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Performance of Bandlimited Gaussian Noise as a Spreading Code in AWGN and Single Tone Interference Channels

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PERFORMANCE OF BANDLIMITED GAUSSIAN NOISE AS A SPREADING CODE IN AWGN AND SINGLE TONE INTERFERENCE CHANNELS

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ABSTRACT: This paper examines the bit error rate (BER) of a direct sequence spread spectrum (SS) system that uses bandlimited white gaussian noise (BLWGN) as a spreading code. The performance of this "bandlimited-SS" system is examined in both additive white gaussian noise (AWGN) and single tone jamming environments. The BLWGN spreading code is inferior to conventional codes in AWGN channels, but is more robust when a narrow band interference signal is present.

I. INTRODUCTION

Direct sequence spread spectrum (DS-SS) techniques were originally developed for low probability of intercept and anti-jamming applications. Once limited to military applications, interest in utilizing DS-SS techniques in commercial applications continues to increase. However most commercial applications require some measure of jamming resistance while imposing a bandwidth constraint on the transmitted signal. This may make it difficult to the use conventional modulation techniques such as bi-level pseudonoise (PN) spreading codes typically found in DS-SS systems. This paper examines a new type of spreading code that generates bandlimited signals that are reasonably robust in tone jamming environments.

If one knew the power spectral density of the interfering signal, it would be possible to carefully shape the spectrum of the transmitted signal to maximize its AJ capability. Unfortunately this signal would perform very poorly in other types of interference. To work well, this approach requires the receiver to determine the characteristics of the interfering signal, and relay this information to the transmitter. This paper focuses on a simpler, but hopefully robust, technique that does not require feedback from the receiver. Using the maximum entropy philosophy, we assume the transmitter knows nothing about the interfering signals. The best approach is arguably then to uniformly spread the transmitted signal power over the entire allotted bandwidth. Simply bandlimiting the output of a conventional DS-SS modulator does not accomplish this, as shown in Figure 1.

The system proposed in this paper uses a BLWGN source as a spreading code. If the bandwidth of the spreading code is far larger than the information bandwidth, the transmitted signal PSD will closely resemble the spreading code PSD. This means the transmitted signal should be reasonably constant over the specified bandwidth, as shown in Figure 1. This will be called a BLWGN DS-SS system throughout this paper. For simplicity, only coherent receivers will be used in this work. Although synchronization is an important concern, it is possible to construct delay-locked loops that can track BLWGN signals [1].

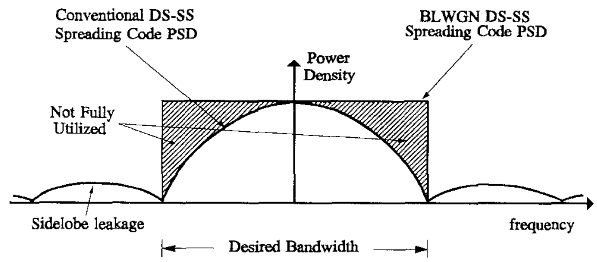

Figure 1 Power spectra of conventional and BLWGN codes.

This paper compares the BLWGN DS-SS system to conventional QPSK, conventional DS-SS and a main-lobe-only DS-SS system. The main-lobe-only system is a conventional DS-SS system that has been converted to a fixed bandwidth system by adding a filter to the output that only passes the main lobe of the original signal. These systems were compared using single tone interference and an AWGN channel. The BLWGN SS system has the lowest BER when the jamming tone is close to the carrier frequency. The BER of the BLWGN SS system is nearly independent of the jamming frequency, a property not enjoyed by any of the other systems investigated. A drawback of the new system is that it is inferior when only AWGN is present in the channel.

Detailed system descriptions and the concept of bandwidth efficiency are addressed in Section II. The BER performance for all four systems in both single tone jamming and AWGN channels is provided in Section III. Section IV summarized the benefits and disadvantages of the new system.

