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Indrajit Chowdhury
Petrofac Int. Ltd., United Arab Emirates

Shambhu P. Dasgupta
Indian Institute of Technology, India

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ESTIMATION OF LATERAL LOAD CAPACITY OF SHORT PILES UNDER EARTHQUAKE FORCE

Indrajit Chowdhury
Head of the department
Civil & Structural Engineering
Petrofac Int Ltd
Sharjah, United Arab Emirates
e-mail Indrajit.Chowdary@petrofac.com

Prof. Shambhu P. Dasgupta
Professor of Civil Engineering
Indian Institute of Technology
Kharagpur-721302
West Bengal India
email: dasgupta@civil.iitkgp.ernet.in

ABSTRACT

Lateral load induced in piles (both long and short) under earthquake is a problem of serious complexity that has been plaguing professional engineers and researchers alike for quite some time. The practice in vogue is to ensure that fixed base shear of the column does not exceed static shear load capacity of the piles. Inertial and stiffness effects of pile are usually ignored in dynamic earthquake analysis. The present paper proposes a method where, based on modal response or time history analysis, load on short piles may be estimated under earthquake considering its stiffness, inertia, effect of material and geometric damping properties. The results are compared with the conventional methods.
Effect of partial embedment, a situation that may develop under soil liquefaction during earthquake has also been derived.
Pile loads are estimated for two cases:
a) When the structure is a lumped mass system having infinite stiffness: like a machine foundation or a heavy short vessel supported directly on the pile cap.
b) Superstructure has finite stiffness and mass like a frame (building /pipe rack etc)
The paper assumes that for all cases when slenderness ratio L/r is less than 20 the pile behaves as short pile when failure or yielding of soil precedes the structural failure of the pile.
The major advantage with this method is that it does not warrant a sophisticated software to be developed for the analysis. A simple spread sheet is sufficient to produce an accurate result.

INTRODUCTION

Vibration of piles under lateral load is an important study for piles supporting machines and structures under earthquake loading. In majority of the cases, of all modes, lateral vibration is most critical and often governs the design during an earthquake. Thus, a study of such motion is of paramount importance for piles supporting important installations.

Many researchers have proposed solution to the problem of pile dynamics, namely, Parmelee et al. (1964), Tajimi (1966), Penzien (1970), Novak et al. (1974, 1983), Banerjee and Sen (1987), Dobry and Gazetas (1988) only to name the pioneering few. However, most of these solutions are based on harmonic analysis and are valid for design of machine foundations, where dynamic stiffness and damping of pile remain frequency dependent, and have all been worked out based on long pile theory where, structural failure of pile precedes soil failure and governs the design. Application of these theories are though well established for design of machine foundations except for an approximate method proposed by Chandrashekaran (1974) and Prakash (1973) for long piles, a comprehensive analytical tool to predict pile response under earthquake load still remains uncertain.

Chowdhury & Dasgupta(2008) proposed a semi analytical method for analysis of long piles under earthquake force. A similar procedure has been extended in this case for analysis of short piles.

PROPOSED METHOD

The present paper deals with a semi-analytic solution for predicting lateral load on a short pile under earthquake forces. For obtaining the time period *vis-a vis* the stiffness and mass of the system, one may start with a pile embedded in homogeneous elastic medium under plane strain condition as shown in Figure 1. To start with, the pile is taken as short with L/r < 20 when soil failure precedes structural failure. Under static condition, the equation of equilibrium in x-direction is given by:

$$E_p I_p \frac{d^4 u}{dz^4} = -k_s D u \quad (1)$$

Here E_p= Young's modulus of the pile I_p= moment of inertia of the pile cross section; k_s=dynamic subgrade modulus of soil (kN/m^3), u=displacement in the x direction and D = diameter of the pile.
The general solution of Equation (1) for a finite beam on elastic foundation may be written as (Bojtsov 1982):

$$u = C_0 \cosh pz \cos pz + C_1 \cosh pz \sin pz + C_2 \sinh pz \sin pz + C_3 \sinh pz \cos pz$$

where $p = \sqrt[4]{k_s D / 4E_p I_p}$ (2)

