Missouri University of Science and Technology
Scholars' Mine

International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics

2010 - Fifth International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics

26 May 2010, 4:45 pm - 6:45 pm

Dynamic Response of Pile Groups Embedded in Transversely Isotropic Media Using Hybrid Numerical Method

B. Ghadimi
Sharif University of Technology, Iran

Follow this and additional works at: https://scholarsmine.mst.edu/icrageesd

Part of the Geotechnical Engineering Commons

Recommended Citation

Ghadimi, B., "Dynamic Response of Pile Groups Embedded in Transversely Isotropic Media Using Hybrid Numerical Method" (2010). *International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*. 11.
https://scholarsmine.mst.edu/icrageesd/05icrageesd/session05/11

This work is licensed under a Creative Commons Attribution-Noncommercial-No Derivative Works 4.0 License.

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact scholarsmine@mst.edu.


Fifth International Conference on
Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics
and Symposium in Honor of Professor I.M. Idriss
May 24-29, 2010 • San Diego, California

DYNAMIC RESPONSE OF PILE GROUPS EMBEDDED IN TRANSVERSELY ISOTROPIC MEDIA USING HYBRID NUMERICAL METHOD

B. Ghadimi
MSc. Holder
Sharif University of Technology
Tehran-Iran, Azadi ST.

ABSTRACT

In this paper, the dynamic response of pile groups embedded in 3-D homogeneous transversely isotropic media subjected to time-harmonic vertical and horizontal loading is investigated. The response of pile groups is calculated by a novel method. This method is less complicated than boundary elements method (BEM) used in the most previous studies in this field. In the method the pile groups is discretized to some beam-column elements and radiation discs. The radiation discs represent the propagation of wave from piles to the unbounded soil medium. By calculating the dynamic stiffness of radiation discs and setting with beam-column elements, the dynamic stiffness of pile groups embedded in soil is properly determined. In this paper, the response of the transversely isotropic 3-D half-space subjected to time-harmonic vertical and horizontal excitations is presented in analytical form.
.

INTRODUCTION

Many structures exposed to dynamic loading such as machine foundations, bridges, or offshore structures are supported by piles arranged in a group, one of the primary purposes of which is to limit deformations to an acceptable level. If the spacing between the piles is very wide, the group stiffness may be evaluated by summing the contributions from the single piles. On the contrary, when the piles are closely spaced, which most often is the case, pile–soil–pile interaction occurs and has to be accounted for in the analyses. Due to this interaction, the load acting on a pile contributes to the motion of the other piles and wave propagation phenomena generally occur. As a result, the group efficiency (in other words, the ratio of the group stiffness to the sum of the individual pile stiffness), which for static loading is always smaller than unity if the pile installation effects are ignored, under dynamic excitation generally exhibits a strong oscillatory behavior when it is plotted versus the excitation frequency, and may even exceed unity depending on pile spacing, group size, frequency, and soil properties.
Many methods were developed to achieve a direct and complete analysis of pile groups under dynamic conditions (Sen et al., 1985; Mamoon et al., 1990; Cairo et al., 2005; Padro´n et al, 2006). These methods are essentially of a numerical nature and involve discretization of the domain (FEM) or its boundary (BEM). Generally, significant computational efforts are required, and large systems of equations have to be solved, especially when the group consists of a great number of piles.

In this paper, a simple method is presented to carry out the analysis of pile groups under time-harmonic vertical and horizontal vibration. In this hybrid numerical method, piles are modeled by FEM as rods and beams under vertical and horizontal loads respectively; moreover, the affect of the wave propagation through soil is considered by using radiation discs. (Fig. 1)

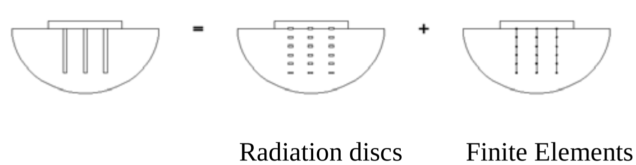


Fig. 1. Discretize a pile group to Finite Elements & Radiation Discs

PILE FINITE ELEMENTS EQUATIONS

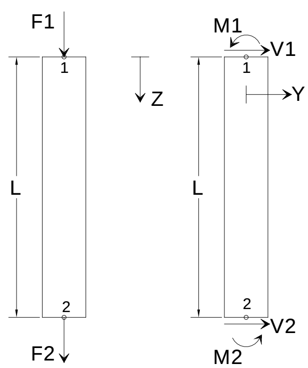
The behavior of a pile submitted to dynamic vertical loads can be described by the following differential equation. In this case, the pile is modelled by FEM as a rod. (Graff, 1991)

$$\frac{\partial^2 w(z,t)}{\partial z^2} = \frac{1}{C_0^2}\frac{\partial^2 w(z,t)}{\partial t^2} \tag{1}$$

