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Quantum gravitational corrections to black hole geometries

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We calculate perturbative quantum gravity corrections to eternal two-dimensional black holes. We estimate the leading corrections to the AdS_2 black hole entropy and determine the quantum modification of N-dimensional Schwarzschild spacetime.

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I. INTRODUCTION

In recent years, the investigation of two-dimensional black holes has earned a great deal of attention because of its relevance to the study of classical and quantum properties of (higher-dimensional) black holes, branes, string theory, and gravitational collapse. (For recent reviews, see, e.g., [1] and [2].)

Black holes in two dimensions are usually described by dilaton gravity theories. In the absence of matter fields[1] two-dimensional dilaton gravity describes eternal black holes. Matterless dilaton gravity enjoys a number of interesting properties [3]: (a) The field equations are completely integrable; (b) all solutions are static, i.e., depend on a single coordinate (generalized Birkhoff theorem); (c) a local gauge-invariant integral of motion M exists; (d) the moduli space of the theory is one dimensional and is described by the modular parameter M. The properties (a)–(d) imply that matterless two-dimensional dilaton gravity is a topological theory with no propagating degrees of freedom. Owing to the Birkhoff theorem the physics on the gauge shell is completely determined by the degrees of freedom on the spacetime boundary. The bulk field modes describe pure gauge degrees of freedom: All physical information is contained in the boundary, where the conserved charge is defined. If matter fields are present, the properties (a)–(d) do not generally hold.

It is worth noticing that a nontrivial relation between local pure gauge degrees of freedom in the bulk and topological degrees of freedom on the boundary has been suggested for topological theories [4,5]. In particular, Carlip has found an explicit realization of this relation for three-dimensional gravity [4]. The idea is that the gauge invariance of three-dimensional gravity is broken by the presence of the boundary. As a consequence, pure gauge degrees of freedom may become dynamical, i.e., off shell, and induce physical degrees of freedom on the boundary. A similar result may be valid for two-dimensional matterless dilaton gravity.

At the quantum level, two main different lines of research have been pursued in the literature. In the *semiclassical approach* matter fields are quantized and gravity is classical. The source term in the Einstein equations is the vacuum expectation value of the stress energy tensor of matter fields evaluated on the classical geometry. Perturbative quantum corrections to the classical metric are obtained by solving the equations at the lowest order in $\hbar$. (See, for instance, [6,7].) Following Hawking's discovery that black holes emit thermal radiation [8], a large number of semiclassical toy models have been used to describe the evolution of evaporating black holes. (See, e.g., [9–11].) In the *quantum gravity formalism* the gravitational sector of the theory is quantized. This approach has been followed by a number of authors [12–15], with seemingly contradictory results. On one hand, Poisson sigma models [12] and first-order path integral quantization [13] of dilaton gravity in the absence of matter fields seem to indicate that quantum corrections to the effective action on shell are zero at all orders of perturbation theory. On the other hand, nonlinear sigma model methods [14,15], quantization of gravity as an effective field theory [16,17], and matrix theory [18] lead to nonzero quantum deformations of the classical metric. Although a comparison between the different approaches is difficult for technical reasons, the origin of the disagreement is most likely due to the different treatment of boundary terms: If pure gauge degrees of freedom become dynamical, as suggested in [4,5], the quantum corrections to the effective action are generally nonzero already at one loop [19–21]. (See also [2].)

In this paper we follow a third approach to the quantization of two-dimensional black holes. Since our aim is to compute pure quantum gravity corrections to the classical geometry of (eternal) black holes, i.e., to the mass M, we quantize the moduli space of the theory. The moduli space approximation is a standard procedure that can be used when there is a continuous family of static solutions. (See, e.g., [22] and references therein.) The action for the moduli space is obtained by substituting the static solutions in the original action with the moduli represented as spacetime fields. The moduli action for two-dimensional black holes coincides with the nonlinear sigma model formulation of two-dimensional dilaton gravity [3]. Thus the quantization of

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[1]Here, *matter field* means any field but the gravitational and dilaton fields.

two-dimensional black hole geometries is formally equivalent to the quantization of the string in curved spacetime [23]. By applying the perturbative quantization algorithm described in Ref. [15] we evaluate pure quantum gravity corrections to the geometry of eternal dilatonic black holes. Quantum corrections to the classical solutions are obtained by a perturbative expansion in powers of the curvature. Quantum gravitational effects vanish in the limit of large Arnowitt-Deser-Misner (ADM) mass and in the asymptotic region far away from the black hole horizon where the black hole behaves classically. Quantum effects become significant instead at finite distances from the black hole horizon. This leads to a modification of the physical quantities that are associated with the classical geometry.

