

26 May 2010, 4:45 pm - 6:45 pm

## Analytical Method for Seismic Bearing Capacity of Stone-Column Reinforced Shallow Foundations

Javad Nazari Afshar  
*Islamic Azad University, Iran*

Mahmoud Ghazavi  
*K. N. Toosi University of Technology, Iran*

Khashayar Hemmati  
*Asil Sazan Abnieh Co., Iran*

Follow this and additional works at: <https://scholarsmine.mst.edu/icrageesd>



Part of the [Geotechnical Engineering Commons](#)

---

### Recommended Citation

Afshar, Javad Nazari; Ghazavi, Mahmoud; and Hemmati, Khashayar, "Analytical Method for Seismic Bearing Capacity of Stone-Column Reinforced Shallow Foundations" (2010). *International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*. 7.  
<https://scholarsmine.mst.edu/icrageesd/05icrageesd/session05/7>



This work is licensed under a [Creative Commons Attribution-Noncommercial-No Derivative Works 4.0 License](#).

This Article - Conference proceedings is brought to you for free and open access by Scholars' Mine. It has been accepted for inclusion in International Conferences on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics by an authorized administrator of Scholars' Mine. This work is protected by U. S. Copyright Law. Unauthorized use including reproduction for redistribution requires the permission of the copyright holder. For more information, please contact [scholarsmine@mst.edu](mailto:scholarsmine@mst.edu).



## ANALYTICAL METHOD FOR SEISMIC BEARING CAPACITY OF STONE-COLUMN REINFORCED SHALLOW FOUNDATIONS

**Javad Nazari afshar**

Islamic Azad University – Shahr-e-Qods branch  
Tehran-Iran

**Mahmoud Ghazavi**

K.N.Toosi University of Technology  
Tehran-Iran

**Khashayar Hemmati**

Asil sazan abnieh Co  
Tehran-Iran

### ABSTRACT

Stone-columns is a useful method for increasing bearing capacity and reducing settlement of foundation soil subjected to structure loading. For stone-column construction, 15 to 35 percent of weak soil volume is usually replaced with stone-column material. Such columns may be constructed with various diameters, lengths, and center-to-center distances. This paper presents a simple method to determine the seismic bearing capacity of stone-column reinforced shallow foundation. For this purpose, a simple failure surface is assumed to characterize the failure stage of the stone column and soil materials using the concept of lateral active and passive earth pressures. The well known Mononobe-Okabe approach is used to represent seismic effects of soil lateral earth pressures. The results show that with increasing the earthquake intensity, the foundation bearing capacity decreases. Parametric studies will be presented to illustrate the role of contributing parameters such as geotechnical data of stone column material, foundation geometry, native soil specification, and earthquake details.

### INTRODUCTION

The use of stone-columns is a useful method for increasing bearing capacity and also for reducing settlement of soil under structures. In stone-column construction, usually 15 to 35 percent of weak soil volume is replaced by stone-column that usually has a special diameter and length and center-to-center distance. Design loads on stone-columns normally vary between 20 to 50 tons. The confinement of stone-column material is provided by the lateral stress induced by the surrounding weak soil. Upon application of the vertical stress at the ground surface, the stone and soil move downward together, resulting in stress concentration in the stone-column due to higher stiffness induced into the stone material than that induced in the soil. Stone-columns are constructed usually in triangular pattern or sometimes in square pattern. The equilateral triangle pattern gives more dense packing of stone-columns in a given area as shown in Figure 1.

Three type of failure mechanism may occur in stone-columns. These are bulging failure, shear failure, and punching shear failure. In end bearing or free floating stone-columns, bulging failure extends to or greater than about than three times the stone diameter in length (Huges et al., 1974 &1976). The shear failure mechanism occurs in very short columns resting on a firm support either a general or local bearing capacity type failure at the surface (Madhav et al., 1978). The punching

shear failure occurs in floating stone-column at a length of less than about two to three times the stone diameter. This failure type may occur in end bearing stone columns embedded in weak soil underlying layer before a bulging failure can develop (Aboshi et al, 1979).