Where

$$C_0 = \sqrt{\frac{E}{\rho}}$$

where E is Young's modulus, ρ is the mass density of the pile, and $w(z,t)$ is the longitudinal displacement of a cross-section of the pile. (Fig. 2.)



(a) (b)

Fig. 2. Pile Finite Element under (a) vertical load, (b) horizontal load

By solving the equation (1), and satisfying the boundary conditions of the pile element, the force-displacement relationship is derived as:

$$\begin{bmatrix} F1 \\ F2 \end{bmatrix} = \frac{EA\gamma}{\sin(\gamma L)}\begin{bmatrix} \cos(\gamma L) & -1 \\ -1 & \cos(\gamma L) \end{bmatrix}\begin{bmatrix} w1 \\ w2 \end{bmatrix} \tag{2}$$

$$\gamma = \frac{\omega}{C_0}$$

where A is the area of the section of the pile and L is the element length. As can be seen, the dynamic stiffness matrix of the element depends on the external load frequency (ω).

Under time-harmonic horizontal loading, piles are modelled by beam elements. In this case, the governing equation of the Pile element is expressed as: (Graff, 1991)

$$EI\frac{\partial^4 y(z,t)}{\partial z^4} + \rho_p A\frac{\partial^2 y(z,t)}{\partial t^2} = 0 \tag{3}$$

where I is the moment of inertia of the section of the pile, and $p(z,t)$ is a distributed force on the element. Also, for horizontal loading the force-displacement relationship is derived as:

$$\begin{Bmatrix} V1 \\ M1 \\ V2 \\ M2 \end{Bmatrix} = D\begin{bmatrix} -\alpha^3\Lambda_1 & -\alpha^2\Lambda_2 & \alpha^3\Lambda_4 & \alpha^2\Lambda_5 \\ -\alpha^2\Lambda_2 & \alpha\Lambda_3 & -\alpha^2\Lambda_5 & \alpha\Lambda_6 \\ \alpha^3\Lambda_4 & -\alpha^2\Lambda_5 & -\alpha^3\Lambda_1 & \alpha^2\Lambda_2 \\ \alpha^2\Lambda_5 & \alpha\Lambda_6 & \alpha^2\Lambda_2 & \alpha\Lambda_3 \end{bmatrix}\begin{Bmatrix} y1 \\ \theta 1 \\ y2 \\ \theta 2 \end{Bmatrix}$$

$$D = \frac{EI}{\cos(\alpha L)\cosh(\alpha L) - 1} \quad , \quad \alpha^4 = \frac{\omega^2 A}{C_0 L^4 I} \tag{4}$$

$$\Lambda_1 = \cos(\alpha L)\sinh(\alpha L) + \sin(\alpha L)\cosh(\alpha L)$$
$$\Lambda_2 = \sin(\alpha L)\sinh(\alpha L)$$
$$\Lambda_3 = \cos(\alpha L)\sinh(\alpha L) - \sin(\alpha L)\cosh(\alpha L)$$
$$\Lambda_4 = \sin(\alpha L) + \sinh(\alpha L)$$
$$\Lambda_5 = \cos(\alpha L) - \cosh(\alpha L)$$
$$\Lambda_6 = \sin(\alpha L) - \sinh(\alpha L)$$

where y is a transverse displacement, and θ is a rotational angle at each node of the element. As we will see in the next section, the dynamic stiffness matrix of the radiation discs corresponds to transverse displacement degrees of freedom. Therefore, the dynamic stiffness of beam elements must be condensed. For this purpose, the stiffness matrix is partitioned into transverse and rotational DOFs as

$$\begin{Bmatrix} V \\ M \end{Bmatrix} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix}\begin{Bmatrix} Y \\ \theta \end{Bmatrix} \quad \Rightarrow \tag{5}$$

$$\begin{cases} K_{11}Y + K_{12}\theta = V & (a) \\ K_{21}Y + K_{22}\theta = M & (b) \end{cases}$$

