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# Cosmological and wormhole solutions in low-energy effective string theory

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We derive and study a class of cosmological and wormhole solutions for low-energy effective string field theory. We consider a general four-dimensional string effective action where moduli of the compactified manifold and the electromagnetic field are present. The cosmological solutions of the two-dimensional effective theory obtained by dimensional reduction of the former are discussed. In particular we demonstrate that the two-dimensional theory possesses a scale-factor duality invariance. Euclidean four-dimensional instantons describing the nucleation of the baby universes are found and the probability amplitude for the nucleation process is given.

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## I. INTRODUCTION

During the last years low-energy effective string field theory has been widely investigated in connection with its black hole [1–6] and cosmological [7–9] solutions. In particular, critical and noncritical string cosmological solutions have been discovered and analyzed in detail. The typical features of these solutions such as the scale-factor duality or the relationship with the corresponding black hole geometries have been also investigated [7–9]. The general idea underlying these investigations is that the short-distance modifications of string theory to general relativity could be crucial in order to understand long-standing problems of quantum gravity such as the loss of information in the black hole evaporation process or the nature of the singularities in the Einstein theory. After all, string theory is the only consistent framework which we presently have for quantizing gravity and it seems very natural to look at it in order to solve these problems.

If the gravitational field is correctly described by a quantum theory, such as string theory, the topology of spacetime is expected to fluctuate on Planck length scales. This is the old idea of *spacetime foam* [10]. Microscopic connections between large regions of spacetime [wormholes (WH's)] and the chaotic formation of Planck scale universes [baby universes (BU's)] branching off, or joining onto, a region of spacetime can have important effects on physics, even at low-energy scales [11]. Semiclassically, WH's are described by instantons which represent

a tunneling between two asymptotic four-dimensional regions of spacetime. If a WH can be joined at the throat to a hyperbolic universe whose spatial section is compact, then the instanton can be interpreted as nucleating a BU with a well-defined semiclassical amplitude.

Even though a large number of Euclidean instantons, and the corresponding cosmological solutions describing BU's, have been found in the context of the Einstein gravity theory [12–14], up to now very little is known about WH's and BU's nucleation in the framework of the low-energy effective string theory. This situation is somehow uncomfortable because if string theory has to solve the puzzles of quantum gravity it should also provide us the natural framework for studying processes such as the formation of WH's and BU's. A first attempt in this direction was made by Giddings *et al.* in Ref. [12]. Using a lowest-order effective string action they found an axionic WH which, however, has an infinite Euclidean action. More recently an instanton has been found which describes a tunneling between zero-energy vacua of string theory [15]. Finally, the existence of quantum WH solutions in the framework of the Jordan-Brans-Dicke theory predicted by string effective actions has been demonstrated in Ref. [16].

In this paper we shall derive and study four-dimensional cosmological solutions of the low-energy string effective action of Refs. [5,6]. This action takes into account, apart from the dilaton and the EM field, a modulus field which acquires nonminimal couplings to the gauge fields owing to string one-loop effects. The theory can be identified as a Jordan-Brans-Dicke gravity theory with Brans-Dicke parameter  $\omega$  taking values in  $[-1, \infty[$  and contains, as particular cases, both the dilaton-gravity theory of Refs. [2,3] and the Einstein-Maxwell theory. The cosmological solutions describe universes filled with the EM and the dilaton field. Even though these solutions do not seem to correspond, at least in the general

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case, to exact conformal string backgrounds, they are interesting from two different points of view. First, the dimensional reduction of the four-dimensional theory on the background defined by the magnetically charged solutions produces a two-dimensional effective theory whose solutions have all the features of two-dimensional string cosmological solutions. Second, the four-dimensional cosmological solutions can be interpreted as BU's nucleated by Euclidean WH's.

The outline of the paper is the following. In the next section we find and discuss the cosmological solutions of the four-dimensional action with both a purely magnetic and electric field. In Sec. III, we study the two-dimensional effective theory obtained by dimensional reduction of the four-dimensional one. In particular, we demonstrate that this theory possesses a scale-factor duality invariance and that its time-dependent solutions describe the region between the horizon and the singularity of the corresponding black hole geometry. In Sec. IV, we find the Euclidean instantons of the four-dimensional theory. These solutions can be joined at  $t = 0$  to their analytic continuations in the hyperbolic spacetime described by the cosmological solutions of Sec. II and then we are led to interpret the instantons as nucleating BU's. The probability amplitude for this process is calculated and using Coleman's mechanism we find that the probability distribution for the coupling constant  $\omega$  is strongly peaked at the value  $\omega = \infty$  corresponding to general relativity. Finally we state our conclusions in Sec. V.

## II. COSMOLOGICAL SOLUTIONS

Let us consider a four-dimensional hyperbolic manifold  $\Omega$  with metric  $g_{\mu\nu}$  and topology  $R \times H$ , where  $H$  is a three-dimensional compact Riemannian hypersurface with metric  $h_{ij}$  (here and in the following Greek indices run from 0 to 4 and Latin indices run from 1 to 3). Our starting point is the four-dimensional low-energy string effective action of [5,6] which generalizes the usual low-energy string effective action [2,3,17] for the case when a modulus is taken into account. Owing to string one-loop threshold effects this modulus is coupled to the EM field.

