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Brane Factories

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Brane factories

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Abstract

We propose that higher-dimensional extended objects (*p*-branes) are created by super-Planckian scattering processes in theories with TeV scale gravity. As an example, we compute the cross section for p -brane creation in a $(n + 4)$ -dimensional spacetime with asymmetric compactification. We find that the cross section for the formation of a brane which is wounded on a compact submanifold of size of the fundamental gravitational scale is larger than the cross section for the creation of a spherically symmetric black hole. Therefore, we predict that branes are more likely to be created than black holes in super-Planckian scattering processes in these manifolds. The higher rate of *p*-brane production has important phenomenological consequences, as it significantly enhances possible detection of non-perturbative gravitational events in future hadron colliders and cosmic rays detectors.

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The fundamental Planck scale may be of the order TeV as in some models of extra dimensions [1–7]. In these theories, processes at energies \gtrsim TeV may experimentally test quantum gravitational effects. In a series of recent papers [8–13] it has been proposed that particle collisions with center-of-mass energy larger than a few TeV and sufficiently small impact parameter might generate black holes. The formation

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of super-Planckian black holes and their subsequent evaporation would be detectable in future hadron colliders [10–12] and in high energy cosmic ray detectors as black holes would form in the Earth's atmosphere $[13–17]$. ² The detection of black hole formation can open an era in both experimental and theoretical highenergy physics. Thus the recent explosion of papers on the phenomenological aspects of black hole production is of no surprise. If super-Planckian scattering probes non-perturbative quantum gravity, then forma-

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² See Refs. [18,19] for criticisms and Refs. [20–22] for counter criticisms.

tion of spherically symmetric black holes is just the simplest of a plethora of possible high-energy physical processes. At super-Planckian energies, we expect the creation of *any* non-perturbative gravitational object which is predicted by a given theory of quantum gravity. In particular, in the presence of extra dimensions, one should expect the creation of higher-dimensional objects (*p*-branes) [23,24]. Thus far, this exciting possibility has been overlooked in the literature.

In this Letter we propose that *p*-branes are created by super-Planckian scattering processes. To make our claim quantitative, we compute the cross section for *p*-brane creation in a simple model. We assume a flat asymmetric compactification for the extra dimensions. Asymmetric compactifications are suggested by some string theory models [25,26]. Specifically, we consider *m* flat compact extra dimensions with size of order of the fundamental scale $L_{\star} = M_{\star}^{-1}$ and $n - m$ flat extra dimensions with size of order $L' \gg M_{\star}^{-1}$. (The generalization to more than two different compactification scales is trivial.) Setting $n = m$ we obtain the standard flat symmetric compactification of [2,3]. We find that the cross section for the formation of a *p*-brane whose dimensions are wound ("wrapped") around the *m* extra dimensions is larger than the cross section for the formation of spherically symmetric black holes. In this case, if super-Planckian scattering processes lead to non-perturbative formation of gravitational objects, the rate of formation of higherdimensional branes is higher than the rate of formation of spherically symmetric black holes.

We consider for simplicity uncharged, non-spinning *p*-brane solutions of $(n + 4)$ -dimensional Einstein gravity. The generalization of the model to either the charged case or to string theory is straightforward and leads to no significantly different results. Using standard notations, we write the $(n + 4)$ -dimensional Einstein–Hilbert action

$$
S_{\rm EH} = \frac{M_{\star}^{n+2}}{16\pi} \int d^{n+4}x \sqrt{-g} \, \mathcal{R}(g). \tag{1}
$$

The fundamental Planck scale M_{\star} is related to the observed Planck scale $M_{\text{obs}} \approx 10^{16}$ TeV by the relation

$$
M_{\star} = M_{\text{obs}} V_n^{-1/2},\tag{2}
$$

where V_n is the volume of the extra dimensions in fundamental Planck units. If $V_n \approx 10^{32}$, M_{\star} is of order TeV.

An uncharged, static *p*-brane living in a $(n + 4)$ dimensional spacetime is described by the ansatz [27]

$$
ds^{2} = A(r)\left(-dt^{2} + dz_{i}^{2}\right) + B(r) dr^{2} + C(r) d\Omega_{q}^{2},
$$
\n(3)

where z_i $(i = 1, ..., p)$ are the brane coordinates and $d\Omega_q^2$ (*q* = *n* − *p* + 2) is the line element of the *q*-dimensional unit sphere.

