment trigger price in a particular volatility range. Of course, an increase in electricity price uncertainty will not always lead to an earlier investment decision.

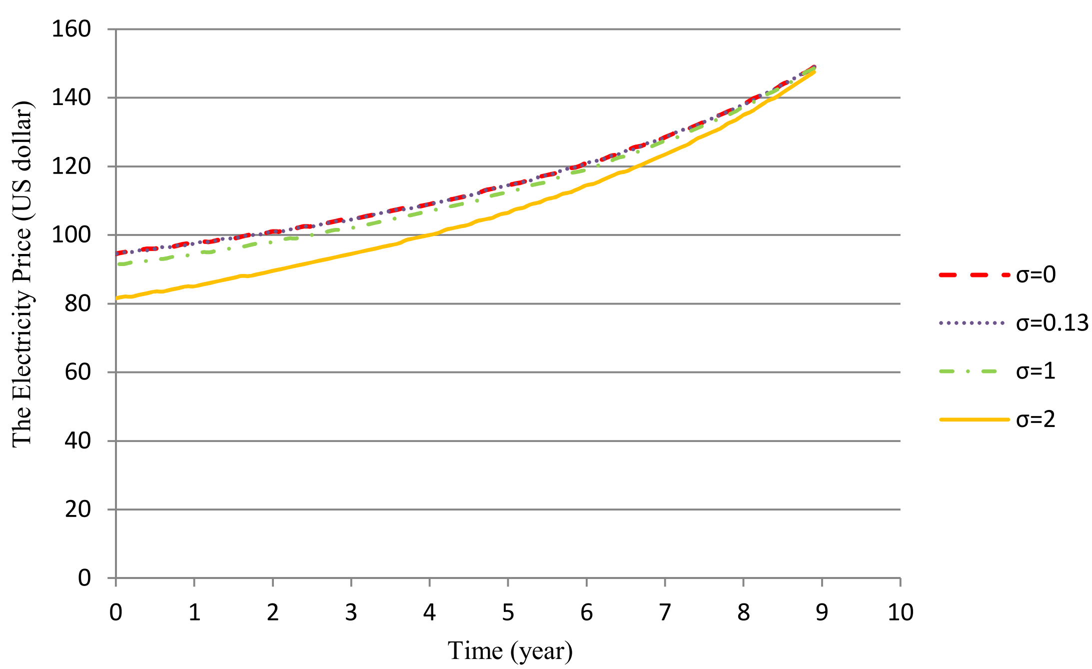


Figure 3.7. Free Boundary for Investment with Different $\sigma$

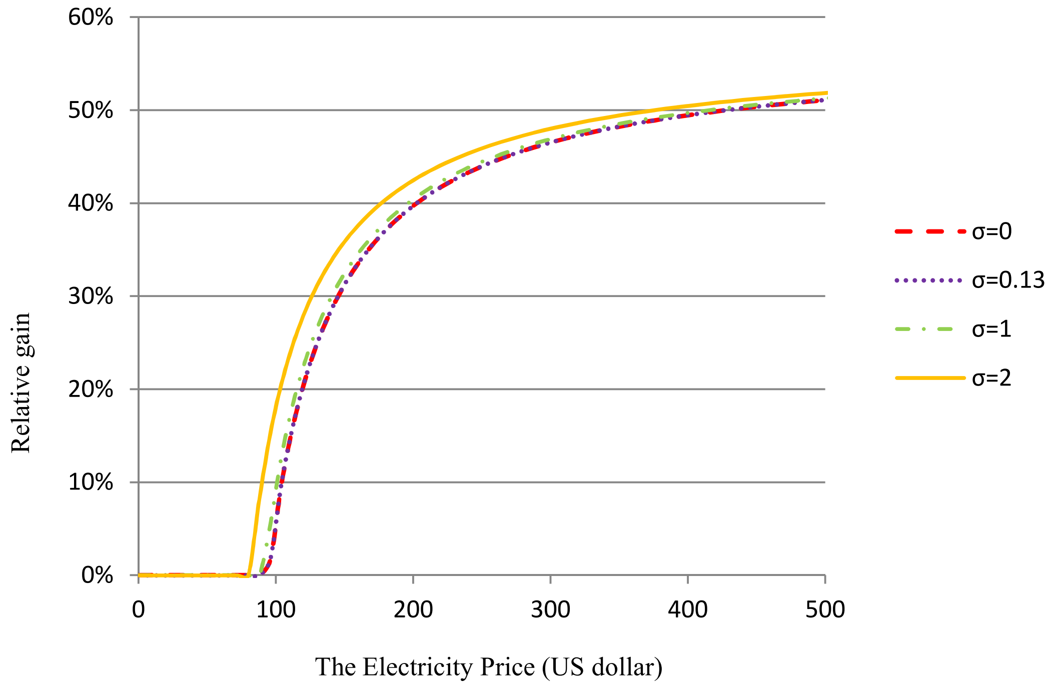


Figure 3.8. Relative Gain with Different $\sigma$

**3.3.3.4 Sensitivity analysis for $c_2$.** According to the sensitivity analysis for $c_2$ (Figure 3.9 and Figure 3.10), it's concluded that the production rate of alternative generating method $c_2$ affects the free boundary and relative gain line. The free boundary moves up when the production rate $c_2$ decreases, but, it moves down when the production rate $c_2$ increases. On the other hand, the relative gain line goes up when the production rate $c_2$ increases, and the relative gain line goes down when the production rate $c_2$ decreases.

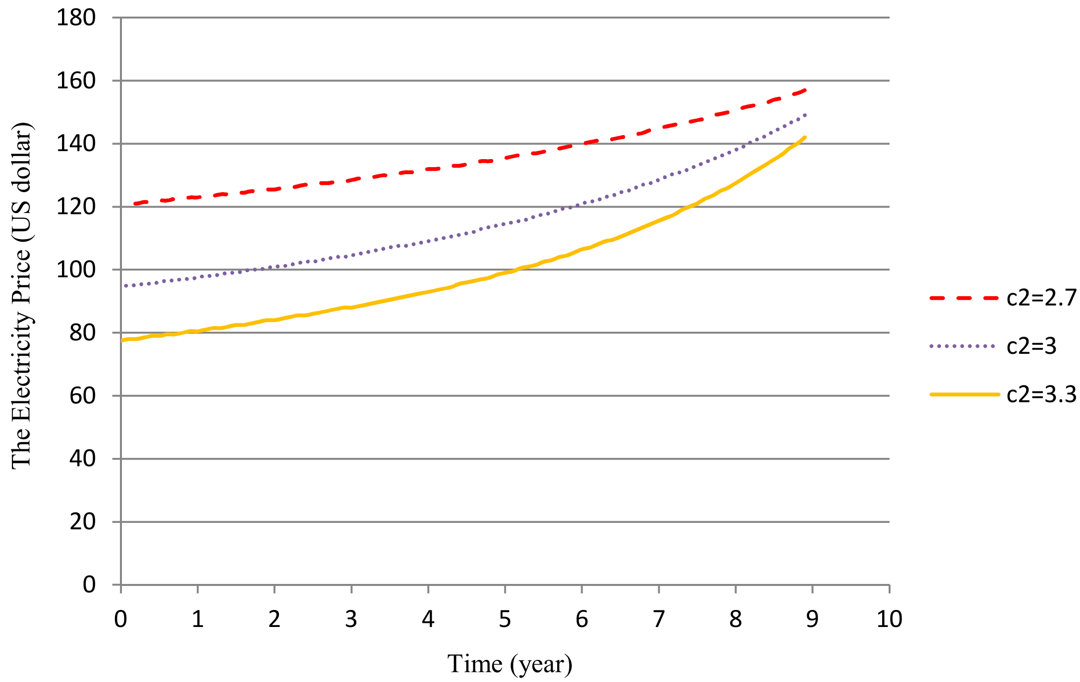


Figure 3.9. Free Boundary for Investment with Different $c_2$

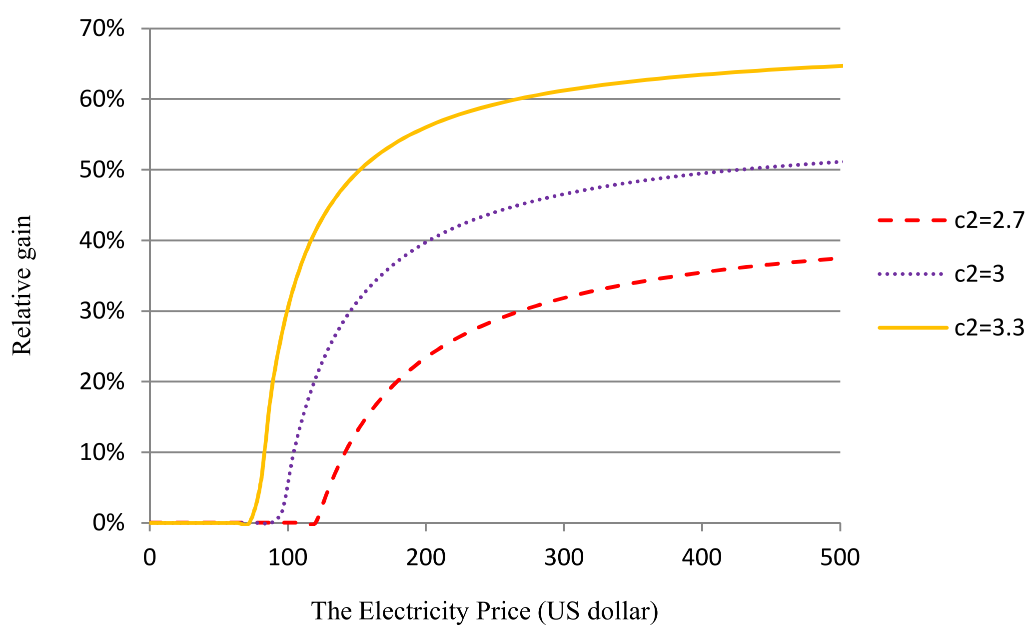


Figure 3.10. Relative Gain with Different $c_2$

**3.3.3.5 Sensitivity analysis for *$D_2$*.** Figure 3.11 and Figure 3.12 show that cost $D_2$ also affects the free boundary and relative gain line. When the cost $D_2$ increases, the free boundary moves up and relative gain line goes down; otherwise, when the cost $D_2$ decreases, the free boundary moves down and the relative gain line goes up. So, it can be concluded that decreasing the operation cost of the new power plant can increase the possibility to invest a new power plant at a given time under the uncertainty of price of electricity, which follows the GMR stochastic process.

