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Black Hole Multiplicity at Particle Colliders (Do Black Holes Radiate Mainly on the Brane?)

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Black hole multiplicity at particle colliders
(Do black holes radiate mainly on the brane?)

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Abstract

If gravity becomes strong at the TeV scale, we may have the chance to produce black holes at particle colliders. In this Letter we revisit some phenomenological signatures of black hole production in TeV-gravity theories. We show that the bulk-to-brane ratio of black hole energy loss during the Hawking evaporation phase depends crucially on the black hole greybody factors and on the particle degrees of freedom. Since the greybody factors have not yet been calculated in the literature, and the particle content at trans-Planckian energies is not known, it is premature to claim that the black hole emits mainly on the brane. We also revisit the decay time and the multiplicity of the decay products of black hole evaporation. We give general formulae for black hole decay time and multiplicity. We find that the number of particles produced during the evaporation phase may be significantly lower than the average multiplicity which has been used in the past literature.

1. Introduction

The inauguration of the Large Hadron Collider (LHC) at CERN [1] could well coincide with the grand opening of the first man-made black hole (BH) factory. Some models of high-energy physics indeed predicts the fundamental scale of gravity to be as low as a few TeVs [2–6]. If this is the case, events at energies above this threshold could trigger nonperturbative gravitational effects, such as the creation of BHs [7] and other extended objects predicted by quantum gravity theories [8,9]. (For recent reviews, see [10–13].)

Proton–proton collisions at LHC, with center-of-mass energy of 14 TeV and luminosity $L = 10^{34}$ cm^{-2} s^{-1}, could produce BHs at a very high rate [14–22]. The detection of these BHs would proceed through the observation of the decay products of the ensuing Hawking thermal emission on the brane. Hawking radiation provides distinct experimental signatures that would allow discrimination between gravitational events and other perturbative nongravitational events [14,23,24].

A number of papers have been devoted to the study of the phenomenological signatures of BH formation at particle colliders. The smoking gun of BH creation would be the detection of events with large multiplicity, high

sphericity, large visible transverse energy and a hadron-to-lepton ratio of about 5 : 1 [14], at least in the simplest compactification models with infinitesimal brane thickness. (For an alternative scenario, see [24].) However, an accurate estimate of the physical observables is often hindered by the poor knowledge of Hawking mechanism and by the use of a number of crude approximations. Although the solution of the theoretical conundrums requires the quantum gravity theory, the accuracy of the predictions can be improved by computational process refinement.

Here we focus on three specific physical observables of BH evaporation: the multiplicity of the decay products, the decay time, and the energy loss. The number of particles which are produced during the radiation process can be estimated from dimensional arguments to be of the order of $N \sim M_{BH}/T_{BH}$, where M_{BH} and T_{BH} are the initial mass and the initial Hawking temperature of the BH, respectively. The average multiplicity $\langle N \rangle$ is usually evaluated in the literature by assuming a Boltzmann statistics for the decay products and the degrees of freedom of the emitted quanta, and neglecting the change in the BH mass during the radiation process [15]. This gives a factor $1/2$, leading to the result $\langle N \rangle = M_{BH}/2T_{BH}$. The Boltzmann statistics approximation and the neglecting of particle degrees of freedom are also assumed in the computation of the BH decay time [25] and energy loss [26]. These are crude assumptions, and it is surprising that all the past literature relies unconditionally on the ensuing results. For instance, a common argument in support of the claim that BHs emit mainly on the brane is that fewer particles are emitted in the bulk than on the brane: in the standard model (SM) only the gravitons are emitted in the bulk, whereas all other SM fields are emitted on the brane. However, the emission rate per degree of freedom of a graviton in the d-dimensional bulk (the graviton greybody factor) could be higher than that of n_i-dimensional brane modes ($n_i < d$). Analytic calculations of BH emissivity in the low-frequency limit have shown that the increase in the number of extra dimensions strongly enhances the absorption cross section of spin-1/2 and spin-1 fields [27–29]. The greybody factor enhancement for six extra dimensions is ~ 10 for the lowest partial waves, and rapidly increasing for higher partial waves. If a similar enhancement holds for spin-2 fields, the graviton might carry more energy into the bulk than the SM fields on the brane.

