An Analytical Solution for Pile-Soil-Pile Interaction with Unequal Embedded Lengths Under Vertical Harmonic Vibrations

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ABSTRACT

Piles are normally constructed in groups such that they are located in the vicinity of each other. As a result, the response of piles is different from that of an isolated pile. In this paper, the interaction between two piles with unequal lengths in the group is investigated using an analytical solution. The elastic theory model and dynamic winkler model are use to characterise vertical isolated prismatic piles subjected to vertical harmonic vibrations. The results obtained for pile-soil-pile interaction from this solution account for unequal lengths for the two piles. In addition, it has been found that upon loading the first pile (called source pile), the presence of the neighbouring pile (called receiver pile) is important, leading to lesser ground movement at the receiver pile head location. The results also indicate that pile-pile spacing and the soil type are important. The general finding in this research indicates that ignoring the presence of the second pile in the calculation of the interaction between a pair of piles can significantly overestimates the pile-soil-pile interaction, resulting in underestimation of the group stiffness.

INTRODUCTION

At static working loads, the displacement of a pile increases if this pile is located within the deformation field of a neighbouring pile. As a result, the overall displacement u^G of the group of piles is greater than the individual displacement u^S which each pile would experience were it is left alone to carry the average load. The static group efficiency u^S/u^G is thus always below unity, and it tends to decline when the distance between piles is shortened or when the number of piles in the group increases.

Rational analyses of pile group displacements were pioneered by Poulos (1968, 1971), who introduced the concept of 'interaction factors' and showed that pile group effects can be assessed by superimposing the effects of only two piles.
Dynamically loaded piles under vertical vibrations have been investigated using various methods.

These methods are lumped mass (Penzien,1975; El Naggar and Novak, 1994) continuum approach (Novak, 1974: Novak and Aboul-Ella, 1978; Novak and Nogami, 1980), boundary element method (Kaynia and Kausel, 1982; Sen et.al., 1985), and finite element solutions (Blaney et. al., 1976; Wolf and von Arx, 1982; Chow, 1985).

All these approaches are used for isolated piles. The interaction between piles vibrating harmonically also been investigated. For example Kaynia and Kausel (1982) used elastic boundary element method to determine interaction factors.

In spite of signification progress achieved in pile dynamic analyses, there is still a serious need to develop simple procedures enabling users to calculate the dynamics interaction factor between piles. This paper present a simple approach for determination of interaction effect between closely spaced unequal embedded lengths piles that to be used in ground with slope subjected to harmonic vibrations (Fig. 1). All studies in the interaction factor are investigated for horizontal ground. But the study for piles which are located in sloping ground, are very limited.

PROBLEM DEFINITION

The problem studied in this paper is that of a two end-bearing, unequal lengths, cylindrical piles embedded in a half space and subjected to an arbitrary vertical harmonic excitation at the top. The applied dynamics forces are transmitted on to each pile and uniform stratum.

The soil is modelled as a linear hysteretic material and each pile is a solid cylinder made of a linear elastic material.

Fig. 2 sketches the two specific soil profiles for which parametric results will be presented.

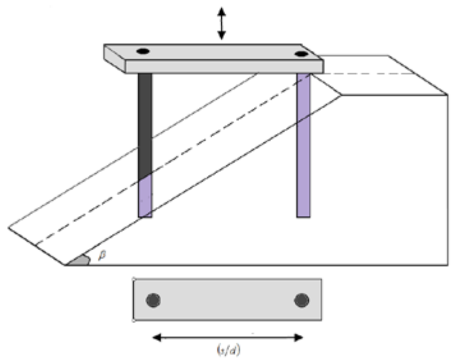


Fig. 1. Profiles of pile groups with unequal embeded length and plan of pile groups studied in this paper

For each particular harmonic excitation of frequency ω, the complex-value dynamic impedance K^G is defined as the ratio of excitation (vertical force H) over the corresponding motion of rigid cap (vertical displacement w). For example, for vertical excitation:

$$K_w = k + i.(c.\omega) = H / w \tag{1}$$

Where $a_0 = \omega.d/V_S$ is a dimensionless frequency. In all cases $i = \sqrt{-1}$. The terms k and c are interpreted as equivalent 'spring' and 'dashpot' coefficient at head of the pile group. Character d represents the pile diameter, $V_S = \sqrt{G/\rho_S}$ is the shear wave velocity, G is the soil shear modulus and ρ_s is the soil mass density. They are both function of the circular frequency of excitation $\omega = 2\pi\nu$.

OUTLINE OF PROPOSED GENERAL METHOD

A general approximate method is proposed that involves the following three consecutive steps, schematically illustrate in Fig. 2 for vertical harmonic force at top of the piles.

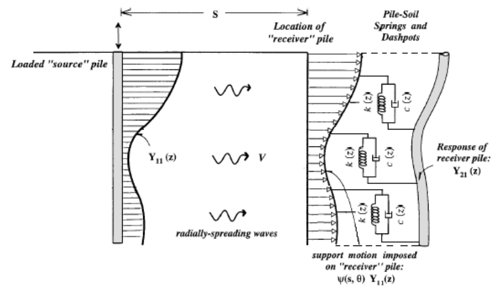


Fig. 2. Schematic illustration of Propose Model for Computing Influence of Head-Loaded Source Pile on Adjacent Receiver Pile Carrying No Load at its Head.

The complex vertical stiffness related to a unit length of the cylinder is found to be (Novak and Beredugo, 1972):

$$k_s = 2\pi G(1+i2\beta)a_0^* \frac{K_1(a_0^*)}{K_0(a_0^*)} \qquad a_0^* = \frac{ia_0}{\sqrt{1+i2\beta}} \tag{2}$$

where K_0, K_1 denotes the zero-order and first-order second-kind modified Bessel function.