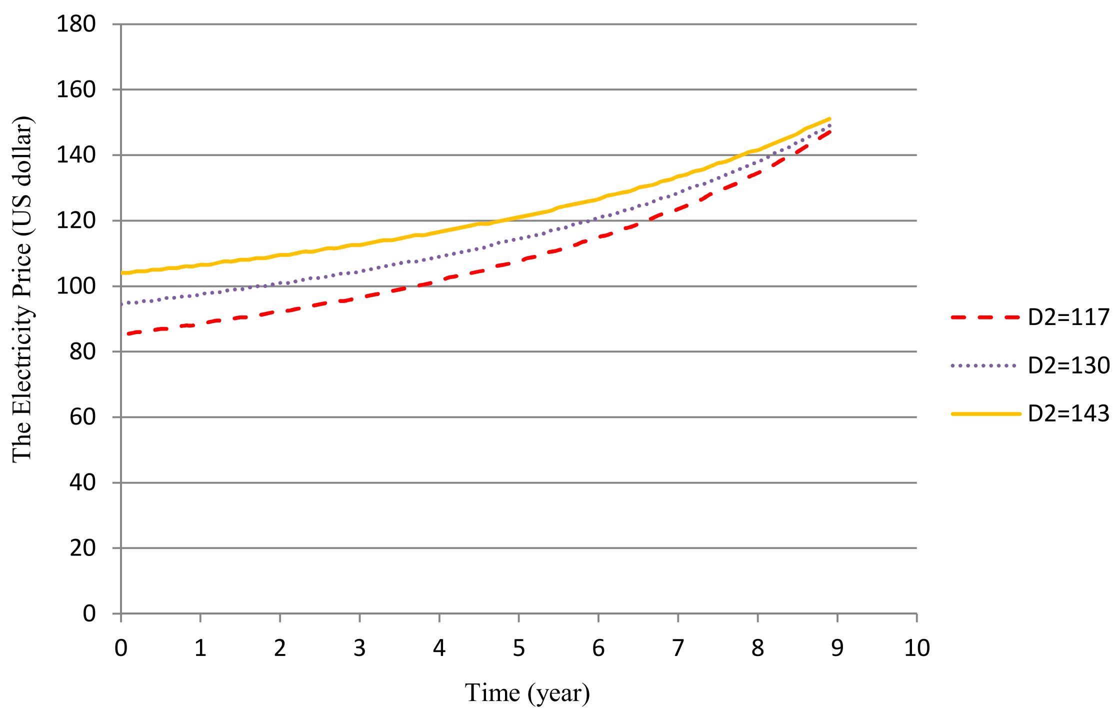


Figure 3.11. Free Boundary for Investment with Different $D_2$

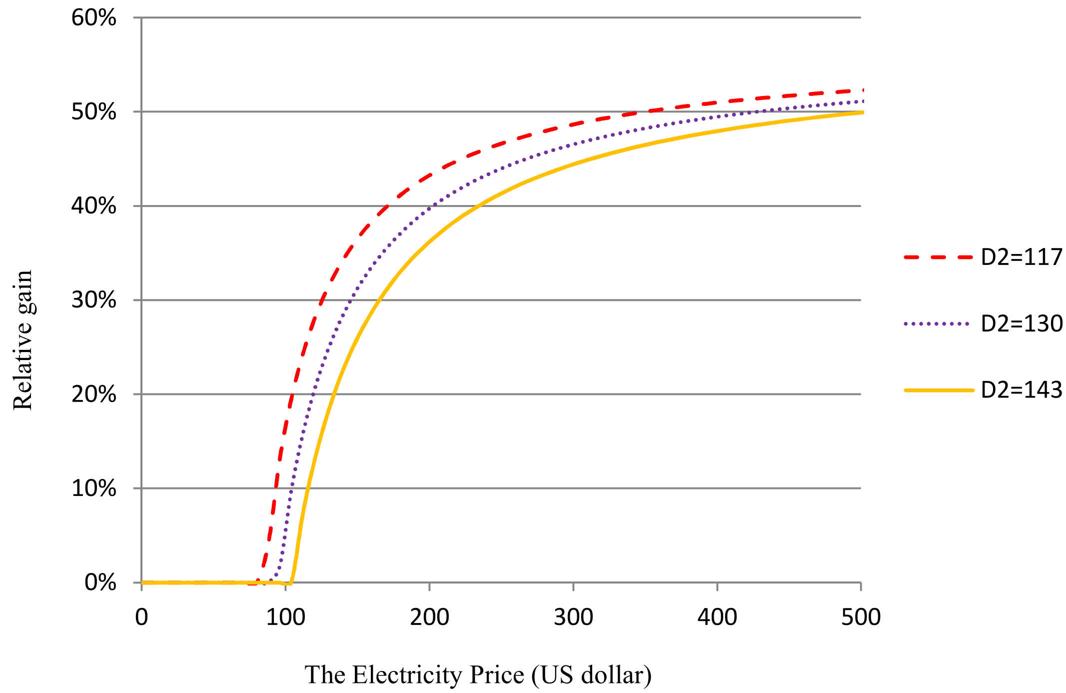


Figure 3.12. Relative Gain with Different $D_2$

Since the new power plant employs new technology to generate electricity, the cost could decrease over time. So, the effect to entry decisions if operation cost decreases must be studied. In this section, parameter $D_2$ is assumed to be a deterministic variable: $D_2 = 130 - 2t$ or $D_2 = 130 - 4t$. The free boundary and relative gain can be obtained as shown below in Figure 3.13 and Figure 3.14, which indicates show how the decreasing of cost over time affects entry decisions and relative gain:

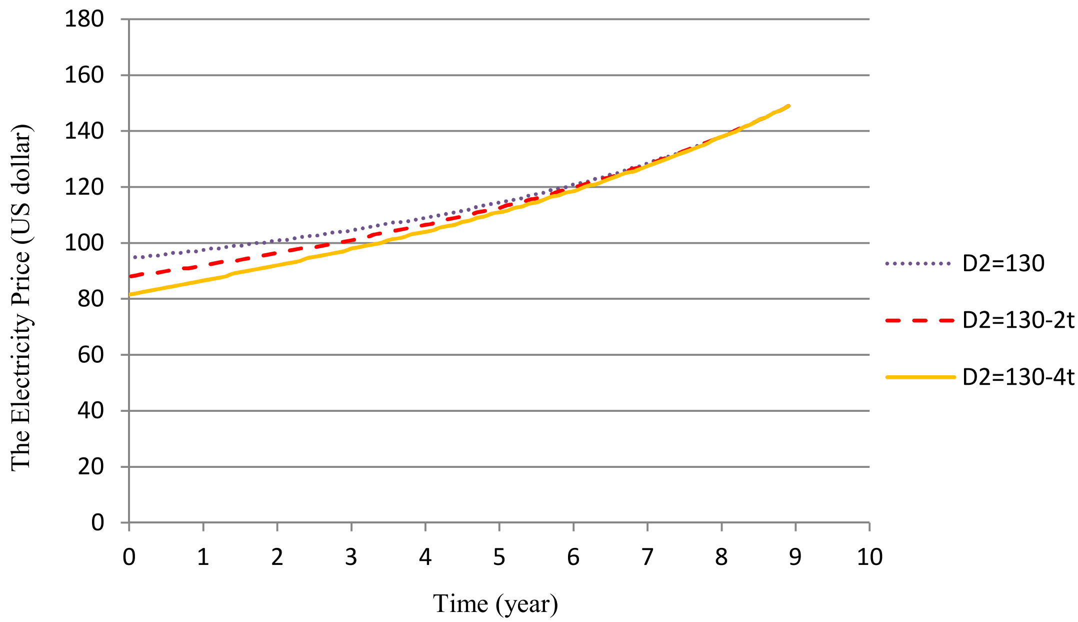


Figure 3.13. Free Boundary for Investment with Different $D_2(t)$

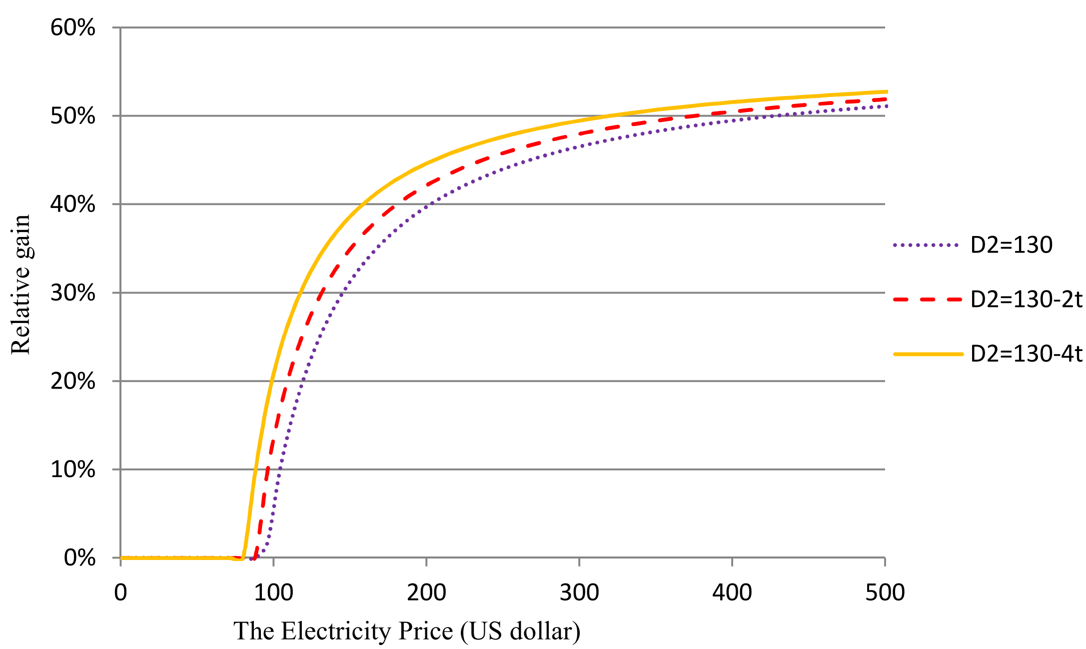


Figure 3.14. Relative Gain with Different $D_2(t)$

These results are intuitive and consistent with Figure 3.11 and Figure 3.12, respectively. The decreasing cost of the new power plant over time lowers the free boundary, on the other hand, the increases relative gain curve. This means that the expected cost decrease would trigger an early investment decision for a new power plant.

**3.3.3.6 Characteristics of production rate and cost.** The values of parameters $c_1, c_2, D_1, D_2$ in the case study section are just for the theoretic model; they are not real data of power plants. This section will discuss all cases with different *c&D* combinations and more realistic data for the production rate and cost. Here, the existing power plant is assumed to be a coal power plant and the new power plant is a natural gas power plant, in order to learn how different combinations of $c_1, c_2, D_1, D_2$ would affect the free boundary and relative gain. First, it is assumed that unit monetary input is \$100; the coal price is either \$28/MWh or \$35/MWh, as a result of the definitions of parameters, $\mathrm{c}_1$ would be either 3.571 or 2.857; natural gas price is \$29/MWh, $\mathrm{c}_2$ would be 3.448; the total cost of the coal power plant is assumed to be \$60/MWh, so, $D_1$ is either \$214.286/MWh or \$171.429/MWh, depending on different production rate. The total costs of the natural gas power plant are assumed to be \$66/MWh, \$59/MWh, \$52/MWh, or \$46/MWh, so, $D_2$ would be \$227.586/MWh, \$203.448/MWh, \$179.310/MWh, or \$158.621/MWh, depending on different total costs of the natural gas power plant (Bloomberg, 2012). So, four cases with different combinations of *c* and *D* are displayed in Table 3.2 as well as the free boundaries and relative gain curves shown in Figure 3.15 and Figure 3.16:

Table 3.2. Different Combinations of *c* and *D*

| Different Combinations | Case 1 $c_1 > c_2$ $D_1 < D_2$ | Case 2 $c_1 > c_2$ $D_1 > D_2$ | Case 3 $c_1 < c_2$ $D_1 < D_2$ | Case 4 $c_1 < c_2$ $D_1 > D_2$ |
|---|---|---|---|---|
| Coal price($/MWh) | 28 | 28 | 35 | 35 |
| Natural gas price($/MWh) | 29 | 29 | 29 | 29 |
| Total cost of coal power plant ($/MWh) | 60 | 60 | 60 | 60 |
| Total cost of natural gas power plant ($/MWh) | 66 | 59 | 52 | 46 |
| $c_1$ | 3.571 | 3.571 | 2.857 | 2.857 |
| $c_2$ | 3.448 | 3.448 | 3.448 | 3.448 |
| $D_1$ | 214.286 | 214.286 | 171.429 | 171.429 |
| $D_2$ | 227.586 | 203.448 | 179.310 | 158.621 |