The aim of this Letter is to revisit the above mentioned aspects of BH phenomenology. We give general formulae for the bulk-to-brane energy loss ratio and the decay time, and compute the BH multiplicity to a better precision by dropping the Boltzmann statistics and the instantaneous evaporation assumptions. We also give a general result for the distribution of decay products vs. the spin, or "flavor multiplicity", which is one of the fundamental observational signatures of high-energy scattering gravitational events. Here it should be stressed that our results are limited by two factors: (i) the BH greybody factors in a generic number of dimensions have not been calculated in the literature, and (ii) the particle content at trans-Planckian energies is model-dependent. The computation of the BH greybody factors and the analysis of the particle content at trans-Planckian energies are beyond the purpose of this Letter. However, using the SM and known results on four-dimensional [30] and higher-dimensional [27–29] greybody factors as guidance, we show that the bulk energy loss may not be negligible in models which are relevant to LHC physics, and that the total multiplicity may significantly differ from the average value, depending on the brane dimension. This has important consequences on the phenomenology of BH creation at particle colliders.

Notations: throughout the Letter we use natural Planck units with $G_d = M_{Pl}^{-2}$, where G_d and M_{Pl} are the d-dimensional Newton constant and the Planck mass, respectively.

2. Black hole energy loss and multiplicity

The emission rate for a particle of spin s_i and mass $m_i \ll M$ "from" a BH of mass M into a n_i-dimensional slice of the d-dimensional spacetime is described by the *greybody* distribution

$$\frac{d\mathcal{N}_i}{dt} = \frac{A_i(M, n_i, d)c_i(n_i)\Gamma_i(E, n_i, s_i)}{(2\pi)^{n_i-1}} \frac{d^{n_i-1}k}{e^{E/T} - (-1)^{2s_i}}, \tag{1}$$

where $A(M, n_i, d)$ is the BH area which is induced on the n_i-dimensional subspace, and $c_i(n_i)$ and $\Gamma_i(E, n_i, s_i)$ are the number of degrees of freedom and the greybody factor of the species i, respectively. Two remarks are in

order. First, we assume that the BH induced area depends only on n_i, on the BH mass M, and on the number of spacetime dimensions d. In this Letter we are interested in spherically symmetric BHs, so this condition is always verified. Second, Eq. (1) is very well approximated by considering the thermally-averaged greybody factors [30]. Therefore, we drop the E-dependence in the Γ_i's and use the BH geometric optics area [26]

$$A_i(M, n_i, d) = \Omega_{n_i-2} r_c^{n_i-2}, \tag{2}$$

where Ω_{n_i-2} is the area of the unit $(n_i - 2)$-dimensional sphere and

$$r_c = \left(\frac{d-1}{2}\right)^{1/(d-3)} \left(\frac{d-1}{d-3}\right)^{1/2} r_s \tag{3}$$

is the optical radius of the d-dimensional Schwarzschild BH of radius r_s. The emitted energy density distribution in n_i-dimensions, $d\mathcal{E}_{em}/dt$, is related to the blackbody energy density distribution $d\mathcal{E}_{BB,i}/dt = E\,d\mathcal{N}_i/dt$ by

$$\frac{d\mathcal{E}_{em,i}}{dt} = \frac{\Omega_{n_i-3}}{(n_i - 2)\Omega_{n_i-2}} \frac{d\mathcal{E}_{BB,i}}{dt}. \tag{4}$$

By integrating Eq. (4) over the phase space and summing over all the particle species we obtain the total emitted energy per unit time of a BH with mass M (generalized Stefan–Boltzmann equation):

$$\frac{dE_{em}}{dt} = \sum_i \sigma_{n_i} c_i(n_i) \Gamma_i(n_i, s_i) f_i(n_i) A_i(M, n_i, d) T^{n_i}. \tag{5}$$

The n_i-dimensional Stefan–Boltzmann constant is

$$\sigma_{n_i} = \frac{\Omega_{n_i-3} \Gamma(n_i) \zeta(n_i)}{(n_i - 2)(2\pi)^{n_i-1}}, \tag{6}$$

where Γ is the gamma Euler's function (not to be confused with the greybody factor Γ_i) and $f_i(n_i) = 1\ (1 - 2^{1-n_i})$ for bosons (fermions). Using Eq. (2) and the relation between the BH temperature and the Schwarzschild radius [25],

$$T = \frac{d-3}{4\pi r_s}, \tag{7}$$

it follows that the energy emitted per unit time is proportional to the temperature square

$$\frac{dE_{em}}{dt} = T^2 \sum_i \sigma_{n_i} c_i(n_i) \Gamma_i(n_i, s_i) f_i(n_i) \mu_i(n_i, d), \tag{8}$$

where

$$\mu_i(n_i, d) = \Omega_{n_i-2} \left(\frac{d-1}{2}\right)^{(n_i-2)/(d-3)} \left(\frac{d-1}{d-3}\right)^{(n_i-2)/2} \left(\frac{d-3}{4\pi}\right)^{n_i-2}. \tag{9}$$