The action reads

$$S = \int_{\Omega} d^4x \sqrt{|g|} e^{-2\phi} \left[ R + 4(\nabla\phi)^2 - \frac{2}{3}(\nabla\psi)^2 - F^2 - e^{2\phi - (2/3)q\psi} F^2 \right] + 2 \int_{\partial\Omega} d^3x \sqrt{h} e^{-2\phi} (\mathbf{K} - \mathbf{K}_0). \quad (2.1)$$

Here  $R$  is the curvature scalar,  $\phi$  is the dilaton field,  $\psi$  a modulus field,  $F_{\mu\nu}$  is the usual EM field tensor, and  $q$  is a coupling constant. We have put  $16\pi G \equiv M_{\text{Pl}}^2/16\pi = 1$ , then measuring all dimensional quantities in these units. The boundary term is required by unitarity [18];  $\mathbf{K}$  is the trace of the second fundamental form  $\mathbf{K}_{ij}$  of  $H$  and  $\mathbf{K}_0$  is that of the asymptotic three-surface embedded in flat space. The latter contribution must be introduced if one requires the spacetime to be asymptotically flat. We will see that the surface term in (2.1) plays a very important

role in the computation of the probability amplitude for BU formation.

Following [5,6] we choose for the modulus field the ansatz

$$e^{-(2/3)q\psi} = \frac{3}{q^2} e^{-2\phi}. \quad (2.2)$$

Using (2.2), the action (2.1) reduces to the form

$$S = \int_{\Omega} d^4x \sqrt{|g|} e^{-2\phi} \left[ R - \frac{8k}{1-k} (\nabla\phi)^2 - \frac{3+k}{1-k} F^2 \right] + 2 \int_{\partial\Omega} d^3x \sqrt{h} e^{-2\phi} (\mathbf{K} - \mathbf{K}_0), \quad (2.3)$$

where

$$k = \frac{3 - 2q^2}{3 + 2q^2}, \quad -1 \leq k \leq 1. \quad (2.4)$$

We have several interesting cases according to the value of  $k$ . For  $k = -1$  (i.e.,  $q \rightarrow \infty$ ) the action reduces to the usual low-energy string action when the modulus  $\psi$  is not taken into account; (2.3) describes then the four-dimensional dilaton-gravity theory considered in [2,3]. For  $k = 0$  we have a four-dimensional action whose two-dimensional reduction gives the Jackiw-Teitelboim theory [6]. The case  $k = 1$  looks singular. However, inserting  $q = 0$  in the action (2.1) and using the equations of motion that force the dilaton to be constant, we recover the usual Einstein-Maxwell theory. Using the ansatz (2.2) exact solutions can be obtained for any value of the coupling constant  $k \in [-1, 1]$ .

One can easily realize that the action (2.3) describes a Brans-Dicke theory coupled to the electromagnetic field. Indeed the redefinition  $\Phi = \exp(-2\phi)$  brings the action (2.3) in the Jordan-Brans-Dicke form with a Brans-Dicke parameter  $\omega = 2k/(1-k)$ . As expected we recover general relativity ( $\omega = \infty$ ) for  $k = 1$ .

Since the EM field prevents spatially homogeneous and isotropic solutions of the field equations, we look for solutions of the form

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\chi^2 + b^2(t)d\Omega_2^2, \quad (2.5)$$

where  $\chi$  is the coordinate of the one-sphere,  $0 \leq \chi < 2\pi$ , and  $d\Omega_2^2$  represents the line element of the two-sphere.  $N(t)$  is the lapse function. The line element (2.5) is known as of the Kantowski-Sachs type [19] and describes a hyperbolic spacetime  $\Omega$  whose three-dimensional spatial hypersurfaces  $H$  have topology  $S^1 \times S^2$ .

Now, we have to consider a form of the EM field compatible with the topology of the spacetime. A suitable configuration is given by the magnetic monopole on the two-sphere:

$$F = Q_m \sin\theta d\theta \wedge d\varphi, \quad (2.6)$$

where  $Q_m$  is the magnetic charge. Equation (2.6) describes a purely magnetic field. Later in this section we will consider a purely electric field with only nonvanishing components along the  $\chi$  direction [14]:

$$A = A(t)d\chi, \quad (2.7)$$

where  $A$  is the EM potential one-form. As we shall see, the solutions corresponding to (2.6) and (2.7) are related by a duality transformation.

It is straightforward (but not so easy; see the Appendix) to find the magnetic charged solution of the equations of motion derived from the action (2.3):

$$ds^2 = -dt^2 + Q^2 \sin^2(t/Q) \left[ \frac{1 + \cos(t/Q)}{2} \right]^{k-1} d\chi^2 + Q^2 d\Omega_2^2, \quad (2.8a)$$

$$e^{2(\phi-\phi_0)} = \left[ \frac{1 + \cos(t/Q)}{2} \right]^{(k-1)/2}, \quad (2.8b)$$

where we have redefined the magnetic charge  $Q_m$  through

$$Q_m = \frac{1}{2} \sqrt{1-k} Q. \quad (2.9)$$

The previous solution exists and is well defined for any  $-1 \leq k < 1$ . For  $k = 1$  the redefinition of the magnetic charge (2.9) becomes singular. This is not surprising because the ansatz (2.2) is singular for  $k = 1$  (i.e.,  $q = 0$ ); so, the solutions for this particular case have to be determined by starting directly from the action (2.1) with  $q = 0$ . One can easily verify that the corresponding solution is described by (2.6)–(2.8) where now  $k = 1$  and  $Q_m = Q$ .

Let us study the properties of the solution (2.8). The line element (2.8a) describes a universe whose spatial sections are compact with topology  $S^1 \times S^2$ . The scale factor of the two-sphere is constant, while the radius of the one-sphere is periodic in time. The behavior of the line element (2.8a) depends on  $k$ , so it is convenient to study separately the following cases.