The structure of the paper is as follows. In the next section we briefly review the classical theory and find the moduli action for two-dimensional eternal black holes. The latter is quantized in Sec. III. As two illustrative examples we estimate the leading quantum gravity corrections to the Bekenstein-Hawking entropy of the AdS_2 black hole and to N-dimensional spherically symmetric gravity. In Sec. IV we state our conclusions.

II. CLASSICAL SETTING AND MODULI SPACE

In the Schwarzschild gauge two-dimensional eternal black holes are described by the metric

$$ds^2_{(2)} = -[N(x)-M]dt^2 + [N(x)-M]^{-1}dx^2, \qquad (1)$$

where M is a constant parameter (modulus) and $N(x)$ is a function of the spacelike coordinate x. The simplest theory that admits Eq. (1) as a general solution is

$$S_G = \int d^2x \sqrt{-g}[\phi R + V(\phi)]. \qquad (2)$$

R is the two-dimensional Ricci scalar constructed from the metric $g_{\mu\nu}(x)$ and ϕ is a scalar field, which is usually called the ''dilaton'' in the literature. In light cone conformal coordinates the general solution of Eq. (2) is [24,3]

$$ds^2_{(2)} \equiv g_{\mu\nu}dx^\mu dx^\nu = 4[N(\phi)-M]du\,dv, \qquad (3)$$

where M is a constant (modulus) and ϕ is a function of $x \equiv u+v$ defined by the differential equation

$$\frac{dx}{d\phi} = \frac{1}{N(\phi)-M}, \quad N(\phi) = \int^\phi d\phi' V(\phi'). \qquad (4)$$

Equation (3) coincides, in the Schwarzschild gauge, with Eq. (1). In the following we will restrict attention to the power-law dilaton potential

$$V(\phi) = b\phi^{b-1}, \qquad (5)$$

where b is a positive real number. The constant parameter M is proportional to the ADM mass. The geometry is singular at $\phi=0$ and asymptotically flat as $\phi\to\infty$. The black hole event horizon is located at $\phi=\phi_h=M^{1/b}$.

The action (2) can also be used to describe the two-dimensional effective theory of N-dimensional spherically symmetric gravity. Setting

$$ds^2_{(N)} = \phi^{-b} g_{\mu\nu}dx^\mu dx^\nu + \phi^{2(1-b)}d\Omega^2_{N-2}, \qquad (6)$$

the general solution in the Schwarzschild gauge reads [15]

$$ds^2_{(N)} = -(1-J/r^{N-3})dt^2 + \frac{dr^2}{(1-J/r^{N-3})} + r^2 d\Omega^2_{N-3}, \qquad (7)$$

where the parameter J is related to the ADM mass by

$$J = \frac{16\pi l_{\rm Pl}^{N-2}}{(N-2)V_{N-2}} M_{\rm ADM}, \qquad (8)$$

and V_{N-2} is the volume of the $(N-2)$-dimensional unit sphere. ($l_{\rm Pl}$ is the N-dimensional Planck length.)

In Ref. [3] it is shown that the theory (2) can be formulated as a nonlinear sigma model. We introduce the new field

$$M(\phi, g_{\mu\nu}) = N(\phi) - \partial_\mu\phi\partial^\mu\phi. \qquad (9)$$

$M(\phi, g_{\mu\nu})$ is invariant under gauge transformations and is locally conserved on shell, where it coincides with the parameter M in Eq. (1). Neglecting inessential surface terms Eq. (2) can be cast in the form

$$S_\sigma = \frac{1}{2}\int d^2x\, G_{ij}(\xi)\partial_\mu\xi^i\partial^\mu\xi^j, \qquad (10)$$

where $\xi^i = (M, \phi)$ and the metric of the target space is

$$G_{ij} = \frac{1}{N(\xi^1)-\xi^0}\Omega_{ij}, \quad \Omega_{ij} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \qquad (11)$$