The ratio of the emitted energy for two different species is

$$\frac{dE_{em,i}/dt}{dE_{em,j}/dt} = \frac{\sigma_{n_i} c_i(n_i) \Gamma_i(n_i, s_i) f_i(n_i) \mu_i(n_i, d)}{\sigma_{n_j} c_j(n_i) \Gamma_j(n_j, s_j) f_j(n_j) \mu_j(n_j, d)}. \tag{10}$$

The previous equation can be compared with the analogous result in Ref. [26]. The latter is obtained from Eq. (10) by setting $f_i(n_i) = f_j(n_j)$ (same statistics for all quanta), and $\Gamma_i(n_i, s_i) = \Gamma_j(n_j, s_j) = 1$ (blackbody approximation). Using these approximations, the ratio of energy loss for two species is simply a function of the geometry and of the degrees of freedom of the species. This leads to the naive expectation that brane emission dominates over bulk emission because the number of bulk modes is much smaller than the number of brane modes, i.e.,

$dE_{\rm em,brane}/dt \gg dE_{\rm em,bulk}/dt$ because $\sum_{\rm brane} c_i \gg \sum_{\rm bulk} c_i$. However, the presence of the greybody factors affects the value of the energy ratio. Apart from the geometrical factors, the relevant quantity to evaluate the ratio of energy loss of two species is $\Gamma_i(n_i, s_i)c_i(n_i)$, rather than the number of modes $c_i(n_i)$. If $\Gamma_i(n_i, s_i) \gg \Gamma_j(n_j, s_j)$, the energy loss due to the species i may be higher than the energy loss of the species j, even though $c_i(n_i) \ll c_j(n_j)$ and the ratio of geometrical factors is smaller than one. It follows that the knowledge of the greybody factors is essential to establish whether the brane emission due to the species dominates on the bulk emission. A large ratio of the bulk-to-brane greybody factors could contradict the naive estimate, commonly used in the literature, that brane emission dominates over bulk emission because the number of brane modes is larger than that of bulk modes. A careful estimate of the bulk-to-brane BH energy loss thus requires the knowledge of the greybody factors of generic spin-s_i fields in a generic dimension n_i. We will illustrate this point with a specific example in Section 3.

Using Eq. (8), the BH mass loss $dM/dt = -dE_{\rm em}/dt$ can be expressed as a function of the BH mass

$$\frac{dM}{dt} = -\mu\big(\{n_i, s_i\}, d\big)M^{-2/(d-3)}, \tag{11}$$

where

$$\mu\big(\{n_i, s_i\}, d\big) = \left(\frac{d-3}{4\pi}\right)^2 \left[\frac{(d-2)\Omega_{d-2}}{16\pi}\right]^{2/(d-3)} \sum_i \sigma_{n_i} c_i(n_i) \Gamma_i(n_i, s_i) f_i(n_i) \mu_i(n_i, d). \tag{12}$$

Integrating Eq. (11) we obtain the decay time

$$\tau = \mu^{-1} \frac{d-3}{d-1} M_{\rm BH}^{(d-1)/(d-3)}. \tag{13}$$

We can compare this result to the decay time of Ref. [25], which is often used in the literature. The latter is calculated by taking into account only the graviton loss in d-dimensions. Our result differs from the result of Ref. [25] because the latter is calculated by using the actual area of the BH rather than the optical area and omitting the geometric emission factor of Eq. (4). As is expected, the decay time Eq. (13) is longer than that of Ref. [25] because of the extra particle evaporation channels. The exact estimate of the decay time may be relevant in physical situations where the BH may accrete, such as in compact astrophysical objects [31]. In this case, a longer decay time translates into stronger constraints on the fundamental gravitational scale.