According to equations (a) and (b), we obtain the force-displacement relationship as

$$(K_{11} - K_{12}K_{22}^{-1}K_{21})Y = V - K_{12}K_{22}^{-1}M \tag{6}$$

FUNDAMENTAL SOLUTION FOR THE TRANSVERSELY ISOTROPIC HALF-SPACE

In this section the displacement functions for a transversely isotropic half-space subjected to a time-harmonic load are presented. The general solutions of the governing equations of motion (brought in Noorzad et al., 2002), in the frequency domain are given as follows:

$$u,v,w(r,\theta,z,t)=u,v,w(r,\theta,z)e^{i\omega t} \tag{7}$$

where u, v, and w are displacements in r, θ, and z directions respectively. The displacement components in the frequency domain can be expressed in terms of two scalar potential functions as: (Noorzad et al., 2002)

$$u=-\alpha_3\frac{\partial^2\psi}{\partial r\partial z}-\frac{1}{r}\frac{\partial X}{\partial\theta} \tag{8}$$

$$v=-\frac{\alpha_3}{r}\frac{\partial\psi}{\partial\theta\partial z}+\frac{\partial X}{\partial r} \tag{9}$$

$$w=(1+\alpha_1)(\nabla_{r\theta}{}^2+\beta\frac{\partial^2}{\partial z^2}+\frac{\rho_0\omega^2}{1+\alpha_1})\psi \tag{10}$$

Where

$$\alpha_1=\frac{(A_{11}+A_{12})}{(A_{11}-A_{12})}\ ,\ \alpha_2=\frac{2A_{44}}{(A_{11}-A_{12})}$$

$$\alpha_3=\frac{2(A_{13}+A_{44})}{(A_{11}-A_{12})}\ ,\ \beta=\frac{\alpha_2}{1+\alpha_1}$$

$$\rho_0=\frac{2\rho}{(A_{11}-A_{12})}\ ,\ \nabla_{r\theta}{}^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}$$

Also, A_{ij} are elastic moduli, and ρ is mass density of the media. Substituting equations 8-10 in the equations of motion yields all the equations of motion described in terms of the potential functions:

$$\nabla_0{}^2X=0 \tag{11}$$

$$\nabla_1{}^2\nabla_2{}^2\psi+B\rho\omega^2\frac{\partial^2\psi}{\partial z^2}=0 \tag{12}$$

where

$$\nabla_0{}^2=\nabla_{r\theta}{}^2+\frac{1}{S_0{}^2}\frac{\partial^2}{\partial z^2}+\rho_0\omega^2$$

$$S_0{}^2=\frac{1}{\alpha_2}\qquad ,\quad \mu_1=\alpha_2$$

$$\nabla_i{}^2=\nabla_{r\theta}{}^2+\frac{1}{S_i{}^2}\frac{\partial^2}{\partial z^2}+\frac{1}{\mu_i}\rho_0\omega^2$$

$$i=1,2\qquad ,\quad \mu_2=1+\alpha_1$$

$$B=\frac{1}{2A_{11}A_{44}}(\frac{b_1+2A_{44}{}^2-b_2}{A_{44}}+\frac{b_1+2A_{11}A_{33}+b_2}{A_{11}})$$

$$b_1=A_{13}{}^2+2A_{13}A_{44}-A_{11}A_{33}$$

$$b_2=\sqrt{(A_{13}{}^2-A_{11}A_{33})(b_1+2A_{13}A_{44}+4A_{44}{}^2)}$$

$S_1{}^2$ and $S_2{}^2$ are the roots of the following equation (Lekhnitskii, 1981):

$$A_{33}A_{44}S^4+(A_{13}{}^2+2A_{13}A_{44}-A_{11}A_{33})S^2+A_{11}A_{44}=0 \tag{13}$$

In past studies in this field, the coefficient of B was disregarded, while we proved that this coefficient has considerable effect on the response of the media. We, therefore, consider this coefficient in our solution, which caused this solution differs from the previous solutions.