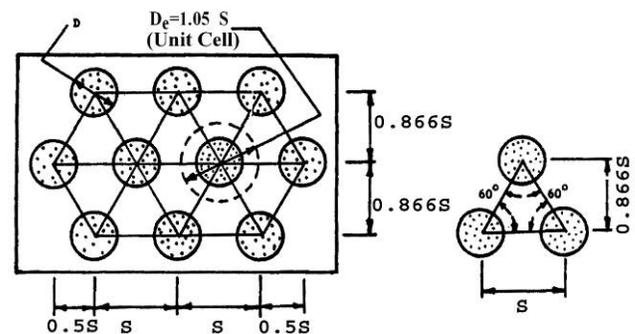


Fig1. Equilateral triangle pattern of stone columns

In this research by using an "imaginary retaining wall assumption", it has been tried to develop a simple analytical method for estimation of the seismic bearing capacity of stone-columns assuming bulging failure mechanism.

## BULGING FAILURE MECHANISIM

Stone-columns have length to diameter ratios equal to or greater than 4 to 6 and are embedded in homogeneous soil, the bulging failure occurs at a depth of about 2-3 times the diameter of the stone-column. This failure type was explored by performing field tests on stone columns (Hughes et al., 1974). A number of theories have been presented for predicting the ultimate capacity of an isolated, single stone-column supported by soft soil. Most of the early analytical solutions assume a triaxial state of stress which exists in the stone-columns while both surrounding soil and the stone-column material are at failure. The lateral confining stress ( $\sigma_3$ ) which supports the stone-columns is usually taken in this methods as the ultimate passive resistance offered by the surrounding soil. This passive stage is reached upon mobilization of the stone-column bulges which occur outward against the soil. Since the column is assumed to be in a state of failure, the ultimate vertical stress ( $\sigma_1$ ) which the stone-column (stone-column assumes to be cohesion less) can tolerate is equal to the coefficient of passive pressure on the stone-column ( $K_p$ ) times the lateral confining stress ( $\sigma_3$ ). This means:

$$\sigma_1 = \sigma_3 \tan^2(45 + \frac{\phi_s}{2}) = \sigma_3 \frac{1 + \sin \phi_s}{1 - \sin \phi_s} = \sigma_3 K_p \quad (1)$$

where  $\phi_s$  is the internal friction angle of stone-column material.

A number of relations for estimation bearing capacity of stone-columns have been presented by Greenwood (1970), Vesic (1972), Hughes et al. (1974), Datye et al. (1975), and Madhav et al. (1979) in the form of  $\sigma_1 = \sigma_3 K_p$ . These relations are similar to Eq. (1). Most of researchers have only tried to enhance the ability and reliability of relations in predicting the surrounding confinement pressure ( $\sigma_3$ ) in Eq. (1). For example, Vesic (1972) introduced the following expression for determination of the ultimate lateral resistance of the soil:

$$\sigma_3 = c_c F'_c + q F'_q \quad (2)$$

where  $c_c$  is cohesion of the surrounding soil and  $q$  is the mean (isotropic) stress  $(\sigma_1 + \sigma_2 + \sigma_3)/3$  at the equivalent failure depth,  $F'_q$  and  $F'_c$  are cavity expansion factors given in a graph as functions of the angle of internal friction angle of the surrounding soil and the rigidity index,  $I_r$ . According to Vesic (1972), the rigidity index is expressed as:

$$I_r = \frac{E}{2(1+\nu)(c_c + q \tan \phi_c)} \quad (3)$$

where  $E$  is the modulus of elasticity of the surrounding soil in which cavity expansion occurs and  $c_c$  is cohesion of the

surrounding soil,  $\nu$  is the Poisson's ratio of surrounding soil,  $q$  is mean stress within the zone of failure.