In terms of Puzrevsky function (Karnovsky and Lebed 2002), Equation (2) can be expressed as

$$u = C_0 V_0(pz) + C_1 V_1(pz) + C_2 V_2(pz) + C_3 V_3(pz) \quad (3)$$

Here

$$V_0(pz) = \cosh pz \cos pz \quad (4)$$

$$V_1(pz) = \frac{1}{\sqrt{2}}\left(\cosh(pz)\sin(pz) + \sinh(pz)\cos(pz)\right) \quad (5)$$

$$V_2(pz) = \sinh pz \sin pz \quad (6)$$

$$V_3(pz) = \frac{1}{\sqrt{2}}\left(\cosh(pz)\sin(pz) - \sinh(pz)\cos(pz)\right) \quad (7)$$

Puzrevsky functions as defined above have some unique functional properties that will be used subsequently for derivation of the stiffness, mass and damping properties of the pile.

$$V_0(0) = 1;\ V_0'(0) = 0;\ V_0''(0) = 0;\ V_0'''(0) = 0 \quad (8)$$

$$V_1(0) = 0;\ V_1'(0) = p\sqrt{2};\ V_1''(0) = 0;\ V_1'''(0) = 0 \quad (9)$$

$$V_2(0) = 0;\ V_2'(0) = 0;\ V_2''(0) = 2p^2;\ V_2'''(0) = 0 \quad (10)$$

$$V_3(0) = 0;\ V_3'(0) = 0;\ V_3''(0) = 0;\ V_3'''(0) = 2\sqrt{2}p^3 \quad (11)$$

And

$$V_0'(pz) = p\sqrt{2}V_3(pz);\ V_1'(pz) = p\sqrt{2}V_0(pz) \quad (12)$$

$$V_2'(pz) = p\sqrt{2}V_1(pz);\ V_3'(pz) = p\sqrt{2}V_0(pz) \quad (13)$$

For solution of short pile one may use the mathematical model as shown in Figure.1.
With reference to the figure the following boundary conditions are assumed:

a) At z=0 Moment and shear at pile tip=0 $\Rightarrow u'' = 0$ and $u''' = 0$ (After Broms 1965).

b) At z=L u=u_0=1 and θ=θ_0=1/L.(After Novak 1974).

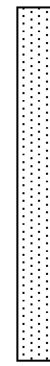

Fig.1 Conceptual Model of short Pile (dashed line shows soil undergone liquefaction).

Implementing the boundary condition (a) we have C_2=C_3=0, for boundary condition (b) when z=L u_0=1 gives

$$C_0 V_0(pL) + C_1 V_1(pL) = 1 \quad (14)$$

and for z=L, u_0'=1/L we have

$$C_0 V_3(pL) + C_1 V_0(pL) = 1/pL\sqrt{2} \quad (15)$$

The above can be expressed in matrix form as

$$[C] = [V]^{-1}\{p\} \quad (16)$$

Performing the above operation gives

$$\begin{Bmatrix} C_0 \\ C_1 \end{Bmatrix} = \frac{1}{\Delta}\begin{bmatrix} V_0(pL) & -V_1(pL) \\ -V_3(pL) & V_0(pL) \end{bmatrix}\begin{Bmatrix} 1 \\ 1/pL\sqrt{2} \end{Bmatrix} \quad (17)$$

Equation (17) on expansion gives

$$C_0 = \frac{1}{\Delta}\left(V_0(pL) - \frac{V_1(pL)}{pL\sqrt{2}}\right) \quad (18)$$

$$C_1 = \frac{1}{\Delta}\left(\frac{V_0(pL)}{pL\sqrt{2}} - V_3(pL)\right) \quad (19)$$

$$\Delta = V_0^2(pL) - V_1(pL).V_3(pL) \quad (19a)$$

The displacement can thus be expressed as

$$u = u_0[C_0 V_0(pz) + C_1 V_1(pz)] \quad (20)$$

In dimensionless form considering β=pL, general shape function of the pile can thus be expressed as

$$\phi = C_0 V_0 (\frac{\beta z}{L}) + C_1 V_1 (\frac{\beta z}{L}) \quad (21)$$

A typical shape function profile for short pile for E_p/G_s=2500 is as shown hereafter in Figure2.