II. SYSTEM DESCRIPTION

In the early development of DS-SS system, a variety of wide bandwidth noise sources were used to generate the spreading signal [2,3]. Some of these wide bandwidth noise sources can be model as a white gaussian noise (WGN) [4]. Since the emergence of bi-level PN sequences [5], the majority of research and development in direct sequence SS systems has used these codes [6,7], and WGN has been largely

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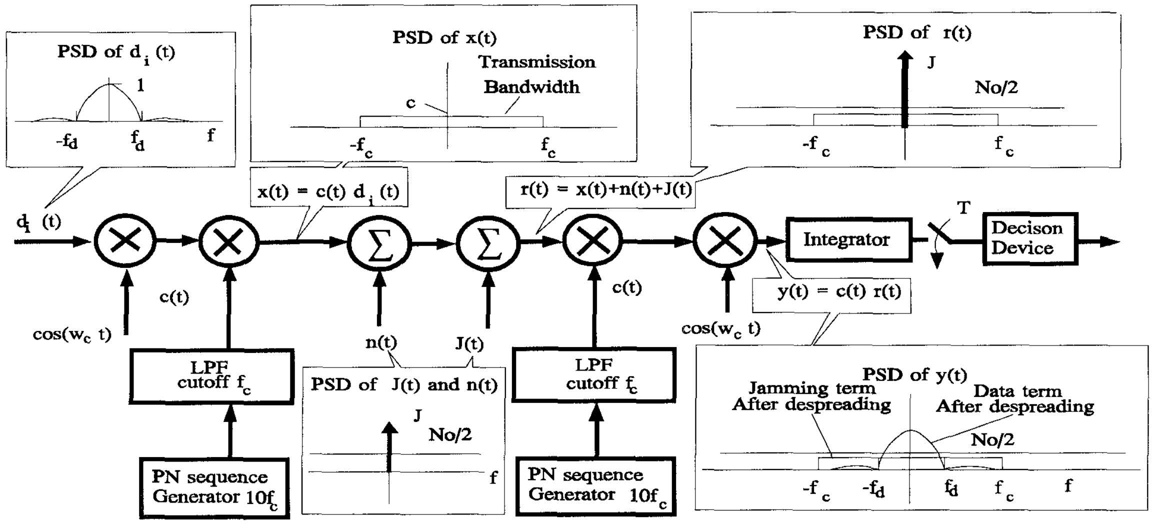

Figure 2 BLWGN DS-SS system block diagram.

abandoned. Advances in digital technology allows us to overcome many of the problems originally encountered with WGN systems. These same advances have been responsible for a dramatic increase in demand for digital radio systems. This generates much more severe bandwidth constraints than early systems faced. This was the motivation for examining WGN as a spreading code.

The system block diagram of BLWGN DS-SS system is shown in Figure 2. To simplify the analysis, the carrier frequency fc is set to zero, i.e. the complex envelope representation is used. There is no loss of generality for such simplification. The BLWGN DS-SS system block diagram closely resembles a conventional DS-SS system, with two noticeable exceptions: (1) The chip rate of the PN code generators has been increased by a factor of 10 and (2) There is a lowpass filter (LPF) placed between the PN code generator and the multiplier. This LPF has a cutoff frequency equal to the chip rate of the conventional DS-SS system. As Figure 3 demonstrates, this output of the LPF will not be perfectly flat over the range $-f_c$ to f_c. This is because the PN sequence generator produces a signal with a power spectral density of $\text{sinc}^2(f/10f_c)$. Filtering this to the range $-f_c$ to f_c will produce a noise sequence with a slightly convex power spectral density.

A conventional DS-SS system theoretically transmits an infinite bandwidth signal, the BLWGN DS-SS system is far more bandwidth efficient. Conventional modulation schemes such as QPSK, MSK, etc. are even more bandwidth efficient. The goal of new system is to generate a signal with a bandwidth between that of QPSK and conventional DS-SS systems. This is done because we expect jammers or interference to be present in the channel. These unwanted signals can disrupt the demodulation of narrow band signals. As Figure 2 demonstrates, the BLWGN DS-SS receiver has a despreading stage that will combat the effects of unwanted interference. This stage will spread a narrow band interference signal over a bandwidth equal to the original spreading code bandwidth. At the same time it will compress the spectrum of the desired signal back to its original bandwidth. This increases the SNR and is commonly known as "processing gain" in the SS literature. The new system is a hybrid that attempts to maximize the processing gain while limiting the bandwidth of the transmitted signal. Since the transmitted spectrum is constant from $-f_c$ to f_c, we expect the demodulator performance to be independent of the frequency of a jamming tone. The computer simulation results presented in the next section support this claim.