In the above equations, in addition to the ultimate lateral stress ( $\sigma_3$ ) and some geotechnical parameter such as  $c$  and  $\phi_c$ , other parameters such as the modulus of elasticity and Poisson's ratio of the surrounding soil are also required to determine the stone column bearing capacity. In addition, all above methods consider the static ultimate bearing capacity of stone columns. The authors are unaware of any developed analytical methods to determine the seismic bearing capacity of stone columns. In this article, a very simple expression is developed for calculating the seismic ultimate bearing capacity of single stone-columns undergoing a bulging failure type only by having common shear strength parameters of soils. The developed method is simply used for static analyses as well.

## NEW SIMPLE ANALYTICAL METHOD

An imaginary retaining wall (AB) is used for estimation the bearing capacity of shallow foundation rested on sand as shown in Fig. 2. An active zone beneath the footing on the left hand side of the wall and a passive zone on the right hand side of the wall are assumed. This method was first introduced by Richards et al. (1993) for determination of the bearing capacity of shallow foundations resting on a homogeneous sand.

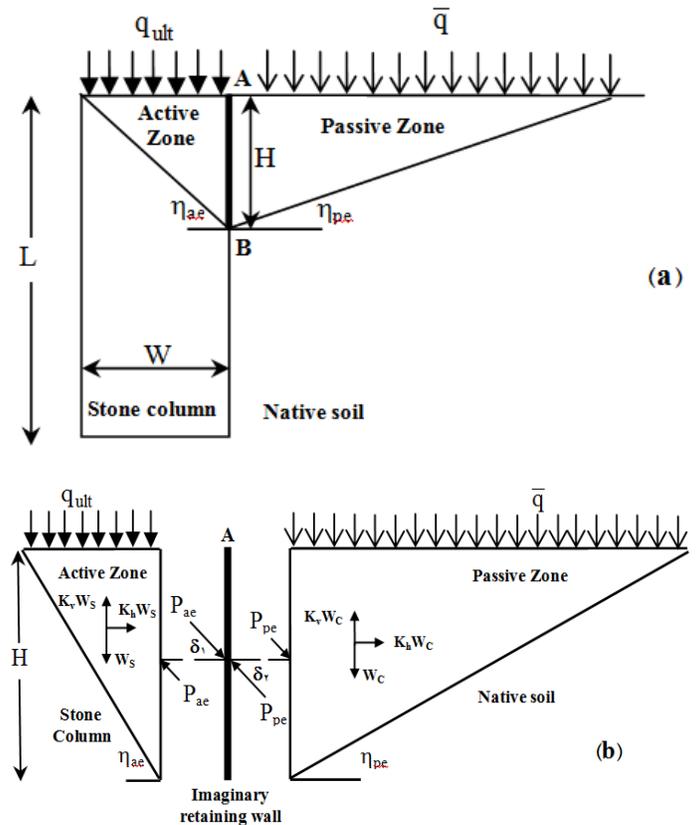


Fig2. Imaginary retaining wall conception

In this paper, as shown in Fig. 2, the active zone consists of stone frictional material and the passive zone consists of natural soil to be improved by stone columns. The imaginary wall AB is between these active and passive zones. Due to exerting load on the stone-column, granular material of stone-column tends to move down and outward. Because of this movement, the lateral stress in the surrounding weak soil increases and the soil will go to the passive state (Fig. 2.a). It is assumed that rigid imaginary retaining wall moves only horizontally and thus the stability equations may be written for this wall.

In seismic condition, the active induced force on the imaginary wall is determined using the pseudo static approach (Fig. 2b). In this method, the horizontal and vertical seismic forces defined as  $F_h=k_h W$  and  $F_v=k_v W$  where  $w$  is the weight of active wedge shown by  $W_s$ . Characters  $k_h$  and  $k_v$  represent the horizontal and vertical seismic coefficients, respectively. The passive wedge weight shown by  $W_c$ . Characters  $k_h$  and  $k_v$  are seismic coefficients in the horizontal and vertical direction, respectively (Fig.2b).