The general solution of Eq. (1) with ansatz (3) has been found in Ref. [27] and later generalized to non boost-symmetric configurations in Ref. [28]. The metric is

$$
ds^{2} = R^{\frac{\Delta}{p+1}}\left(-dt^{2} + dz_{i}^{2}\right) + R^{\frac{2-q-\Delta}{q-1}}dr^{2} + r^{2}R^{\frac{1-\Delta}{q-1}}d\Omega_{q}^{2},
$$
\n(4)

where

$$
R(r) = 1 - \left(\frac{r_p}{r}\right)^{q-1}.\tag{5}
$$

∆ is a constant parameter related to the brane dimension *p* and to the sphere dimension *q* by

$$
\Delta = \sqrt{\frac{q(p+1)}{p+q}}.\tag{6}
$$

The spherically symmetric solution is recovered for $p = 0$. In this case $\Delta = 1$, and Eq. (4) reduces to the $(n + 4)$ -dimensional Schwarzschild black hole.

Let us briefly discuss two interesting features of Eq. (4). When $p = 0$ (black hole case) $r = r_p$ defines the Schwarzschild horizon. For $p \neq 0$ (*p*-brane case) the metric (4) possesses a naked singularity at $r = r_p$ which is the higher-dimensional analogue of a cosmic string conical singularity [27]. r_p sets the curvature scale of the geometry. Therefore r_p can be interpreted as the physical radius of the brane though the proper area of the *p*-brane per unit brane-volume V_p is infinite. The interpretation of the curvature singularity has been discussed in Ref. [27]. The metric (4) is interpreted as *vacuum* exterior solution to the *p*-brane, with the curvature singularity being smoothed out by the core of the *p*-brane. In analogy to the black hole case, we expect that a scattering process with impact parameter $b \leq r_p$ will produce a *p*-brane which is described by a suitable localized energy field configuration. Being an extended object endowed with tension (mass/unit *p*-volume), the *p*-brane (4) is unstable [29]. The decay process of the *p*-brane depends on the type of instability and is presently very speculative. String field theory arguments [30–36] suggest that the *p*-brane decays into lower-dimensional branes, and eventually into gauge radiation. On the other hand, analogy to cosmic strings [37–39] suggest a nonperturbative instability, which would unlikely lead to a final fragmentation into 0-branes. For the bosonic uncharged *p*-brane (4), the main difference from the black hole scenario is that the absence of an event horizon does not immediately lead to Hawking evaporation, though we expect the *p*-branes to eventually evaporate by emission of observable, possibly thermal, particles. The intermediate states of the *p*-brane decay are highly dependent on the details of the theory considered. However, we do not expect significant qualitative differences as far as the final evaporation stage is concerned. For instance, when the model is embedded in string theory, the presence of charges usually lead to *p*-branes with horizons. As long as the solution does not saturate the Bogomol'ny bound [23,24] the *p*-brane will have non-zero entropy and will evaporate by Hawking radiation.

p-branes form when two partons *i, j* with centerof-mass energy $E_{ij} = \sqrt{s}$ scatter with impact parameter $b \lesssim r_p$. The geometrical cross section for this process can be approximated by an absorptive black disk with area πr_p^2 , i.e.,

$$
\sigma_{ij \to br}(s; n, p) = F(s)\pi r_p^2,\tag{7}
$$

where $F(s)$ is a dimensionless form factor of order one. By analogy to the black hole case, in the following we will assume $F(s) = 1$. This is a rather conservative choice that has been widely discussed in the literature (see Refs. [14,21,22]). From Eq. (4) the radius of a *p*-brane with mass M_p is

$$
r_p = \frac{1}{\sqrt{\pi} M_{\star}} \gamma(n, p) V_p^{-\frac{w}{n+1}} \left(\frac{M_p}{M_{\star}}\right)^{\frac{w}{n+1}}.
$$
 (8)

 V_p is the volume of the extra dimensions in fundamental Planck units where the p -brane wraps, $w =$ $[1 - p/(n + 1)]^{-1}$, and

$$
\gamma(n, p) = \left[\frac{8\Gamma((n+3-p)/2)}{(2+n)\sqrt{(p+1)^{-1}(1-p/(n+2))}} \right]^{\frac{w}{n+1}}.
$$
\n(9)

