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Efficient Modeling of Discontinuities and Dispersive Media in Printed Transmission Lines

R. Araneo, C. Wang, X. Gu, J. Drewniak, and S. Celozzi

***Abstract*—The finite-difference time-domain method is applied to the analysis of transmission lines on printed circuit boards. The lossy, dispersive behavior of the dielectric substrate is accurately accounted for by means of several algorithms whose accuracy is discussed and compared. Numerical results are validated by comparisons with measurements and an equivalent circuit of slot in the ground plane is proposed.**

***Index Terms*—Dispersive dielectric, FDTD, multiconductor line, printed circuit.**

I. Introduction

THE FINITE-DIFFERENCE time-domain (FDTD) method has been widely applied to solve electromagnetic problems since its first application [1]. In this paper, the method is applied to investigate the effects of lossy, dispersive dielectric substrates and those of discontinuities in the reference plane on high-speed digital signals transmitted in printed circuit boards. In fact, a general agreement exists on the key role played by dielectric losses in limiting the possibility to increase the clock frequency of digital circuits.

In the past, various efficient algorithms [2]–[8] have been presented to simulate in the time domain the frequency-dependence of the dielectric constant; their accuracy in this kind of configurations is compared and discussed. To a much lesser extent, the influence of the dielectric dispersive behavior on the signal propagation has been analyzed, and, to this end, different commercial substrates are compared.

A perfectly matched uniaxial medium is used to terminate the computational domain [9].

Moreover, FDTD results concerning the scattering parameters of a differential line above a dispersive dielectric are compared with measurements confirming the accuracy of numerical predictions as well as the influence of dielectric losses on the overall performance of high-speed digital circuits.

Finally, an equivalent circuit of the typical discontinuity represented by a slot in the ground plane is extracted from the FDTD results and used in a computer-aided design circuit simulator, assessing its validity through measurements conducted on real structures.

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II. FDTD Algorithms for Lossy Dielectric Media

In the frequency domain, the substrate (FR-4 type) is modeled as a single-pole Debye medium whose relative permittivity can be expressed as

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + j\omega\tau} \tag{1}$$

where ε_s and ε_∞ are, respectively, the zero-frequency relative permittivity and the relative permittivity at infinite frequency, and τ is the pole relaxation time. In the time domain, two main strategies have been followed in the past: the first one is based on the use of a recursive convolution algorithm for the evaluation of the electric flux density D, as

$$D(t) = \varepsilon_0\varepsilon_s E(t) + \varepsilon_0 \int_0^t \chi t(t-\tau)E(\tau)\,d\tau. \tag{2}$$

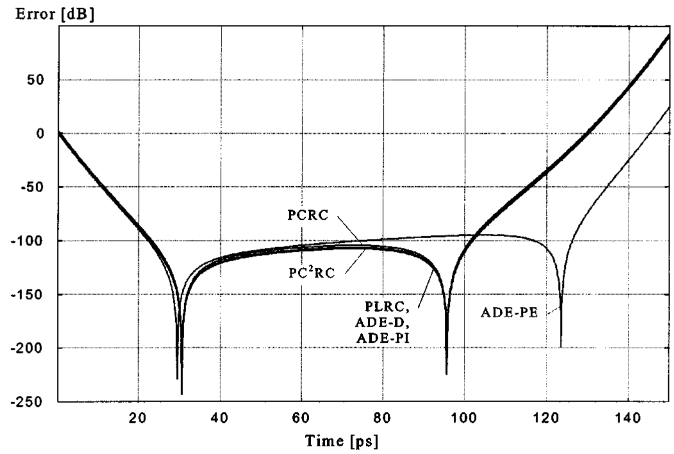
In particular, the convolution integrals are approximated assuming the electric field piecewise constant (PCRC) [2] or piecewise linear (PLRC) [3] between the discrete times at which is calculated, or it is assumed to be constant over each time segment t centered around E^n ($\mathrm{PC^2RC}$) [4]. It has been demonstrated that the PCRC scheme is first-order accurate, whereas the PLRC and $\mathrm{PC^2RC}$ ones are second-order accurate.