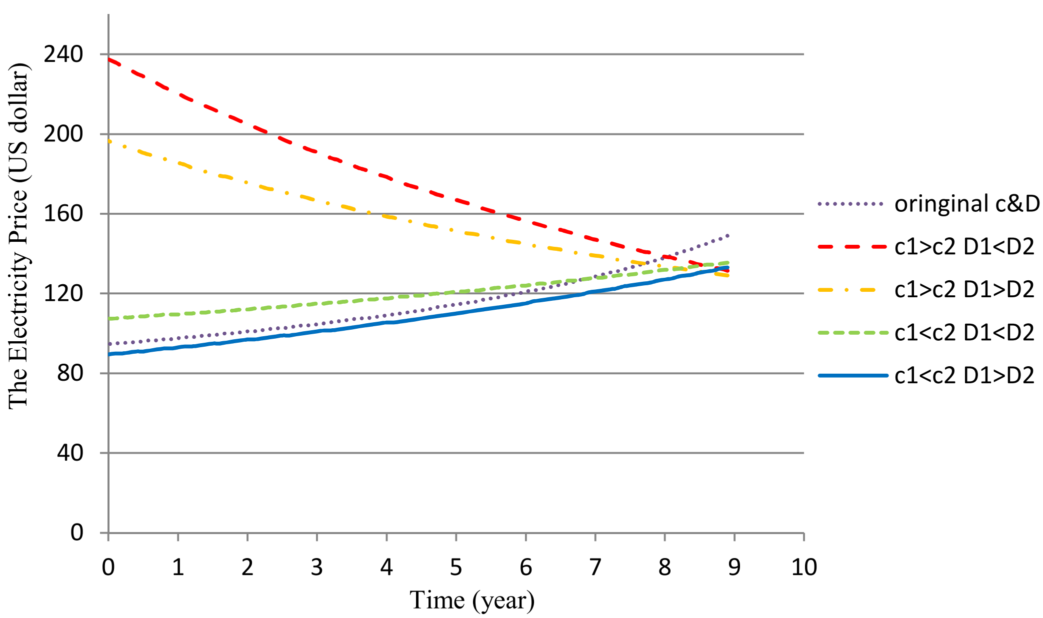


Figure 3.15. Free Boundary for Investment with Different *c&D*

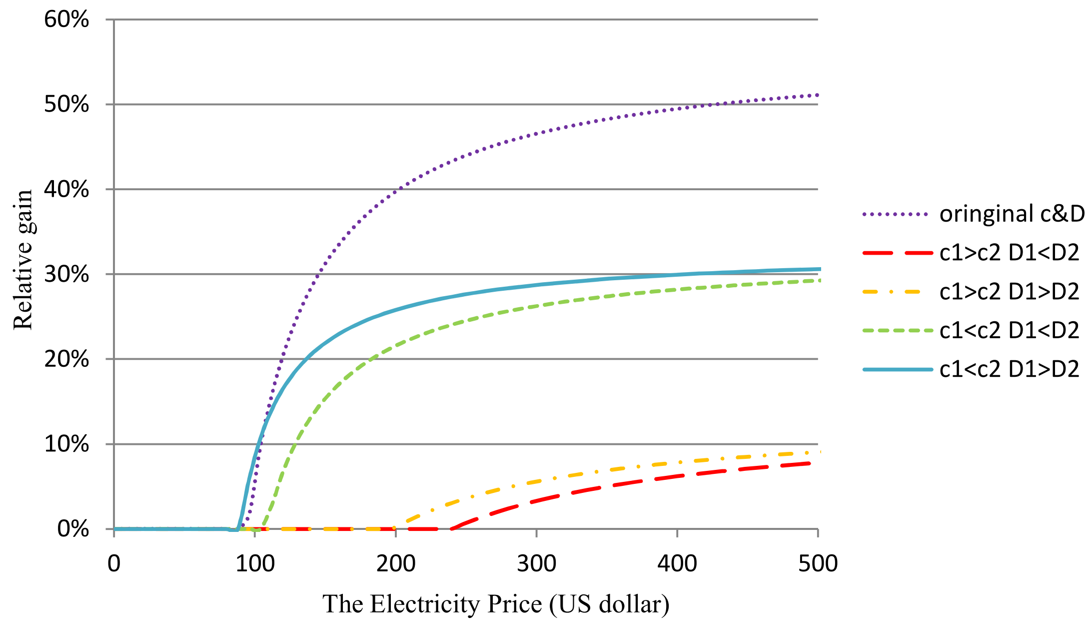


Figure 3.16. Relative Gain with Different *c&D*

Figure 3.15 reveals that the free boundary moves down when the *c&D* combinations are changed from case 1 to case 4. Case 1 represents the new technology which has a lower production rate but a higher cost; in reality, generators will not accept this kind of new technology to generate electricity. Case 2 shows that the new technology has a lower production rate as well as a lower cost, as a result, this new technology is not advantageous enough to be adopted. Case 3 give a normal example in reality, which indicates that the new technology has a higher production rate as well as a higher cost; some combinations in case 3 can be considered as a "good" investment by the investor if its free boundary is an increasing line before year 9. Our original parameters for the *c&D* belong to case 3. Case 4 is a perfect case for an investor: the new technology has a higher production rate but a lower cost. All combinations in case 4 would have the results of increasing free boundaries, which indicates “good” investment and should be invested as soon as possible when the electricity price jumps above the free boundary. Finally, it is concluded that a lower free boundary means “better” investment potential, which is also shown in Figure 3.16 in terms of relative gain.

### 3.4. CONCLUSIONS

In this section, studies are made of the optimal entry decisions for a new plant, given a fixed capital investment with a construction delay, and the optimal dispatch decisions between the existing power plant and a new power plant. The objective function is to maximize the long-term profit under the geometric mean reversion process for electricity price. Optimal control and finite difference methods are used to solve the entry decision problem in the energy portfolio investment.

The results provide some valuable data for the generators to make decisions. The free boundary can be employed to decide when the generator should build a new plant. It is determined that the new power plant should be built when the electricity price is above the free boundary at a particular time. After the new plant is built, the new plant should get the maximum proportion of monetary input ($\bar{\alpha}$) when the electricity price is higher than $\frac{D_2-D_1}{c_2-c_1}$, otherwise, it would be kept idle.

The relative gain of the portfolio investment at the beginning of the finite planning horizon is increased as the price of electricity; but, the increase rate decreases when the electricity price increases. The sensitivity analysis shows that many parameters affect the free boundary as well as the optimal entry time decisions.

Sensitivity analysis gives the decision-makers in energy industry more information about investment entry decisions. A less investment requirement capital or operation cost can increase the possibility to make the investment decision at a given time; on the other hand, a greater construction delay, volatility, or production rate can result less possible to make entry decision. Free boundary also can be used to value power plant investment: lower free boundary indicates better investment.

# 4. OPTIMAL ABANDONMENT DECISION MODEL

## 4.1. MOTIVATIONS

This section models the optimal time to abandon an old power plant of a firm, which has a portfolio of new power plant and an old traditional power plant, with the objective to maximize the long-term expected profit. Assume that the firm owns a power plant portfolio, and considers shutting down of the old traditional power plant while maximizing the expected long-term profit. Under the Mean Reversion Stochastic process of electricity prices, the decision problem is formulated as a mixed stochastic control problem. Due to the intractability of the mixed problem, it is decomposed into two auxiliary problems: one is a regular stochastic control problem, and the other one is an optimal stopping problem.

The rest of this section is organized as follows. Section 4.2 describes a mathematical model of the problem. Section 4.3 employs finite difference methods to solve the partial differential equations (PDEs) using forward and backward difference approximations. To ensure a stable and accurate solution, explicit finite difference methods with a proper step size are used to calculate the PDEs.

## 4.2. MODELS AND FORMULATIONS

Assume that an energy firm is operating two power plants as an energy portfolio, and that the firm's decision makers are considering shutting down one of the power plants within a certain period due to the lifetime of the plant, or the tight regulation from government. Thus, the optimal operation policies of the energy portfolio and the optimal time for abandoning one of the power plants have to be determined. This problem is formulated as a mixed stochastic control and optimal stopping time problem to maximize the long-term profit of the company. In this section, $\beta$ represents the proportion of total monetary input in the old power plant; $M$ is the liquidation value of the old power plant, which includes government subsidies and salvage values. Other notations are the same notations used in Section 2.

Many factors can affect the decisions of abandonment time and operation. To simplify the models, only the price of the electricity price is considered as a stochastic process. In this section, electricity prices are also modeled as a mean reversion stochastic process.

Assuming that the decision to shut down the old plant is made at time τ when the power plant can gain a liquidation value of $M$ dollars.

Thus, the long-term profit functional is:

$$J(t,x;\tau,\alpha) \equiv E_x[\int_t^{\tau} (\beta(c_1X_s - D_1)+(1-\beta)(c_2X_s - D_2))e^{-\rho(s-t)}ds +$$

$$Me^{-\rho(\tau-t)} + \int_{\tau}^{T}(c_2X_s - D_2)e^{-\rho(s-t)}ds]. \tag{4.1}$$

All values are discounted to the present value at time $t$. Here, $\tau$ and $\beta$ are control variables.

Function $u_0(t,x)$ represents optimal value of the objective at given $(t,x)$. So, the value function $u_0(t,x)$ is defined as:

$$(\mathrm{P}_0)u_0(t,x) = \sup_{\tau\in\Gamma_{t,T},\beta\in[0,\bar{\beta}]} J(t,x;\tau,\beta). \tag{4.2}$$

That is

$$u_0(t,x) = \sup_{\tau\in\Gamma_{t,T},\beta\in[0,\bar{\beta}]} E_x[\int_t^{\tau} \beta(c_1X_s - D_1)+(1-\beta)(c_2X_s - D_2)e^{-\rho(s-t)}ds +$$

$$Me^{-\rho(\tau-t)} + \int_{\tau}^{T}(c_2X_s - D_2)e^{-\rho(s-t)}ds] \tag{4.3}$$

where $\Gamma_{t,T,}$ denotes the set of stopping times in $[t,T]$.

Due to the intractability of this mixed stochastic control and optimal stopping time problem, the following auxiliary function is introduced.

**4.2.1. Auxiliary Function.** Suppose the old power plant has already been shut down at time $\tau$. Therefore, there is only one new power plant in operation. The value function $v(t,x)$ is defined as:

$$v(t,x) \equiv E_x[\int_t^T(c_2X_s - D_2)e^{-\rho(s-t)}ds]. \tag{4.4}$$

According to the dynamic programming, the following H-J-B equation is obtained:

$$\frac{\partial}{\partial t}v + (c_2X_s - D_2) + L_v - \rho v = 0 \tag{4.5}$$

where

$$L_v \equiv (\mu(\lambda - lnX_t) + \frac{1}{2}\sigma^2)x\frac{\partial v}{\partial x} + \frac{1}{2}\sigma^2x^2\frac{\partial^2 v}{\partial_{x^2}}, \tag{4.6}$$

with the terminal condition:

$$v(x,T) = 0\ . \tag{4.7}$$

From equation (4.5), the maximal discounted values for different $t$ and $x$ after shutting down the old power plant can be obtained, and these results can be used in the next section.

**4.2.2. Optimal Time to Abandon the Old Power Plant.** According to previous calculations, the value of the operation is given by $v(\tau,x_\tau)$ when the old traditional power plant has been shut down; therefore, the value function $w(t,x)$ is defined as follows:

$$w(t,x) \equiv \sup_{\tau\in\Gamma_{t,T},\beta\in[0,\overline{\beta}]}\mathrm{E_x}[\int_t^\tau(\beta(c_1X_s - D_1) + (1-\beta)(c_2X_s - D_2))e^{-\rho(s-t)}ds +$$

$$(v(\tau,X_\tau) + M)e^{-\rho(\tau-t)}],\ t \in [0,T)\ . \tag{4.8}$$

The solution to this optimal stopping time problem *w* satisfies the following linear complementarity problem:

$$\begin{cases} \frac{\partial}{\partial t} w + L_w + \beta(c_1 X_s - D_1) + (1-\beta)(c_2 X_s - D_2) - \rho w \leq 0 \\ w \geq v + M \\ (\frac{\partial}{\partial t} w + L_w + \beta(c_1 X_s - D_1) + (1-\beta)(c_2 X_s - D_2) - \rho w)\ (w - v - M) = 0 \\ L_w = \ \mu \frac{\partial w}{\partial t} x + \frac{1}{2}\sigma^2 \frac{\partial^2 w}{\partial x^2} x^2 \end{cases} . \quad (4.9)$$

That is,

$$Min\left(-\frac{\partial}{\partial t} w - L_w - \beta(c_1 X_s - D_1) - (1-\beta)(\mathrm{c_2 X_s - D_2}) + \rho w, w - v - M\right) = 0$$

(4.10)

with the terminal condition:

$$w(x, T) = 0. \quad (4.11)$$

It is obvious that the optimal proportion $\beta^*$ is:

$$\beta^*(t, x) = \begin{cases} 0 & , if\ (x < \frac{D_1 - D_2}{c_1 - c_2}) \\ \bar{\beta} & , if\ (x \geq \frac{D_1 - D_2}{c_1 - c_2}) \end{cases}. \quad (4.12)$$

So, equation (4.10) is transformed to:

$$Min\left(-\frac{\partial}{\partial t} w - L_w - \beta^*(c_1 X_s - D_1) + (1-\beta^*)(\mathrm{c_2 X_s - D_2}) + \rho w, w - v - M\right) = 0.$$

(4.13)

Following the same procedures, the original problem is decomposed into a stochastic control problem (4.12), and an optimal stopping time problem, which can be solved by the PDE equation (4.13).

## 4.3. NUMERICAL SOLUTION AND RESULTS

**4.3.1. Numerical Solution.** Finite difference methods are used to solve these PDE equations. The following are the solution results of the problems by using finite difference methods.