The equations 11 and 12 are analytically solved by using Fourier series in the tangential direction of the coordinate, and using Hankel transform in the radial direction. Therefore, the potential functions are obtained as

$$X_m^m(\xi,z)=A_m(\xi)e^{-\alpha_0'z}+B_m(\xi)e^{+\alpha_0'z}$$
$$\alpha_0'=S_0\sqrt{\xi^2-\rho_0\omega^2} \tag{14}$$

$$\psi_m^m(\xi,z)=C_m(\xi)e^{-\alpha_1'z}+D_m(\xi)e^{+\alpha_1'z}$$
$$+E_m(\xi)e^{-\alpha_2'z}+F_m(\xi)e^{+\alpha_2'z}$$
$$\alpha'=\sqrt{\frac{-M\pm\sqrt{M^2-4LN}}{2N}} \tag{15}$$

where

$$N=\frac{1}{S_1^2S_2^2}\quad ,\quad L=(\frac{\rho_0\omega^2}{\mu_1}-\xi^2)(\frac{\rho_0\omega^2}{\mu_2}-\xi^2)$$

$$M=\frac{1}{S_1^2}(\frac{\rho_0\omega^2}{\mu_2}-\xi^2)+\frac{1}{S_2^2}(\frac{\rho_0\omega^2}{\mu_1}-\xi^2)+B\rho\omega^2$$

For satisfying radiation condition, we divide the media in two parts (Fig. 3)

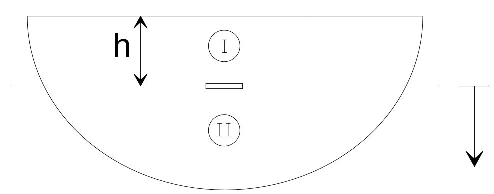


Fig. 3. Divided media embedding the loaded disc

In equation 14 and 15 for part (II), the terms of $e^{\alpha_i'z}$ must be omitted because only outwardly propagation waves are considered.

By substituting potential functions in transformed stress and displacement functions and satisfying boundary conditions, the coefficients of $X_m^m(\xi,z)$ and $\psi_m^m(\xi,z)$ for the two parts of the media are obtained. The displacement functions is expressed in terms of the potential functions as follows

$$w=(1+\alpha_1)\int_0^\infty \xi\left[-\xi^2+\frac{\alpha_2}{1+\alpha_1}\frac{d^2}{dz^2}+\frac{\rho_0\omega^2}{1+\alpha_1}\right]\psi_m^m J_0(\xi r)d\xi \qquad (16)$$

$$u=\frac{\alpha_3}{2}\left[\int_0^\infty (J_2(\xi r)-J_0(\xi r))\,\xi^2\,\frac{\partial \psi_m^m}{\partial z}d\xi\right] -\frac{i}{2}\left[\int_0^\infty (J_2(\xi r)-J_0(\xi r))\,\xi^2\,\frac{\partial X_m^m}{\partial z}d\xi\right] \qquad (17)$$

$$v=\frac{-i\alpha_3}{2}\left[\int_0^\infty (J_2(\xi r)-J_0(\xi r))\,\xi^2\,\frac{\partial \psi_m^m}{\partial z}d\xi\right] -\frac{1}{2}\left[\int_0^\infty (J_2(\xi r)-J_0(\xi r))\,\xi^2\,\frac{\partial X_m^m}{\partial z}d\xi\right] \qquad (18)$$

These integrals cannot be evaluated analytically. It is, thus, required to employ a suitable numerical scheme to evaluate these displacement functions. The singularities and oscillatory nature of the integrand require careful consideration in constructing the integration scheme and in setting the increment of variable of integration as well as upper limit of the integral at some appropriate values. Reviewing the oscillatory nature of different displacement functions, the trapezoidal method is used as the main scheme for numerical integration with the increment of $\Delta\xi$ set at a particular value less than 0.2. The upper limit of various integrals has been determined reviewing the characteristics of integrands for different frequencies. Because of the decaying functions $e^{-\alpha_i' z}$, the deeper the point is located, as is included in the integrand, the faster is the convergence.