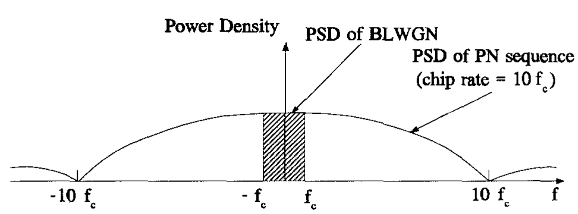

Figure 3 PSD of approximated BLWGN.

III. PERFORMANCE ANALYSIS

The primary performance measure for digital communication systems is the BER vs the signal-to-noise ratio (SNR) and jammer-to-signal ratio (JSR). This was determined for four modulation types: QPSK, conventional DS-SS, main-lobe-only DS-SS and BLWGN DS-SS. The BER for QPSK in AWGN and conventional DS-SS in AWGN are both analytically tractable problems with a well-known solution.

$$P_E = Q\left(\sqrt{\frac{E_s}{2N_0}}\right)$$

$$Q(x) = \int_x^{\infty} \frac{\exp(-y^2/2)}{\sqrt{2\pi}}\, dy$$

Where E_s is the average energy per symbol and N_0 is single-sided power spectrum density of the white noise. If a tone jammer is presented, then the analysis becomes more complex. For QPSK with a tone jammer at the carrier frequency the probability of error can be expressed as

$$P_E = \frac{1}{4}\left[Q\left(\sqrt{\frac{2}{N_o}}\left(\sqrt{E_s} - 2J_{ich}\right)\right) + Q\left(\sqrt{\frac{2}{N_o}}\left(\sqrt{E_s} + 2J_{ich}\right)\right) + Q\left(\sqrt{\frac{2}{N_o}}\left(\sqrt{E_s} - 2J_{qch}\right)\right) + Q\left(\sqrt{\frac{2}{N_o}}\left(\sqrt{E_s} + 2J_{qch}\right)\right)\right]$$

where J is the jamming power, θ is the jamming phase relative to the carrier, $J_{ich} = \sqrt{2J}\ \cos(\theta)$ and $J_{qch} = \sqrt{2J}\ \sin(\theta)$.) From Schilling and Sigh [8,9], for a conventional DS-SS system facing a tone jammer at the carrier frequency the BER can be approximated:

$$P_e(\theta) = \frac{1}{2} Q\left[\left(\frac{N_o}{2E} + \frac{J}{N}\cos^2\theta\right)^{-1/2}\right] + \frac{1}{2} Q\left[\left(\frac{N_o}{2E} + \frac{J}{N}\sin^2\theta\right)^{-1/2}\right]$$

This approximation assumes the PN sequence has a very long period and the chip rate is much grater than the data rate (identical assumptions to the ones made in this work). The BER of the main-lobe-only and BLWGN DS-SS systems for the AWGN channel was derived from approximating the output probability density function given by [10].

$$P(z) \approx \frac{1}{\sqrt{K_2}} \sum_{j=0}^{\infty} \alpha_j\, \phi^j\left(\frac{z - K_1}{\sqrt{K_2}}\right)$$

where

$$\phi^j(x) = \frac{d^j}{dx^j}\left[\frac{1}{\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right)\right]$$

and

$$\alpha_0 = 1 \quad ;\ \alpha_1 = \alpha_2 = 0 \quad ;\ \alpha_3 = \frac{-K_3}{6\, K_2^{3/2}}$$

$$K_1 = N_o\,(1 - X) = 1$$

$$K_2 = \frac{N_o^2}{\sqrt{1+2\gamma^2}}\left[1 + 2X\left(\frac{1+2\gamma^2}{1+\gamma^2}\right)\right]$$

$$K_3 = \frac{4N_o^3}{2+3\gamma^2}\left[1 + 3X\left(\frac{2+3\gamma^2}{2+\gamma^2}\right)\right]$$

where z is the instantaneous received power, $X = S/N_0$ (signal to noise ratio) where S is the average signal energy per bit. γ is the spreading code bandwidth to data bandwidth ratio. The BER of these two systems in AWGN can be approximated by averaging the instantaneous BER evaluated by a Q function.

$$P_E = \int_{-\infty}^{+\infty} P(z)\ Q\left(\frac{z}{N_o}\right) dz$$

The BER of these two proposed systems in tone jamming are difficult to calculate. This is partially due to the unequal amount of energy that is transmitted in each bit of information. Hence, for an initial evaluation of the performance of these systems, computer simulation was used. The data rate for all systems is 1 Kbit/sec, the transmitted bandwidth is 10KHz and the sampling rate is 1 Msample/sec. The single tone jamming case is described from Figures 4 through 6. The JSR for all tests was set to 0 Db, and the BER performance was represented as a function of the SNR.