The total multiplicity is obtained by integrating $E^{-1}\mathcal{E}_{\rm em,i}/dt$ over the phase space and summing over all the particle species. Using Eqs. (7) and (11) we find

$$N = \frac{d-3}{d-2} \frac{M_{\rm BH}}{T_{\rm BH}} \frac{\sum_i \sigma_{n_i} c_i(n_i) \Gamma_i(n_i, s_i) f_i(n_i - 1) \mu_i(n_i, d) \zeta(n_i - 1)[(n_i - 1)\zeta(n_i)]^{-1}}{\sum_j \sigma_{n_j} c_j(n_j) \Gamma_j(n_j, s_j) f_j(n_j) \mu_j(n_j, d)}. \tag{14}$$

The multiplicity per particle species is

$$N_i = N \frac{\sigma_{n_i} c_i(n_i) \Gamma_i(n_i, s_i) f_i(n_i - 1) \mu_i(n_i, d) \zeta(n_i - 1)[(n_i - 1)\zeta(n_i)]^{-1}}{\sum_j \sigma_{n_j} c_j(n_j) \Gamma_j(n_j, s_j) f_j(n_j - 1) \mu_j(n_j, d) \zeta(n_j - 1)[(n_j - 1)\zeta(n_j)]^{-1}}. \tag{15}$$

Eq. (15) gives the statistical number of particles per species produced during the evaporation process.

3. Black hole evaporation at LHC

Using the previous equations, we can estimate the energy loss, the decay time, and the multiplicity of Schwarzschild BHs at particle colliders. Eqs. (10) and (13)–(15) are exact, modulo the greybody thermal average approximation. These equations can be evaluated for a given particle model (standard model, SUSY, etc.) and a given geometry of the spacetime (ADD [3], Randall–Sundrum [5,6], fat brane [32], universal extra dimensions

[33], etc.) if the corresponding greybody factors $\Gamma_i(n_i, s_i)$ are known. The thermally-averaged greybody factors for particles with spin 0, 1/2, 1 and 2 in four dimensions have been known for a long time [30]. The greybody factors for fields with spin 0, 1/2, and 1 in higher-dimensions have recently been calculated in the low-frequency limit [27–29]. To our knowledge, the spin-2 greybody factors in higher dimensions have not been calculated. Therefore, some kind of approximation is often required to evaluate Eqs. (10) and (13)–(15).

We illustrate the relevance of the greybody factors by considering a model which has been widely investigated in connection with LHC phenomenology. The SM brane is four-dimensional and infinitesimally thin, with only the graviton propagating into the 10-dimensional bulk. Therefore, we can use the four-dimensional greybody factors for all the fields with the exception of the graviton, which "feels" a 10-dimensional greybody factor. Using Eq. (10) we can compare the energy loss due to graviton emission in the bulk with that of a field on the brane as a function of the unknown 10-dimensional greybody factor for a spin-2 field. As an example, let us compare the graviton emission to the *total* spin-1 field emission on the brane. Eq. (10) gives the result

$$\frac{dE_{\text{em,grav}}/dt}{dE_{\text{em,spin-1}}/dt} \approx 0.64 \frac{\Gamma_{\text{spin-2}}(10, 2)}{\Gamma_{\text{spin-1}}(4, 1)}, \tag{16}$$

where the spin-1 greybody factor is $\Gamma_{\text{spin-1}}(4, 1) \approx 0.24$. The graviton emission in the bulk is negligible compared to the *total* spin-1 emission on the brane if the r.h.s. of Eq. (16) is $\ll 1$. In the blackbody approximation, i.e., neglecting the greybody factors ($\Gamma_{\text{spin-2}}(10, 2) = \Gamma_{\text{spin-1}}(4, 1) = 1$), the graviton energy loss is approximately two thirds of the total spin-1 energy loss. The situation is quite different if we take into account the greybody factors. If the ten-dimensional spin-2 greybody factor is ≈ 0.38, the graviton emission in the bulk is comparable to the *total* spin-1 emission on the brane. As previously stressed, the main point here is that a large spin-2 greybody factor in the bulk can dramatically enhance the bulk emission compared to the emission on the brane, even if the number of brane modes is larger than the number of bulk modes. Is $\Gamma_{\text{spin-2}}(10, 2) \approx 0.38$ a realistic possibility? The thermally-averaged spin-2 greybody factor in four dimensions is $\Gamma_{\text{spin-2}}(4, 2) = 0.028$ [30]. Although the thermally-averaged greybody factors in higher dimensions are not known, analytic calculations in the low-frequency limit show that the greybody factors for the lowest partial waves of spin-1/2 and spin-1 fields are enhanced by a factor ~ 10 as the number of extra dimensions increases from four to ten, with the enhancement rapidly increasing for higher partial waves [27–29]. Therefore, it is no unreasonable to expect a similar increase for spin-2 fields. A definitive answer requires the knowledge of the graviton greybody factor in higher dimensions. However, *it is premature to conclude that BHs emit mainly on the brane*.