G_{ij} is singular at the black hole horizon and asymptotically flat for $\phi\to\infty$. Equation (10) coincides with the moduli action of the theory. The latter is defined as [remember Eq. (3)]

$$S_{\rm moduli} \equiv S_G[g_{\mu\nu} = \rho(x)\eta_{\mu\nu}], \qquad (12)$$

where $\rho(x) = N[\phi(x)] - M(x)$. Using Eq. (9) in Eq. (12) we find $S_{\rm moduli} = S_\sigma$. The moduli action (10) is our starting point in the calculation of quantum corrections to black hole geometries.

III. QUANTIZATION

The moduli action can be quantized perturbatively by expanding the metric of the target space in Riemann normal coordinates ζ [25]:[2]

[2]For the derivation and a detailed discussion of Riemann normal coordinates see, e.g., Ref. [26].

$$G_{ij}(\xi)=G_{ij}[\xi(0)]-\frac{1}{3}R_{ikjl}[\xi(0)]\zeta^k\zeta^l - \frac{1}{3!}R_{ikjl;m}[\xi(0)]\zeta^k\zeta^l\zeta^m+\cdots, \qquad (13)$$

where R_{ikjl} is the Riemann tensor. The Riemann coordinates are (up to second order in the Riemann expansion)

$$\xi^i=\xi^i(0)+\zeta^i-\frac{1}{2}\Gamma^i_{jk}[\xi(0)]\zeta^j\zeta^k - \frac{1}{3!}\Gamma^i_{jkl}[\xi(0)]\zeta^j\zeta^k\zeta^l+\cdots, \qquad (14)$$

where

$$\Gamma^i_{jkl}\equiv\tfrac{1}{3}P(\Gamma^i_{jk,l}-2\Gamma^i_{\alpha j}\Gamma^\alpha_{kl}), \qquad (15)$$

and P indicates the sum of terms obtained by permuting the subscripts cyclically. The corrections to the fields are defined as

$$\delta\xi^i\equiv\zeta^i-\frac{1}{2}\Gamma^i_{jk}[\xi(0)]\zeta^i\zeta^k-\frac{1}{3!}\Gamma^i_{jkl}[\xi(0)]\zeta^j\zeta^k\zeta^l+\cdots. \qquad (16)$$

The perturbative action follows by inserting the Riemann expansion (13) in Eq. (10):

$$S_{\sigma,\mathrm{pert}}=\frac{1}{2}\int d^2x[\partial_\mu\zeta^i\partial^\mu\zeta^j\Omega_{ij}+g(\zeta)\epsilon_{ij}\epsilon_{kl}\partial_\mu\zeta^i\partial^\mu\zeta^k\zeta^j\zeta^l], \qquad (17)$$

where ϵ_{ij} is the two-dimensional Levi-Civita tensor density and $g(\zeta)$ is the (ζ-dependent) coupling constant,

$$g(\zeta)=\sum_{m,n=0} g_{mn}(\phi_0,M_0)(\zeta^1)^m(\zeta^0)^n. \qquad (18)$$

Because of the curvature singularity of the target metric (11), all terms in the coupling constant expansion $g(\zeta)$ diverge on the black hole horizon, the latter corresponding to the strongly coupled regime of the theory. Therefore the black hole horizon cannot be described by the perturbative expansion (17).

In Eq. (17) we have rescaled the fields ζ^i by a factor:

$$\zeta\rightarrow\zeta'=\zeta[N(\phi_0)-M_0]^{-1/2}, \qquad (19)$$

and have dropped the primes for convenience. At the leading order in the perturbative region ($\phi_0\rightarrow\infty$) the coupling constants g_{mn} are

$$g_{mn}\approx\phi_0^{-bn/2+m(b/2-1)-1}. \qquad (20)$$

Equation (17) describes an interacting string which propagates in a curved two-dimensional target space. The first-order coupling constant $g_{00}\approx\phi_0^{-1}$ can be interpreted as the (coordinate-dependent) coupling constant of the theory (2). Thus the perturbative expansion (17) is both a weak-coupling expansion in terms of the coordinate-dependent gravitational coupling of the model and an expansion near the boundary at $\phi_0\rightarrow\infty$.