DYNAMIC STIFFNESS MATRIX OF RADIATION DISCS

This matrix is obtained by inverting the dynamic flexibility matrix of Radiation Discs. By applying a dynamic unit force ($e^{i\omega t}$) at each disc ($P_i=e^{i\omega t}$ & $P_j=0,\ i\neq j$), and calculating displacements of that disc and the other radiation discs, the flexibility matrix will be achieved. As an example, the dynamic flexibility matrix of a square pile group is schematically brought (Fig. 4)

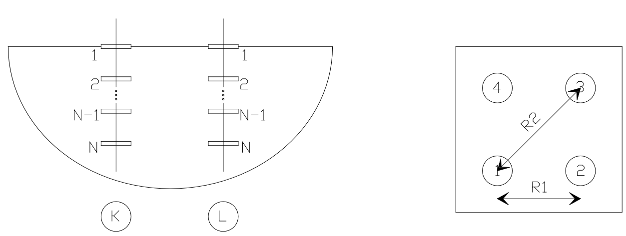


Fig. 4. Radiation discs in a square pile group

$$\begin{aligned} &For\ R=0 \ \rightarrow\ [C_{11}]\\ &For\ R=R1 \ \rightarrow\ [C_{12}]\\ &For\ R=R2 \ \rightarrow\ [C_{13}] \end{aligned}$$

$$[C]=\begin{bmatrix} [C_{11}] & [C_{12}] & [C_{13}] & [C_{12}]\\ & [C_{11}] & [C_{12}] & [C_{13}]\\ & & [C_{11}] & [C_{12}]\\ Sym. & & & [C_{11}] \end{bmatrix} \qquad (19)$$

where $[C]$ is dynamic flexibility matrix, and $[C_{KL}]$ is the displacement matrix of the radiation discs of pile L when unit force is applied to the radiation discs of pile K.

COUPLING FINITE ELEMENT AND RADIATION DISC STIFFNESS MATRICES

According to the fact that displacements of one radiation disc must be equal with the displacement of the corresponding node of the finite element, and because of the fact that Total external forces subjected to a pile group-soil are equal to the sum of the nodal forces subjected to finite elements and radiation discs, the dynamic pile group-soil stiffness matrix will be achieved by summing the Finite Element and Radiation Disc stiffness matrices

$$[K_T]=[K_P]+[K_s] \qquad (20)$$

In formulating the pile-soil interaction problem, it should be noted that the soil flexibility matrices are valid only for the whole medium without plies, i.e. the free field. Therefore, a modified pile density must be used to account for the added soil column mass

$$\overline{\rho}=\rho_p-\rho_s \qquad (21)$$

in which ρ_p and ρ_s are the pile and soil densities respectively.

RATE OF CONVERGENCE

Increasing number of the Elements, the impedance of pile groups converges to a limit value. In prediction of the response of pile groups, especially when the group consists of a great number of piles, it is very important that how fast this rate is. In Fig. 5 and 6 the rate of convergence of the impedances of a 2*2 square pile group embedded in Tr. Isotropic Clay subjected to dynamic vertical and horizontal loads has been shown where L and d is the length and diameter of the piles, respectively. Also, R is the side lengths of the 2*2 square pile group (Fig. 4)

Table 1. Parameters for the Pile Group

E_P	R	L/d	ρ
24820 *MPa*	3m	15	7 ton/m^3

Table 2. Parameters for Tr. Isotropic Clay

ρ	$E_H(Mpa)$	$E_H(Mpa)$	υ_{HH}	υ_{VH}	$n = E_H / E_V$
1.5 ton/m^3	59.6	47.68	0.37	0.49	1.25

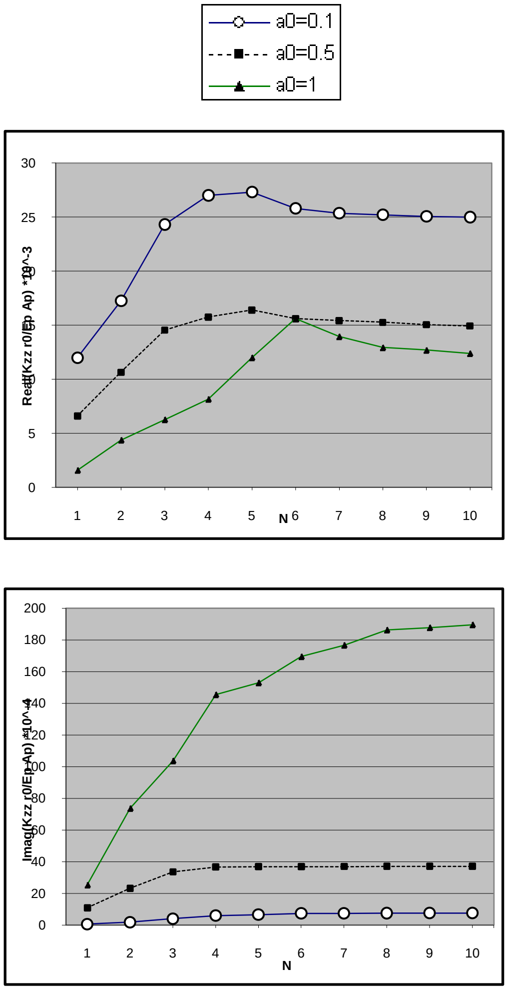


Fig 5. Convergence of the Impedance (Vertical Loading)

In these figures, a0 is the dimensionless frequency, Kyy is the horizontal impedance of the pile group head, and Kzz is the vertical impedance of the pile group head.