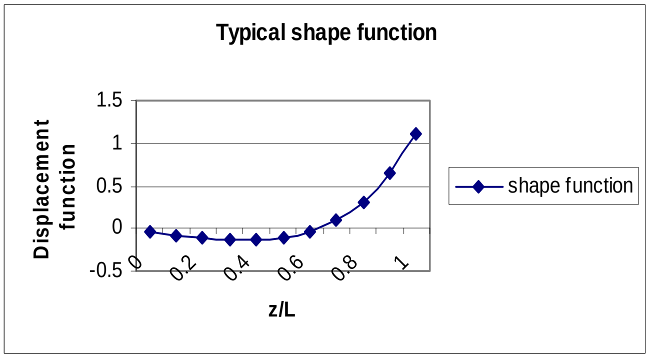

Fig.2 Typical shape function for short pile for E_p/G_s=2500

Differentiating equation (21) and using the properties of Puzrevsky as mentioned earlier, one could have

$$\phi'' = \frac{2\beta^2}{L^2}\left[C_0 V_2\left(\frac{\beta z}{L}\right) + C_1 V_3\left(\frac{\beta z}{L}\right)\right] \quad (22)$$

Potential energy $d\Pi$ of an element of depth dz as shown in Figure 1 is then given by (Shames and Dym 1995)

$$d\Pi = \frac{E_p I_p}{2}\left[\frac{d^2 u}{dz^2}\right]^2 + \frac{K_h u^2}{2} \quad (23)$$

Here K_h= lateral dynamic stiffness of soil in kN/m and the displacement u may be written as $u = \phi(z)q(t)$.

For a rigid circular disc embedded in soil of depth h the stiffness under earthquake force can be expressed as (Wolf-1988):

$$K_x = \frac{8G_s r_0}{2-\nu}\left(1 + \frac{h}{r_0}\right) \quad (24)$$

were K_x= static foundation stiffness in horizontal direction in kN/m, G_s= dynamic shear modulus of soil, r_0=radius of foundation, h = depth of embedment of the foundation and ν=Poisson's ratio.

Ignoring the first term within bracket in equation (24) which contributes to base resistance and substituting the same in Equation (23), for a cylindrical element of depth dz embedded in soil the potential energy Π for a pile of length L may be expressed as :

$$\Pi = \frac{E_p I_p}{2}\int_0^L \left[\frac{d^2 u}{dz^2}\right]^2 dz + \frac{8G_s}{2(2-\nu)}\int_0^L u^2 dz \quad (25)$$

Considering $u(z,t) = \phi(z)q(t)$ it can be shown (Hurty & Rubenstein 1967) that

$$K_{ij} = E_p I_p \int_0^L \phi_i''(z)\phi_j''(z)dz + \frac{8G_s}{(2-\nu)}\int_0^L \phi_i(z)\phi_j(z)dz \quad (26)$$

Here the shape function ϕ(z) is expressed by equation (21). For the fundamental mode stiffness of the pile is given by

$$K_{ij} = E_p I_p \int_0^L \phi''(z)^2 dz + \frac{8G_s}{(2-\nu)}\int_0^L \phi(z)^2 dz \quad (27)$$

Expansion of Equation (27) finally gives

$$K_{pile} = \frac{4\beta^4 E_p I_p}{L^4}\int_0^L \left[C_0 V_2\left(\frac{\beta z}{L}\right) + C_1 V_3\left(\frac{\beta z}{L}\right)\right]^2 dz + \frac{8G_s}{(2-\nu)}\int_0^L \left(C_0 V_0\left(\frac{\beta z}{L}\right) + C_1 V_1\left(\frac{\beta z}{L}\right)\right)^2 dz \quad (28)$$