Consider the H-J-B Equation (4.5):

$$\frac{\partial}{\partial t}v + (c_2X_s - D_2) + L_v - \rho v = 0$$

$$L_v = \mu' \frac{\partial v}{\partial x}x + \frac{1}{2}\sigma^2 \frac{\partial^2 v}{\partial x^2}x^2 \tag{4.14}$$

$$\mu' = \mu(\lambda - lnX_t) + \frac{1}{2}\sigma^2. \tag{4.15}$$

To calculate the equation, the terminal condition needs to be changed to initial condition by considering $t' = T - t$. But this model still uses $t$ to represent $t'$ for convenience. So, the following PDE is obtained:

$$-\frac{\partial}{\partial t}v + (c_2X_s - D_2) + L_v - \rho v = 0 \tag{4.16}$$

the boundary conditions:

$$v(X_{min}, t) = -\frac{D_1}{\rho}(1 - e^{-\rho t}) \qquad (X_{min} = 0) \tag{4.17}$$

$$v(X_{max}, t) = \frac{c_2 * X_{max} - D_2}{\rho}(1 - e^{-\rho t}) \ \ (\tau = t, X_{max} \text{ is a big number}) \tag{4.18}$$

the initial condition:

$$v(x, T) = 0. \tag{4.19}$$

To overcome the artificial oscillations, an upwind scheme is used to represent $\frac{\partial v}{\partial x}$.

The formula for $v_n^{m+1}$:

$$v_n^{m+1} = \frac{\Delta t}{(\Delta x)^2}\frac{\sigma^2}{2}x^2 v_{n+1}^m - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta x}x\mu' + \frac{\Delta t}{(\Delta x)^2}\sigma^2 x^2\right)v_n^m + \left(\frac{\Delta t}{(\Delta x)^2}\frac{\sigma^2}{2}x^2 - x\mu'\frac{\Delta t}{\Delta x}\right)v_{n-1}^m + \Delta t(c_2 x - D_2). \tag{4.20}$$

In order to improve the calculation efficiency, set $Y = lnX$,

$$v_n^{m+1} = \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} v_{n+1}^m - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right) v_n^m +$$

$$\left(\frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu'\frac{\Delta t}{\Delta y}\right) v_{n-1}^m + \Delta t(c_2 e^y - \mathrm{D}_2). \tag{4.21}$$

Consider the H-J-B Equation (4.13)

$$Min\left(-\frac{\partial}{\partial t} w - L_w - \beta^*(c_1 X_s - D_1) - (1-\beta^*)(\mathrm{c_2 X_s - D_2}) + \rho w, w - v - M\right) = 0.$$

First step is considering the first part of (4.13) by introducing w′:

$$\frac{\partial}{\partial t}\mathrm{w}' + L_{\mathrm{w}'} + \beta^*(c_1 X_s - D_1) + (1-\beta^*)(\mathrm{c_2 X_s - D_2}) - \rho \mathrm{w}' = 0. \tag{4.22}$$

$$L_{w'} = \mu\frac{\partial w'}{\partial x}x + \frac{1}{2}\sigma^2\frac{\partial^2 \mathrm{w}'}{\partial_{x^2}}x^2 \ . \tag{4.23}$$

The following PDE is obtained by changing the terminal condition to initial condition.

$$-\frac{\partial}{\partial t}\mathrm{w}' + L_{\mathrm{w}'} + \beta^*(c_1 X_s - D_1) + (1-\beta^*)(\mathrm{c_2 X_s - D_2}) - \rho \mathrm{w}' = 0 \tag{4.24}$$

with the boundary conditions:

$$\mathrm{w}'(X_{min}, t) = -\frac{D_2 - \beta^*(D_2 - D_1)}{\rho}(1 - e^{-\rho t}) \qquad (X_{min} = 0) \tag{4.25}$$

$$\mathrm{w}'(X_{max}, t) = \frac{(c_2 + \beta^*(c_1 - c_2)) * X_{max} - (1-\beta^*)D_2 - \beta^* D_1}{\rho}(1 - e^{-\rho t}) \quad (X_{max}\text{is a big number})$$

(4.26)

with the initial condition:

$$w'(x, T) = 0. \tag{4.27}$$

The formula for ${w'}_n^{m+1}$ is obtained using an upwind scheme:

$${w'}_n^{m+1} = \left(\frac{\Delta t}{\Delta x}x\mu + \frac{\Delta t}{(\Delta x)^2}\frac{\sigma^2}{2}x^2\right){w'}_{n+1}^m - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta x}x\mu + \frac{\Delta t}{(\Delta x)^2}\sigma^2 x^2\right){w'}_n^m +$$

$$\frac{\Delta t}{(\Delta x)^2}\frac{\sigma^2}{2}x^2 {w'}_{n-1}^m + \Delta t((c_1 + \beta^*(c_2 - c_1))x - (1-\beta^*)D_1 - \beta^* D_2). \tag{4.28}$$

In order to improve the calculation efficiency, set $Y = lnX$, equations (4.28) is transferred as following:

$$w'^{m+1}_{n} = \left(\frac{\Delta t}{\Delta y}\mu + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right) w'^{m}_{n+1} - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right) w'^{m}_{n} + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} w'^{m}_{n-1} +$$

$$\Delta t((c_1 + \beta^*(c_2 - c_1))e^y - (1 - \beta^*)D_1 - \beta^* D_2) . \quad (4.29)$$

As the equation for $v_n^{m+1}, w'^{m+1}_{n}$, in terms of $v_n^m, w'^{m}_{n}$, are explicit, these processes can be solved by MATLAB. Thus, the numerical solutions of *v* and *w* are found. Thus, the numerical solution of $w{=}max\{w',\, v + M\}$ is found.

According to equation (4.13), once $w'^{m}_{n}$ are obtained, the evolution for $w_n^m = max\,\{w'^{m}_{n}, v_n^m + M\}$ can be obtained. Also define the free boundary present $w'^{m}_{n} = v_n^m + M$ at a given point $(n, m)$. Now, the optimal abandonment problem becomes clear: when the electricity price $X_s$ makes $v_n^m + M > w'^{m}_{n}$, $w_n^m$should equal to $v_n^m + M$. In other words, the old power plant should be shut down. The free boundary can be used to help the decision makers in the energy companies to make abandonment decisions.

**4.3.2. Case Study and Results.** To illustrate this technique for abandonment decision model, the parameters established in Table 4.1 are used in the case study section. The parameters about the price of electricity are the same the parameters used in the previous models. Other values of parameters come from literature reviews and experience on energy industry.

Table 4.1. The List of Parameters for Abandonment Model

| Notation | Value | Unit |
|---|---|---|
| $\mu$ | 0.053 | |
| $\sigma$ | 0.13 | |
| $\lambda$ | 4.788 | |
| $\rho$ | 6% | |
| $\bar{\beta}$ | 50% | |
| $c_1$ | 3 | MWh/unit monetary input |
| $c_2$ | 2 | MWh/ unit monetary input |
| $D_1$ | 130 | $/c_1$MWh |
| $D_2$ | 100 | $/c_2$MWh |
| $M$ | 150 | $/unit monetary input |
| $T$ | 10 | Year |

The equation $w'^m_n = v^m_n + M$ can be used to form the free boundary in abandonment model, which shows when the old power plant should be shut down at a given time in the planning horizon, with respect to the price of electricity; the free boundary of $w$ is shown in Figure 4.1.

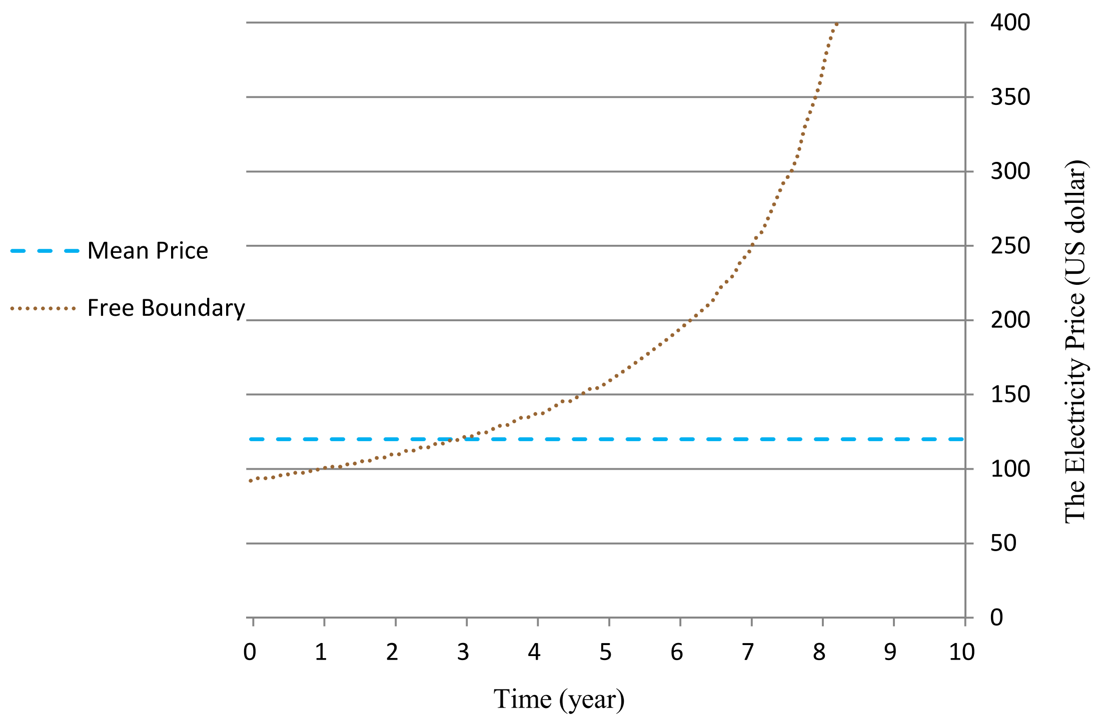


Figure 4.1. Free Boundary for Abandonment

According to the case study, $w_n^m = v_n^m + M$ when electricity prices are below the free boundary as in Figure 4.1, and $w_n^m = {w'}_n^m$ when electricity prices are above the free boundary. Thus, the abandonment decisions become very clear according to the free boundary of $w$. The decision maker should shut down the old power plant when the electricity price is below the free boundary at that given time, since the company's long-term profit, without an energy portfolio plus liquidation value, is more than the profit with an energy portfolio. Otherwise, the old power plant should not be shut down, but should keep operating the energy portfolio to await better timing. Figure 4.2 shows how to use free boundary to make abandonment decisions for the generator.

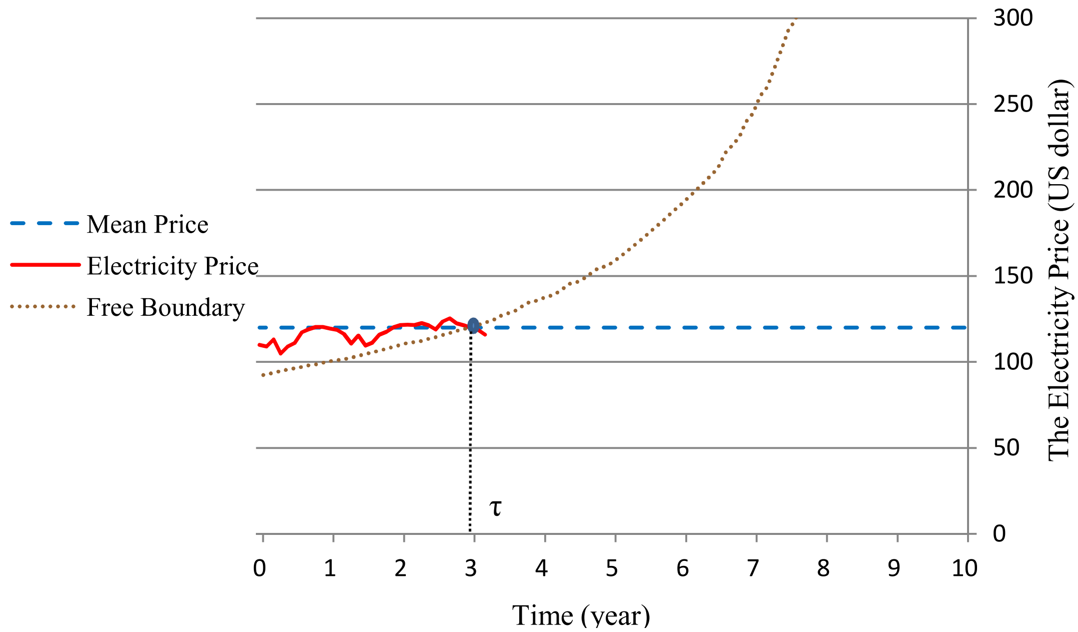


Figure 4.2. Abandonment Decision by Using Free Boundary

**4.3.3. Sensitivity Analysis.** Sensitivity analysis of free boundary for liquidation value, volatility, production rate and operation cost of the old power plant are conducted as following:

**4.3.3.1 Sensitivity analysis for liquidation value.** Different liquidation values $M$ derive different free boundaries, as Figure 4.3 shows, and the free boundaries move up when the liquidation value $M$ increases. This means that the abandonment decision (shutting down the old power plant) is easier (easier to drop below the free boundary), or sooner when the liquidation value $M$ increases.

