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Vitor Cardoso

Marco Cavaglia

Missouri University of Science and Technology, cavagliam@mst.edu

Mario Pimenta

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**Gravitational diffraction radiation**Vitor Cardoso<sup>\*</sup>*Department of Physics and Astronomy, University of Mississippi, University, Mississippi 38677-1848, USA*Marco Cavaglia<sup>†</sup>*Department of Physics and Astronomy, University of Mississippi, University, Mississippi 38677-1848, USA*Mário Pimenta<sup>‡</sup>*Departamento de Física, IST, Avenida Rovisco Pais, 1049-001 Lisboa, Portugal**LIP, Avenida Elias Garcia, 14-1, 1000-149 Lisboa, Portugal*

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We show that if the visible universe is a membrane embedded in a higher-dimensional space, particles in uniform motion radiate gravitational waves because of spacetime lumpiness. This phenomenon is analogous to the electromagnetic diffraction radiation of a charge moving near to a metallic grating. In the gravitational case, the role of the metallic grating is played by the inhomogeneities of the extra-dimensional space, such as a hidden brane. We derive a general formula for gravitational diffraction radiation and apply it to a higher-dimensional scenario with flat compact extra dimensions. Gravitational diffraction radiation may carry away a significant portion of the particle's initial energy. This allows to set stringent limits on the scale of brane perturbations. Physical effects of gravitational diffraction radiation are briefly discussed.

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**I. INTRODUCTION**

Larmor's formula of electromagnetism [1] states that an electric charge in uniform motion does not radiate. However, there are two ways to have radiation from a charge moving with constant velocity. The first way is to have a particle moving in a medium with velocity exceeding the phase velocity of light in that medium. This gives rise to the well-known Vavilov-Cherenkov radiation [1–3]. The second way is to consider charge motion in inhomogeneous media. Ginzburg and Frank [2] first discussed this effect by investigating a particle in uniform motion which crosses a planar interface between two media with dissimilar refractive index. This kind of radiation is known as transition radiation [1]. More generally, any motion near finite-size objects also induces radiation, in a process called diffraction radiation [4]. One of the first experimental verifications on this effect was provided by Smith and Purcell [5]. The Smith-Purcell experimental setup consisted of an electron moving close to the surface of a metal diffraction grating at right angles to the rulings. The theory of the Smith-Purcell effect has been discussed by several authors [6].

The aim of this paper is to show the existence of gravitational diffraction radiation (GDR) and discuss its physi-

cal effects. The model under consideration is a braneworld scenario [7] such as, for example, the Randall-Sundrum I model [8]. Braneworld models have attracted a lot of interest in recent years, revolutionizing our view of how our universe may be described. The central idea of braneworld scenarios is that the visible universe is restricted to a four-dimensional brane inside a higher-dimensional space, called the bulk. The additional dimensions are taken to be compact and other branes may be moving through the bulk. Interactions of the visible brane with the bulk and hidden branes introduce effects not seen in standard physics.

In this setup, a particle in uniform motion on the visible brane radiates gravitational waves due to the presence of a second (hidden) brane at finite distance, which plays the role of the metal diffraction grating of the Smith-Purcell experiment. GDR on the visible brane is generated by inhomogeneities on the hidden brane due, for example, to bulk-brane interactions and brane fluctuations [9]. We will show that the amount of GDR depends on the size of extra dimensions and the length scale of brane perturbations. This result is general and independent on the fine details of the model. Without loss of generality, in our computations we will consider a flat five-dimensional spacetime with the extra dimension taking values within the interval  $[0, L]$ . The distance between the particle and the diffraction grating,  $h = L/2$ , is the distance between the two branes located at the orbifold fixed points. The model is illustrated pictorially in Fig. 1. The generalization to higher-dimensions is trivial and is discussed at the end of this paper.

