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WTHD-Optimal Staircase Modulation of Single-Phase Multilevel Inverters

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Abstract-**This paper considers the problem of achieving the minimum frequency-weighted THD of the output voltage of multilevel inverters, when staircase modulation is utilized. Since the single-phase ac case is being addressed, the triplen harmonics need to be included in the analysis. The results show that using unequal, non-integer ratio, dc source voltages is significantly better than using equal ones, as expected. They also indicate that imposing the requirement of eliminating the lowest harmonics leads to slightly worse distortion than the minimum that is achievable.**

I. INTRODUCTION

The first multilevel power converter circuit introduced was the series cascaded H-bridge topology, which was patented almost 30 years ago [1]. Modern power semiconductor devices make this design practical for use as medium-voltage industrial drives, static VAr compensators, etc. And, in general, multilevel inverters with various topologies have become increasingly popular due to their advantages of higher-voltage capability, higher power quality, lower switching losses, and improved electromagnetic compatibility [2–9].

One advantage of the series H-bridge circuit over the others is that this topology is comprised of similar cells leading to a modular design. The original series cascaded H-bridge inverter patents prescribed the same value of dc source voltage being applied to each cell [1, 2]. Later research has shown that the overall number of output voltage levels can be increased for a given number of semiconductor devices if a binary (1:2) ratio between the dc source voltage values is used [3], to achieve lower total harmonic distortion (THD). Around the same time, the ternary (1:3) ratio was investigated and a patent obtained for general integer ratios between the dc source voltages of the H-bridge cells [4]. Other binary and ternary source voltage ratio [5, 6] designs have been proposed, and another patent has been issued for these same integer ratios [7]. Even more recently, for staircase modulation operation, [8] and [9] have proposed using non-integer dc source voltage ratios for multilevel inverters to achieve minimal total harmonic distortion (THD) and frequency-weighted total harmonic distortion (WTHD), respectively, where the latter may be a more appropriate measure than THD for inductive load applications such as motor drives. But since their focus was on three-phase applications, those two works excluded the effect of the triplen harmonics, which however cannot be ignored for single-phase applications.

In this paper, we consider the problem of achieving the minimum output voltage waveform distortion in multilevel inverters, under staircase modulation control and used for single-phase applications, when the ratios between their dc source voltages are not restricted to integer values. Although non-integer voltage ratios are typically not desirable in conjunction with PWM outputs, it can work well for staircase outputs in applications where a high-frequency fundamental component and/or a high voltage are required. The case when the lowest harmonics are to be eliminated and the case when WTHD is to be truly minimized are analyzed and compared.

II. ANALYSIS

Fig. 1 shows the familiar 2-cell series cascaded H-bridge inverter topology, which was utilized as the basic circuit for developing the results herein. Two examples of its output waveform (under staircase control) are shown in Fig. 2; when the two dc source voltages (E_1 and E_2) are equal (to E) and when they are unequal. For an output voltage waveform that is quarter-wave symmetric (as in Fig. 2) with s steps of generally unequal magnitudes E_i, $i = 1, \ldots, s$, its Fourier series expansion is given by

$$v_o(t) = \sum_{\text{odd } h} \{ V_h \sin(h\omega t) \} \qquad (1)$$

with $V_h = \frac{4}{h\pi}[E_1\cos(h\theta_1) + E_2\cos(h\theta_2) + \ldots + E_s\cos(h\theta_s)]$, where the θ_i, $i = 1, \ldots, s$, are the angles at which the s steps within the first quarter of each waveform cycle occur. Then the problem of synthesizing a stepped waveform that has a desired level of V_1 (the fundamental component) with some of the higher harmonics possibly equal to zero, is equivalent to choosing the source levels E_i, $i = 1, \ldots, s$, and the step angles $0 \le \theta_1 < \theta_2 < \ldots < \theta_s \le \pi/2$ such that

$$\tfrac{4}{\pi}[\, E_1 \cos(\theta_1) + E_2 \cos(\theta_2) + \ldots + E_s \cos(\theta_s)] = V_1 \qquad \text{(2a)}$$

$$\tfrac{4}{3\pi}[\, E_1 \cos(3\theta_1) + E_2 \cos(3\theta_2) + \ldots + E_s \cos(3\theta_s) = V_3 \qquad \text{(b)}$$

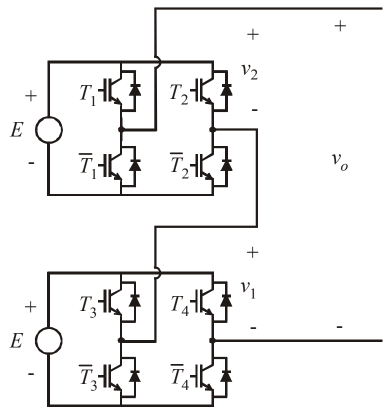


Fig. 1. Cascaded (2-cell) series H-bridge multilevel inverter with equal dc sources