The second class of methods is based on the introduction of an auxiliary differential equation (ADE) for the update of the electric field. In particular, ADE methods substitute the integral constitutive relation (2) with a time-domain auxiliary differential equation which, in the different direct integration techniques, links either the electric flux density $D(t)$ [5] (ADE-D), or the polarization currents $J_p(t)$ [6] (ADE-J_p) or the polarization vector $P(t)$ to the electric field, with an explicit [7] (ADE-PE) or semi-implicit [8] (ADE-PI) scheme. The ADE is time stepped in synchronism with Ampere's equation, thus yielding to a system of second-order accurate equations. According to the choice of the numerical algorithm, different time-marching explicit equations, available in the above mentioned literature, hold for the update of the unknown electric field.

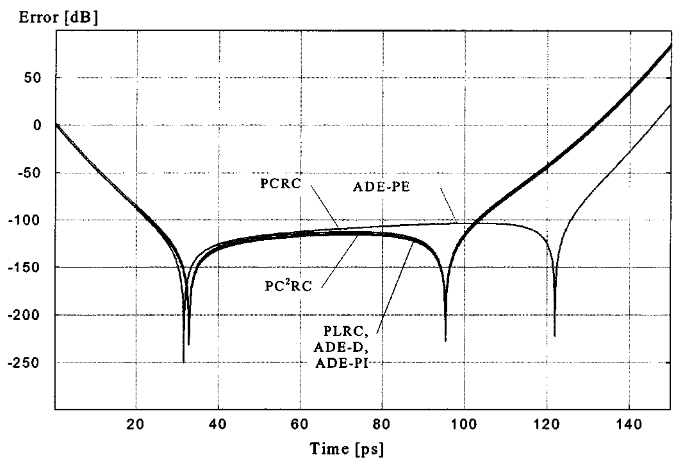
In order to investigate the accuracy of the available different implementations, an auxiliary integral equation is considered, in which the curl of the magnetic field is assumed to be known and the integral equation is solved in terms of the unknown electric field. Thus, the following equation has to be solved with respect to the electric field:

$$\nabla \times H = \frac{\partial D(t)}{\partial t} + \sigma E. \tag{3}$$

Assuming a Gaussian pulse as electric field excitation ($E = A\exp[-(\alpha(t-t_0)^2]$, $t_0 = 7.329\,36E-11$ s, $\alpha = 4.140\,68E+$



(a)



(b)

Fig. 1. Time trends of the errors on the electric field evaluated inside dielectric materials of type (a) 1 and (b) 3, for different methods.

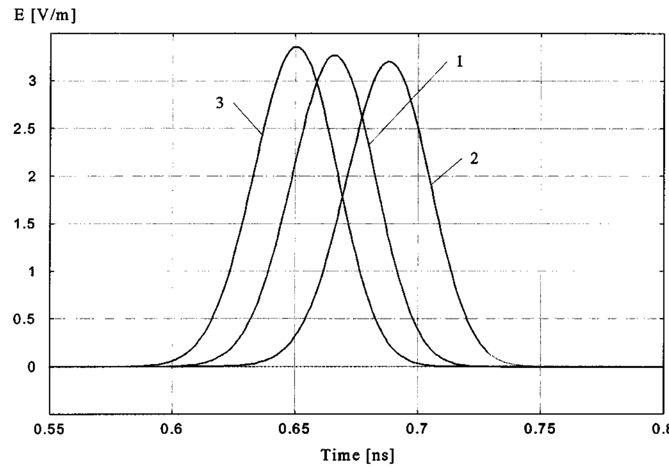


Fig. 2. Time trends of the transmitted electric field in different dielectric materials, at $x = 9$ cm (origin at air-dielectric interface).

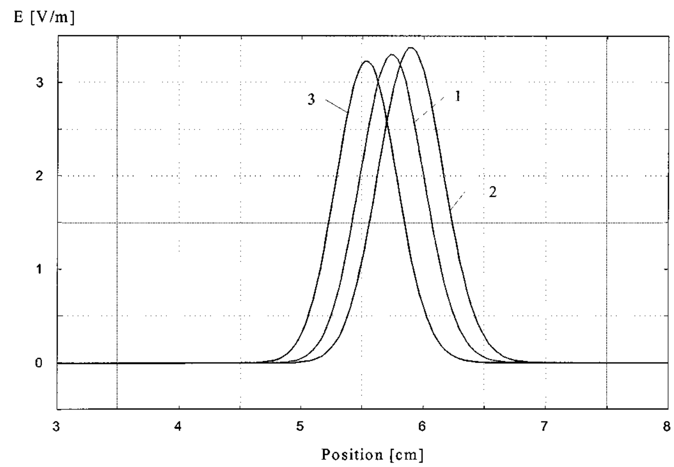


Fig. 3. Spatial profiles of the electric field in different dielectric media, at $t = 83.365$ ps.