Now considering $\xi = z/L$ Ldξ=dz and as z→ 0 , $\xi \to 0$ and as z→ L , $\xi \to 1$, when Equation(28) can be expressed in natural co-ordinates as

$$K_{pile} = \frac{4\beta^4 E_p I_p}{L^3}\int_0^1 [C_0 V_2(\beta\xi) + C_1 V_3(\beta\xi)]^2 d\xi + \frac{8G_s L}{(2-\nu)}\int_0^1 (C_0 V_0(\beta\xi) + C_1 V_1(\beta\xi))^2 d\xi \quad (29)$$

or $$K_{pile} = \frac{4\beta^4 E_p I_p}{L^3} I_1 + \frac{8G_s L}{(2-\nu)} I_2 \quad (30)$$

in which

$$I_1 = \int_0^1 [C_0 V_2(\beta\xi) + C_1 V_3(\beta\xi)]^2 d\xi \quad (31)$$

$$I_2 = \int_0^1 (C_0 V_0(\beta\xi) + C_1 V_1(\beta\xi))^2 d\xi \quad (32)$$

are integral functions that need to be determined numerically. However, prior to that relationship between dynamic subgrade modulus k_s and Wolf's parameter as shown in Equation (24) needs to be established.

Observing equation(30) it is seen that the first term represents the structural stiffness of pile and the second term expresses the contributing soil stiffness. Thus in terms of k_s the soil part can be expressed as

$$k_{soil} = k_s D L I_2 \quad (33)$$

Equating Equation (33) to second term of (30), we have

$$k_s = 8G_s / [(2-\nu)D] \quad (34)$$

This gives $\beta = pL = \sqrt[4]{2G_s L^4 / (2-\nu)E_p I_p}$ (35)

Based on β as mentioned above and dynamic modulus of soil G_s Equation (30) can be expressed as

$$K_{pile} = \frac{8G_s L}{(2-\nu)}\chi_{12} \quad (36)$$

Here $\chi_{12} = I_1 + I_2$ is pile stiffness coefficient.

For a short pile when L/r is less than 20 and Ep/Gs varying from 1000 to 10,000 (the usual range when piles are deployed), the value of β usually varies from 2.2 to 4.
Thus considering β varying from 2.0 to 4.0, the values of χ_{12} are furnished in Table-1 for ready reference.

Table-1 Stiffness coefficient for short pile ($\alpha = L_1/L=1$)

β	χ_{12}
2	22.878
2.25	8.213
2.5	1.101
2.75	0.421
3.0	0.236
3.25	0.167
3.5	0.143
3.75	0.151
4.0	0.26

In the above formulation it is observed that static effect of the soil spring is only considered. The dynamic part which is frequency dependent has been ignored. This is justified in this case since it has been observed by Wolf et. al (2004) that for vertical and horizontal motion, spring constants are almost independent of the dimensionless frequency a_0(= ωr/vs). Same conclusion has also been arrived at by Hall (1976) and Kramer (2002) wherein it is suggested that static soil spring adequately serves the purpose of earthquake analysis.

For a partially embedded pile when some part near the surface of soil has lost its strength due to liquefaction, the pile stiffness is calculated by ignoring this portion Equation (29) changes to

$$K_{pile} = \frac{4\beta^4 E_p I_p}{L^3}\int_0^{\alpha}\left[C_0V_2(\beta_1\xi) + C_1V_3(\beta_1\xi)\right]^2 d\xi + \frac{8G_s L}{(2-\nu)}\int_0^{\alpha}\left(C_0V_0(\beta_1\xi) + C_1V_1(\beta_1\xi)\right)^2 d\xi + \frac{12E_p I_p}{L^3(1-\alpha)^3} \quad (37)$$

As shown in Figure 1, $\alpha = L_1/L$ and $0 \le \alpha \le 1$ and

$$\beta_1 = \sqrt[4]{2G_s\alpha^4 L^4/(2-\nu)E_p I_p} \quad (38)$$

<u>Calculation of pile mass and damping</u>

The pile mass consists of two parts, i) the self weight and ii) the lumped mass as its head. The contribution of self weight of the pile can be expressed as (Meirovitch 1967) :

$$M_x = m_x\int \phi_i(z)\phi_j(z)dz \quad (39)$$

For the present case Equation (39) can be expressed as

$$M_x = \frac{\gamma_p A_p}{g}\int_0^L \phi(z)^2 dz \quad (40)$$

Here γ_p = unit weight of pile material, A_p= cross sectional area of the pile, g= acceleration due to gravity.
The above in natural co-ordinates can be expressed as

$$M_x = \frac{\gamma_p A_p L}{g} I_2 \quad (41)$$

Here I_2 is the integral function explained in Equation (32).
Table-2 gives typical values I_2 for short piles having L/r<20.