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<sup>\*</sup>Electronic address: [vcardoso@phy.olemiss.edu](mailto:vcardoso@phy.olemiss.edu)

<sup>†</sup>Electronic address: [cavaglia@phy.olemiss.edu](mailto:cavaglia@phy.olemiss.edu)

<sup>‡</sup>Electronic address: [pimenta@lip.pt](mailto:pimenta@lip.pt)

<sup>§</sup>Also at Centro de Física Computacional, Universidade de Coimbra, P-3004-516 Coimbra, Portugal

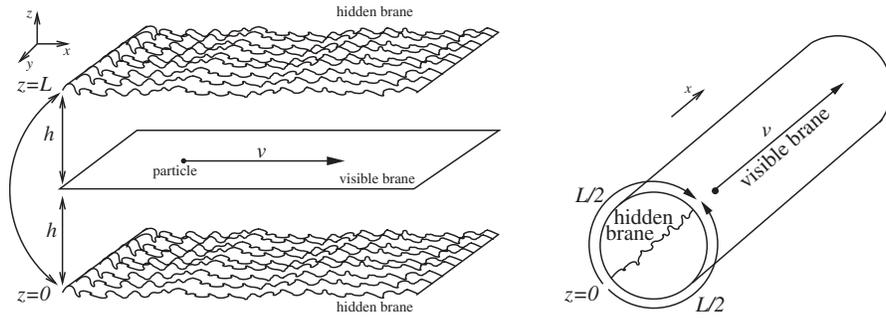


FIG. 1. Pictorial representation of the braneworld scenario considered in the text. The hidden brane at  $z = 0$  is identified with the brane at  $z = L$ . The visible brane is located at  $z = h = L/2$ . The particle moves on the visible brane along  $x$  with constant velocity  $v$ . Only one transverse coordinate,  $y$ , is shown. The right panel shows the wrapping around the extra dimension  $z$ . If the two halves of the cylinder are identified, the Randall-Sundrum model I [8] is obtained.

## II. THEORY OF GRAVITATIONAL DIFFRACTION RADIATION

In the harmonic gauge, the five-dimensional linearized Einstein equations are

$$\square h_{\mu\nu} = -16\pi G_5 S_{\mu\nu}, \quad (1)$$

where  $h_{\mu\nu}$  represents small corrections to the flat background and  $S_{\mu\nu}$  is the modified energy-momentum tensor of the source [10]. For sake of computational simplicity, we replace the the gravitational field components with a single scalar degree of freedom, i.e. rewrite the above equation as  $\square\varphi = S(x)$ . This is a common procedure [11]. If we carefully select  $S(x)$ , the scalar field  $\varphi$  can mimic all aspects of gravitational waves (except polarization) [12]. The source of the field is a minimally coupled pointlike particle with nonzero mass  $m$ . If we represent its worldline by  $x^\mu = x_p^\mu(\tau)$ , where  $\tau$  is the proper time,  $\varphi$  satisfies the inhomogeneous wave equation

$$\square\varphi = g \int d\tau \delta^5(x^\mu - x_p^\mu(\tau)), \quad (2)$$

where  $g$  is the coupling constant. We consider a particle in uniform motion on the visible brane. Denoting with  $z$  the coordinate transverse to the brane and with  $y_1, y_2$  the longitudinal coordinates perpendicular to the particle direction of motion  $x$ , the particle worldline in the brane reference frame is  $x^\mu(\tau) = (t = \gamma\tau, x = v\tau, y_1 = 0, y_2 = 0, z = h)$ , where  $\gamma$  is the Lorentz factor of the particle. The time-averaged energy radiated per unit time in the direction  $\mathbf{n}$  is

$$\frac{dE_{\mathbf{n}}}{dt} = -\frac{1}{2} \text{Re} \left[ \int dA \dot{\varphi} \frac{\partial \varphi^*}{\partial n} \right], \quad (3)$$

where  $dA$  is the surface element with normal  $\mathbf{n}$ . If the tensor structure of the gravitational perturbations is taken into consideration, the source term in Eq. (2) must be changed into

$$S^{\mu\nu} \sim m \int d\tau \dot{x}^\mu \dot{x}^\nu \delta^5(x^\mu - x_p^\mu(\tau)). \quad (4)$$

The results for the scalar field can be translated to the gravitational case by setting  $g = \sqrt{G_5} m \gamma^2$  [12].