Using Eqs. (11), (16), and (19), the asymptotic corrections to the physical fields can be expressed as functions of ϕ_0 and ζ^i. For instance, at the third order in the Riemann expansion we have

$$\delta M\equiv\delta\xi^0\approx\phi_0^{b/2}\zeta^0+A(\zeta^0)^2+B\phi_0^{-b/2}(\zeta^0)^3 + C\phi_0^{b/2-1}(\zeta^0)^2\zeta^1+D\phi_0^{-b}(\zeta^0)^4+E\phi_0^{-1}(\zeta^0)^3\zeta^1 + F\phi_0^{b-2}(\zeta^0)^2(\zeta^1)^2+\cdots \qquad (21)$$

and

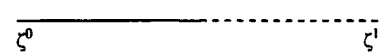
$$\delta\phi\equiv\delta\xi^1\approx\phi_0^{b/2}\zeta^1+A'\phi_0^{b-1}(\zeta^1)^2+B'\phi_0^{3b/2-2}(\zeta^1)^3 + C'\phi_0^{b/2-1}(\zeta^1)^2\zeta^0+D'\phi_0^{2b-3}(\zeta^1)^4+E'\phi_0^{b-2}(\zeta^1)^3\zeta^0 + F'\phi_0^{-1}(\zeta^1)^2(\zeta^0)^2+\cdots, \qquad (22)$$

for M and ϕ, respectively.

From Eq. (17) it is straightforward to read the Feynman rules of the theory. The free propagator is antidiagonal in the fields:

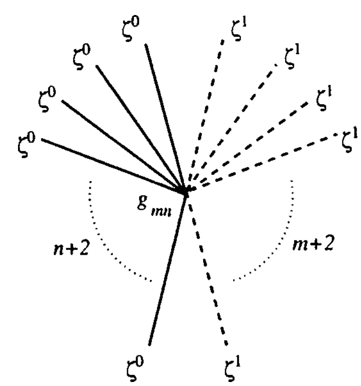
$$\langle\zeta^i(x_1)\zeta^j(x_2)\rangle=-\Omega^{ij}\frac{1}{4\pi}\ln[(x_1-x_2)^2]. \qquad (23)$$

Graphically,



(24)

For each interaction term in Eq. (17) we have a vertex



(25)

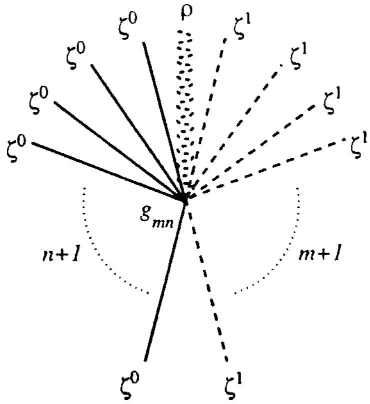
which is generated by the interaction term of order $k=n+m+1$ in the Riemann expansion.

Classical conformal invariance of the moduli action is broken at the quantum level. In the conformal gauge in $d=2-\epsilon$ dimensions the (renormalized) moduli action is

$$S=\int d^dx\, e^{-\epsilon\rho/2}\mathcal{L}_{\mathrm{ren}}+S_\rho+S_{\mathrm{ghost}}, \qquad (26)$$

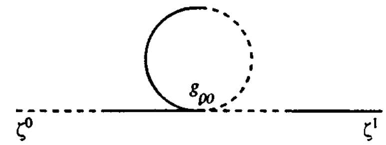
where S_ρ is the effective action due to the Weyl anomaly and $\mathcal{L}_{\mathrm{ren}}$ is the renormalized Lagrangian.