The column material is granular and the surrounding native soil is cohesive. The active and passive wedges make  $\eta_{ae}$  and  $\eta_{pe}$  angles with the horizontal direction, respectively as shown in Fig. 2.a). This angles are given by:

$$\eta_{ae} = \varphi_s - \psi + \tan^{-1} \left[ \frac{-\tan(\varphi_s - \psi) + C_1}{C_2} \right] \quad (4a)$$

where:

$$C_1 = \sqrt{\tan(\varphi_s - \psi)[\tan(\varphi_s - \psi) + \cot(\varphi_s - \psi)][1 + \tan(\delta_1 + \psi)\cot(\varphi_s - \psi)]}$$

$$C_2 = 1 + [\tan(\delta_1 + \psi)[\tan(\varphi_s - \psi) + \cot(\varphi_s - \psi)]]$$

where  $\psi$  is seismic inertia angle and can be calculated by following equation:

$$\psi = \tan^{-1} \frac{k_h}{1 - k_v} \quad (4b)$$

For surrounding native soil  $\eta_{pe}$  can be calculated by following equation:

$$\eta_{pe} = \psi + \tan^{-1} \left[ \frac{\tan(-\psi)}{C_3} \right] \quad (5a)$$

where :

$$C_3 = 1 + [\tan(\psi)[\tan(\psi) + \cot(-\psi)]] \quad (5b)$$

In above equations, the column material internal friction angle is  $\varphi_s$ .

The active force exerted by the stone-column material on the wall is equal to:

$$P_{ae} = \frac{1}{2} K_{aes} \gamma_s H^2 (1 - K_v) + q_{ult} K_{aes} H \quad (6)$$

The passive force exerted by the native soil on the rigid retaining wall is equal to:

$$P_{pe} = \frac{1}{2} \gamma_c H^2 (1 - K_v) + \bar{q} H (1 - K_v) + 2cH \quad (7)$$

Where  $\gamma_s$  is unit weight of the stone-column material,  $\gamma_c$  is unit weight of the native soil,  $K_{aes}$  is active seismic pressure coefficient,  $\bar{q}$  is surcharge on passive region,  $c$  is cohesion of the native soil, and  $H$  is the failure wedge height (Fig. 2.a). The value of  $K_{aes}$  can be calculated by:

$$K_{aes} = \frac{\cos^2(\varphi_s - \psi)}{\cos \psi \cos(\delta_1 + \psi) \left[ 1 + \sqrt{\frac{\sin(\varphi_s + \delta_1) \sin(\varphi_s - \psi)}{\cos(\delta + \psi)}} \right]^2} \quad (8)$$

where  $\delta_1$  is the angle between the stone-column material and wall and  $\delta_2$  is the friction angle between the native soil and rigid retaining wall native soil on the one hand and with imaginary rigid retaining wall. Richard et al. (1993) suggested that  $\delta = 0.5\varphi$ .

Stone-columns constructed with a special center to center (S) distance, for analysis in a plane strain condition (similar the condition of imaginary rigid retaining wall), it is necessary to convert one column to an equivalent continuous stone-column strip with width,  $W$  (Fig.3).

$$W = \frac{A_s}{S} \quad (9)$$

where:  $A_s$  is horizontal cross section area of stone-column and  $S$  is center to center distance of stone-column. By using  $W$  parameter, it is stated that:

$$H = W \tan \eta_{ae} = \frac{A_s}{S} \tan \eta_{ae} \quad (10)$$

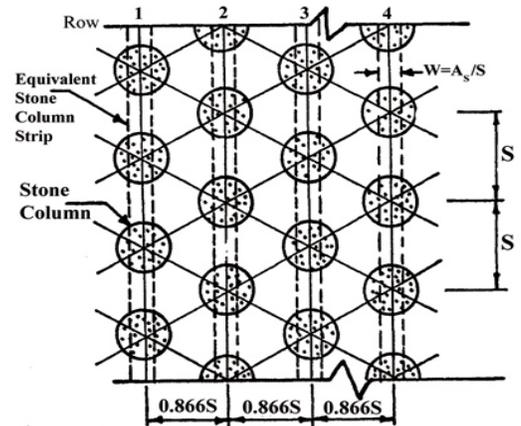


Fig 3. Stone-column strip idealization

If equilibrium equation in the horizontal direction is written on the face of the imaginary rigid retaining wall, then:

$$P_a \cos \delta_1 = P_p \cos \delta_2 \quad (11)$$

Substituting Eqs. (6) and (7) into Eq. (11) gives:

$$q_{ult} = \frac{\cos \delta_2}{\cos \delta_1} \frac{\left( \frac{1}{2} \gamma_c H^2 (1 - K_v) + \bar{q} H (1 - K_v) + 2cH \right)}{K_{aes} H} - \frac{1}{2} \gamma_s H (1 - K_v) \quad (12)$$

Since the native soil is cohesive, then  $\delta_2 = 0$ .

Having  $H = W \tan \eta_{ae}$  gives:

$$q_{ult} = \frac{1}{\cos \frac{\phi_s}{2}} \frac{\left( \frac{1}{2} \gamma_c H (1 - K_v) + \bar{q} (1 - K_v) + 2c \right)}{K_{aes}} - \frac{1}{2} \gamma_s H (1 - K_v) \quad (13)$$

Making a simplification in Eq. (13) yields:

$$q_{ult} = c \frac{2}{K_{aes} \cos \frac{\phi_s}{2}} + \bar{q} \frac{(1 - K_v)}{K_{as} \cos \frac{\phi_s}{2}} + \frac{1}{2} W \gamma_c \left( \frac{1}{K_{aes} \cos \frac{\phi_s}{2}} - \frac{\gamma_s}{\gamma_c} \right) \tan \eta_{ae} (1 - K_v) \quad (14)$$

Eq. (14) is similar to conventional shallow foundation ultimate bearing capacity expression. Thus, it can be re-written as:

$$q_{ult} = c N_{cE} + \bar{q} N_{qE} + \frac{1}{2} W \gamma_c N_{\gamma E} \quad (15a)$$

where:

$$N_{cE} = \frac{2}{K_{aes} \cos \frac{\phi_s}{2}} \quad (15b)$$

$$N_{qE} = \frac{(1 - K_v)}{K_{aes} \cos \frac{\phi_s}{2}} \quad (15c)$$

$$N_{\gamma E} = \left( \frac{1}{K_{aes} \cos \frac{\phi_s}{2}} - \frac{\gamma_s}{\gamma_c} \right) \tan \eta_{ae} (1 - K_v) \quad (15d)$$

As mentioned before, the stone column material is only granular with the internal friction angle ( $\phi_s$ ). However, if the surrounding native soil is assumed to be cohesionless with internal friction angle of  $\phi_c$ , in Fig. 2.a, the value of  $\eta_{pe}$  is calculated from:

$$\eta_{pe} = \psi - \phi_c + \tan^{-1} \left[ \frac{\tan(\phi_c - \psi) + C_4}{C_3} \right] \quad (16)$$

where:

$$c_3 = 1 + [\tan(\delta_2 + \psi) [\tan(\phi_c - \psi) + \cot(\phi_c - \psi)]]$$

$$c_4 = \sqrt{\tan(\phi_c - \psi) [\tan(\phi_c - \psi) + \cot(\phi_c - \psi)] [1 + \tan(\delta_2 + \psi) \cot(\phi_c - \psi)]}$$

The total passive force is obtained from:

$$P_{pe} = \frac{1}{2} \gamma_c H^2 K_{pe_c} (1 - K_v) + \bar{q} H (1 - K_v) \quad (17)$$

Where  $K_{pe_c}$  is passive seismic pressure coefficient and given by:

$$K_{pe_c} = \frac{\cos^2(\phi_c - \psi)}{\cos \psi \cos(-\delta_2 - \psi) \left\{ 1 - \sqrt{\frac{\sin(\phi_c - \delta_2) \sin(\phi_c - \psi)}{\cos(-\delta_2 - \psi)}} \right\}^2} \quad (18)$$

where  $\delta_1$  and  $\delta_2$  are defined as before.