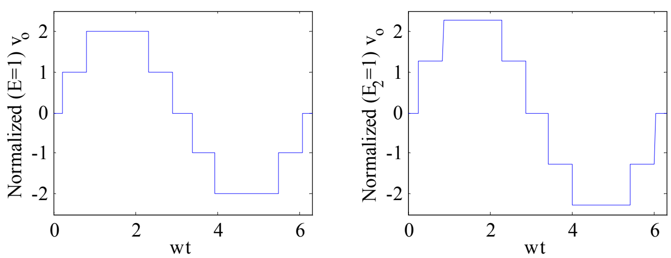


Fig. 2. 2-step 5-level waveforms with
(a) equal step levels (b) unequal step levels

Next, applying the identities $\cos(3\theta) = 4\cos(\theta)^3 - 3\cos(\theta)$, $\cos(5\theta) = \ldots$, etc., as in [10–13], and defining c_i as $\cos(\theta_i)$ and $\rho_i = E_i / E_s$, transforms (2) from a set of trigonometric equations to the set of multivariate polynomial equations

$$\sum_{i=1,..,s} \rho_i\, c_i = V_1 / \tfrac{4 E_s}{\pi} = m_1 \qquad (3a)$$

$$\sum_{i=1,..,s} \rho_i \{ 4 c_i^3 - 3 c_i \} = m_3 \qquad (b)$$

where m_1 is defined as the modulation index of the fundamental component (with respect to E_s), etc. This set of equations can now be solved exactly (to yield multiple solutions in general) using procedures based on, for example, resultant polynomials or Gröbner bases as described in [10–13]. Note that a necessary condition for the existence of nontrivial solutions to (3) is that the number of steps s per quarter cycle be greater than or equal to the number of constraint equations. Therefore, as has been typically advocated, $s-1$ of the lowest harmonics can be eliminated to reduce the waveform's distortion.

To quantify waveform distortion, let the frequency-weighted THD of the output voltage be defined (being more appropriate than THD for motor drive applications) as

$$WTHD = \sqrt{\sum_{h=2}^{\infty} (V_h / h)^2} \,/\, V_1 \qquad (4)$$

Note that in three-phase applications, the triplen harmonics do not appear in the line-line voltages under balanced conditions so (4) can be modified to exclude those harmonics [9]. In the following, the minimal distortion as measured by (4) obtained when the lowest harmonics are eliminated is compared to the case where those harmonics are not so constrained.

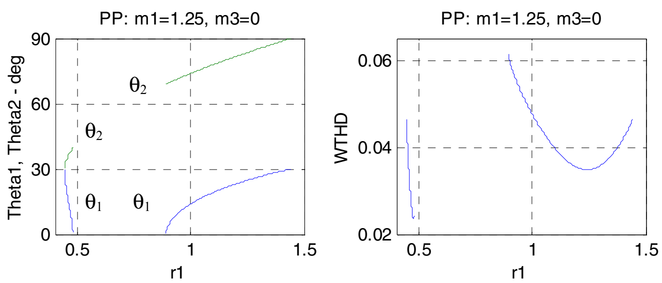
A. 5-level (2-step) waveform

1. Harmonic elimination

Considering initially the five-level output voltage case with $s = 2$ steps per quarter cycle, analysis of (3a–b) with $m_3 = 0$ (to eliminate the 3rd harmonic) yielded the following results: solutions obtained for the 2 step-angles θ_1 and θ_2 as ρ_1 varies (with $\rho_2 = 1$) for each m_1 and determination of the specific ρ_1 yielding the minimal WTHD for each m_1, as illustrated with $m_1 = 1.25$ in Fig. 3(a) and 3(b), respectively. This then leads to the plots of minimal WTHD and corresponding ρ_1, as functions of m_1, shown in Fig. 4. The minimum WTHD (based on 49 harmonics) achievable for the five-level (two-step) case, with elimination of the 3rd harmonic, is 0.014152 or 1.4152% for $\rho_1 = 1.2618$, $\theta_1 = 15.8°$, $\theta_2 = 49.5°$, so that $m_1 = 1.8632$. Note that the discontinuous slope in the WTHD plot occurring at about $m_1 = 0.975$ is due to the change from having solutions of (3a–b) over one range of ρ_1 to having solutions over two ranges of ρ_1 (as for example with $m_1 = 1.25$). This result can be contrasted to the equal source five-level output case, where the minimal WTHD (with elimination of the 3rd harmonic) of 1.4999% was found to be achieved for $m_1 = 1.6607$ using $\theta_1 = 13.5°$, $\theta_2 = 46.5°$, which is worse by 5.86%.