10 s^{-1}), through (2) and (3) an *exact* value for the curl of H is evaluated, then (3) is solved *numerically* in order to find the electric field. Fig. 1 shows the time trend of the errors due to the above algorithms. The errors (in decibels) are evaluated on the electric field, $E^{(n)}$, at a given position, as

$$\Delta E_{\mathrm{dB}} = 20 \cdot \log_{10} \frac{|E^{\mathrm{approx}} - E^{\mathrm{exact}}|}{E^{\mathrm{exact}}} \tag{4}$$

for two dielectric materials [10] having, respectively: $\varepsilon_{s\_1} = 4.5$, $\varepsilon_{\infty\_1} = 3.89$, $\tau_1 = 1.634$ ns, $\sigma_1 = 4.159$ μS/m and $\varepsilon_{s\_3} = 3.9$, $\varepsilon_{\infty\_3} = 3.693$, $\tau_3 = 0.861$ ns, $\sigma_3 = 1.89$ μS/m.

For all the above error evaluations, a time step below the Courant stability condition has been considered, obtained by multiplying the limit by a factor 0.8, because in practical configurations several discontinuities are dealt with. It should be noted that after $t = 140$ ps the pulse is practically extinguished and thus, the significant (10% of the peak value) time interval is in the range between 40 ps and 110 ps.

From the results, it can be concluded that, in this class of problems and for the characteristics of actual dielectric substrates, the various methods available in literature are *equivalent* as to their accuracy in representing the dispersive nature of materials. In fact, the signal propagation characteristics are affected in a much more appreciable way by the dielectric properties rather than by the method adopted for the simulation and special care has to be paid to the simulation of the substrate, as shown in Section III.

III. Influence of Dielectric Characteristics on the Signal Propagation

Despite to the wide literature covering the subject of the numerical time-domain modeling of lossy dielectrics, much less attention has been devoted to the analysis of the signal propagation characteristics exhibited by different types of substrates. In the following, the transmitted electric field inside a dielectric half-space is analyzed, considering three different dielectrics: the two already considered and the third one having $\varepsilon_{s\_2} = 4.5$, $\varepsilon_{\infty\_2} = 4.19$, $\tau_2 = 0.949$ ns, $\sigma_2 = 4.539$ μS/m. In Fig. 2, the time trend of the electric field in the dielectric material is shown, at a distance $x = 9$ cm from the interface. The incident electromagnetic field is a plane wave (Gaussian pulse), as considered previously.

In Fig. 3, the spatial distribution of the electric field in the dielectric material is shown, at a given time $t = 83.365$ ps, assuming the same field source as before. It is evident that dielectric characteristics affects signal propagation considerably and

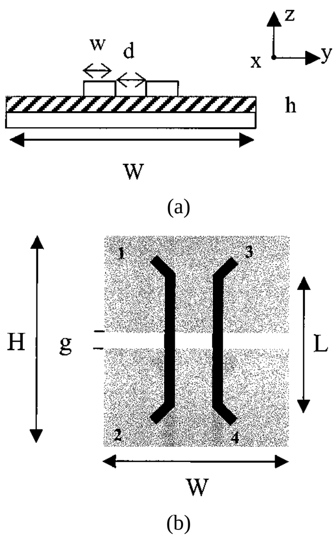


Fig. 4. System configuration: (a) cross-section, (b) top view.

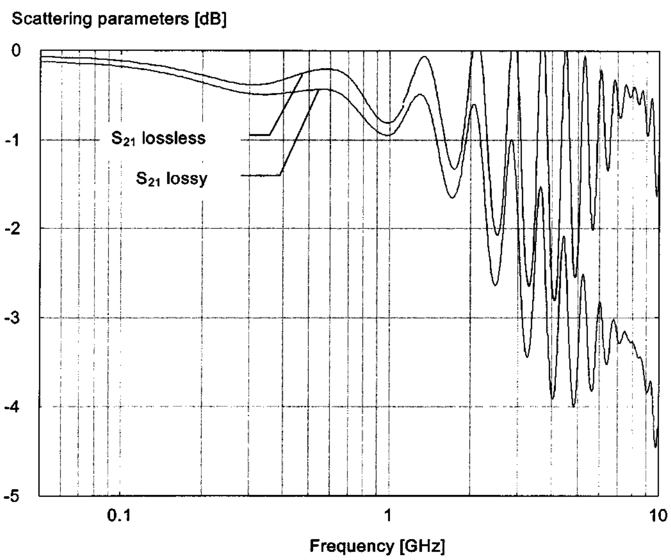


Fig. 5. Frequency spectra of the scattering parameter S_{21} for either a lossy dispersive dielectric substrate and a lossless one, in case of ground plane without a gap.