If the branes are smooth, a particle with constant velocity does not radiate. This can be checked by deriving the Larmor's formula for the field. In order to avoid complications due to the higher-dimensional nature of the space-time [10,11,13], we temporarily assume that there is only one transverse spatial dimension. The total power emitted per unit of solid angle in the direction  $\mathbf{n}$  is

$$\frac{dP}{d\Omega} = \frac{g^2}{16\pi^2} \frac{[\dot{\mathbf{v}} \cdot (\gamma^2(1 - \mathbf{v} \cdot \mathbf{n})\mathbf{v} - \mathbf{n})]^2}{\gamma^2(1 - \mathbf{v} \cdot \mathbf{n})^3} \mathbf{n}. \quad (5)$$

Thus a particle in uniform motion in empty space (on the brane) does not radiate. If a hidden brane is present, the Green's function has to be modified to take into account its effects. The hidden brane can be thought as a wall parallel to the particle direction of propagation at distance  $h$ . By repeating the steps leading to Eq. (5) it is straightforward to show that there is no radiation for uniform motion parallel to the wall. This result can be understood by boosting the solution to the rest frame of the particle. The problem is reduced to a static problem with one image particle on the other side of the wall. Clearly, the reduction to the static configuration is only possible if the brane is smooth and infinite in the  $x$  direction. If the brane is inhomogeneous in the particle direction of propagation, diffraction radiation is generated. In that case, the image configuration is time dependent and the system is equivalent to a set of oscillating charges in the particle reference frame. Diffraction radiation can be understood as being generated by the reflection of the boosted static field on the nearby wall.

We now compute the radiated power in the presence of a brane with typical longitudinal perturbations of length scale  $l$  and transverse perturbations of length scale  $b$ . These perturbations are modeled with a  $l$ -periodic lamellar grating with rulings of width  $a$  perpendicular to the particle direction of motion as in Fig. 2. (The Smith-Purcell effect

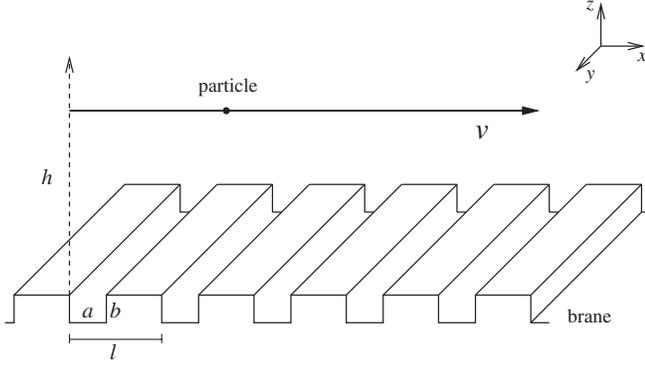


FIG. 2. Pictorial representation of brane perturbations. The longitudinal brane perturbations are periodic in  $x$  with length  $l$ . The transverse perturbations have length  $b$ . Only one transverse coordinate,  $y$ , is shown.

for a single grating with these characteristics has been discussed in Ref. [14].) Although this model is clearly an oversimplification, its main features do not depend on the choice of the perturbation structure: The existence of diffraction radiation is due to the excitation of propagating modes by evanescent waves and is independent of the particular mechanism by which the propagating modes are excited. The calculation with a perturbed brane can be conveniently solved by considering the Fourier transform of the field  $\phi$ :

$$\phi = \frac{1}{(2\pi)^3} \int d\omega d^2\eta e^{-i\omega t + i\mathbf{n}\cdot\mathbf{y}} \phi(x, z; \omega, \eta). \quad (6)$$