At one loop the breaking of conformal invariance leads to the following anomalous vertices:



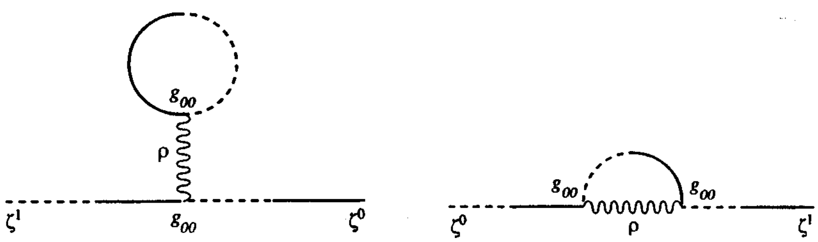
(27)

We can calculate the corrections to the free propagator due to the interaction vertices of the string (25) and (27). At first order in the Riemann curvature expansion the topology of the interaction implies that the propagator is antidiagonal at any loop. The one-loop correction to the two-point Green function is given by the Feynman graph



(28)

and by the two anomalous graphs



(29)

The two anomalous graphs are subdominant with respect to the contribution in Eq. (28). Following Ref. [15] the one-loop renormalized two-point function is

$$\langle \zeta^i(x_1)\zeta^j(x_2)\rangle = -\Omega^{ij}\frac{1}{4\pi}\left[1-\frac{g_{00}}{4\pi}\ln\left(\frac{\mu}{m}\right)^2+O(g_{00}^2)\right]\times\ln[(x_1-x_2)^2]. \tag{30}$$

(The leading nonzero diagonal contribution to the propagator is obtained at two loops and at third order in the Riemann curvature expansion.) It is straightforward to evaluate the contributions to higher-order Green functions. Here we will give the results for three- and four-point functions at one loop and up to third order in the Riemann expansion. The leading contribution to the three-point Green functions is given at one loop by Feynman graphs with a single five-point vertex:

$$\langle \zeta^0(x_1)\zeta^0(x_2)\zeta^1(x_3)\rangle \approx O(\phi_0^{b/2-2}),$$

$$\langle \zeta^0(x_1)\zeta^1(x_2)\zeta^1(x_3)\rangle \approx O(\phi_0^{-b/2-1}). \tag{31}$$

(The leading nonvanishing contribution to the three-point Green functions with identical external legs is obtained at two loops and at fourth order in the Riemann expansion.) At one loop the nonvanishing four-point Green functions are

$$\langle \zeta^0(x_1)\zeta^0(x_2)\zeta^0(x_3)\zeta^1(x_4)\rangle \approx O(\phi_0^{b-3}),$$

$$\langle \zeta^0(x_1)\zeta^0(x_2)\zeta^1(x_3)\zeta^1(x_4)\rangle \approx O(\phi_0^{-1}),$$

$$\langle \zeta^0(x_1)\zeta^1(x_2)\zeta^1(x_3)\zeta^1(x_4)\rangle \approx O(\phi_0^{-b-1}). \tag{32}$$

The (one-loop) perturbative quantum corrections to the classical geometry follow from Eqs. (21), (22) and Eqs. (30)–(32). At third order in the Riemann expansion we find

$$\langle \delta M\rangle_{\text{one-loop}} \approx C\phi_0^{b/2-1}\langle(\zeta^0)^2\zeta^1\rangle + E\phi_0^{-1}\langle(\zeta^0)^3\zeta^1\rangle + F\phi_0^{b-2}\langle(\zeta^0)^2(\zeta^1)^2\rangle \approx O(\phi_0^{b-3}), \tag{33}$$

$$\langle \delta\phi\rangle_{\text{one-loop}} \approx C'\phi_0^{b/2-1}\langle(\zeta^1)^2\zeta^0\rangle + E'\phi_0^{b-2}\langle(\zeta^1)^3\zeta^0\rangle + F'\phi_0^{-1}\langle(\zeta^1)^2(\zeta^0)^2\rangle \approx O(\phi_0^{-2}). \tag{34}$$

Finally, from the free perturbative action we have

$$\langle \delta M\ \delta\phi\rangle_{\text{one-loop}} \approx O(\phi_0^b). \tag{35}$$