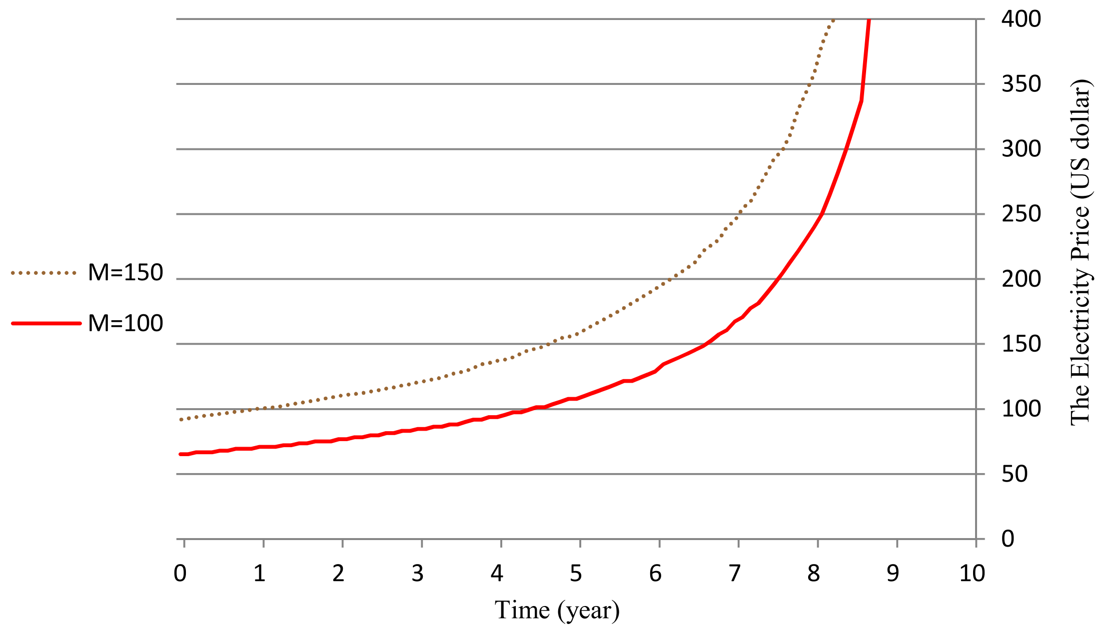


Figure 4.3. Free Boundary for Abandonment with Different *M*

**4.3.3.2 Sensitivity analysis for volatility.** Figure 4.4 shows that the changing of volatility $\sigma$ also affects the decisions (free boundaries). The free boundary moves down when volatility σ increases, which reveals that the uncertainty of future electricity prices can increase the possibility of keeping the old power plant to avoid an abandonment decision. These results reveal that a portfolio is a better way to deal with the higher uncertainty of the market.

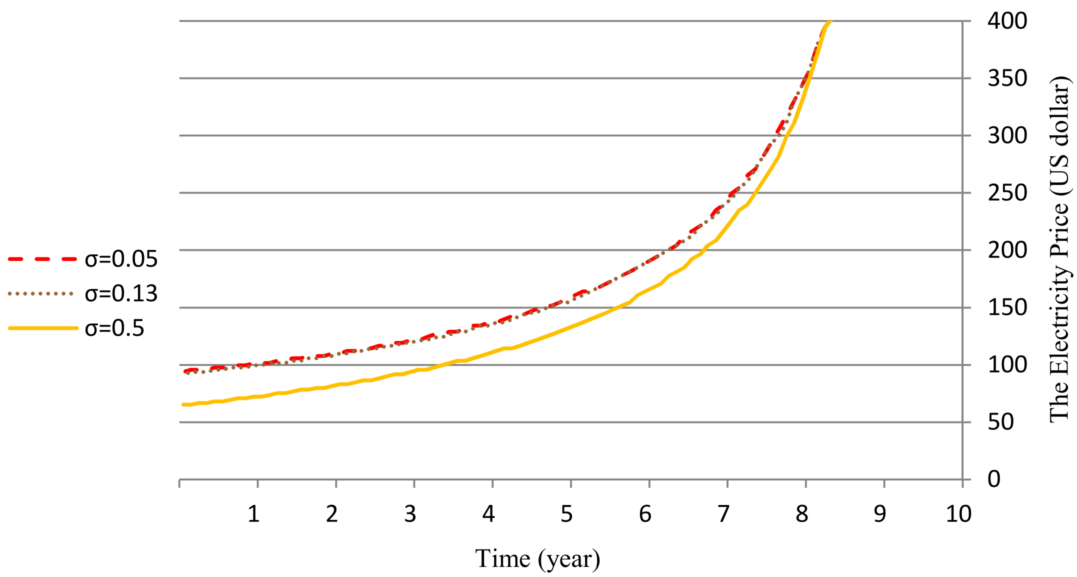


Figure 4.4. Free Boundary for Abandonment with Different $\sigma$

**4.3.3.3 Sensitivity analysis for $c_1$.** According to the sensitivity analysis for $c_1$ in Figure 4.5, the production rate of the old power plant $c_1$ affects the free boundary. The free boundary moves down when the production rate $c_1$ increases, which means that the electricity price needs to drop down to a lower level to excise the abandonment option. In other words, it is expected that the old power plant can be kept longer.

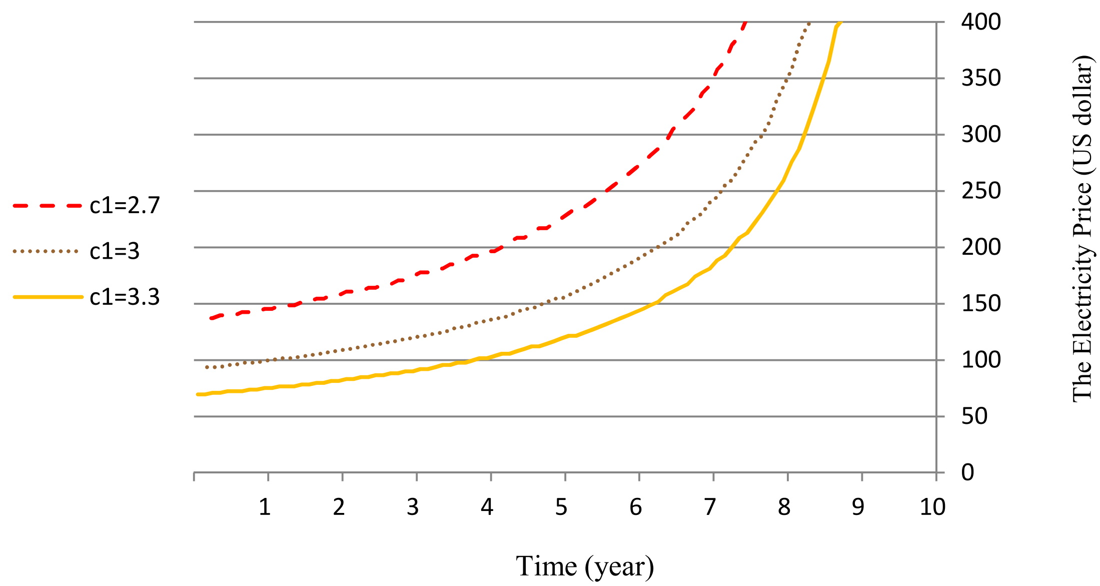


Figure 4.5. Free Boundary for Abandonment with Different $c_1$

**4.3.3.4 Sensitivity analysis for $D_1$.** Figure 4.6 shows that cost $D_1$ also affects the free boundary. When the cost $D_1$ decreases, the free boundary moves down; otherwise, if the cost $D_1$ increases, the free boundary moves up. This means that the old power plant is expected to shut down earlier (the abandonment option can be easily exercised) when its cost goes up.

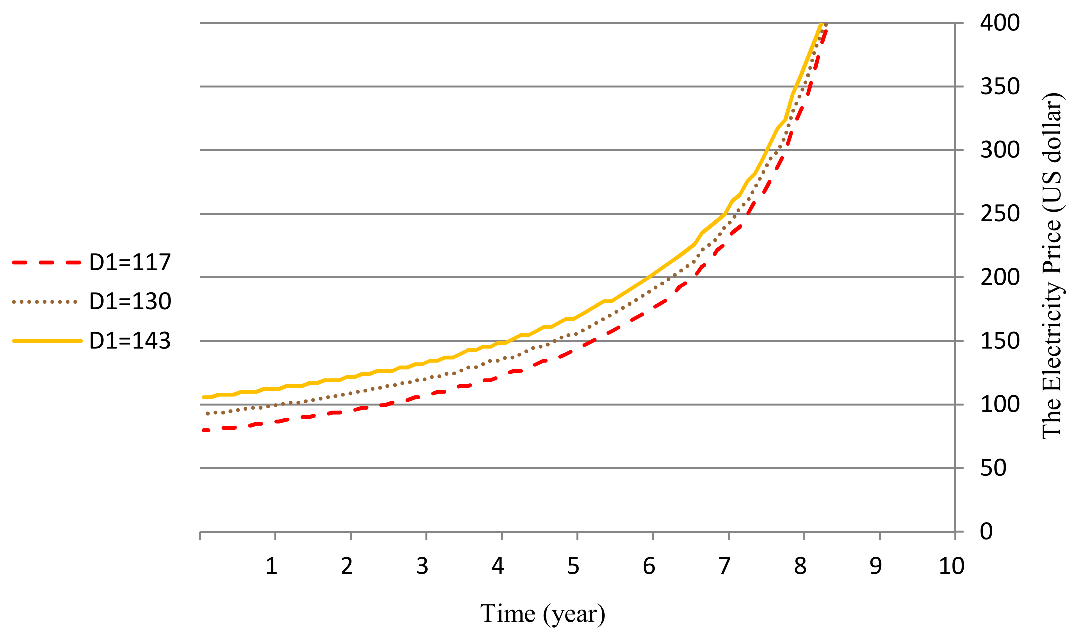


Figure 4.6. Free Boundary for Abandonment with Different $D_1$

## 4.4. CONCLUSIONS

In this section, the optimal abandonment decision for the old power plant and the optimal dispatch decision between the two power plants are studied. The objective function is to maximize the long-term profit under the mean reversion stochastic process for electricity prices. Optimal stochastic control and finite difference methods are used to solve the abandonment decision problem in this section.

The results of abandonment model also provide valuable policy for the decision-makers of the generators to make abandonment decisions. The free boundary can be employed to decide when the generator should shut down the old traditional power plant: the generator should shut down the old power plant if the electricity price is below the free boundary at a given time. Liquidation value, volatility, production rate and cost of the old traditional power plant all make effect on the abandonment decisions.

# 5. DYNAMIC INVENTORY MANAGEMENT MODEL

## 5.1. MOTIVATIONS

This section formulates a dynamic inventory optimal control problem in a finite planning horizon, with consideration of debt financing and tax, which are two important factors that influence inventory decisions. The retailer, who raises funds from the financial market at the beginning of the planning horizon and pays off the debt at the end of the horizon, replenishes the stock under constraint of the cash flow over each period of the planning horizon. The retailer faces random demand and the unmet demand in each period is lost. It is assumed that tax losses are not allowed for tax carry-backs or carry-forwards. The objective is to maximize the expected terminal wealth at the end of the planning horizon. Finally, the optimal inventory policy and the optimal debt financing decision can be found with the capital constraint and the effect of tax.

Many small retailers face problems about capital constraints when they order to maintain their inventory. Financing ability is a critical factor for start-up and growing retailers, whose developments heavily depend on the venture capital or debt. In most cases, they do not have enough capital to do what they want to in their operations. Though operational and financing decisions have a strong relationship, the dynamic inventory management literature considers little about the financial constraints. Therefore, it is very important to combine the ordering decisions and the financing decisions together in order to obtain the long term profit for the retailers.