Note that the hybrid numerical method presented in this paper converges to a limit point with a high rate. Moreover, by increasing dimensionless frequency, we should increase the number of elements to achieve a solution with special accuracy.

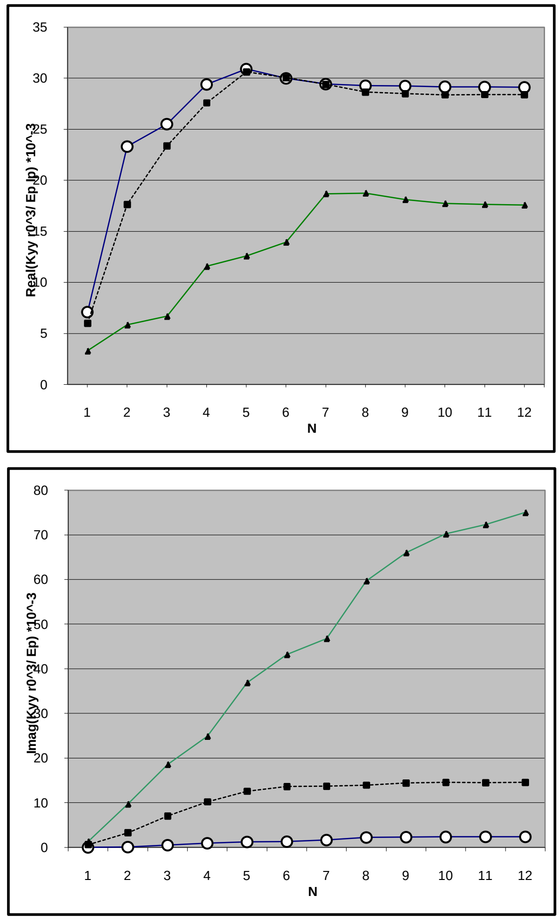


Fig. 6. Convergence of the Impedance (Horizontal Loading)

CONCLUSION

A simple method has been proposed for the analysis of pile groups subjected to time-harmonic vertical and horizontal loading. In this hybrid numerical method, piles are modelled by FEM as rods and beams under vertical and horizontal loads, respectively. According to high rate of convergence, this method is suggested especially when pile groups consist of a great number of piles.

REFERENCES

Cairo, R., Conte, E., Dente, G. [2005]. “Analysis of pile groups under vertical harmonic vibration.” *Computers and Geotechnics*, No.32, 545–554.

Graff, K. [1991]. “*Wave Motion in Elastic Solids*.” Dover Publications, Inc., New York.

Kaynia, AM, Kausel, E. [1982]. *"Dynamic behaviour of pile groups." 2nd International conference on numerical methods of offshore piling, Austin, Texas. p. 509–32.*

Lekhnitskii, S. G. [1981]. *"Theory of Elasticity of an Anisotropic Body"*, Mir, Moscow

Mamoon, SM., Kaynia, AM., Banerjee, PK. [1990]. "Frequency-domain dynamic analysis of piles and pile groups." *J Eng Mech ASCE; 116(10):2237–57.*

Noorzad, A., Eskandari Ghadi, M., and Konagai, K. [2002]. "Fundamental steady-state solution for the transversely isotropic half-space." *International Journal of Engineering. 105-122.*

Padro´n, L.A, Azna´rez, J.J., Maeso, O. [2006]. "BEM–FEM coupling model for the dynamic analysis of piles and pile groups" *Engineering Analysis with Boundary Elements*

Poulos, HG. (1968). "Analysis of the settlement of pile groups." *Ge´otechnique; 18(4):449–71.*

Sen, R., Kausel, E., Banerjee, PK., (1985). "Dynamic analysis of piles and pile groups embedded in non-homogeneous soils." *Int J Numer Anal Meth Geomech; 9:507–24.*