2. WTHD optimization

Consider next the analysis of (3) with a possibly non-zero 3rd harmonic to minimize the five-level output voltage waveform's WTHD, i.e., the problem was to determine the (ρ_1, θ_1, θ_2) yielding the lowest WTHD for any m_1 with no constraints on the higher harmonics. The optimization procedure, based on the Newton-Raphson method, used the solutions obtained from the harmonic elimination case as the initial guesses. For equal sources, the minimum WTHD (based on 49 harmonics) is 1.4989% for $\theta_1 = 13.4°$ and $\theta_2 = 46.3°$, with $m_1 = 1.6630$. For unequal sources, the minimum WTHD is 1.4099% for the dc source voltage ratio of E_1:E_2 = 1.275:1 and $\theta_1 = 15.7°$, $\theta_2 = 49.3°$, with $m_1 = 1.8796$. This represents a 5.93% reduction in the minimum achievable distortion. Note that these solutions result in a non-zero but small 3rd harmonic.



(a) (b)

Fig. 3. (a) Step-angle solutions for varying ρ_1
(b) Corresponding WTHD for varying ρ_1

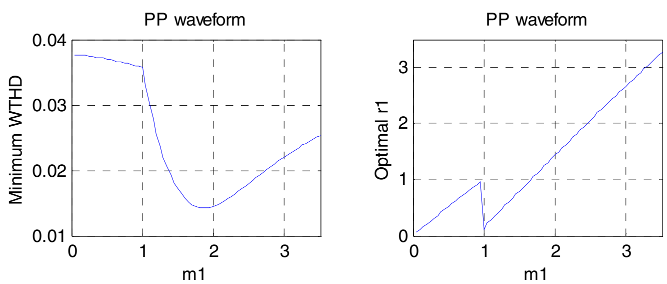


Fig. 4. Minimum WTHD and corresponding optimal ρ_1 versus m_1, harmonic eliminating case

B. 7-level (3-step) waveform

1. Harmonic elimination

Considering next the seven-level output voltage case with $s = 3$ steps per quarter cycle, analysis of (3) has resulted in the following: solutions for the 3 step-angles θ_1, θ_2 and θ_3, as ρ_1 and ρ_2 vary (with $\rho_3 = 1$) for each m_1 (with $m_3 = m_5 = 0$ to eliminate the 3rd and 5th harmonics) and determination of the specific ρ_1 and ρ_2 yielding the minimal WTHD for each m_1. Requiring elimination of the 3rd and 5th harmonics yields minimal WTHD of 0.68478% achieved with $\rho_1 = 1.4762$, $\rho_2 = 1.3364$, and $\theta_1 = 10.8°$, $\theta_2 = 33.5°$, $\theta_3 = 59.2°$, so $m_1 = 3.075$. This result can be contrasted to the equal source seven-level output case, where the minimal WTHD (with elimination of the 3rd and 5th harmonics) of 0.77184%, which is worse by 12.7%, can be achieved using $\theta_1 = 9.08°$, $\theta_2 = 28.5°$, $\theta_3 = 55.1°$, so $m_1 = 2.439$.

2. WTHD optimization

Without the harmonic elimination requirement, it was determined that the minimal WTHD of 0.68088% for the unequal seven-level waveform can be achieved with $\rho_1 = 1.4744$, $\rho_2 = 1.3406$, and $\theta_1 = 10.9°$, $\theta_2 = 33.4°$, $\theta_3 = 58.7°$, so $m_1 = 3.087$. On the other hand, having equal sources yields minimal WTHD of 0.76565%, which is achieved with $\theta_1 = 9.26°$, $\theta_2 = 28.6°$, $\theta_3 = 54.5°$, for $m_1 = 2.446$; so the minimal WTHD is worse by 12.5%.

C. 9-level (4-step) waveform

1. Harmonic elimination

Considering next the nine-level output voltage case with $s = 4$ steps per quarter cycle, analysis of (3) has resulted in the following: solutions for the 4 step-angles θ_1, θ_2, θ_3 and θ_4, as ρ_1, ρ_2 and ρ_3 vary (with $\rho_4 = 1$) for each m_1 (with $m_3 = m_5 = m_7 = 0$) and determination of the specific ρ_1, ρ_2 and ρ_3 yielding the minimal WTHD for each m_1. Requiring elimination of the 3rd, 5th and 7th harmonics yields minimal WTHD of 0.401344% achieved with $\rho_1 = 1.6568$, $\rho_2 = 1.5866$, $\rho_3 = 1.3566$, and $\theta_1 = 8.47°$, $\theta_2 = 25.6°$, $\theta_3 = 43.7°$, $\theta_4 = 64.8°$, so $m_1 = 4.4766$. This result can be contrasted to the equal source nine-level output case, where the minimal WTHD (with elimination of the 3rd, 5th and 7th harmonics) of 0.488647%, which is worse by 21.8%, can be achieved using $\theta_1 = 7.38°$, $\theta_2 = 21.7°$, $\theta_3 = 36.8°$, $\theta_4 = 60.2°$, so $m_1 = 3.2188$.