Table -2 Integral coefficient for mass and damping of pile (α=1)

β	I_2
2	6.931
2.25	1.567
2.5	0.17
2.75	0.094
3.0	0.089
3.25	0.096
3.5	0.108
3.75	0.129
4.0	0.192

Now the question is what will be the lumped mass to be considered at the top of the pile?
The most logical inference is that it must be equal to static vertical design load of the pile, for this is what a designer would always restrict his load on pile to.
Hence total contributing mass of the pile may be expressed as

$$M_{pile} = \frac{\gamma_p A_p L}{g} I_2 + \frac{P_d}{g} \quad (43)$$

Here P_d is the allowable static vertical load on the pile. For partial embedment case, I_2 as given in the second part of Equation (37) needs to be considered.
Damping of pile embedded in soil medium will consist of two parts: material and radiation damping. Material damping of soil is also a part of the vibrating system, however, it has been found that for translational motion this effect is insignificant and may be ignored. As a first step for calculating the total damping one may ignore material damping of pile for the time being.
For a rigid circular disc embedded in soil for a depth h Wolf (1988) has shown that radiation damping may be expressed as:

$$c_x = \left(\frac{r_0 K_x}{V_s}\right)\left[0.68 + 0.57\sqrt{\frac{h}{r_0}}\right] \quad (44)$$

where K_x=lateral stiffness of the embedded disc; V_s= shear wave velocity of the soil.
Thus for an infinitesimally thin circular disc of thickness dz Equation (44) can be expressed as

$$c_x = \left(\frac{r_0 K_x}{V_s}\right)\left[0.68 + 0.57\sqrt{\frac{dz}{r_0}}\right] \qquad (45)$$

Now considering $y = \sqrt{\varepsilon}$ where $\varepsilon = dz / r_0$ one can write taking logarithm on both sides and then expanding $\log_e \varepsilon$ as a series of ε where higher orders of ε are ignored for being very small.

$$\log_e y = (1.5\varepsilon - 0.92) \qquad (46)$$

$$\Rightarrow y = e^{(1.5\varepsilon - 0.92)} \qquad (47)$$

Expanding the right hand side of Equation (47) in power series and ignoring higher orders of ε being exceedingly small since it contains higher order of dz one can finally arrive at

$$y = 1.5\varepsilon + 0.083 \qquad (48)$$

Substituting this value in equation (45) and ignoring the first term within the parenthesis which is due to base resistance, one can have

$$c_x = 0.855\left(\frac{r_0 K_x}{V_s}\right)\frac{dz}{r_0} \qquad (49)$$

For systems having continuous response function, the damping may be expressed as (Paz 1987):

$$C_x = c_x \int \phi_i(z)\phi_j(z)dz \qquad (50)$$

Equation (50) for pile, partially or fully embedded in soil, can be generally expressed as

$$C_x = 0.855\frac{K_{pile}}{V_s}\int_0^{\alpha} \phi(\xi)^2 \, d\xi \qquad (51)$$

Here $0 \le \alpha \le 1$, when fully embedded α=1 and for partial embedment α<1.

The damping ratio of the pile is given by $\varsigma_x = C_x / C_c$ where $C_c = 2\sqrt{K_{pile} . M_{pile}}$, based on above one finally arrives at an expression

$$\varsigma_x = \left(\frac{0.43 L \omega_n}{V_s}\right) I_2 \qquad (52)$$

In equation (52) ω_n is the natural frequency of the pile ($\sqrt{K_{pile} / M_{pile}}$) and I_2 are the integral functions furnished in Table-2.

To Equation (52) now, a suitable material damping ratio of pile (ς_m), depending on what constitutes the pile (concrete or steel), may be added to arrive at total damping ratio of the system.