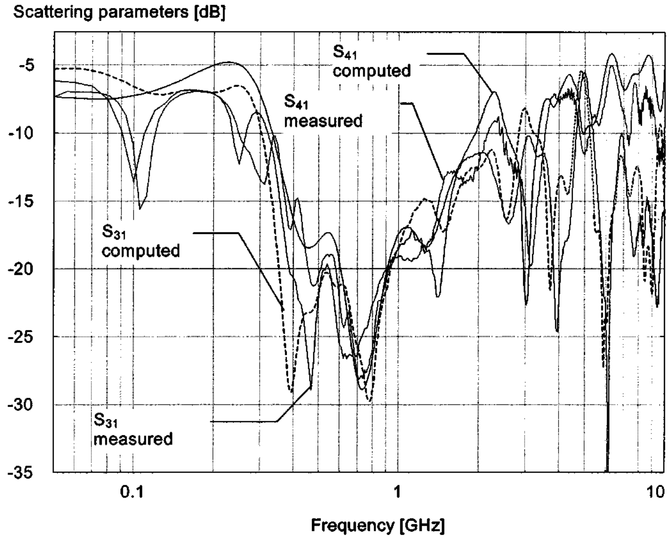


Fig. 6. Measured and computed frequency spectra of the scattering parameters S_{31} and S_{41}, in case of ground plane without a gap.

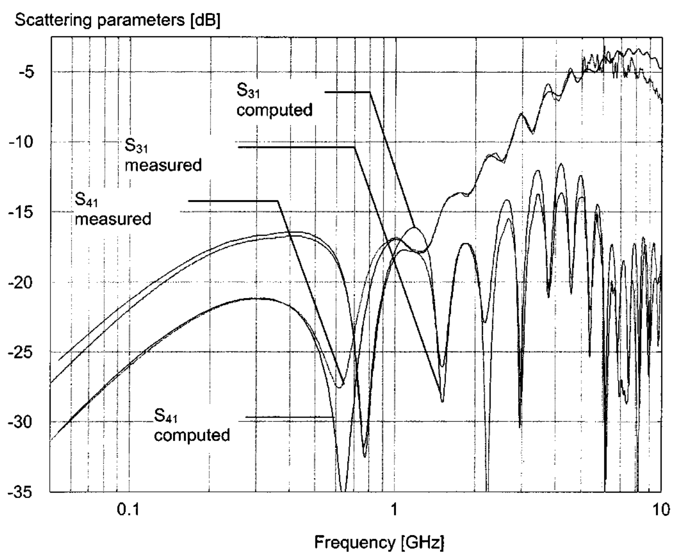


Fig. 7. Measured and computed frequency spectra of the scattering parameters S_{31} and S_{41}, in case of ground plane with a gap.

that, in this class of problems, focus has to be put on the material modeling rather than on the numerical algorithms.

IV. Numerical Analysis of Printed Transmission Lines

First, the simple differential configuration shown in Fig. 4 and considered without gap (geometric dimensions: $H = 16$ cm; $W = 9$ cm; $L = 11.5$ cm; $h = 1.5$ mm; and $w = d = 1$ mm) is investigated. The dielectric substrate is modeled either considering losses ($\varepsilon_s = 4.3$, $\varepsilon_\infty = 4.1$, $\tau = 0.033$ ns, and $\sigma = 200$ μS/m) or lossless ($\varepsilon_r = 4.11$ at the central frequency 5.025 GHz), in order to underline the influence of dielectric losses in the analysis of high-frequency signal transmission propagation. In Fig. 5, the scattering parameters S_{21}, computed reading the ground-to-line voltage at the port 2 opposite to the driving ports 1 and 3, between which a differential signal is applied, is shown in the lossless and lossy cases.