Figure 4 shows the performance of the systems when the jamming tone is at the carrier frequency. As expected at this JSR, the QPSK performance was dominated by the tone jammer amplitude. Conventional DS-SS and main-lobe-only DS-SS have nearly the same result. The BLWGN system had the best performance. This was expected since the BLWGN system has a lower transmitted PSD at the carrier frequency.

Figure 5 investigates the performance of the conventional and main-lobe-only DS-SS systems as the frequency of the jammer is changed relative to the carrier frequency. Both systems are very sensitive to the jammer frequency. As the jammer approaches the null at the chip rate the performance improves to match the no-jamming case. In contrast, Figure 6 documents the performance of the BLWGN DS-SS system against a variable frequency jammer. As expected, the system is insensitive to the jammer frequency.

A drawback of the BLWGN DS-SS system is shown in Figure 7. The system suffers from having an unequal amount of energy in each transmitted symbol. This means that for a given average power level, there will be some bits that are very prone to error because of the low amount of transmitted energy. This substantially increases the BER for this system in AWGN over the more conventional techniques. Figure 7 also shows the analytical as well as simulation results. The approximation made in the analysis account for the difference between the BLWGN curves.

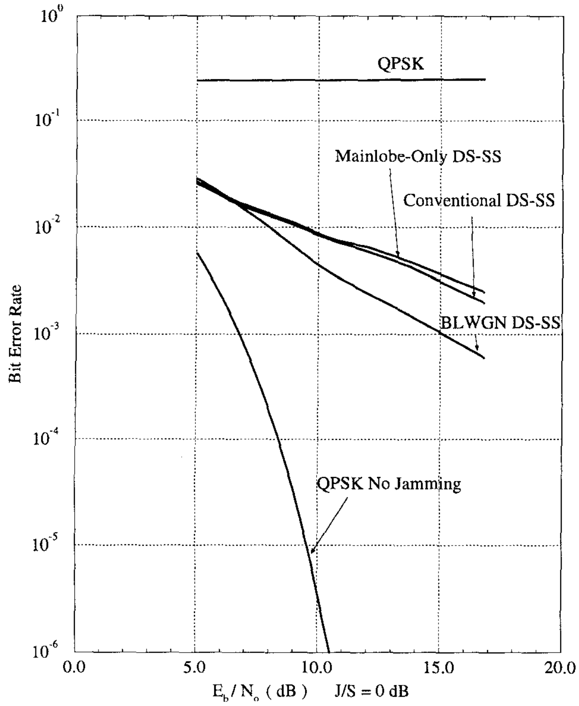

Figure 4 Performance in tone jamming. (Jamming at carrier frequency).

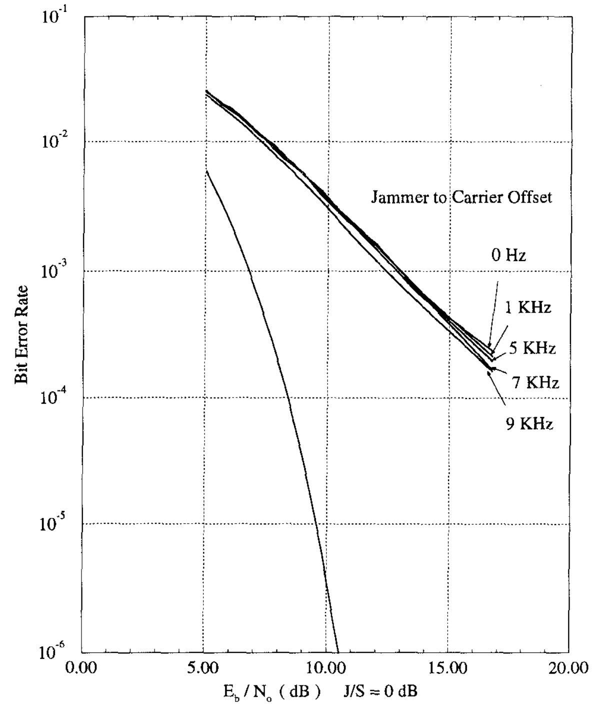

Figure 6 Performance of BLWGN SS system in tone jamming.