The field equation in the Fourier space is

$$[\nabla^2 + (\omega^2 - \eta^2)]\phi = g/(\gamma v) e^{i\alpha_0 x} \delta(z - h), \quad (7)$$

where  $\alpha_0 = \omega/v$ . The general solution for the field  $\phi$  in the bulk is the sum of the inhomogeneous solution of the above equation plus a superposition of plane wave solutions of the two-dimensional Helmholtz equation:

$$\phi_{\text{bulk}} = A \left\{ e^{i\alpha_0 x + i\gamma_0 |z-h|} + \sum_{n=-\infty}^{+\infty} [B_n e^{i\gamma_n z} + C_n e^{-i\gamma_n z}] e^{i\alpha_n x} \right\}, \quad (8)$$

where  $A = -ig/(2\gamma\gamma_0 v)$ ,  $\gamma_n = (\omega^2 - \eta^2 - \alpha_n^2)^{1/2}$ , and  $\alpha_n = \alpha_0 + 2\pi n/l$ . Since the parameter  $\gamma_0 = i[\omega^2/(v\gamma)^2 + \eta^2]^{1/2}$  is pure imaginary, the first term in Eq. (8) describes an evanescent wave that decays exponentially with increasing distance from the particle trajectory. This evanescent wave is the nonpropagating part of the spectrum due to the particle in uniform motion. The second term in Eq. (8) represents a superposition of propagating plane waves. The physical interpretation of these propagating modes is that of radiation arising from the modes of the brane perturbations, which are excited by the evanescent waves. The amplitude  $\mathcal{A}(\omega)$  of the GDR is thus expected

to be of the order of the amplitude of the nonpropagating modes, i.e.,

$$\mathcal{A}(\omega) \sim |A| e^{i\gamma_0 h} \sim (g/\omega) e^{-\omega h/\gamma}, \quad (9)$$

where we have taken the relativistic limit  $\gamma \gg 1$ . This can be shown by computing the amplitudes of the radiating modes as function of the evanescent wave amplitude  $A$ . The solution in the bulk must match the solution on the brane

$$\phi_{j,\text{br}} = A \sum_{m=0}^{+\infty} D_{j,m} [e^{-i\kappa_m z} - E_{j,m} e^{i\kappa_m z}] \sin\left[\frac{\pi m x_j}{\Delta_j}\right], \quad (10)$$

where  $\kappa_m = (\omega^2 - \eta^2 - (\pi m/a)^2)^{1/2}$  and  $E_{j,m} = (e^{2i\kappa_m b}, e^{-2i\kappa_m L})$ ,  $x_j = (x, x-a)$  and  $\Delta_j = (a, l-a)$  in the intervals  $x \in [0, a]$  ( $j=1$ ) and  $x \in [a, l]$  ( $j=2$ ), respectively. Choosing Dirichlet boundary conditions, the amplitudes of the propagating waves satisfy the system of linear inhomogeneous algebraic equations: small

$$\sum_{n=-\infty}^{+\infty} [B_n (l\delta_{nk} + V_{1,nk}) + C_n (l\delta_{nk} - V_{1,nk})] + e^{i\gamma_0 L/2} (l\delta_{0k} - V_{1,0k}) = 0, \quad (11)$$

$$\sum_{n=-\infty}^{+\infty} [B_n e^{i\gamma_n(L-b)} (l\delta_{nk} + V_{2,nk}) + C_n e^{-i\gamma_n(L-b)} \times (l\delta_{nk} - V_{2,nk})] + e^{i\gamma_0(L/2-b)} (l\delta_{0k} + V_{2,0k}) = 0, \quad (12)$$

normalize where

$$V_{j,nk} = 2\Delta_j \epsilon_j \gamma_n \sum_{m=0}^{+\infty} \kappa_m^{-1} \tan(\kappa_m b) \Phi_{j,mk} \Phi_{j,mn}^*, \quad (13)$$

$$\Phi_{j,mk} = \Delta_j^{-1} \int_0^{\Delta_j} dx e^{-i\alpha_k x} \sin(\pi m x / \Delta_j)$$

and  $\epsilon_j = (1, e^{2i\pi(n-k)a/l})$ . The power emitted in the bulk follows from Eq. (3) after solving Eqs. (12) for the propagating wave amplitudes  $B_n$  and  $C_n$ . Generalizing to  $D$  spacetime dimensions and taking the relativistic limit, we have