The cross section for *p*-brane formation is

$$
\sigma_{ij \to br}(s; p, n, V_p) \approx \frac{1}{s_\star} \gamma(n, p)^2 V_p^{-\frac{2w}{n+1}} \left(\frac{s}{s_\star}\right)^{\frac{w}{n+1}},
$$
\n(10)

where $s_{\star} = M_{\star}^2$. Let us compare Eq. (10) to the cross section for the production of a spherically symmetric black hole with mass M_p (see, e.g., Ref. [13]). The latter is recovered for $p = 0$ and is explicitly given by

$$
\sigma_{ij \to bh}(s; n) \approx \frac{1}{s_{\star}} \gamma(n, 0)^2 \left(\frac{s}{s_{\star}}\right)^{\frac{1}{n+1}}.
$$
 (11)

The ratio of the two cross sections is

$$
\Sigma(s; n, p, V_p) \equiv \frac{\sigma_{ij \to br}}{\sigma_{ij \to bh}} \approx V_p^{-\frac{2w}{n+1}} \frac{\gamma(n, p)^2}{\gamma(n, 0)^2} \left(\frac{s}{s_\star}\right)^{\frac{w-1}{n+1}}.
$$
 (12)

Since $w > 1$ for any $n \geq p > 0$, Σ becomes larger for higher energy. At fixed *s*, the value of *Σ* depends on the dimensionality of the brane and on the size of the extra dimensions. In the scenario with *m* extra dimensions compactified on the *L* scale and $n - m$ dimensions compactified on the L' scale, Eq. (2) gives

$$
\left(\frac{L}{L_{\star}}\right)^m \left(\frac{L'}{L_{\star}}\right)^{n-m} = \left(\frac{M_{\rm obs}}{M_{\star}}\right)^2. \tag{13}
$$

If we assume that the *p*-brane wraps on *r* small dimensions ($r \leq m$) and on $p - r$ large dimensions, the volume V_p is

$$
V_p = \left(\frac{L}{L_\star}\right)^r \left(\frac{L'}{L_\star}\right)^{p-r}
$$

$$
= \left(\frac{L}{L_\star}\right)^{\frac{nr - mp}{n}} \left(\frac{M_{\text{obs}}}{M_\star}\right)^{\frac{2(p-r)}{n-m}}.
$$
(14)

Substituting Eq. (14) in Eq. (12) we find

$$
\Sigma(s; n, m, p, r)
$$
\n
$$
\approx \left(\frac{M_{\text{obs}}}{M_{\star}}\right)^{-\alpha} \left(\frac{L}{L_{\star}}\right)^{-\beta} \frac{\gamma(n, p)^2}{\gamma(n, 0)^2} \left(\frac{s}{s_{\star}}\right)^{\frac{w-1}{n+1}}, \quad (15)
$$
\nwhere

where

$$
\alpha = \frac{4(p-r)}{(n-m)(n-p+1)} \ge 0,
$$

$$
\beta = \frac{2(nr-mp)}{(n-m)(n-p+1)} \ge 0.
$$
 (16)

In theories with TeV scale gravity, $M_{\text{obs}}/M_{\star} \approx 10^{14}$ (10¹⁶) for $M_{\star} \approx 100$ TeV (1 TeV). Since $0 \le$ $(w-1)/(n+1) \leq 1$, for physically interesting energy scales the *p*-brane cross section is suppressed w.r.t. spherically symmetric black hole cross section by a factor $\approx 10^{14\alpha}$ (10^{16 α}). The largest cross section is obtained for $p = r$, i.e., when the *p*-brane is completely wrapped on small-size dimensions:

$$
\Sigma(s; n, m, p \leq m)
$$

$$
\approx \left(\frac{L}{L_{\star}}\right)^{-\frac{2p}{n-p+1}} \frac{\gamma(n, p)^2}{\gamma(n, 0)^2} \left(\frac{s}{s_{\star}}\right)^{\frac{w-1}{n+1}}.
$$
 (17)

Since $L \leq L_{\star}$, the *p*-brane formation process dominates the black hole formation process. When the *p*-brane is wrapped on some of the large extra dimensions, the *p*-brane cross section is instead suppressed w.r.t. black hole cross section. *Σ* slightly increases with the dimension of the brane. Therefore, in a spacetime with *m* fundamental-scale extra dimensions and *n* − *m* large extra dimensions a *m*-brane is the most likely object to be created.