(a)  $k = 1$ , the Einstein-Maxwell theory. In this case the line element reduces to the one found in [14] for the Einstein gravity. The radius  $a$  of the one-sphere takes values in the range  $[0, Q]$  and the line element is singular at  $t = n\pi Q$ ,  $n = 0, \pm 1, \pm 2, \dots$ , where the scale factor  $a$  vanishes. This singularity can be removed by a different choice of coordinates [14]. This can be easily seen noting that in the neighborhood of  $t = 0$  the line element (2.8a) reduces to the form

$$ds^2 = -dt^2 + t^2 d\chi^2 + Q^2 d\Omega_2^2. \quad (2.10)$$

The topology is locally  $R^{1,1} \times S^2$  and the three-

dimensional spatial hypersurface becomes homotopic to  $S^2$  and a point. Thus (2.8a) represents a universe which periodically reproduces itself with period  $\pi Q$ . In this case the dilaton is constant.

(b)  $0 < k < 1$ . Contrary to the previous case, when  $k$  takes values in the interval  $]0, 1[$ , the metric has a curvature singularity for  $t = (2n+1)\pi Q$ , where the dilaton diverges and the theory becomes strongly coupled. At  $t = 2n\pi Q$  there is a coordinate singularity analogous to case (a). The radius of the one-sphere vanishes for  $t = n\pi Q$  and has a maximum for  $\cos(t/Q) = (k-1)/(k+1)$ . In this case (2.8a) describes a universe whose two-sphere scale factor remains constant, whereas the radius of the one-sphere vanishes at  $t = 0$ , grows until a maximum value and becomes again zero after a time  $t = \pi Q$ .

(c)  $k = 0$ . In this case (2.8a) describes a periodic universe with period  $2\pi Q$ . The scale factor  $a$  vanishes for  $t = 2n\pi Q$ , where there is a coordinate singularity analogous to case (a), and takes its maximum value  $a_{\max} = 2Q$  when  $t = (2n+1)\pi Q$ . Note that even though there are no curvature singularities, the dilaton diverges and the theory becomes strongly coupled for  $t = (2n+1)\pi Q$ .

(d)  $-1 \leq k < 0$ . The scale factor  $a$  vanishes for  $t = 2n\pi Q$ , where the metric shows a coordinate singularity, and goes to the infinity for  $t = (2n+1)\pi Q$  where the line element has a curvature singularity. Hence the radius of the one-sphere starts with zero at  $t = 0$  and grows to infinity at  $t = \pi Q$ . Note that  $k = -1$  corresponds to the usual dilaton-gravity theory.

As we shall see in Sec. IV,  $\forall k \in [-1, 1]$  the solution (2.8) can be joined at  $t = 0$  with a Euclidean asymptotically flat instanton and then (2.8a) can be interpreted as the line element of a BU nucleated starting from an asymptotically flat region.

To conclude this section, let us briefly discuss the solutions obtained from the purely electric field (2.7). It is well known that the equations of motion for four-dimensional dilaton gravity coupled to the EM field are invariant under the discrete duality transformation [3]

$$F_{\mu\nu} \rightarrow \frac{1}{2} e^{-2\phi} \epsilon_{\mu\nu}{}^{\lambda\rho} F_{\lambda\rho}, \quad \phi \rightarrow -\phi, \quad g_c \rightarrow g_c, \quad (2.11)$$

when expressed in terms of the canonical metric  $g_c = e^{-2\phi} g_s$ . Transformation (2.11) relates magnetically to electrically charged solutions. This invariance also holds for the theory described by (2.3) when the action is expressed in terms of the canonical metric  $g_c$  [6]. Using the duality invariance (2.11) of the equations of motion, it is straightforward to obtain in the string frame the electrically charged solution

$$ds^2 = e^{4\phi_0} \left[ \frac{1 + \cos(t/Q)}{2} \right]^{1-k} \left\{ -dt^2 + Q^2 \sin^2(t/Q) \left[ \frac{1 + \cos(t/Q)}{2} \right]^{k-1} d\chi^2 + Q^2 d\Omega_2^2 \right\}, \quad (2.12a)$$

$$F = \frac{1}{2} \sqrt{1-k} e^{2\phi_0} \sin(t/Q) dt \wedge d\chi, \quad (2.12b)$$

$$e^{2(\phi-\phi_0)} = \left[ \frac{1 + \cos(t/Q)}{2} \right]^{(1-k)/2}, \quad (2.12c)$$

where  $Q$  is related to the electric charge by the same relation as in (2.9). Note that for  $k = 1$  (2.8a) and (2.12a) coincide after taking  $\phi_0 = 0$ ; indeed, in this case the dilaton is constant and the duality invariance holds also in the string frame. Solution (2.12) has properties analogous to solution (2.8). The line elements (2.8a) and (2.12a) differ only for a conformal factor because of the duality relation. The most striking difference between the two solutions resides in the fact that differently from (2.8) the scale factor for the two-sphere in (2.12a) is not constant. As we shall see in the following, the two-dimensional section of the magnetic solution (2.8) can be described in terms of an effective two-dimensional theory obtained by dimensional reduction of the action (2.3). This is of course not possible for the electric solution (2.12).