$$P \sim \frac{g^2 b^2}{\gamma^2 l^D} (1 + \mathcal{O}(b/l)) e^{-2\pi L/(\gamma l)}, \quad (14)$$

where we have set  $l = 2a$  and assumed small  $b/l$ . Since we assumed the source to be pointlike, Eq. (14) is valid for particle size  $\ll L, b$  and  $l$ , otherwise it describes the diffraction radiation of a relativistic particle under very general assumptions. The GDR energy loss per unit distance is

$$\frac{d \ln E}{dx} \sim E \frac{G_D b^2}{l^D} e^{-2\pi L/(\gamma l)}, \quad (15)$$

where  $G_D$  is the  $D$ -dimensional Newton constant.

### III. BOUNDS ON BRANE INHOMOGENEITIES

As an example of the physical consequences of GDR, let us consider the scenario with  $D - 4$  large extra dimensions of size  $L$  [15]. In this model  $G_D$  is related to the four-dimensional Newton constant  $G_4$  by  $G_D \sim L^{D-4}G_4$ . There are two natural length scales, the Planck length,  $L_{\text{Pl}} \sim 10^{-17}$  cm, and the size of the large extra dimensions,  $L \sim 10^{30/(D-4)-17}$  cm. The perturbation length scales  $l$  and  $b$  are expected to be larger than the Planck length. This implies  $L/l \leq 10^{30/(D-4)}$  and  $b/L \geq 10^{-30/(D-4)}$ . The largest  $\gamma$  factors are observed in ultra high-energy cosmic rays, reaching  $\gamma \sim 10^{11}$  [16]. In this case, the exponential in Eq. (15) can be set to unity for all  $D > 6$ . Using the above constraints, we obtain

$$\frac{d \ln E}{dx} \geq 10^{19-120/(D-4)} \left(\frac{L}{l}\right)^D \left(\frac{E}{10^{20} \text{ eV}}\right) \text{ Mpc}^{-1}. \quad (16)$$

Equation (16) imposes stringent limits on the smoothness of the brane. Upper bounds on  $L/l$  range from  $\sim 10^3$  for  $D = 7$  to  $\leq 1$  for  $D \geq 9$ . Therefore, GDR implies that the brane must be smooth on scales of order of the size of the large extra dimensions, i.e., on scales much larger than the Planck length. This fact has important consequences for the problem of initial conditions in models of brane cosmology. (See, e.g., Ref. [17].)

### IV. DISCUSSION

In higher-dimensional models of our universe, particles radiate gravitational waves due to the structure of the extra dimensions, specifically the presence of an inhomogeneous hidden brane. We have called this phenomenon ‘‘Gravitational Diffraction Radiation’’ because of its similarity with the electromagnetic diffraction radiation which is emitted by a charge moving near a metallic grating. GDR is a general phenomenon of all higher-dimensional brane-

world scenarios. It applies to particles propagating both on the brane and in the bulk. As an example of its physical effects, we have discussed the GDR emitted by a particle on the visible brane and derived lower bounds on the scale of brane perturbations.

The above results can be extended in several directions. For instance, this paper considers only inhomogeneities on the hidden brane. The visible brane is also inhomogeneous. Therefore, free motion on the visible brane is not uniform from the five-dimensional perspective. This effect could be comparable to the GDR effect studied here. The derivation of GDR presented above is purely classical. Although the scale of brane inhomogeneities suggests that the system can be treated as classical, the importance of quantum effects should be assessed. It would also be interesting to consider GDR emitted by photons or bulk particles. In the latter case, GDR energy loss would manifest itself as a background of stochastic gravitational waves on the visible brane.

GDR is originated by the lumpiness of the spacetime at small scales. Thus GDR effects may show up in a variety of physical situations besides braneworld models. Emission of radiation by free particles in a cosmic string background was discussed in [18]. GDR could also play an important role in the very early universe or near the horizon of black holes. Direct signatures are unlikely to be observed in these cases. However, GDR emission could lead to interesting predictions at theoretical level and possibly indirect observable effects.

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