Dynamic inventory problems have been studied by many researchers. Pioneering works include Arrow, Harris and Marschak (1951), Scarf (1960), Iglehart (1963), and Weinott and Wagner (1965) for a single warehouse, Clark and Scarf (1960) for multi-echelon systems; and Eppen and Schrage (1981) and Federgruen and Zipkin (1984) for distribution systems. More recently, the works of Zheng (1991), and Chen and Zheng (1994) have revealed new insights and have provided more efficient algorithms for these problems.

Several papers have recognized the relationship between operational decisions and financing decisions. The seminal work by Modigliani and Miller (1958) has showed that a firm's investment and financing decisions could be made separately within a perfect capital market. Due to market imperfections, such as taxes, agency costs, and asymmetric information, however, the choice of a firm's capital structure may in fact be closely related to its production decisions. Xu and Birge (2004) developed models to make production and financing decisions, simultaneously, in the presence of demand uncertainty and market imperfections. Their models illustrated how a firm's production decisions were affected by the existence of financial constraints. Li (1997) considered a single-product firm that made production decisions, borrowing decisions, and dividend policies during each period while facing uncertain demand. The firm could obtain an unbounded single-period loan with a constant interest rate. Archibald (2002) focused on start-up firms with the probability of long-term survival.

Chao et al. (2008) introduced a self-financing retailer model with financial constraints. The retailer periodically replenishes its stock from a supplier and sells it to the market. Excess demand in each period is lost. They derived the optimal inventory policy for each period, and characterized the dependence of the firm's optimal operational policy on its financial status. They also analyzed the relationship between the optimal control parameters and system parameters.

In this section, the optimal ordering policy and the optimal financing decision are considered simultaneously. This model incorporates the holding cost and the effect of tax that was not considered by Chao et al. (2008). The section is organized as follows. A mathematical model is first set up for the inventory problem of the retailer. By solving this problem, the optimal ordering solution, without considering the debt financing, can be found out. Finally, the debt financing, which can be done only at the beginning of the finite planning horizon and be paid back at the end of the finite planning horizon, is incorporated into the model. It is concluded at the last section.

## 5.2. MATHEMATIC MODELS

It is assumed that a retailer sells a single product to the market in a finite planning horizon. Due to financial constraints, the retailer have to decide about how much fund need to raise from the financial market at the beginning of the finite planning horizon, which will be paid back at the end of the finite planning horizon. The retailer has an initial inventory $x_1$ and raised an initial capital $s_1$. For simplicity, only the demand is considered as a stochastic process; the sale and purchase prices, tax and interest rate, holding cost and salvage value rate are all assumed to be constant.

The retailer makes replenishment decisions over the planning horizon of *N* periods. Assume unmet demand in each period is lost and that the ordering lead time (delay) is zero.

The periods are numbered from 1 to *N*, the demands $D_n (1 \leq n \leq N)$ are independent, and identically distributed nonnegative random variables. Let $p$ be the unit sales price, and $c$ is the unit ordering cost. Any inventory left at the end of the planning horizon has a salvage value $\gamma$ per unit, where $-\infty < \gamma < c < p$. The holding cost per unit per period is $h$.

Let $S_n$, $1 \leq n \leq N$ be the capital level at the beginning of period *n*, $x_n$ and $y_n$, $1 \leq n \leq N$ ,be the inventory levels, before and after the replenishment at the beginning of period n, respectively, and $\mathrm{S_{N+1}}$ be the terminal wealth at the end of the planning horizon. The interest rate $d$ is charged by debt holders for the whole horizon, and $r_f$ is the risk-free interest rate per period. Assume $c(1+r_f) < p$ . Otherwise, the operations will not have been necessary, since the retailer can just put the money into the bank to make higher income. Also assume $d > r_f$. At end of each period, the retailer receives its revenue from sales and interest on deposits.

Because the retailer only finances once, at the beginning of the planning horizon, the ordering decision have to satisfy the cash flow constraints $c(y_n - x_n) \leq S_n$. And the remaining capital will be deposited in the bank in order to get the interest $r_f$. The sales revenue in period n is $pMin\{\, y_n,\, D_n\}$; the holding cost is $h(y_n - D_n)^+$; so, the total capital level at the end of period n is:

$$S_{n+1} = pMin\{\, y_n,\, D_n\} + (1 + r_f)\big(S_n - c(y_n - x_n)\big) - h(y_n - D_n)^+, \quad n=1, 2, 3\ldots, N. \tag{5.1}$$

The inventory level, which considers unmet demand as lost, at the beginning of the period *n+1* is:

$$x_{n+1} = (y_n - D_n)^+,\ n=1, 2, 3\ldots, N. \tag{5.2}$$

For simplicity, gains are assumed to be taxed at a constant rate $\tau$, while tax losses are not allowed for tax carry-backs or carry-forwards (Xu and Birge, 2008). The terminal wealth net of tax is:

$$\begin{cases} S_{N+1} - (1+d)S_1 - \tau(S_{N+1} - dS_1) & if\ S_{N+1} > (1+d)S_1 \\ 0 & if\ S_{N+1} \leq (1+d)S_1 \end{cases}. \tag{5.3}$$

Therefore, the problem for the retailer is to decide on an ordering policy $y_1, y_2, y_3, \dots y_n \dots y_N$, and an initial debt level $S_1$ to maximize the expected terminal wealth at the end of the planning horizon, given the initial inventory level $x_1$, subject to the cash flow constraints in each period. That is, the decision problem is:

$$Max_{y_1 \dots y_n \dots y_N, S_1}\ E[1_{\{S_{N+1} > (1+d)S_1\}}((1-\tau)S_{N+1} - (1+(1-\tau)d)S_1 \tag{5.4}$$

Subject to (5.1), (5.2) and

$$0 \leq c(y_n - x_n) \leq S_n \quad n=1,\ 2,\ 3\dots,\ N. \tag{5.5}$$

The value function of the objective problem $v(x,S)$ represents the maximum expected terminal wealth by changing the control variable $y_n$ and $S_1$ at the given x and S. So, the dynamic programming Bellman optimality equation is:

$$v_n(x,S) = \underset{x \leq y \leq x + \frac{S}{c}}{Max}\ E[v_{n+1}((y_n - D_n)^+, pMin\{y_n, D_n\} +$$

$$\left(1 + r_f\right)\left(S_n - c(y_n - x_n)\right) - h(y_n - D_n)^+)] \tag{5.6}$$

with the boundary condition:

$$v_{N+1}(x,S) = \begin{cases} (1-\tau)(S+\gamma x) - (1+(1-\tau)d)S_1 & , S+\gamma x > (1+d)S_1 \\ 0 & , S+\gamma x \leq (1+d)S_1 \end{cases}. \tag{5.7}$$

**5.2.1. Raised Initial Capital is Given.** A result similar to that of Chao et al. (2008) can be obtained if the $S_1$ is considered as a constant. In order to solve the problem, two propositions are proposed first:

Proposition 1:

For any period *n* and fixed *A* and $B$, $v_n(A - \theta, B + (p+h)\theta)$ is increasing in $\theta$.

Proof:

Note that $v_{N+1} = (1-\tau)(S+\gamma x) - (1+(1-\tau)d)S_1$ is increasing in *S*. So, $v_N(x,S) = \underset{x \le y \le x+\frac{S}{c}}{Max} E[v_{N+1}(x_{N+1}, S_{N+1})]$ is increasing in S; from this terminal value, it can be concluded that the general format$v_n(x,S) = \underset{x \le y \le x+\frac{S}{c}}{Max} E[v_{n+1}(x_{n+1}, S_{n+1})$ is increasing in *S*.

Note the relationship:

$$v_n(A-\theta, B+(p+h)\theta) = \underset{A-\theta \le y \le A+\frac{B}{c}+\frac{p-c}{c}\theta}{Max} E[v_{n+1}((y_n - D_n)^+, pMin\{\, y_n,\ D_n\} +$$

$$\left(1+r_f\right)\left((p+h-c)\theta + B + cA - cy_n\right) - h(y_n - D_n)^+)].$$

From the previous proof, it can be concluded that:

$$v_{n+1}((y_n - D_n)^+, pMin\{\, y_n,\ D_n\} + \left(1+r_f\right)\left((p+h-c)\theta + B + cA - cy_n\right) - h(y_n - D_n)^+)$$

is increasing in $\theta$. So, $v_n(A-\theta, B+(p+h)\theta)$ is increasing in $\theta$.

Proposition 2:

For any period *n*, $v_n(x,S)$ is jointly concave in *x* and *S*.

Proof:

In order to simplify the proof, let:

$\bar{x} = \lambda x_1 + (1-\lambda)x_2,$

$\tilde{x} = min\,\{\lambda x_1 + (1-\lambda)x_2, D_n\},$

$\hat{x} = \lambda min\,\{x_1, D_n\} + (1-\lambda)min\,\{x_2, D_n\};$

$\bar{y} = \lambda y_1 + (1-\lambda)y_2,$

$\tilde{y} = min\,\{\lambda y_1 + (1-\lambda)y_2, D_n\},$

$\hat{y} = \lambda min\,\{y_1, D_n\} + (1-\lambda)min\,\{y_2, D_n\};$

$\bar{S} = \lambda S_1 + (1-\lambda)S_2,$

$\tilde{S} = min\,\{\lambda S_1 + (1-\lambda)S_2, D_n\},$

$\hat{S} = \lambda min\{S_1, D_n\} + (1-\lambda)min\{S_2, D_n\}$;

Note that: $\tilde{x} \geq \hat{x}$; $\tilde{y} \geq \hat{y}$; $\tilde{S} \geq \hat{S}$.

Induction is used to prove this proposition.

First, prove $v_{N+1}(x, S)$ is jointly concave in *x* and *S*. Note that $v_{N+1}(x, S)$ is jointly concave in *x* and *S* when $S + \gamma x > (1+d)S_1$, and $v_{N+1}(x, S) = 0$ is also jointly concave in *x* and *S* when $S + \gamma x < (1+d)S_1$.

Note that if $S_{N+1} - (1+d)S_1 \leq \tau(S_{N+1} - dS_1)$, $v_{N+1}(x, S)$ is jointly concave in *x* and *S* in the whole area. So, let $\tau \geq 1 - \frac{S+\gamma x - S_1}{S+\gamma x - dS_1}$ in order to make sure that jointly concave in *x* and *S*.

Second, assume $v_{n+1}(x, S)$ is jointly concave in *x* and *S*, then it need to be proved that $v_n(x, S)$ is jointly concave in *x* and *S*.

$v_n(\lambda x_1 + (1-\lambda)x_2, \lambda S_1 + (1-\lambda)S_2)$

$= \underset{x \leq y \leq x + \frac{S}{c}}{\text{Max}} \; E[v_{n+1}((\lambda y_1 + (1-\lambda)y_2 - D_n)^+, pMin\{\lambda y_1 + (1-\lambda)y_2,\ D_n\} +$

$\left(1+r_f\right)\left(\lambda S_1 + (1-\lambda)S_2 - c(\lambda y_1 + (1-\lambda)y_2 - \lambda x_1 - (1-\lambda)x_2)\right) -$

$h(\lambda y_1 + (1-\lambda)y_2 - D_n)^+]$

$= \underset{x \leq y \leq x + \frac{S}{c}}{\text{Max}} \; E[v_{n+1}\left(\bar{y} - \tilde{y}, \left(1+r_f\right)\left(\bar{S} - c(\bar{y} - \bar{x})\right) - h\bar{y} + (p+h)\tilde{y}\right)]$

$\geq \underset{x \leq y \leq x + \frac{S}{c}}{\text{Max}} \; E[v_{n+1}\left(\bar{y} - \hat{y}, \left(1+r_f\right)\left(\bar{S} - c(\bar{y} - \bar{x})\right) - h\bar{y} + (p+h)\hat{y}\right)]$

(Proposition1)

$= \underset{x \leq y \leq x + \frac{S}{c}}{\text{Max}} \; E[v_{n+1}(\lambda(y_1 - D_n)^+ + (1-\lambda)(y_2 - D_n)^+, \lambda(pMin\{y_1,\ D_n\} +$

$\left(1+r_f\right)\left(S_1 - c(y_1 - x_1)\right) - h(y_1 - D_n)^+ + (1-\lambda)(pMin\{y_2,\ D_n\} +$

$\left(1+r_f\right)\left(S_2 - c(y_2 - x_2)\right) - h(y_2 - D_n)^+]$

$$\geq \underset{x \leq y \leq x+\frac{S}{c}}{\text{Max}} \; E[\lambda v_{n+1}((y_1 - D_n)^+, pMin\{y_1,\ D_n\} + (1+r_f)(S_1 - c(y_1 - x_1)) -$$

$$h(y_1 - D_n)^+) + (1-\lambda)v_{n+1}((y_2 - D_n)^+, pMin\{y_2,\ D_n\} +$$

$$(1+r_f)(S_2 - c(y_2 - x_2)) - h(y_2 - D_n)^+)]$$

(Since $v_{n+1}(x, S)$ is jointly concave)

$=\lambda v_n(x_1, S_1) + (1-\lambda)v_n(x_2, S_2)$

So, $v_n(x, S)$ is jointly concave in *x* and *S*.