### III. THE TWO-DIMENSIONAL EFFECTIVE THEORY AND DUAL SOLUTIONS

The metric part of the magnetic solution (2.8) has the form of a direct product of a two-dimensional solution and a two-sphere of constant radius. Thus, it is useful to study the two-dimensional effective theory obtained by retaining only the time-dependent modes of the four-dimensional theory. This two-dimensional theory is expected to describe the essential four-dimensional physics for perturbations around the background solution (2.8). The action (2.3) can be dimensionally reduced by taking the angular coordinates to span a two-sphere of constant radius  $Q$ . The resulting two-dimensional action is

$$S = \int d^2x \sqrt{|g|} e^{-2\phi} \left[ R - \frac{8k}{1-k} (\nabla\phi)^2 + \lambda^2 \right], \quad (3.1)$$

where  $\lambda^2 = (1-k)/2Q^2$ . This two-dimensional action has been studied in connection with its black hole solutions and its duality invariances in [6,20]. In this section we will study (3.1) from the cosmological point of view. As shown in [20] considering space-dependent field configurations, action (3.1) possesses a duality symmetry. It is easy to see that this duality invariance also holds for time-dependent configurations. Let us consider the metric and the dilaton field of the form

$$ds^2 = -dt^2 + e^{2\rho(t)} dx^2, \quad \phi = \phi(t), \quad (3.2)$$

where  $0 \leq x < \infty$ . The action becomes

$$S = \int dt e^{-2\phi+\rho} \left( 2(\ddot{\rho} + \dot{\rho}^2) + \frac{8k}{1-k} \dot{\phi}^2 + \lambda^2 \right), \quad (3.3)$$

where the overdots represent time derivatives. One can easily check that the transformation

$$\rho \rightarrow k\rho - 2(k+1)\phi, \quad \phi \rightarrow \frac{k-1}{2}\rho - k\phi \quad (3.4)$$

leaves the action invariant modulo a total derivative. The duality transformation (3.4) is the generalization for the action (3.1) of the scale-factor duality symmetry of string

theory [7,9]. Indeed, for  $k = -1$  we get the standard scale-factor duality transformation  $\rho \rightarrow -\rho$ ,  $\phi \rightarrow \phi - \rho$  which exchanges the radius of the two-dimensional universe with its inverse.

Let us now discuss the cosmological solutions of the two-dimensional theory and their behavior under the duality transformation (3.4). The time-dependent solution of action (3.1) is easily found to be

$$ds^2 = -dt^2 + \sin^2(t/2Q) [\cos^2(t/2Q)]^k dx^2, \quad (3.5a)$$

$$e^{2(\phi-\phi_0)} = [\cos^2(t/2Q)]^{(k-1)/2}. \quad (3.5b)$$

Considering a periodic space, i.e., setting  $x = 2Q\chi$ ,  $0 \leq \chi < 2\pi$ , the solution (3.5) coincides with the two-dimensional section of solution (2.8). The effect of the duality transformation (3.4) on the solution (3.5) is to exchange the sine and cosine everywhere:

$$ds^2 = -dt^2 + \cos^2(t/2Q) [\sin^2(t/2Q)]^k dx^2, \quad (3.6a)$$

$$e^{2(\phi-\phi_0)} = [\sin^2(t/2Q)]^{(k-1)/2}. \quad (3.6b)$$

The dual solution corresponds of course to a solution of the four-dimensional theory (see the Appendix). Moreover, for  $k = 1$  (3.5) and (3.6) are the same, i.e., the solution is self-dual. Comparing Eq. (3.5) with Eq. (3.6) and keeping in mind the discussion of the previous section, one easily realizes that the effect of the duality transformation (3.4) on the solutions with  $k \neq 1, 0$  is to exchange the coordinate singularities at  $t = 2n\pi Q$  with the curvature singularities at  $t = (2n+1)\pi Q$ . For  $k = 0$  there are no curvature singularities and the duality transformation simply exchanges strong string couplings with weak ones.

The previous cosmological solutions are a further example of the two-dimensional string cosmologies studied in [7–9]. They exhibit all the peculiar properties of string cosmological solutions such as the above-discussed duality invariance. In particular, for  $k = -1$  solutions (3.5) and (3.6) correspond to well-known  $D = 2$  cosmological conformal string backgrounds [7–9]. However, for generic  $k$  we do not know if the interpretation of (3.5) and (3.6) as conformal string backgrounds can be maintained. In this context the case  $k = 0$  seems very interesting. As we have seen, the cosmological solution describes a universe which periodically reproduces itself without encountering a singularity, thus avoiding the singularity problem which affects the models with  $k \neq 0, 1$ .

To conclude this section, let us discuss the relationship between the two-dimensional cosmological solutions and the corresponding two-dimensional black hole geometries. For the particular case  $k = -1$ , it has been already shown that the cosmological solution (3.5) describes the region between the horizon and the singularity of the black hole geometry [9] derived from (3.1):

$$ds^2 = -4Q^2 \tanh^2(x/2Q) d\tau^2 + dx^2, \quad (3.7a)$$

$$e^{2(\phi-\phi_0)} = [\cosh(x/2Q)]^{-2}. \quad (3.7b)$$

This construction can be easily generalized for arbitrary  $k$ . Consider the metric (3.5a) expressed in terms of the

periodic coordinate  $\chi = x/2Q$  and choose the new coordinates

$$u = e^{t^* + \chi}, \quad v = e^{t^* - \chi},$$

$$t^* = \frac{1}{2Q} \int \frac{dt}{\sin(t/2Q) \cos^k(t/2Q)}$$

$$= \frac{y^{(1-k)/2}}{k-1} F\left(\frac{1-k}{2}, 1, \frac{3-k}{2}, y\right), \quad (3.8)$$

where  $F$  is the hypergeometric function and  $y = \cos^2(t/2Q)$ . The line element (3.5a) becomes

$$ds^2 = -4Q^2 \frac{(1-y)y^k}{uv} du dv. \quad (3.9)$$

An identical form for the metric can be obtained starting from the black hole solution of the action (3.1) [6],