The horizontal equilibrium of forces exerted on the imaginary wall yields:

$$q_{ult} = \frac{\cos \delta_2}{\cos \delta_1} \frac{\left( \frac{1}{2} \gamma_c H^2 K_{pe_c} (1 - K_v) + \bar{q} H K_{pe_c} (1 - K_v) \right)}{K_{aes} H} - \frac{1}{2} \gamma_s H (1 - K_v) \quad (19)$$

Assuming  $\delta_1 = 0.5\phi_s$  and  $\delta_2 = 0.5\phi_c$  and  $H = W \tan \eta_{ae}$ , the ultimate seismic bearing capacity of the stone column will be expressed by:

$$q_{ult} = \bar{q} \frac{K_{pe_c} \cos \frac{\phi_c}{2}}{K_{aes} \cos \frac{\phi_s}{2}} (1 - K_v) + \frac{1}{2} W \gamma_c \left( \frac{K_{pe_c} \cos \frac{\phi_c}{2}}{K_{aes} \cos \frac{\phi_s}{2}} - \frac{\gamma_s}{\gamma_c} \right) \tan \eta_{ae} (1 - K_v) \quad (20)$$

Eq. (20) is similar to common shallow foundation ultimate bearing capacity relation. Thus, again:

$$(21a)$$

$$q_{ult} = \bar{q} N_{qE} + \frac{1}{2} W \gamma_c N_{\gamma E}$$

where:

$$N_{qE} = \frac{K_{pe_c} \cos \frac{\phi_c}{2}}{K_{aes} \cos \frac{\phi_s}{2}} (1 - K_v) \quad (21b)$$

$$N_{\gamma E} = \left( \frac{K_{pe_c} \cos \frac{\phi_c}{2}}{K_{aes} \cos \frac{\phi_s}{2}} - \frac{\gamma_s}{\gamma_c} \right) \tan \eta_{ae} (1 - K_v) \quad (21c)$$

## APPLICATION EXAMPLE

To show how the developed method is used to determine the seismic bearing capacity of stone columns, it is assumed  $k_v = 0$ . The horizontal seismic force assumed for four different values of 0, 0.15, 0.25 and 0.35 for  $k_h$ . It is further assumed  $\gamma_c = 17 \text{ kN/m}^3$ ,  $\gamma_s = 19 \text{ kN/m}^3$ . The stone-column diameter is  $D = 1 \text{ m}$ , the center to center distance of stone-columns is  $S = 3 \text{ m}$ , and the internal friction angle of the stone column material varies  $35\text{--}45^\circ$ . The undrained shear strength of the native cohesive saturated clay is assumed to be  $40 \text{ kPa}$ . Because native soil is cohesive then for calculation (Equation 15a) is used. The results are shown in Fig. 4. As seen, with increasing the horizontal seismic force, the ultimate bearing capacity of the stone column decreases. The effect of internal friction angle of the stone column material has little effect on the ultimate bearing capacity of the stone column for higher seismic force values.