We also illustrate the computation of the multiplicity with a model which has been widely used in LHC phenomenology, though Eq. (14) and Eq. (15) hold in general. Let us evaluate Eq. (14) for a n-dimensional infinitesimally thin brane in a d-dimensional spacetime. We consider only standard model fields and we neglect the graviton energy loss into the bulk. Setting $n_i = n_j = n$, Eq. (14) simplifies to

$$N = \frac{d-3}{d-2} \frac{\zeta(n-1)}{(n-1)\zeta(n)} \frac{M_{\text{BH}}}{T_{\text{BH}}} \frac{\sum_i c_i(n) \Gamma_i(n, s_i) f_i(n-1)}{\sum_j c_j(n) \Gamma_j(n, s_j) f_j(n)}. \tag{17}$$

Setting $n = 4$, we can evaluate exactly the statistical multiplicity of BH events at LHC as function of the BH mass and of the spacetime dimensions by using the four-dimensional greybody factors. Choosing the fundamental Planck scale to be $M_{\text{Pl}} = 1$ TeV, the total number of emitted particles is $N = 4, 6, 7$ for $M_{\text{BH}} = 8, 10$ and 12 TeV, respectively. Let us compare these values to the average multiplicity $\langle N \rangle$ which is used in the literature. For the same choices of M_{BH} we find $\langle N \rangle = 8, 10$, and 12. The multiplicity calculated by taking into account the particle statistics and the greybody factors is reduced by a factor $\approx 43\%$ w.r.t. average multiplicity. The reduction is stable as the number of spacetime dimensions changes, varying from $\approx 48\%$ for $d = 7$ to $\approx 42\%$ for $d = 11$. If we take into account possible emission of gravitons in the bulk, the visible multiplicity could be further reduced. How are the decay products distributed among the particle flavors? Using Eq. (15) we find that the $M_{\text{BH}} = 8$ TeV BH is likely to decay into three quarks plus either one charged lepton or one gluon, and the $M_{\text{BH}} = 10$ (12) TeV BH

decays into four (five) quarks, one charged lepton and one gluon, each quark and gluon eventually producing a hadronic jet. The hadron-to-lepton ratio is still about $5 : 1$.

4. Conclusion

In this Letter we have derived generic expressions for the energy loss, the decay time, the statistical multiplicity and the flavor multiplicity of spherically symmetric BHs by taking into account the Boltzmann statistics and the greybody factors of the decay products. We have found that these quantities depend crucially on the greybody factors of the species and on the particle content of the model. Unfortunately, both the greybody factors and the particle content at trans-Planckian energies are not yet known. The computation of the greybody factors for a generic spin-s_i field in n_i dimensions is quite a difficult task and is beyond the purpose of this Letter. The aim of this Letter is to emphasize that the phenomenology of BH decay depends strongly on unknown parameters. The main message is that it is premature to accept unconditionally results which rely heavily on these unknowns. In particular, the multiplicity of BH decay may be significantly smaller than that considered in the past literature. The large multiplicity has long been considered one of the main signatures of BH formation at particle colliders. If the fundamental Planck scale is of order of the TeV scale, we expect creation of BHs with mass of a few TeVs at LHC. The multiplicity of these BHs could be, however, far from being large, depending on the value of the BH greybody factors. This is the case, for instance, in the simplest model of a brane with infinitesimal thickness. Assuming $M_{\text{Pl}} = 1$ TeV and ten dimensions, BHs in the range 8–10 TeV produce at most four or five hadronic jets. This could have important consequences on the possible detection of BHs events, making the discrimination between standard model events and gravitational events harder. The introduction of the greybody factors and of the particle statistics also affects the bulk-to-brane energy loss ratio. Usually, the bulk emission is considered negligible w.r.t. emission into the brane. However, the exact amount of energy radiated into the bulk vs. the brane crucially depends on the greybody factor of the graviton even in the simplest models. Analytic calculations for lower-spin fields seem to suggest that the greybody factors increase with the number of dimensions, at least in the low-frequency limit [27–29]. This would lead to an increase in the emission of bulk modes vs. brane modes. Since no thermally-averaged greybody factors in higher dimensions have been calculated, the answer to the question whether the emission in the bulk is negligible remains open. If the BH energy loss into the bulk is comparable with the energy loss on the brane, the possibility of BH detection in particle colliders is further reduced.

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