$$ds^2 = -4Q^2 \sinh^2(x/2Q) [\cosh(x/2Q)]^{2k} d\tau^2 + dx^2, \quad (3.10a)$$

$$e^{2(\phi - \phi_0)} = [\cosh(x/2Q)]^{k-1}, \quad (3.10b)$$

and introducing the coordinates

$$u = e^{x^* + \tau}, \quad v = e^{x^* - \tau},$$

$$x^* = \frac{1}{2Q} \int \frac{dx}{\sinh(x/2Q) \cosh^k(x/2Q)}$$

$$= \frac{y^{(1-k)/2}}{k-1} F\left(\frac{1-k}{2}, 1, \frac{3-k}{2}, y\right), \quad (3.11)$$

but now with  $y = \cosh^2(x/2Q)$ . In (3.8),  $y$  takes values in the interval  $[0,1]$ , whereas in (3.11)  $1 \leq y < \infty$ . Hence, solution (3.9) describes the region between the horizon and the singularity of the black hole solution (3.10). Strictly speaking, because we are working with  $\chi$  periodic, this correspondence holds for a wedge in the region between the horizon and the singularity (see [9]).

As shown in Ref. [9] for the case  $k = -1$ , one could continue the time past the singularity at  $t = \pi Q$  to get an identical copy of the interior of the black hole where the Universe now starts at the singularity and evolves until it reaches zero size at  $t = 2\pi Q$ . By continuing this procedure, i.e., not identifying  $t \rightarrow t + 2\pi Q$ , we end up with a universe which undergoes infinitely many oscillations. This construction cannot be taken too seriously because near the singularity, where the size of the Universe becomes infinitely large, one cannot trust anymore the low-energy string effective action (3.1) and one should consider the exact theory. We will not discuss this point further, we just note that our model with  $k = 0$  avoids the singularity problem. Indeed, for this value of  $k$  the scalar curvature stays everywhere finite and only the dilaton diverges at  $t = (2n+1)\pi Q$  indicating that the theory becomes strongly coupled.

#### IV. EUCLIDEAN INSTANTONS

Let us now discuss the action (2.3) in the Riemannian space. In order to do this, we have to deal with the ambiguity in the choice of the sign for the action of the EM field. Indeed, the EM field in the Euclidean space is not analytically related to the EM field in the hyperbolic space; i.e., the two fields are not related by the coordinate transformation  $t \rightarrow i\tau$ : a real electric field in the hyperbolic spacetime gives, once continued in the Euclidean space, an imaginary electric field. Using a more or less *ad hoc* rule, one normally requires the Euclidean EM field and its action  $S_E$  to be real; i.e., one uses the hyperbolic expression of the action with an overall minus sign (the discussion about the Euclidean formulation of the Maxwell theory can be found in Ref. [21]). Of course, other prescriptions are possible. We will consider the Euclidean version of the action (2.3) in the general form

$$S_E = \int_{\Omega} d^4x \sqrt{|g|} e^{-2\phi} \left[ -R + \frac{8k}{1-k} (\nabla\phi)^2 + \varepsilon \frac{3+k}{1-k} F^2 \right]$$

$$- 2 \int_{\partial\Omega} d^3x \sqrt{h} e^{-2\phi} (\mathbf{K} - \mathbf{K}_0), \quad (4.1)$$

where  $\varepsilon = \pm 1$ . The meaning of the parameter  $\varepsilon$  can be understood looking at the EM field. According to the sign of  $\varepsilon$ , the electric and the magnetic fields in the hyperbolic and Euclidean space are related by

$$E_{\text{hyp}}^2 = \varepsilon E_{\text{Eucl}}^2, \quad H_{\text{hyp}}^2 = -\varepsilon H_{\text{Eucl}}^2.$$

Thus, if we require the analytical continuations of real hyperbolic fields to be real fields in Euclidean space, we must choose  $\varepsilon = -1$  for the purely magnetic configuration (2.6) and  $\varepsilon = 1$  for the purely electric configuration (2.7). Another argument that supports the above choice relies on the duality invariance (2.11). One can easily see that (2.11) does not hold for the Euclidean EM theory. This is essentially due to the fact that the Euclidean EM energy is proportional to  $\exp(-2\phi)(E^2 - H^2)$ , thus changing sign under the transformation (2.11). Hence, in order to maintain the duality relation (2.11) in the Euclidean space we must reverse the sign of  $\varepsilon$  in the action in passing from the purely magnetic configuration (2.6) to the purely electric configuration (2.7).

Let us first consider the purely magnetic ansatz. We have to choose  $\varepsilon = -1$  in (4.1) and the Euclidean equations of motion are solved by

$$ds^2 = d\tau^2 + 2^{(1-k)} Q^2 \frac{\tau^2}{\tau^2 + Q^2} \left( 1 + \frac{Q}{\sqrt{\tau^2 + Q^2}} \right)^{k-1} d\chi^2$$

$$+ (\tau^2 + Q^2) d\Omega_2^2, \quad (4.2a)$$

$$F = \frac{\sqrt{1-k}}{2} Q \sin\theta d\theta \wedge d\varphi, \quad (4.2b)$$

$$e^{2(\phi - \phi_0)} = 2^{(1-k)/2} \left( 1 + \frac{Q}{\sqrt{\tau^2 + Q^2}} \right)^{(k-1)/2} \quad (4.2c)$$

Let us study the properties of the solution (4.2). In the asymptotic regions  $\tau \rightarrow \pm\infty$ , the line element becomes

$$ds^2 = d\tau^2 + 2^{(1-k)}Q^2d\chi^2 + \tau^2d\Omega_2^2. \tag{4.3}$$

Thus, the asymptotic Riemannian space is flat with topology  $R^3 \times S^1$ . At  $\tau = 0, \forall k \in [-1, 1]$ , the metric is singular. This singularity is only due to the choice of the coordinates that cover only half of the manifold described by the line element (4.2a). Indeed, it is possible to find a new chart that covers the whole space (for the case  $k = 1$ , see [14]). One can easily verify this, observing that in the neighborhood of  $\tau = 0$ , (4.2a) becomes

$$ds^2 = d\tau^2 + \tau^2d\chi^2 + Q^2d\Omega_2^2; \tag{4.4}$$

hence, the singularity at  $\tau = 0$  can be removed going to Cartesian coordinates in the  $(\tau, \chi)$  plane and adding the point  $\tau = 0$ . Thus, in the neighborhood of  $\tau = 0$  the topology is locally  $R^2 \times S^2$  with  $R^2$  contracting to zero as  $\tau \rightarrow 0$ . This particular case has been classified by Gibbons and Hawking [22] as a ‘‘bolt’’ singularity.