Dynamic Response of pile:

Having established stiffness, mass and damping ratio of pile for the fundamental mode, time period of pile can be generically expressed as

$$T = 2\pi\sqrt{\frac{M_{pile}(2-\nu)}{8GL\chi_{12} + 12E_p I_p (2-\nu) / \{L^3(1-\alpha)^3\}}} \qquad (53)$$

In Equation (53), it is assumed that the super-structure has infinite stiffness ($T \rightarrow 0$) like a rigid generator resting over a pile cap or a heavy rigid Hydro cracker resting over a pile foundation. In such cases fixed base stiffness of the superstructure is far too high and may be ignored. For full embedment the second term in denominator of Equation (53) is to be ignored.

For the case when superstructure has finite stiffness the problem may be analyzed as explained hereafter. Let us assume for a project the functional dimension of a building is known (Height H and overall plan dimensions knopwn), then the fundamental time period of the building as per UBC(1997) is

$$T_s = 0.09H / \sqrt{D} \qquad (54)$$

Based on the above it can be argued that in fundamental mode whole building mass (all parts) is moving with a time period T_s and acceleration thus generated is a function of T_s. Thus for any arbitrary mass which forms the part of the building will be subjected to an acceleration S_a which is a function of this time period T_s. The mass (P_d/g), the static design load at the top of pile, should also move with an acceleration that is a function of T_s. If one assumes a fictitious column above the pile supporting this mass, the stiffness of the column assumed to be carrying this load can be expressed as

$$K_{col} = \frac{4\pi^2 (P_d / g)}{T_s^{\,2}} \qquad (55)$$

Based on the above we can now mathematically model the superstructure and pile as a two mass lumped model as shown in Figure 3.

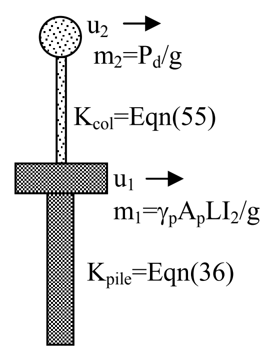

Fig.3 Two Mass lumped model for pile superstructure

The equation of motion in terms of stiffness, mass and damping matrix can be expressed as

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} C_{col} + C_{pile} & -C_{col} \\ -C_{col} & C_{col} \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} K_{col} + K_{pile} & -K_{col} \\ -K_{col} & K_{col} \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = -[M]\{\ddot{u}_g\} \qquad (56)$$

In the above equation $C_{col} = 2\zeta_{col}\sqrt{K_{col} P_d / g}$ where ζ_{col}

is usually 0.02 for steel structure and 0.05 for RCC structures. ζ_{pile} is derived as in Equation(52) plus the material damping ratio of the pile.

In this case the damping being non-classical in nature a time history analysis has to be performed from which the force induced on pile can be established.

Based on modal analysis, the maximum amplitude of the pile head can be expressed as

$$S_d = \kappa_i C_F \left(\frac{S_a}{\omega^2}\right) \qquad (57)$$

Here κ_i is modal mass participation factor, C_F is code factor constituting of importance factor, zone factor and response reduction factor etc. S_a is the acceleration corresponding to the time period of the pile and ω is the natural frequency of the pile. Considering ω=2π/T equation (57) can be expressed as

$$S_d = \kappa_i C_F \frac{W(2-\nu)}{8G_s L\chi_{12}}\left(\frac{S_a}{g}\right) \text{ where } W= M_{pile} x g. \qquad (58)$$

The displacement along pile length may be expressed as

$$u(z) = \kappa_i C_F \frac{W(2-\nu)}{8G_s L\chi_{12}}\left(\frac{S_a}{g}\right)\left[C_0 V_0(\beta\xi) + C_1 V_1(\beta\xi)\right] \qquad (59)$$

For partial embedment case maximum displacement (u_p) at pile head can be estimated as

$$u_p = \frac{\kappa_i C_F W(2-\nu)}{8G_s L\chi_{12} + 12E_p I_p(2-\nu)/[L^3(1-\alpha)^3]}\left(\frac{S_a}{g}\right) \qquad (60)$$

Modal mass participation factor may be expressed as

$$\kappa_i = \sum m_i\phi_i / \sum m_i\phi_i^2 \qquad (61)$$

For the present problem this can be expressed as

$$\kappa_i = \frac{\frac{\gamma_p A_p L}{g}\int_0^1 \phi(z) + \frac{P_d}{g}\phi(L)}{\frac{\gamma_p A_p L}{g}\int_0^1 \phi(z)^2 + \frac{P_d}{g}\phi(L)^2} \qquad (62)$$