2. WTHD optimization

Without the harmonic elimination requirement, it was determined that the minimal WTHD of 0.39862% for the unequal nine-level waveform can be achieved with $\rho_1 = 1.6563$, $\rho_2 = 1.5684$, $\rho_3 = 1.3709$, and $\theta_1 = 8.37°$, $\theta_2 = 25.4°$, $\theta_3 = 43.5°$, $\theta_4 = 64.1°$, so $m_1 = 4.4855$. On the other hand, having equal sources yields minimal WTHD of 0.471508%, which is achieved with $\theta_1 = 6.98°$, $\theta_2 = 21.4°$, $\theta_3 = 37.3°$, $\theta_4 = 59.3°$, for $m_1 = 3.2299$; so the minimal WTHD is worse by 18.3%.

Table 1 summarizes the minimum WTHD achievable for the various cases. In addition, a comparison of the percentage-amplitudes of the lowest harmonics for the unequal nine-level waveform harmonic eliminating case to the corresponding harmonic percentage-amplitudes for the WTHD-optimal case is shown in Fig. 5. It indicates that the lowest harmonics of the WTHD-optimal waveform have non-zero but fairly small amplitudes.

III. CONCLUSIONS

This paper has considered the problem of determining the minimum achievable WTHD by staircase modulation of multilevel inverters, with and without the requirement of (lowest) harmonic elimination. Those WTHD values, together with the corresponding necessary step angles and dc source ratios, have been obtained for the 5-level (2-step), 7-level (3-step), and 9-level (4-step) output voltage waveform cases. The results show that the use of unequal, non-integer ratio, dc source voltages can achieve significantly lower minimal WTHD than if equal source voltages were used, as expected; furthermore, the percentage amount of this improvement increases as the number of waveform levels increases from five to nine. In addition, the results also show that requiring the elimination of the lowest harmonics leads to slightly worse WTHD than if this requirement was not imposed, although this difference does increase as the number of levels increases from five to nine. Finally, the results indicate that the lowest harmonics of the WTHD-optimal waveform have non-zero but fairly small amplitudes and that, notably, these are lower than the amplitudes obtained by optimizing instead with respect to the usual THD measure as indicated in Fig. 5.

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TABLE I

MINIMUM ACHIEVABLE WTHD

	With harmonic elimination		Without harmonic elimination	
	Equal dc sources	Unequal dc sources	Equal dc sources	Unequal dc sources
5-level (2-step)	1.4999%	1.4152%	1.4989%	1.4099%
7-level (3-step)	0.77184%	0.68478%	0.76565%	0.68088%
9-level (4-step)	0.48865%	0.40134%	0.47151%	0.39862%

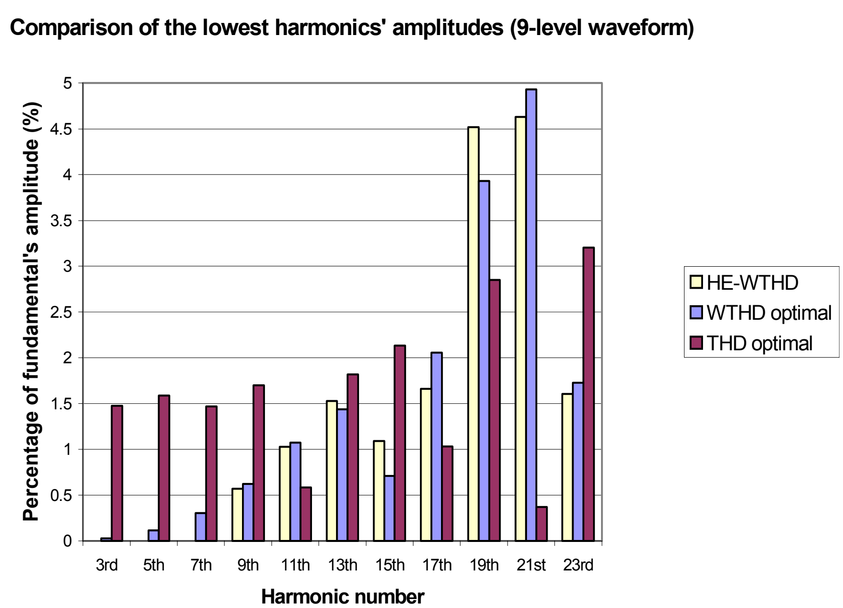

Fig. 5. Comparison of the percentage-amplitudes of the lowest harmonics.