The asymptotic behavior of solution (4.2) and its regularity allow us to interpret the instanton (4.2a) as a WH that connects two asymptotic flat regions. Moreover, the instanton (4.2) can be joined at  $\tau = 0$  with the cosmological solution (2.8). Thus, (4.2) describes the nucleation of a BU starting from an original flat region. Let us see this in detail.

As one can easily see, solutions (4.2) with  $-\infty < \tau < 0$  and (2.8) with  $t > 0$  satisfy Darboux’ conditions for change of signature at  $t = \tau = 0$  [23]; indeed, at  $t = \tau = 0$  both the Euclidean and hyperbolic manifolds are well defined and the first and second fundamental forms of the three-dimensional hypersurfaces coincide smoothly for  $\tau \rightarrow 0^-$  and  $t \rightarrow 0^+$ . The dilaton is continuous with its derivative on the hypersurface  $t = \tau = 0$ , where the change of signature occurs. Therefore, the asymptotic behavior for  $\tau \rightarrow \pm\infty$  of the Euclidean solution (4.2) allows us to interpret (4.2) as an instanton which provides a tunneling between a flat vacuum region and the universe described by (2.8a).

In conclusion, solution (4.2) describes the nucleation of a nonisotropic BU at  $t = 0$  starting from an original flat spacetime and it is the generalization to effective string theory of the solution found in [14]. The hypersurface of signature change is two dimensional: this corresponds to the particular situation of a BU nucleated in a phase of maximum shrinkage of the spatial metric. Once the BU is nucleated then it evolves according to (2.8) and eventually ends in a singularity depending on the parameter  $k$ .

As stated in the Introduction, the existence of WH solutions for Jordan-Brans-Dicke theories have already been demonstrated in [16]. In particular, solutions of the Wheeler-DeWitt equations describing quantum WH’s have been found in the absence of matter, even though an

explicit analytic form is not given. However, it is difficult to compare our solutions with those of [16] because in our case the electromagnetic field plays a crucial role. Solutions (4.2) do not exist for  $Q = 0$ .

Let us now discuss the amplitude probability of nucleation of a BU. In the semiclassical approximation, the amplitude probability in a Planck volume and in a Planck time is given by [24]

$$\Gamma = e^{-|\tilde{S}|}, \tag{4.5}$$

where  $\tilde{S}$  represents the Euclidean action evaluated on the solutions of the classical equations of motion. After a straightforward calculation and taking into account the boundary terms to cancel the divergent contribution coming from the asymptotic region, one finds

$$\Gamma = \exp[-8\pi^2 e^{-2\phi_0} Q^2 (k + 1)]. \tag{4.6}$$

For  $k \neq -1$ , in order to have a probability of the order of unity, the charge  $Q$  appearing in the solution must be of the order of the unity, so the nucleation probability is maximum for BU’s with dimension of order of the Planck length. Conversely, for the usual dilaton-gravity theory ( $k = -1$ ), the semiclassical amplitude probability (4.6) does not fix the dimension of the BU, because one obtains  $\Gamma = 1$  for any value of the charge  $Q$ . In this case, in order to fix the amplitude probability one must consider higher-order contributions in the string tension  $\alpha'$  to the low-energy string effective action.

Following Coleman’s argument for the vanishing of the cosmological constant, in [16] it was argued that the most probable value of the effective Brans-Dicke parameter is  $\omega = \infty$ , i.e., general relativity is the low-energy effective theory of gravity. The demonstration was performed using a Jordan-Brans-Dicke theory with a cosmological constant; however, one would expect this result to hold independently of the details of the matter action to which gravity couples. Let us prove that this result indeed holds for the theory defined by action (2.3). Using a dilute WH approximation and taking into account that the main contributions to the path integral come from the classical Euclidean action evaluated at its saddle point, Coleman’s mechanism gives for the probability distribution  $\mathcal{Z}(k)$  of the parameter  $k$

$$\ln \mathcal{Z}(k) = \exp\left(32\pi^2 e^{-2\phi_0} Q_m^2 \frac{1+k}{1-k}\right), \tag{4.7}$$

where we made use of Eq. (2.9) to reinstate the full dependence of the action on the parameter  $k$ . The probability distribution for the coupling constant  $k$  is strongly peaked at  $k = 1$ ; i.e., the most probable value for the effective Brans-Dicke parameter is  $\omega = \infty$  according to the results of [16].