The leading contribution to $\langle \delta M\ \delta\phi\rangle_{\text{one-loop}}$ is given at zeroth order in the Riemann expansion by the term $\phi_0^b\zeta^0\zeta^1$. Nonzero values of $\langle \delta M\rangle_{\text{one-loop}}$ and $\langle \delta\phi\rangle_{\text{one-loop}}$ are first generated at second order in the Riemann expansion when ζ^3 terms appear in the expansions (21) and (22) and the three-point Green functions are nonvanishing. The corrections to $\langle \delta M\rangle_{\text{one-loop}}$ and $\langle \delta\phi\rangle_{\text{one-loop}}$ that are generated by the third-order term in the Riemann expansion are of the same order as the corrections generated by the second-order term. This is due to the presence of the $(\zeta^0)^2(\zeta^1)^2$ term in the expansions

(21) and (22), which is nonzero at tree level. Higher-order corrections that follow from higher-order terms in the Riemann expansion are subleading.

Using Eqs. (33)–(35) we can compute the corrections to the classical metric (1). In the (t,ϕ) gauge we have

$$\delta g_{\mu\nu}\approx\begin{pmatrix}\phi_0^b\left[\frac{b\ \delta\phi}{\phi_0}-\frac{\delta M}{\phi_0^b}\right] & 0\\ 0 & \frac{1}{\phi_0^b}\left[\frac{b\ \delta\phi}{\phi_0}-\frac{\delta M}{\phi_0^b}\right]\end{pmatrix}. \quad (36)$$

Since no cross term $\delta M\ \delta\phi$ appears in Eq. (36) the leading corrections to the classical geometry are generated at second order in the Riemann expansion. We shall see below that the leading quantum corrections to the N-dimensional Schwarzschild black hole are given instead by Eq. (35).

At one loop the leading quantum corrections to $g_{\mu\upsilon}$ are

$$\langle\delta g_{\mu\nu}\rangle_{1\ \text{loop}}=\begin{pmatrix}O(\phi_0^{b-3}) & 0\\ 0 & O(\phi_0^{-b-3})\end{pmatrix}. \quad (37)$$

These corrections modify the asymptotic structure of the spacetime and have important consequences for physical quantities that are defined on the boundary. The quantum-corrected line element at one loop is

$$ds^2_{1\ \text{loop}}=[\phi^b-M+O(\phi^{b-3})]dt^2 -[\phi^b-M+O(\phi^{b-3})]^{-1}d\phi^2. \quad (38)$$

This metric is a solution of the effective action

$$S_{\text{eff}}=\int d^2x\sqrt{-g}[\phi R+V_{\text{eff}}(\phi)], \quad (39)$$

where

$$V_{\text{eff}}(\phi)=b\phi^{b-1}+O(\phi^{b-4}). \quad (40)$$

Let us now discuss two specific models.

A. $b=2$: $\mathbf{AdS_2}$

The case $b=2$ is rather interesting. If $b=2$ Eq. (2) describes the Jackiw-Teitelboim model [27] and the classical metric is of anti–de Sitter (AdS) type. This describes the near-horizon limit of four-dimensional Reissner-Nordström black holes close to extremality [28–30]. Thanks to the AdS conformal field theory (CFT) correspondence, dilaton gravity on AdS_2 is dual to a conformal field theory defined on the AdS_2 boundary: The gravitational dynamics can be described by the microscopic degrees of freedom of the CFT [31,32]. In particular, the statistical entropy of two-dimensional AdS black holes can be derived by counting the microstates on the one-dimensional (timelike) boundary of AdS_2 [33–35]. The result is

$$S_{\text{AdS}}=2\pi\sqrt{M}. \quad (41)$$

In Ref. [36] Cadoni and Mignemi have generalized the previous calculation to models with asymptotic dilatonic potential

$$V(\phi)=2\phi+O(\phi^{-2}). \quad (42)$$

They find that the black hole entropy coincides with the AdS entropy at the leading order. So the statistical entropy of the two-dimensional AdS black hole $(b=2)$ is not modified by (perturbative) one-loop quantum gravity effects at the leading order. If we trust the thermodynamical derivation of the black hole entropy [37–39] beyond the semiclassical regime, the one-loop thermodynamical entropy of the AdS black hole is, in the large mass limit,

$$S_{\text{AdS,1 loop}}=2\pi\sqrt{M}+O(M^{-1}). \quad (43)$$

Therefore, quantum gravity corrections to the thermodynamical entropy are subdominant to the logarithmic corrections that follow from the Cardy formula [40,41].