It is worth noting the importance of accounting for losses, in order to simulate correctly the behavior of the addressed structure above 2 GHz. In Fig. 6, scattering parameters S_{31} and S_{41} for the configurations without the gap are shown and compared with measured data in order to validate the numerical predictions. The geometric dimensions of the board and the dielectric parameters of the substrate are equal to that of the previous example, with a gap of 2 mm at $H/2$.

In Fig. 7, the same S-parameters are reported when the gap is present ($g = 2$ mm) on the ground plane. The FDTD results agree favorably with the measurements in the whole frequency range of interest.

V. Discontinuity Modeling

An accurate and general way to model the discontinuity is to consider the gap in the ground plane as a transmission line (slot-

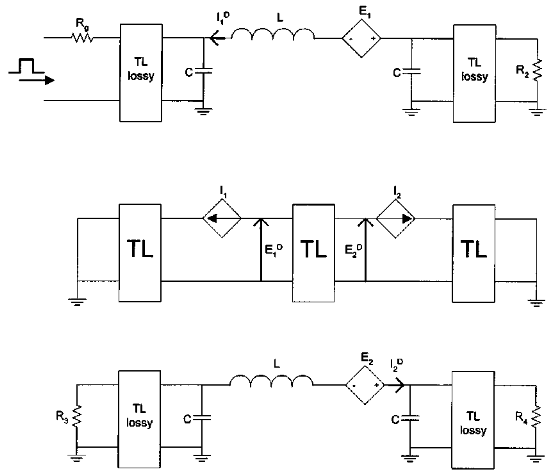


Fig. 8. Equivalent circuit for the two microstrip lines crossing a split in the reference plane.

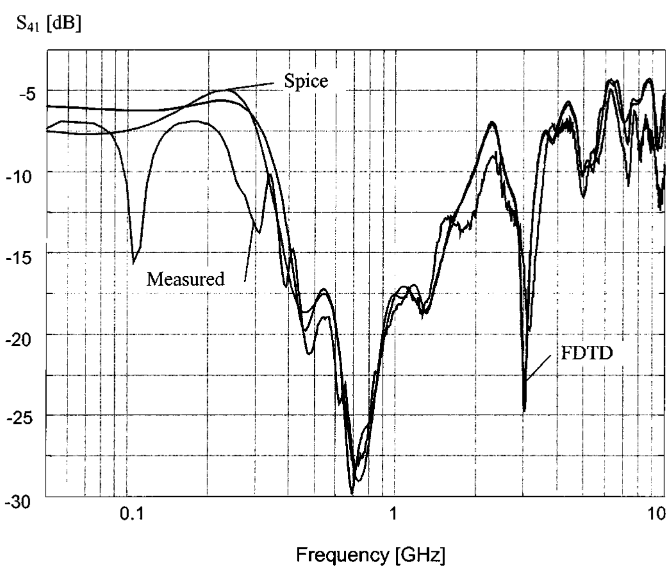


Fig. 9. Comparison among measured data, FDTD, and SPICE simulations.

line) [11], [12]. Without entering into the physics of the conversion from the so-called *microstrip mode* to the *slotline mode*, and *vice versa*, the proposed equivalent lumped circuit is shown in Fig. 8. The excess transverse capacitances C model the effects of the fringing fields which are present between the signal line and the edges of the reference plane. The excess longitudinal inductance L takes into account the effect of the partial absence of the ground plane beneath the stripline. The numerical values of C and L can be obtained following the procedure discussed in [13] and [14]. Moreover, the signal conversion from stripline mode to slotline mode and the reciprocal mode conversion are modeled, respectively, by means of the current-controlled current source $I_{1,2}$ and the voltage-controlled voltage source $E_{1,2}$ which are driven by the corresponding quantities with superscript "D." Fig. 9 demonstrates the accuracy of the results predicted by means of the equivalent circuit.

VI. Conclusion

The FDTD analysis of printed transmission lines has been carried out taking into account the real dispersive behavior of lossy substrates used in practical configurations. The correct modeling of dielectric substratess with the inclusion of the effect of lossess has been shown to be essential in actual printed circuit boards to predict the signal propagation in the gigahertz range. An error analysis of the various algorithms available to simulate the dielectric has been conducted, and the influence of dielectric characteristics on the signal propagation has been assessed. Finally, an equivalent circuit for modeling slots in the ground plane has been proposed and validated. All the numerical predictions have been validated experimentally.

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