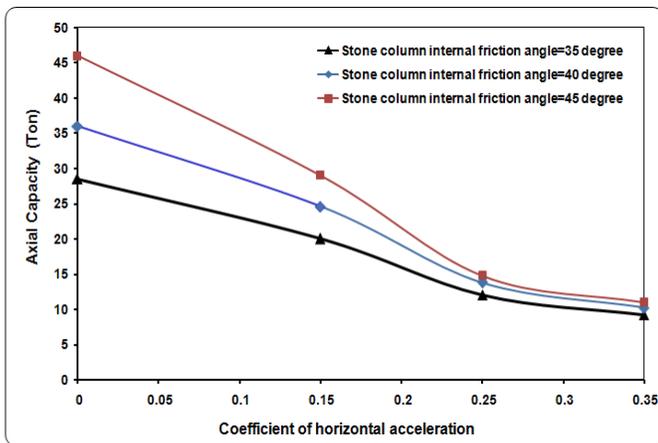


Fig 4. Effect of seismic force on ultimate bearing capacity of stone column

The effect of stone-column diameter is depicted in Figs. 5 and 6 for  $k_h = 0.25$  and  $0.35$ , respectively. In producing Figs. 5 and 6,  $s = 2.5 \text{ m}$  and other parameters are the same as above. Figs. 5 and 6 shows that the ultimate bearing capacity of the stone column increases by increasing the internal friction angle of the stone column material and diameter of the stone-column. Also with increasing the seismic force, the effect of increasing the stone-column diameter is more efficient than increasing the internal friction angle of the stone column material. The effect of  $s$  (stone column spacing) on the bearing capacity is illustrated in Figs. 7 and 8 for which  $k_h = 0.25$  and  $k_h = 0.35$ , respectively. The stone-column diameter is  $D = 1 \text{ m}$  and other parameters are the same as above. As seen, by increasing the column spacing, the bearing capacity tends to decrease slightly. However, it has no effect on the bearing capacity of the stone-column.

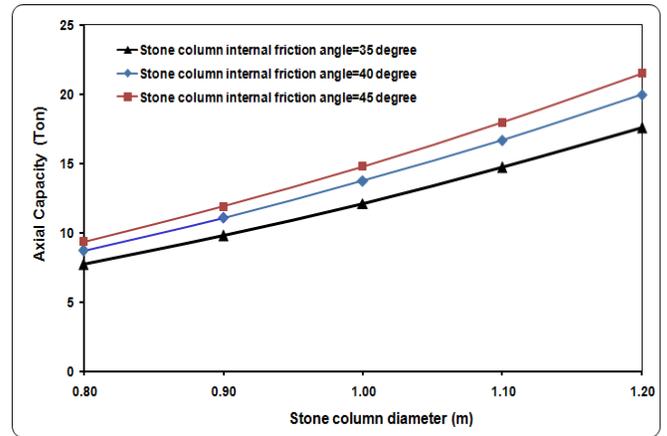


Fig 5. Effect of stone-column diameter on column ultimate bearing capacity ( $k_h = 0.25$ )

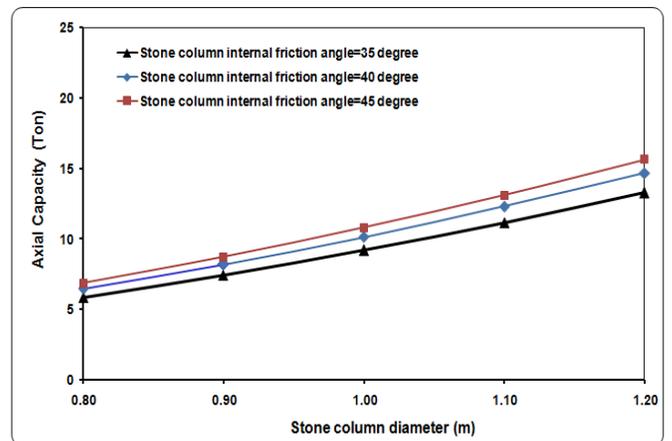


Fig 6. Effect of stone-column diameter on column ultimate bearing capacity ( $k_h = 0.35$ )

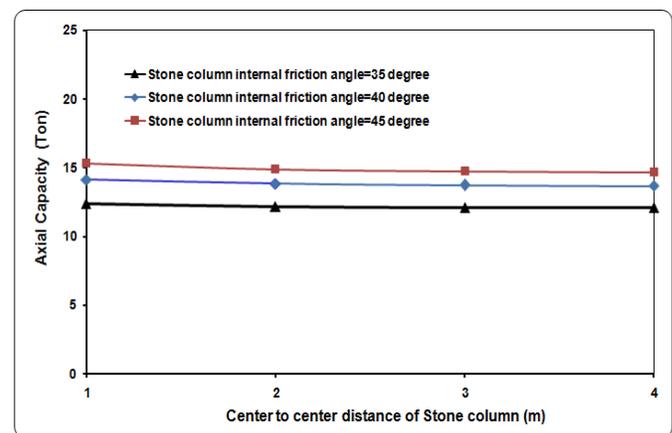


Fig 7. Effect of stone-column spacing on column ultimate bearing capacity ( $k_h = 0.25$ )