To conclude this section, let us discuss the Euclidean instanton for the purely electric field (2.7). Now we have to choose  $\varepsilon = -1$  in action (4.1) and we find

$$ds^2 = e^{4\phi_0} \left(1 + \frac{Q}{\sqrt{\tau^2 + Q^2}}\right)^{1-k} \left[d\tau^2 + 2^{(1-k)}Q^2 \frac{\tau^2}{\tau^2 + Q^2} \left(1 + \frac{Q}{\sqrt{\tau^2 + Q^2}}\right)^{k-1} d\chi^2 + (\tau^2 + Q^2)d\Omega_2^2\right], \tag{4.8a}$$

$$F = \frac{1}{2}\sqrt{1-k}Q^2e^{2\phi_0}\frac{\tau}{(\tau^2+Q^2)^{3/2}}d\tau \wedge d\chi, \quad (4.8b)$$

$$e^{2(\phi-\phi_0)} = 2^{(k-1)/2} \left(1 + \frac{Q}{\sqrt{\tau^2+Q^2}}\right)^{(1-k)/2}. \quad (4.8c)$$

Analogously to the purely magnetic configuration, one can easily verify that solution (4.8) can be joined at  $t = \tau = 0$  to the electric cosmological solution (2.12), so (4.8) can be interpreted as an instanton nucleating a BU with metric (2.12a). The amplitude probability for this process coincides with (4.6).

## V. CONCLUSIONS

In this paper we have derived and studied a class of cosmological solutions for a low-energy effective string field theory which coincides with a Jordan-Brans-Dicke gravity theory coupled with an electromagnetic field. These four-dimensional solutions describe universes filled with the dilaton and a purely magnetic or a purely electric field. They are characterized by a parameter  $k$ , related to the Brans-Dicke parameter  $\omega$ , and their behavior depends crucially on this parameter. In particular, in the purely magnetic case, for  $k = 1$  (the Einstein-Maxwell theory) and for  $k = 0$ , the solution describes a universe which periodically reproduces itself without running in a singularity. For  $k \neq 0, 1$  the solution describes a universe which ends in a singularity. This behavior holds in particular for  $k = -1$  which is the case of the usual dilaton gravity coupled to the EM field. We have also shown the theory can be dimensionally reduced to a two-dimensional theory. The two-dimensional theory exhibits a scale-factor duality symmetry which is a generalization of the duality symmetries found for exact cosmological conformal string backgrounds. For  $k = -1$  the two-dimensional solution reduces to a well-known string conformal background [7-9]. Finally, the two-dimensional time-dependent solutions describe the region between the horizon and the singularity of the black hole solutions of the two-dimensional theory. This feature has been found also for exact conformal string backgrounds.

The four-dimensional cosmological solutions can be also interpreted as BU's nucleated starting from a flat spacetime region. The Euclidean instantons describing this process have been found and the amplitude probability of nucleation has been calculated. Finally, using Coleman's mechanism, we have shown that the probability distribution for the effective coupling constant  $k$  is strongly peaked at  $k = 1$ , i.e., at the value  $\omega = \infty$  of the effective Brans-Dicke parameter corresponding to general relativity.

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## APPENDIX

Here we deduce solution (2.8) and its dual of Sec. III. Since computations are not trivial, we shall show them

in detail. The calculations for the Euclidean solutions of Sec. IV are analogous.

Our starting point is action (2.3). The calculation is much simpler if we express the action in terms of the canonical metric. Rescaling the metric as  $g_{\mu\nu} \rightarrow \exp(2\phi)g_{\mu\nu}$  we get the action in the canonical frame:

$$S = \int_{\Omega} d^4x \sqrt{|g|} \left[ R - 2\frac{3+k}{1-k} [(\nabla\phi)^2 + \frac{1}{2}e^{-2\phi}F^2] \right] + 2 \int_{\partial\Omega} d^3x \sqrt{h} (\mathbf{K} - \mathbf{K}_0). \quad (A1)$$

The equations of motion are

$$G_{\mu\nu} = 2\frac{3+k}{1-k} \left[ \nabla_{\mu}\phi\nabla_{\nu}\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2 + e^{-2\phi} \left( F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}F^2 \right) \right], \quad (A2a)$$

$$\nabla^2\phi = -\frac{1}{2}e^{-2\phi}F^2, \quad (A2b)$$

$$\nabla_{\mu}(e^{-2\phi}F^{\mu\nu}) = 0. \quad (A2c)$$

Using the ansatz (2.6) for the EM field, we can easily see that Eq. (A2c) is identically satisfied. Substituting the line element (2.5) and (2.6) in the other equations, (A2) reduce to

$$\frac{\ddot{b}^2}{\dot{b}^2} + \frac{N^2}{\dot{b}^2} + 2\frac{\dot{a}\dot{b}}{ab} = \frac{3+k}{1-k} \left( \dot{\phi}^2 + Q_m^2 \frac{N^2}{b^4} e^{-2\phi} \right), \quad (A3a)$$

$$2\frac{\ddot{b}}{\dot{b}} + \frac{\dot{b}^2}{\dot{b}^2} - 2\frac{\dot{b}\dot{N}}{bN} + \frac{N^2}{b^2} = -\frac{3+k}{1-k} \left( \dot{\phi}^2 - Q_m^2 \frac{N^2}{b^4} e^{-2\phi} \right), \quad (A3b)$$

$$\begin{aligned} \frac{\ddot{b}}{\dot{b}} + \frac{\ddot{a}}{\dot{a}} + \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{b}\dot{N}}{bN} - \frac{\dot{a}\dot{N}}{aN} \\ = -\frac{3+k}{1-k} \left( \dot{\phi}^2 + Q_m^2 \frac{N^2}{b^4} e^{-2\phi} \right), \end{aligned} \quad (A3c)$$

$$\frac{d}{dt} \left( \frac{ab^2}{N} \dot{\phi} \right) = Q_m^2 \frac{Na}{b^2} e^{-2\phi}, \quad (A3d)$$

where the overdot represents the derivative with respect to the time. The Lagrangian density in the minisuper-



space is (we neglect the surface terms since these do not affect the equations of motion)

$$L = 2 \left[ -\frac{ab^2}{N} - \frac{2\dot{a}b\dot{b}}{N} + Na + \frac{3+k}{1-k} \left( \frac{ab^2}{N} \dot{\phi}^2 - \frac{Na}{b^2} Q_m^2 e^{-2\phi} \right) \right]. \quad (\text{A4})$$