Considering Pd/g>> $\gamma_p.A_pL/g$; $\kappa_i \to 1$

Bending moment and shear force on the pile can now be expressed as

$$M = -E_p I_p u'' = -2C_F \frac{E_p I_p W\beta^2(2-\nu)}{8G_s L^3\chi_{12}}\left[\frac{S_a}{g}\right]\left[C_0 V_2(\beta\xi) + C_1 V_3(\beta\xi)\right] \qquad (63)$$

$$V = -E_p I_p u''' = -2C_F \frac{E_p I_p W\beta^3(2-\nu)}{2\sqrt{2}G_s L^4\chi_{12}}\left[\frac{S_a}{g}\right]\left[C_0 V_1(\beta\xi) + C_1 V_2(\beta\xi)\right] \qquad (64)$$

What has been discussed till now is the kinematical interaction between the soil and pile. Other than this, the free field displacement of the site also influences the stresses in the pile. For a site having a depth H to the bedrock and shear wave velocity V_s, the free field time period in fundamental mode is estimated as $4H/V_s$.Considering a suitable material damping of soil based on say Ishibashi and Zang(1993) one can estimate the free field acceleration of the site.

It has been shown by Chowdhury and Dasgupta(2008) that the shape function of such free field motion of the ground in fundamental mode can be expressed as $\phi(z) = \cos(\pi z/2H)$ in one dimension. It should be noted that in this case z=0 is at the top of the pile and opposite to what has been shown in Figure 1. The displacement of the soil can then be expressed as

$$u_f = \frac{32C_F S_{af}\gamma_s H^2}{\pi^2(\pi+2)G_s g}\cos\left(\frac{\pi z}{2H}\right) \qquad (65)$$

Here γ_s= weight density of soil.

Now considering H=μ L(refer Figure 1) where 0<μ<1, the displacement of the soil free surface can be expressed in terms of pile length L as

$$u_f = \frac{32C_F S_{af}\gamma_s \mu^2 L^2}{\pi^2(\pi+2)G_s g}\cos\left(\frac{\pi z}{2\mu L}\right) \qquad (66)$$

Bending moment and shear force on the pile may be expressed as

$$M_f = \frac{8C_F\gamma_s}{(\pi+2)}\left(\frac{S_{af}}{g}\right)\left(\frac{E_p I_p}{G_s}\right)\cos\left(\frac{\pi z}{2\mu L}\right) \qquad (67)$$

$$V_f = \frac{4\pi C_F\gamma_s}{(\pi+2)\mu L}\left(\frac{S_{af}}{g}\right)\left(\frac{E_p I_p}{G_s}\right)\sin\left(\frac{\pi z}{2\mu L}\right) \qquad (68)$$

Equations (67) and (68) are to be added to Equations (63) and (64) respectively to arrive at the final dynamic response of the short pile. In many cases it will be observed that unless the pile is very short and thick (like a pier or a caisson) the free field moment and shear give quite low values and may be neglected in such cases.

RESULTS AND DISCUSSION

To compare the results, a 8500 kN rigid vessel supported on 10 piles having dimensions 1.2 meter diameter 10 meter long is compared. The vertical capacity of pile is 1000kN.The unit weight of soil is 20 kN/m^3.The dynamic shear wave velocity of soil is 125m/sec. Size of the pile cap supporting the vessel is 5.2 m X 5.2mX 2.1m.The site is Zone IV are as IS-1893 Code of practice for Earthquake resistant design of Structures and Foundation. Here the vessel being very rigid its stiffness is assumed to be infinite when $T_s \to 0$

Table-3 Comparison of basic design parameters:

Design Parameters	Conventional Method	Proposed Method
Time period	0.0sec	0.133
Sa/g	1.0	1.2
Damping ratio (%)	5 %	21 %
Shear at Pile head	59.5 kN	88.23 kN
Moment on pile	152 kN.m	181 kN.m

A comparative study of the moments and shears with conventional analysis considering the structure as fixed base and the proposed method is given in Table-3 and presented in Figs. 4 and 5.