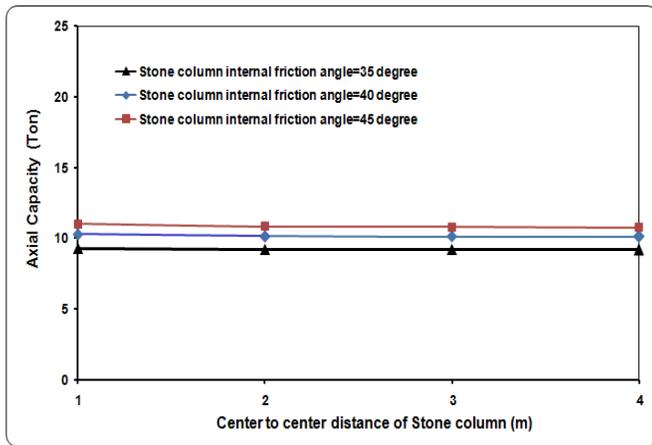


Fig 8. Effect of stone-column spacing on column ultimate bearing capacity ( $k_h = 0.35$ )

## CONCLUSIONS

A simple method has been introduced for determination of the bearing capacity of stone columns. The method is based on the lateral earth pressure theorem and requires conventional shear strength parameters of the stone column material and the native soil to be reinforced. It has been shown that with increasing the seismic force, the ultimate bearing capacity of the stone column decreases. In addition, with increasing the friction angle of the stone column material, the bearing capacity of the column increases particularly at low to moderate seismic intensities. However, this effect is insignificant at higher seismic intensities. It has been shown that the increase of the diameter of the stone-column is more efficient than increasing the internal friction angle of stone-column material or with decreasing center to center distance of stone-columns.

## REFERENCES

- Aboshi, H., Ichimoto, E., Harada, K., and Emoki, M. [1979] "The composer-A method to improve the characteristics of soft clays by inclusion of large diameter sand columns." Proc., Int. Conf. on Soil Reinforcement. E.N.P.C., 1, Paris, pp.211-216.
- Datye, K. R. and Nagaraju, S. S, [1975] "Installation and testing of rammed stone columns," Proc. IGS Specialty Session, 5th Asian Regional Conference on Soil Mechanics and Foundation Engineering, Bangalor, India, pp.101-104.
- Greenwood, D. A. [1970]" Mechanical improvement of soils below ground surface", Proc. Ground Eng'g. Conference, Institute of Civil Engineers, pp.9-29
- Huges, J. M. O and N. J. Withers, [1974] "Reinforcing of soft cohesive soils with stone columns", Ground Engineering, Vol. 7, No. 3, pp. 42-49.

Huges, J. M. O.,Withers, N. J., and Greenwood, D. A. [1976]"A field trial of reinforcing effect of stone column in soil." Proc., Ground Treatment by Deep Compaction, Institution of Civil Engineers, London, pp. 32-44.

Madhav, M. R. and Vitkar, P. P. [1978] "Strip footing on weak clay stabilized with a granular trench or pile." Can. Geotech. J., 15\_4, pp. 605-609.

Madhav, M. R., Iyengar, N.G.R., Vitkar , R.P., and Nandia, A. [1979] "Increased bearing capacity and reduced settlements due to inclusions in soil," Proc. Int. Conf. on Soil Reinforcement: Reinforced and other techniques, Vol. 2, pp. 239-333.

Richards R. Jr, Elms, D. G., and Budhu, M., [1993] "Seismic bearing capacity and settlements of foundations", J. Geotech. Engrg., ASCE, Vol. 119, No. 4. pp. 662-674.

Vesic, A. S., [1972] "Expansion of cavities in infinite soil mass", J. Soil Mech. and Found. Engrg., Div., ASCE, Vol. 98, No. SM3, pp. 265-290.