B. $b=(N-3)/(N-2)$: N-dimensional Schwarzschild black hole

Let us now discuss the quantum corrections to the N-dimensional Schwarzschild black hole. Because of the presence of the two-dimensional conformal factor $[\phi(x)]^{-b}$ in Eq. (6), the expectation values of $g_{tt}^{(N)}$ and $g_{rr}^{(N)}$ involve the term $\langle\delta M\ \delta\phi\rangle$. This term gives the leading quantum corrections in the perturbative limit $\phi_0\to\infty$. The quantum-corrected geometry is

$$ds^2=-\left\{1-\frac{J}{r^{N-3}}\left[1+O\left(\frac{1}{r}\right)\right]\right\}dt^2 +\frac{dr^2}{1-(J/r^{N-3})[1+O(1/r)]}+r^2d\Omega^2_{N-2}. \quad (44)$$

Higher-order corrections in the Riemann expansion are subleading with respect to the $O(1/r)$ terms in Eq. (44). We have evaluated one-loop corrections up to third order. Second- and third-order quantum effects contribute to $\langle g_{tt}\rangle$ and $\langle g_{rr}^{-1}\rangle$ with terms of order $O(r^{-3N+6})$ and $O(r^{-4N+9})$. The spherical part of the $(N-2)$-dimensional metric receives corrections of order $O(r^{-3N+8})$ and $O(r^{-4N+10})$.

Comparing Eq. (44) to the classical metric (7) we can define an effective ADM mass:

$$M_{\text{ADM,eff}}=M_{\text{ADM,cl}}+O\left(\frac{1}{r}\right). \quad (45)$$

Thus the classical ADM mass receives quantum corrections of order $O(1/r)$, in agreement with the four-dimensional analysis of [14] and [16–18]. In four dimensions quantum gravitational effects are formally equivalent to an electric charge: Asymptotically, Eq. (44) describes a Reissner-Nordström black hole. For $N>4$ the $O(r^{-N+2})$ term in Eq. (44) dominates over the Reissner-Nordström charge term of order $O(r^{-2(N-3)})$ [42].

IV. CONCLUSIONS

In this paper we have evaluated the leading quantum gravitational corrections to eternal two-dimensional black hole spacetimes. We have found that quantum gravitational corrections to the classical geometry are finite and nonzero. Let us stress that the existence of nonzero quantum corrections to classical geometries may be crucial in understanding unsolved issues in quantum gravity such as black hole evaporation, loss of coherence, and/or gravitational collapse.

Our results have been obtained by quantizing the moduli action of two-dimensional dilaton gravity. The action for the moduli fields of eternal black holes is described by a conformal nonlinear sigma model with a fixed target metric. The fields are the dilaton and the spacetime-dependent modulus M. The sigma model describes a two-dimensional string propagating in a two-dimensional curved spacetime. Hence, the theory can be quantized perturbatively by expanding the metric of the target space in normal Riemann coordinates. Since the expansion parameter is proportional to the curvature of the manifold, the theory becomes asymptotically free at large distances from the black hole horizon(s), where the perturbative regime is valid (weak-coupled region). Finally, we presented two specific models. We estimated the leading corrections to the Bekenstein-Hawking entropy of the AdS_2 black hole and the quantum gravitational modification of the N-dimensional Schwarzschild black hole. For the Schwarzschild black hole our results confirm the previous four-dimensional analysis of [14] and [16–18].

Let us conclude this paper by mentioning two possible extensions of our results. The perturbative approach presented above breaks down on the black hole horizon where the gravitational theory becomes strongly coupled. Therefore it cannot be used to discuss quantum corrections to the geometry in the near-horizon region. This can possibly be accomplished either by using duality symmetries of the dilaton gravity theory (see, e.g., [43]) or by formulating an alternative sigma model description with nonsingular target metric. Finally, it is of primary importance to explore the effects of the inclusion of matter fields in the action (2), and investigate black hole evaporation in a full quantum gravitational context.

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