Solving the equations of motion in the form (A3) is very hard. Equations (A3) and the Lagrangian density can be greatly simplified using the new variables

$$N = e^{2\rho+\nu}, \quad a = a_0 e^\nu, \quad b = b_0 e^\rho. \quad (\text{A5})$$

The variables defined in (A5) reduce the system to a Toda lattice form [3]. Thus, defining  $\xi = \nu + \rho$  and neglecting an overall constant factor, the Lagrangian density (A4) becomes

$$L = -\dot{\xi}^2 + \dot{\nu}^2 + \frac{e^{2\xi}}{b_0^2} + \frac{3+k}{1-k} \left( \dot{\phi}^2 - \frac{Q_m^2}{b_0^4} e^{2(\nu-\phi)} \right). \quad (\text{A6})$$

Varying (A6) with respect to  $\xi$ ,  $\nu$ , and  $\phi$  we obtain the equations of motion for the new variables:

$$\ddot{\xi} = -\frac{e^{2\xi}}{b_0^2}, \quad (\text{A7a})$$

$$\ddot{\nu} = -\frac{3+k}{1-k} \frac{Q_m^2}{b_0^4} e^{2(\nu-\phi)}, \quad (\text{A7b})$$

$$\ddot{\phi} = \frac{Q_m^2}{b_0^4} e^{2(\nu-\phi)}. \quad (\text{A7c})$$

The Hamiltonian constraint is

$$\dot{\xi}^2 - \dot{\nu}^2 + \frac{e^{2\xi}}{b_0^2} = \frac{3+k}{1-k} \left( \dot{\phi}^2 + \frac{Q_m^2}{b_0^4} e^{2(\nu-\phi)} \right) = 0. \quad (\text{A8})$$

Now, we are able to solve (A7) and (A8). From (A7a) we obtain

$$\xi = \ln(\alpha b_0) - \ln \cosh[\alpha(t - t_0)], \quad (\text{A9})$$

where  $\alpha$  and  $t_0$  are integration constants. From (A7b) and (A7c), defining  $\chi = \nu - \phi$ , we have

$$\dot{\phi} = \frac{1-k}{4} (\gamma t - \chi) + \delta, \quad (\text{A10a})$$

$$\nu = \frac{1}{4} [(1-k)\gamma t + (3+k)\chi] + \delta, \quad (\text{A10b})$$

$$\chi = \ln \beta - \ln \cosh[\beta(t - t'_0)] - \frac{1}{2} \ln \left[ \frac{4Q_m^2}{(1-k)b_0^4} \right], \quad (\text{A10c})$$

where  $\beta$ ,  $\gamma$ ,  $\delta$ , and  $t'_0$  are integration constants. Substituting (A9) and (A10) in the Hamiltonian constraint, we find that  $\alpha$ ,  $\beta$ , and  $\gamma$  must satisfy the relation

$$\alpha^2 - \frac{1-k}{4} \gamma^2 - \frac{3+k}{4} \beta^2 = 0. \quad (\text{A11})$$

Choosing  $\alpha = \beta = \pm \gamma$  (A11) is satisfied  $\forall k \in [-1, 1]$ . Recalling (A5) and setting  $t_0 = t'_0 = 0$  we find

$$N = \alpha Q e^{-\phi_0} e^{\mp \alpha t(1-k)/4} [\cosh(\alpha t)]^{-(5-k)/4}, \quad (\text{A12a})$$

$$a = a'_0 e^{-\phi_0} e^{\pm \alpha t(1-k)/4} [\cosh(\alpha t)]^{-(3+k)/4}, \quad (\text{A12b})$$

$$b = Q e^{-\phi_0} e^{\mp \alpha t(1-k)/4} [\cosh(\alpha t)]^{-(1-k)/4}, \quad (\text{A12c})$$

$$e^{\phi-\phi_0} = e^{\pm \alpha t(1-k)/4} [\cosh(\alpha t)]^{(1-k)/4}, \quad (\text{A12d})$$

where  $a'_0$  and  $\phi_0$  are arbitrary constants and  $Q$  is given in terms of  $Q_m$  by Eq. (2.9). Choosing the positive sign and rescaling the metric to the string frame, (A12a)–(A12c) become

$$N = \alpha Q [\cosh(\alpha t)]^{-1}, \quad (\text{A13a})$$

$$a = a'_0 e^{\alpha t(1-k)/2} [\cosh(\alpha t)]^{-(1+k)/2}, \quad (\text{A13b})$$

$$b = Q. \quad (\text{A13c})$$

To express the line element as a function of the proper time we perform the coordinate transformation

$$\tau = 2Q \arctan(e^{\alpha t}). \quad (\text{A14})$$

The solution becomes

$$ds^2 = -d\tau^2 + a_0'^2 \sin^2(\tau/Q) [1 + \cos(\tau/Q)]^{k-1} d\chi^2 + Q^2 d\Omega_2^2, \quad (\text{A15a})$$

$$e^{2(\phi-\phi_0)} = [1 + \cos(\tau/Q)]^{(k-1)/2}. \quad (\text{A15b})$$

From (A15) we get (2.8) with a suitable choice of the constant  $a'_0$ . Finally, we would emphasize that choosing  $\gamma = -\alpha$  we obtain the four-dimensional dual solution corresponding to (3.6).

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