From proposition 2, the following proposition can be easily obtained:

Proposition 3:

For any period n and given $R = S + cx$ ,

$\pi_n(y, R) = E[v_{n+1}((y - D_n)^+, pMin\{y,\ D_n\} + (1+r_f)(R - cy) - h(y - D_n)^+)]$ is jointly concave in *(y, R)*.

Let $y_n{}^*(R)$be the optimal solution to the problem $\underset{x \leq y \leq \frac{R}{c}}{Max} \; \pi_n(y, R)$. The optimal inventory policy proposed in Theorem 1 naturally follows from Proposition 3.

Theorem 1:

The capital-dependent base stock inventory policy of period *n*:

$$\mu_n{}^*(R) = \begin{cases} \frac{R}{c} & x_n \leq y_n{}^*(R) - \frac{S}{c} \\ y_n{}^*(R) & y_n{}^*(R) - \frac{S}{c} \leq x < y_n{}^*(R) \\ x_n & x_n \geq y_n{}^*(R) \end{cases} \tag{5.8}$$

According to Theorem 1, the optimal inventory policy is to keep the inventory level as close to $y_n{}^*(R)$ as possible. The retailer should use all of the capital to replenish its stock if $R \leq c y_n{}^*(R)$, even though the resulting stock level is not the optimal level due to capital constraint. The stock should be replenished to the optimal level $y_n{}^*(R)$

when there is enough capital, that is, $R > cy_n^*(R)$. The inventory level $x_n$ should be kept unchanged when $x_n \geq y_n^*(R)$.

**5.2.2. Raised Initial Capital is a Decision Variable.** If the initial capital is controllable, the retailer should have earned more profit at the end of the planning horizon, since the retailer can choose the optimal amount to debt financing.

In order to derive the optimal debt financing and optimal ordering decisions. Assume that the cost to finance one unit capital is smaller than the profit generated by one unit capital, that is, $d < \frac{\partial v_n(x,S)}{\partial S}$. It is intuitively clear that the retailer should keep the inventory as the optimal inventory level $y_1^*(R)$ at the beginning of the finite planning horizon if the retailer can control the debt amount $S_1$. And, note that $R_n$ is increasing in *n*.

Theorem 2:

The capital-dependent base stock inventory policy of period *n* with debt financing decision:

$$\mu_n^*(R) = \begin{cases} y_1^*(R) & first\ step \\ \frac{R}{c} & x_n \leq y_n^*(R) - \frac{S}{c} \\ y_n^*(R) & y_n^*(R) - \frac{S}{c} \leq x_n < y_n^*(R) \\ x_n & x_n \geq y_n^*(R) \end{cases} \tag{5.9}$$

And the optimal debt is:

$$S_1 = c(\ y_1^*(R) - x)^+. \tag{5.10}$$

According to theorem 2, considering the debt financing, the optimal decisions are simple: the retailer should finance $S_1 = c(\,y_1^*(R) - x)^+$ at the beginning of the finite planning horizon in order to have the ability to maintain the optimal inventory level throughout the whole finite planning horizon. At the beginning of each period, the retailer should replenish the inventory level to reach the optimal inventory $y_n^*(R)$ if $x <$

$y_n{}^*(R)$; otherwise keep the current inventory level x unchanged at the beginning of the planning horizon.

## 5.3. CONCLUSIONS

This section studies a dynamic multi-period inventory model that is proposed for incorporating financial decisions into ordering decisions with consideration of capital constraints, lost sales, holding cost and tax. First, a basic model is studied, in which the debt financing is not available. For this base model, the optimal ordering policies with capital constraints at each period are proposed. Then, the optimal debt financing decisions, as well as the optimal ordering policies with debt financing, are described.

The optimal financing amount $S_1 = c(\, y_1{}^*(R) - x)^+$ is financed at the beginning of the finite planning horizon only once. That financing amount assures that the retailer has sufficient capital to keep its inventory at the optimal level; the optimal ordering policy is to order up to $y_n{}^*(R)$ at the beginning of period n, when the inventory level is below $y_n{}^*(R)$, otherwise nothing is ordered in that period.

# 6. CONCLUSIONS

This dissertation formulates mathematic optimization models to help decision makers in the energy and retail industries to make optimal entry, optimal operation, and optimal abandonment decisions under stochastic process. These research findings can help generators in energy industry reduce carbon dioxide emissions as well as maximizing long-term profits. It can also benefit retailers in retail industry reducing inventory and financing cost as well as maximizing long-term profits.

## 6.1. CONTRIBUTIONS

The first model described in this dissertation is a basic entry decision model in energy portfolio management, which displays a basic method for formulating and solving optimal entry and optimal operation problems in the energy industry. Electricity price, which follows Geometric Mean Reversion (GMR) stochastic process, is assumed to be the only stochastic process in this model, as well as in the other two models related to the basic model. The long-term unit profit of the firm was maximized over a finite time horizon as the objective function. The original mixed optimal stochastic control and optimal stopping problem is divided into two sub-problems: an optimal control problem and an optimal stopping problem. A numerical method is employed to solve PDEs, which come from H-J-B equations, in order to find free boundaries. The free boundaries are used to help generators make entry decisions with different electricity prices over the finite time horizon. With the results obtained from this model, the investment decision become very clear: the generator should excise the investment option when the electricity price jumps up above the free boundary, but otherwise, just wait until the price of electricity satisfy the requirement. It is obvious that a lower free boundary represents the better investment for the new power plant.

The second model is a more reality model for the entry and operation decision, because it considers construction delay, variability of costs, and different types of power plants. Construction delay brings the delay model with different solution procedures but almost the same solution pattern. Construction delays obviously lower the free boundary, which means it is easy to trigger investment decisions. The relative gains are also displayed in the second model to show the benefit of investment versus no investment at different electricity prices when the free boundaries imply the investment option should be exercised.

The sensitivity analysis also shows that the changing of cost for the new power plant can affect the free boundary as well as the entry decision: when the cost of the new power plant is lowered to a new constant cost all of the time, or assuming it decreasing over time deterministically would easily trigger investment decisions (lower free boundary). Meanwhile, reducing capital investment, improving production rate of new power plant can also lower the free boundary.

Another valuable finding determined by this model is that the uncertainty of the electricity also affects the free boundary: greater volatility (more uncertainty) leads to a lower free boundary, but higher relative gains.

It should be pointed out that the free boundaries in this model can be used to evaluate the investment of new power plant. Various combinations of operation rates and costs represent different types of power plants, which have different free boundaries and relative gain curves. The conclusion is that the lowest free boundary represents the best investment choice, which corresponds with the highest relative gain curve.

The abandonment model studies the optimal abandonment for the generator who owns an energy portfolio, including two power plants. This model is the opposite case of the previous models although a similar modeling methods and solution procedures are used. The free boundary can also be used to make abandonment decisions. The generator should excise the abandonment option (make abandonment decision) when the electricity price drops below the free boundary. The sensitivity analysis of subsidies reveals that more subsidies trigger easier abandonment decisions (a higher the free boundary).

The dynamic inventory model was proposed to provide an optimal control policy and an optimal financing policy. This dynamic multi-period inventory model incorporates financial decisions into ordering decisions while considering capital constraints, lost sales, holding cost and tax. The closed form solution about the optimal financing amount at the beginning of the finite planning horizon is obtained. The optimal ordering policy is also found for each period: orders are placed up to ${y_n}^*(R)$ at the beginning of period $n$, when the inventory level is below ${y_n}^*(R)$, otherwise nothing is ordered for that period.

## 6.2. FUTURE WORKS

Energy portfolio models described in this dissertation can possibly be extended in three ways.

First, switching costs will be incurred when generator considers switching from the existing plant to the new plant, and vice versa; a singular control technique would be employed to study this problem. Second, in reality, the spikes (jumps) of the spot electricity prices are an important characteristic of the stochastic process of electricity prices. Electricity spot prices often jump to 10 or 20 times their current or normal price for a few hours before returning to normal levels. Thus, the spikes of the electricity prices will need to be considered in the future. After considering the jump, the evolution of electricity price can be represented as:

$$dX_t = \left(\mu(\lambda - lnX_t) + \frac{1}{2}\sigma^2\right)X_t dt + \sigma X_t dB_t + \sum_{k=1}^{N} \gamma_k dq_k \tag{6.1}$$

Note that $dq_k$ are Poisson processes with the properties:

$$dq_k = \begin{cases} 0 & p = 1 - \epsilon_k(X,t)dt \\ 1 & p = \epsilon_k(X,t)dt \end{cases} \tag{6.2}$$

Third, the energy portfolio models in this dissertation are based on only one stochastic process (the electricity price); it is obvious that there are other stochastic processes that can affect the decisions, such as the cost of carbon dioxide emissions. Multidimensional optimal control problems should be solved since there are more stochastic processes, in addition to the electricity prices. A possible extension could be formulated as bellow:

Let $Y_t$ represents $CO_2$ emission cost at time *t* (\$/MWh). So, the evolution of $CO_2$ emission cost is represented by:

$$dY_t = \mu_y Y_t dt + \sigma_y Y_t dB_t^y. \tag{6.3}$$

Here, the $CO_2$ emission cost follows the Geometric Brownian Motion (GBM). Where $\mu_y$and $\sigma_y$ are no stochastic functions and $B_t$ is a Wiener processes.

And, $B_t^x$, $B_t^y$ are correlated Wiener processes with:

$$dB_t^x dB_t^y = \rho_{xy} d_s \tag{6.4}$$

Assume that the decision to build a new plant is made at time $\tau$ which requires a capital investment of $K$ dollars which will be paid when the construction is completed in $\delta$ years.

Thus, the long-term profit functional is:

$$J(t,x,y;\tau,\alpha) \equiv E_x[\int_t^{\tau+\delta} (c_1X_s - D_1 - s_1Y_s)e^{-\rho(s-t)}ds - Ke^{-\rho(\tau+\delta-t)} +$$

$$\int_{\tau+\delta}^{T}((1-\alpha)(c_1X_s - D_1 - s_1Y_s) + \alpha(c_2X_s - D_2 - s_2Y_s))e^{-\rho(s-t)}ds]. \quad (6.5)$$

The value function u is defined as

$$u(t,x,y) = \sup_{\tau\in\Gamma_{t,T-\delta,}\alpha\in[0,\bar{\alpha}]} J(t,x,y;\tau,\alpha). \quad (6.6)$$

where $\Gamma_{t,T,}$ denotes the set of stopping times in $[t,T]$.

Decompose above problem into the following two problems:

The first problem is optimal dispatch once the new plant is built. The value function $v$ of the optimal dispatch problem is defined as:

$$v(t,x,y) \equiv \max_{\alpha_t\in[0,\bar{\alpha}]} E_{x,y}[\int_\tau^T((1-\alpha_s)(c_1X_s - D_1 - s_1Y_s) +$$

$$\alpha_s(c_2X_s - D_2 - s_2Y_s))e^{-\rho(s-t)}ds]. \quad (6.7)$$

This stochastic control problem, as defined above, is transformed into a partial differential equation problem by using the principle of dynamic programming. The value function $v$ satisfies the following H-J-B equation:

$$\frac{\partial}{\partial t}v + \underset{\alpha\in[0,\bar{\alpha}]}{Sup}[((1-\alpha_s)(c_1X_s - D_1 - s_1Y_s) + \alpha_s(c_2X_s - D_2 - s_2Y_s))e^{-\rho(s-t)} + L_v - \rho v] = 0,$$

(6.8)

where

$$L_v \equiv \left[\mu_x(m_t^x - lnX_t) + \frac{1}{2}\sigma_x^2\right]\frac{\partial v}{\partial x} + \frac{1}{2}{\sigma_x}^2\frac{\partial^2 v}{\partial_{x^2}} + \mu_y\frac{\partial v}{\partial y} + \frac{1}{2}{\sigma_y}^2\frac{\partial^2 v}{\partial_{y^2}} + \rho_{xy}\sigma_x\sigma_y xy\frac{\partial^2 v}{\partial x\partial y}. (6.9)$$

Therefore, the H-J-B equation is reduced to

$$\frac{\partial}{\partial t}v + (1-\alpha^*)(c_1X_s - D_1 - s_1Y_s) + \alpha^*(c_2X_s - D_2 - s_2Y_s) + L_v - \rho v = 0. \qquad (6.10)$$

Second, consider optimal time to build a new plant with construction delay. The value of the portfolio is given by $v(\tau, X_\tau)$ when the new plant is built; therefore, the value function of this problem w by considering construction time delay is defined as follows:

$$w(t,x,y) \equiv \sup_{\tau\in T_{t,T}} E_{x,y}[\int_t^{\tau+\delta}(c_1X_s - D_1 - s_1Y_s)e^{-\rho(s-t)}ds +$$

$$(v(\tau+\delta, X_{\tau+\delta}) - K)e^{-\rho(\tau+\delta-t)}, t \in [0, T-\delta). \qquad (6.11)$$

The delayed optimal stopping problem can be transformed to a non-delayed optimal stopping problem and solve it by using a similar method once used in previous entry decision model with delay.