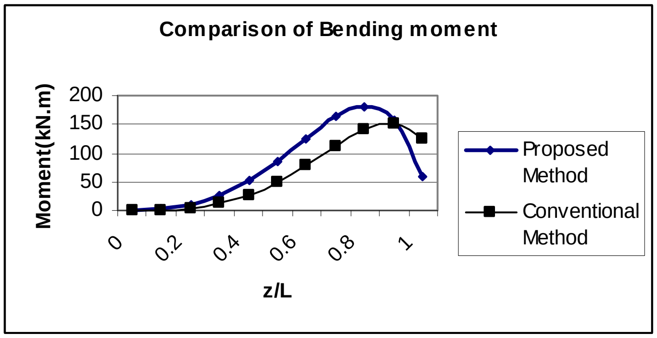

Fig.4 Comparison of Bending Moment in pile, conventional versus proposed method.

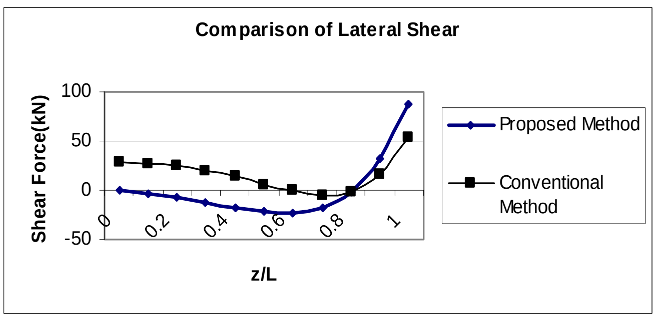

Fig.5 Comparison of Shear force in pile, conventional versus proposed method.

Based on the above data it is observed that dynamic response of pile can undergo significant amplification. As per conventional analysis when time period is considered $T \rightarrow 0$, the shear obtained at pile head is 59.5 kN, the same considering the dynamic response of the pile when the time period is T=0.133 second, the base shear obtained is 88.kN.This increase in pile shear is attributed to the amplification of response due to the finite time period of the pile including the effect of the surrounding soil. Thus it is evident that conventional analysis of fixed base shear of the super structure may or may not give a realistic result and can under or even overestimate the values depending on the type of soil and the superstructure it supports. A proper dynamic analysis of the pile including the effect of the soil and inertial and stiffness effect of superstructure is essential especially for important facilities to arrive at a realistic result.

Based on the above method the design steps for the pile including the algorithm for development of a spreasheet can be summarized as hereafter.

- Read values of Dynamic Shear Modulus (G) and Poisson's ratio(ν) from soil report.
- Read basic pile data like E_p, I_p, L, P_d, γ_p etc. from soil report.
- Determine β from Equation (38).
- Determine χ_{12} and I_2 for a given β from Tables -1 and 2 respectively.
- Determine M_{pile} from Equation (43).
- Determine Time period T and damping ratio ζ from Equation (53) and (52) respectively.
- For the given T and ζ read off Sa/g from the code and select the paremters Z,I and R.
- Determine displacement (u), bending moment (M) and shear (V) in pile from Equation (59),(63) and (64) respectively.
- Determine free field moment and shear in pile from Equation (67) and (68).
- Add free field moment and shear to M and V to get the final Design moment and shear.

For two mass lumped system

- Determine M_1 and M_2 as shown in Fig-2
- Determine K_{col} and K_{pile} as shown in Fig-2 .
- Determine C_{pile} and C_{col} as stated in the paper.
- Form Equation (56) to perform time history to determine the displacement (u), moment (M) and shear (V) in pile.

CONCLUSION

It is evident from the above that lateral load on pile is dependent on the soil- pile-structure stiffness and damping property. And without undergoing a proper dynamic analysis it cannot be estimated as to what is the actual load on the pile. Recommendations furnished in some codes (like IS2911), of considering lateral load as 5% of the axial load may seriously underrate the load at times.

Present method gives a rational and practical way for estimation of such forces on short piles under earthquake force including partial embedment. Formulas for the time period, moment, shear etc are direct and can very well be developed in a spread sheet for dynamic analysis of the pile based on steps as explained above.

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