In the following, we give a 11-dimensional spacetime as a concrete example. Let us consider $m = 5$ fundamental-scale extra dimensions $L = L_{\star}$ 10⁻² (TeV)⁻¹ and two large extra dimensions $L \approx$ 10^{12} (TeV)⁻¹. At $s \approx 10s$, the cross sections for the formation of a 5-brane and a 4-brane completely wrapped on the fundamental-size dimensions are enhanced by a factor ≈ 2 and ≈ 1.5 w.r.t. cross section for creation of a spherically symmetric black hole, respectively. If the 5-brane wraps on four extra dimensions with fundamental scale size and on one large extra dimension, $\Sigma(s \approx 10s_{\star})$ is suppressed by a factor \approx 10⁸ (see Figs. 1, 2).

The cross sections are enhanced if the dimensions where the *p*-brane is wrapped are smaller than the fundamental scale. For instance, assuming $L = 0.5L_{\star}$ $(L = 0.25L_{\star})$ the cross section for the creation of 5-branes in a 11-dimensional spacetime is enhanced by a factor ≈ 10 (100). This result is of special interest to the ultra high energy cosmic ray community, as the enhancement of the cross section should allow a sufficient flux of *p*-branes to be detected by ground array and air fluorescence detectors [40].

Finally, let us consider the case where all extra dimensions are compactified on the same scale *L* . We

Fig. 1. Ratio between the cross section for the creation of *p*-branes $(p \leq m)$ completely wrapped on fundamental-size dimensions and a spherically symmetric black hole in a spacetime with $m = 5$ fundamental-size extra dimensions $L = L_{\star} = 10^{-2}$ *(TeV)*^{−1} and $n - m = 2$ large extra dimensions of size $L^7 \approx 10^{12}$ *(TeV)*^{−1} $\gg L_{\star}$.

Fig. 2. Ratio between the cross section for the creation of *p*-branes ($p \le m$) completely wrapped on fundamental-size dimensions and a spherically symmetric black hole in a spacetime with $m = 5$ fundamental-size extra dimensions $L = L_{\star} = 10^{-1}$ (TeV)⁻¹ and $n - m = 2$ large extra dimensions of size $L' \approx 10^{14}$ (TeV)⁻¹ ≈ 2 × 10⁻³ cm. If the fundamental-size extra dimensions have size $L = 0.25L_{\star}$ the cross sections are enhanced by a factor ≈ 100 , 16, 5, 2, 1.5 for $p = 5$, 4, 3, 2 and 1, respectively.

have

$$
\Sigma \approx \left(\frac{M_{\rm obs}}{M_{\star}}\right)^{-\frac{4wp}{n(n+1)}} \frac{\gamma(n,p)^2}{\gamma(n,0)^2} \left(\frac{s}{s_{\star}}\right)^{\frac{w-1}{n+1}}.\tag{18}
$$

In this case the cross section for *p*-brane formation is subdominant to the cross section for black hole formation. This result is understood qualitatively as follows. If all the extra dimensions have (large) identical characteristic size, the spacetime appears isotropic to the *p*-brane and a spherically symmetric object is likely to form. Conversely, when the compactification is asymmetric, that is, *m* of the extra dimensions are smaller than the others, non-spherically symmetric objects are more likely to be created. The most likely *p*-brane to form is that with the highest symmetry compatible with spacetime symmetries, i.e., a *m*-brane.

To conclude, let us briefly comment on the relevance of our results for short-distance experimental physics. In Ref. [10] it has been argued that the creation of event horizons by relativistic high-energy collisions limits the ability to probe short-distance physics by perturbative hard scattering processes in future colliders. Since neutral *p*-branes do not possess an event horizon, *p*-brane formation does not cloak hard processes. Therefore, different hard super-Planckian processes can still lead to different experimental signatures depending on the physics of the collision and on the structure of the extra dimensions. Rather than representing the end of *experimental investigation of short-distance physics* [10], detection of *p*-branes may represent the *beginning of experimental quantum gravity*.

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