For the dynamic inventory model, it is assumed that the retailer has no equity at the beginning and only acquires capital by one-time debt financing. Future work can relax this condition to make the problem more realistic, e.g., the retailer already has equity. Other conditions also can be extended. For example, the retailer has shortage cost as well as holding costs. The relationship between the optimal ordering policy and the parameters is also very important to study.

# APPENDIX

## THE SOLUTION PROCEDURE OF DELAY MODEL

The following are the details of solution for the PDEs in Section 3.

First, Consider the H-J-B Equation (3.9):

$$\frac{\partial}{\partial t}v + (1-\alpha^*)(c_1x - D_1) + \alpha^*(c_2x - D_2) + L_v - \rho v = 0$$

$$L_v = \mu' \frac{\partial v}{\partial x}x + \frac{1}{2}\sigma^2 \frac{\partial^2 v}{\partial_{x^2}}x^2 \quad \text{(A.1)}$$

$$\mu' = \mu(\lambda - lnx) + \frac{1}{2}\sigma^2. \quad \text{(A.2)}$$

To calculate the equation, the terminal condition needs to be changed to initial condition by considering $t' = T - t$. However, the model still uses $t$ to represent $t'$ for convenience. Thus:

$$-\frac{\partial}{\partial t}v + (1-\alpha^*)(c_1x - D_1) + \alpha^*(c_2x - D_2) + L_v - \rho v = 0 \quad \text{(A.3)}$$

the boundary conditions:

$$v(X_{min}, t) = -\frac{D_1}{\rho}(1 - e^{-\rho t}) \qquad (X_{min} = 0) \quad \text{(A.4)}$$

$$v(X_{max}, t) = \frac{(c_1 + \alpha^*(c_2 - c_1)) * X_{max} - (1-\alpha^*)D_1 - \alpha^* D_2}{\rho}(1 - e^{-\rho t})$$

$$(\tau = t, X_{max} \text{ is a big number}) \quad \text{(A.5)}$$

the initial condition:

$$v(x,T) = \begin{cases} c_1x - D_1 & , if\ (x < \frac{D_2 - D_1}{c_2 - c_1}) \\ (1-\bar{\alpha})(c_1x - D_1) + \bar{\alpha}(c_2X - D_2) & , if\ (x \geq \frac{D_2 - D_1}{c_2 - c_1}) \end{cases}. \quad \text{(A.6)}$$

To overcome the artificial oscillations, an upwind scheme is used to represent $\frac{\partial \mathrm{v}}{\partial \mathrm{x}}$. Thus, equation (2.16) is used when $\mu^{'} < 0$ ; equation (2.17) is used when $\mu^{'} > 0$.

In this model, changing variable technique (*Y=lnX*) is used to improve the calculation efficiency.

Using equations (2.15),(2.16),(2.17) and (2.18), ignoring terms of $O(\Delta t)$ and $O(\Delta x)$, and plugging in equation (A.3), obtain the formula for $v_n^{m+1}$:

If $\mu^{'} < 0$

$$v_n^{m+1} = \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} v_{n+1}^m - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta y}\mu^{'} + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right) v_n^m + \left(\frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu^{'}\frac{\Delta t}{\Delta y}\right) v_{n-1}^m +$$

$$\Delta t\big((c_1 + \alpha^*(c_2 - c_1)\big)e^y - (1 - \alpha^*)D_1 - \alpha^* D_2).$$

If $\mu^{'} > 0$

$$v_n^{m+1} = \left(\frac{\Delta t}{\Delta y}\mu^{'} + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right) v_{n+1}^m - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu^{'} + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right) v_n^m + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} v_{n-1}^m +$$

$$\Delta t\big((c_1 + \alpha^*(c_2 - c_1)\big)e^y - (1 - \alpha^*)D_1 - \alpha^* D_2).$$

Consider the H-J-B Equation (3.13) with terminal condition:

$$\begin{cases} \frac{\partial}{\partial t} g_0 + L_{g_0} + (c_1 X_s - D_1) - \rho g_0 = 0 \\ g_0(\tilde{T}, x) = v(\tilde{T}, x) - K \end{cases}$$

By using the same strategy as before, the following PDE is obtained:

$$-\frac{\partial}{\partial t} g_0 + L_{g_0} + (c_1 X_s - D_1) - \rho g_0 = 0 \tag{A.7}$$

the boundary conditions:

$$g_0(X_{min}, t) = -\frac{D_1}{\rho}(1 - e^{-\rho t}) \quad (\tau = T, X_{min} = 0) \tag{A.8}$$

$$g_0(X_{max}, t) = \frac{c_1 X_{max} - D_1}{\rho}(1 - e^{-\rho t}) \; (\tau = t, X_{max} \text{ is a big number}) \tag{A.9}$$

the initial condition:

$$g_0(x,\delta) = v(x,\delta) - K. \tag{A.10}$$

So, $g_0(t)$ is obtained by using the following formulas in the interval [$t$-$\delta$, $t$].

If $\mu^{'} < 0$

$$g_{0}{}_{\mathrm{n}}^{\mathrm{m+1}} = \frac{\Delta \mathrm{t}}{(\Delta y)^2}\frac{\sigma^2}{2} g_{0}{}_{\mathrm{n+1}}^{\mathrm{m}} - \left(\rho\Delta \mathrm{t} - 1 - \frac{\Delta \mathrm{t}}{\Delta y}\mu^{'} + \frac{\Delta \mathrm{t}}{(\Delta y)^2}\sigma^2\right) g_{0}{}_{\mathrm{n}}^{\mathrm{m}} + \left(\frac{\Delta \mathrm{t}}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu^{'}\frac{\Delta \mathrm{t}}{\Delta y}\right) g_{0}{}_{\mathrm{n-1}}^{\mathrm{m}} +$$

$$\Delta \mathrm{t}(\mathrm{c}_1 e^y - \mathrm{D}_1).$$

If $\mu^{'} > 0$

$$g_{0}{}_{n}^{m+1} = \left(\frac{\Delta t}{\Delta y}\mu^{'} + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right) g_{0}{}_{n+1}^{m} - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu^{'} + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right) g_{0}{}_{n}^{m} + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} g_{0}{}_{n-1}^{m}$$

$$+ \Delta t(\mathrm{c}_1 e^y - D_1).$$

After solving $g_0(n,m)$, considering the H-J-B Equation (3.15)

$$Min(-\frac{\partial}{\partial t}w - L_w - (c_1X_s - D_1) + \rho w, w - g_0) = 0$$

First, consider:

$$\frac{\partial}{\partial t}\mathrm{w}' + L_{\mathrm{w}'} + (c_1X_s - D_1) - \rho\mathrm{w}' = 0 \tag{A.11}$$

$$L_{\mathrm{w}'} = \mu^{'}\frac{\partial w'}{\partial x}x + \frac{1}{2}\sigma^2\frac{\partial^2 w'}{\partial x^2}x^2 \tag{A.12}$$

By using the same strategy as before, the following PDE is obtained:

$$-\frac{\partial}{\partial t}w' + L_{\mathrm{w}'} + (c_1X_s - D_1) - \rho w' = 0 \tag{A.13}$$

the boundary conditions:

$$w'(X_{min},t) = -\frac{D_1}{\rho}(1 - e^{-\rho t}) \qquad (\tau = T, X_{min} = 0) \tag{A.14}$$

$$w'(X_{max},t) = \frac{c_1X_{max} - D_1}{\rho}(1 - e^{-\rho t}) \quad (\tau = t, X_{max} \text{ is a big number}) \tag{A.15}$$

the initial condition:

$$w'(x,\delta) = 0. \tag{A.16}$$

Also the formula for ${w'}_{n}^{m+1}$ is obtained:

If $\mu' < 0$

$${w'}_{n}^{m+1} = \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}{w'}_{n+1}^{m} - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right){w'}_{n}^{m} + \left(\frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu'\frac{\Delta t}{\Delta y}\right){w'}_{n-1}^{m} +$$

$$\Delta t(c_1 e^y - D_1).$$

If $\mu' > 0$

$${w'}_{n}^{m+1} = \left(\frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right){w'}_{n+1}^{m} - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right){w'}_{n}^{m} + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}{w'}_{n-1}^{m}$$

$$+ \Delta t(c_1 e^y - D_1).$$

Using the same method, the numerical solution of *w=max*$\{w', g_0\}$ is found.

Consider the H-J-B Equation (3.18)

$$\frac{\partial}{\partial t}u + (c_1 X_s - D_1) + L_u - \rho u = 0$$

$$L_u \equiv \mu\frac{\partial u}{\partial x}x + \frac{1}{2}\sigma^2\frac{\partial^2 u}{\partial x^2}x^2$$

the boundary conditions:

$$u(X_{min}, t) = -\frac{D_1}{\rho}(1 - e^{-\rho t}) \qquad (X_{min} = 0) \tag{A.17}$$

$$u(X_{max}, t) = \frac{(c_1 X_{max} - D_1)}{\rho}(1 - e^{-\rho t}) \quad (X_{max} \text{ is a big number}) \tag{A.18}$$

the initial condition:

$$u(x, 0) = 0 \tag{A.19}$$

Also the formula for $u_n^{m+1}$ is obtained:

If $\mu' < 0$

$$u_n^{m+1} = \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}u_{n+1}^{m} - \left(\rho\Delta t - 1 - \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right)u_n^m + \left(\frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2} - \mu'\frac{\Delta t}{\Delta y}\right)u_{n-1}^m +$$

$$\Delta t(c_1 e^y - D_1).$$

If $\mu' > 0$

$$u_n^{m+1} = \left(\frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}\right)u_{n+1}^m - \left(\rho\Delta t - 1 + \frac{\Delta t}{\Delta y}\mu' + \frac{\Delta t}{(\Delta y)^2}\sigma^2\right)u_n^m + \frac{\Delta t}{(\Delta y)^2}\frac{\sigma^2}{2}u_{n-1}^m +$$

$$\Delta t(c_1 e^y - D_1).$$

Using the same method, the numerical solution of $u$ is found.

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# VITA

Jianjun Deng, the son of Zuxin Deng and Shaoqiong Lu, was born in Lezhi, Sichuan Province, China in 1977. After graduating from Beijing Institute of Technology (BIT), Beijing, China, with a bachelor's degree in Mechanical Engineering in 2000, he worked for BIT as a Student Advisor for 3 years. In 2003, he was admitted into the MBA program at Tsinghua University, Beijing, China, and two years later he received his Master's Degree in Business Administration from Tsinghua University. From 2005 to 2008, Jianjun worked as a Human Resource Manager in Beijing Tellhow Intelligence Tech. Ltd.

In the spring of 2009, Jianjun began pursuing his Ph.D. in the Department of Engineering Management and Systems Engineering at Missouri University of Science and Technology (formerly University of Missouri-Rolla) under the supervision of Dr. Zhen Liu. His research has focused on dynamic decision making under uncertainty in renewable energy portfolio management and inventory management. In the course of his study and research, he has published papers in several conferences and earned the Outstanding Ph.D. Graduate Student Research Award (2010-2011) from the Department of Engineering Management and Systems Engineering at Missouri S&T. He has also served as President of the Financial Engineering Association (FEA) at Missouri S&T and President of INFOMS Missouri S&T student chapter. He is a member of INFORMS, IIE, and ASEM. He received his Ph.D. from the Missouri University of Science and Technology in May 2012.