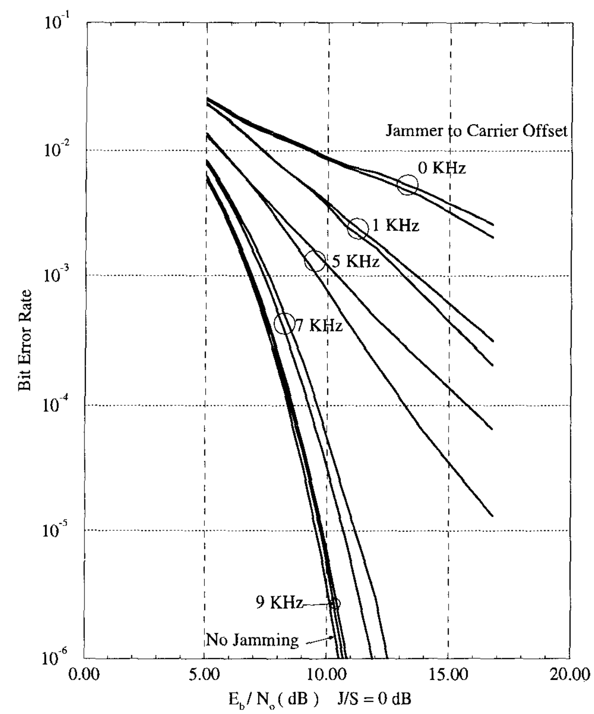

Figure 5 Performance of conventional and main-lobe-only DS-SS system in tone jamming.

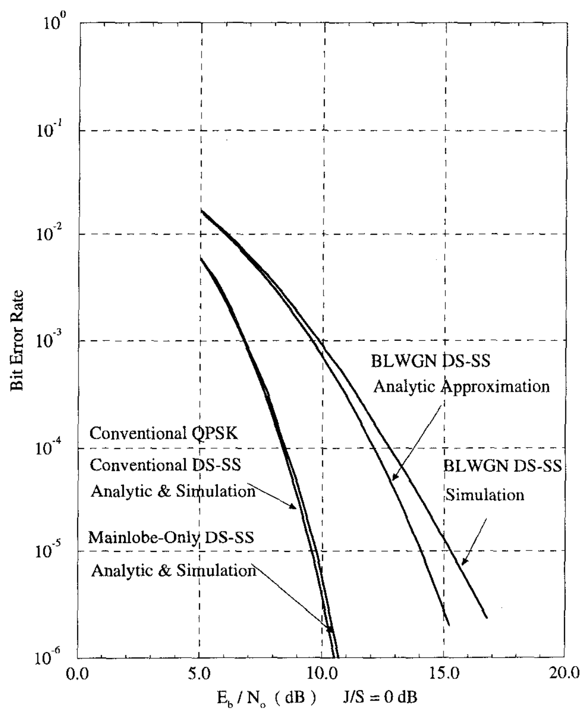

Figure 7 Performance in AWGN channel.

IV. CONCLUSION

Numerous techniques exist for combating narrow band interference in spread spectrum systems. Unlike methods such as adaptive notch filters (that alter the receiver structure) the method presented in this paper focused on altering the transmitted signal. We focused techniques that balance the requirements of finite bandwidth and jamming resistance. Since the modulation and demodulation schemes are non-adaptive, there are no convergence issues or failure modes that sometimes occurs in adaptive structures. This feature is particularly helpful when the jamming tone is produced by a hostile party. If one is willing to insert an adaptive notch filter into the receiver, the performance of the BLWGN DS-SS system will improve, but this topic was not addressed here.

We attempted to minimize the impact of the worst case jammer by shaping the transmitted signal. The result was a system that is insensitive to the jamming frequency. Unlike most conventional systems, there is no worst-case jammer frequency. Another feature of the new system is that the transmitted signal has a finite bandwidth. This will make it easier to use in some commercial applications where signal leakage into adjacent frequency bands is not allowed. The BLWGN DS-SS system is not optimal in any sense, however it is a bandlimited system and appears to be rather robust. Its worst-case performance in narrow band interference appears to be superior to the worst-case performance of some more conventional systems.

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