The complex base stiffness, for interaction between tip of the pile and soil by frequency-dependent, is defined (Bycroft, G. N., 1956):

$$\begin{aligned} &K_b = Gr_0(C_{w1} + iC_{w2}) \\ &\nu = 0.25 \Rightarrow \begin{cases} C_{w1} = 5.33 + 0.364a_0 - 1.41a_0^2 \\ C_{w2} = 5.06a_0 \end{cases} \\ &\nu = 0.5 \Rightarrow \begin{cases} C_{w1} = 8.00 + 2.18a_0 - 12.63a_0^2 + 20.73a_0^3 - 16.47a_0^4 + 4.458a_0^5 \\ C_{w2} = 7.414a_0 - 2.986a_0^2 + 4.324a_0^3 - 1.782a_0^4 \end{cases} \end{aligned} \tag{3}$$

ANALYSIS FOR SINGLE PILE

The equilibrium of pile displacement that is not relative to soil reaction at a certain depth (free length) can be expressed by a single governing equation. For a single pile-soi1 system loaded vertically the solution involves the solution of a second-order differential equation as given below:

$$m\frac{d^2 w_{11}(z,t)}{dt^2} - E_P A_P \frac{d^2 w_{11}(z,t)}{dz^2} + k_s w_{11}(z,t) = 0 \tag{4}$$

Decomposing Eq. (4) results in:

$$w(z,t) = w(z).\exp(i\omega t) \tag{5}$$

where:

$$(m\omega^2)w(z) + E_P A_P \frac{d^2 w(z)}{dz^2} = 0 \tag{6}$$

$$\left(\frac{m\omega^2}{E_P A_P}\right) w(z) + \frac{d^2 w(z)}{dz^2} = 0 \tag{7}$$

$$\Lambda_1 = \sqrt{\frac{m\omega^2}{E_P A_P}} \quad (8)$$

$$w(z) = A_{11}\sin(\Lambda_1 z) + B_{11}\cos(\Lambda_1 z) \quad (9)$$

where:

$$\Lambda_1 = \sqrt{\frac{m\omega^2}{E_P A_P}} \quad (10)$$

The solution is given by:

$$w_{11}(z) = A_{11}\sin(\Lambda_1 z) + B_{11}\cos(\Lambda_1 z) \quad (11)$$

where A_{11}, B_{11} are integration constant calculated using appropriate boundary conditions.
In other words:

$$p(z) = -E_P A_P \frac{dw(z)}{dz} \quad (12)$$

$$p_{11}^{s}(z) = -E_P A_P(\Lambda_1)(A_{11}\cos(\Lambda_1 z) - B_{11}\sin(\Lambda_1 z)) \quad (13)$$

The equilibrium of pile displacement relative to soil reaction at a certain depth (embeded length) can be expressed by a single governing equation. For a single pile-soi1 system loaded vertically, the solution involves solving a second-order differential Eq. (14) given by Novak (1974) where w(z,t) is the pile settlement at depth z. t is time. Character m is the mass of the pile per unit length, c is coefficient of pile inertial damping, E_P is Yong's modulus of the pile, A_P circular equivalent cross section and G is the soil shear modulus.

$$m\frac{d^2 w_{11}(z,t)}{dt^2} + c\frac{dw_{11}(z,t)}{dt} - E_P A_P \frac{d^2 w_{11}(z,t)}{dz^2} + k_s w_{11}(z,t) = 0 \quad (14)$$

Decomposing Eq. (14) results in:

$$w(z,t) = w(z).\exp(i\omega t) \quad (15)$$

where:

$$-m\omega^2 w(z) + ic\omega w(z) + k_s w(z) - E_P A_P \frac{d^2 w(z)}{dz^2} = 0 \quad (16)$$

$$(k_s - m\omega^2 + ic\omega)w(z) - E_P A_P \frac{d^2 w(z)}{dz^2} = 0 \quad (17)$$

$$\left(\frac{(k_s - m\omega^2 + ic\omega)}{E_P A_P}\right)w(z) - \frac{d^2 w(z)}{dz^2} = 0 \quad (18)$$

$$\Lambda_2 = \sqrt{\frac{(k - m\omega^2 + ic\omega)}{E_P A_P}} \quad (19)$$

The solution is as follows:

$$w_{11}^{s}(z) = A_{11}\exp(\Lambda_2 z) + B_{11}\exp(-\Lambda_2 z) \quad (20)$$

where A_{11}, B_{11} are integration constants calculated using appropriate boundary conditions. These are:

$$p(z) = -E_P A_P \frac{dw(z)}{dz} \quad (21)$$

$$p_{11}^{s}(z) = -E_P A_P(\Lambda_2)(A_{11}\exp(\Lambda_2 z) - B_{11}\exp(-\Lambda_2 z)) \quad (22)$$

REDUCTION OF SOIL DISPLACEMENT AWAY FROM 'SOURCE' PILE

P-and S-wave are emitted from the oscillating pile, travelling in all directions and being reflected from the free surface of the soil. Several approximate models, base on 1D, 2D or 3D wave propagation idealization, are nevertheless available. In order to calculate the interaction factor between two piles, the dynamics displacement around a vibrating is described by the following asymptotic cylindrical wave expression pile may be initially calculated using (Morse and Ingard, 1968 as reported by Dobry and Gazetas, 1988):

$$\psi(s/d, a_0) = (2\times(s/d))^{-0.5}\exp(-\beta' a_0(s/d))\exp(-ia_0(s/d)) \quad (23)$$

where s is the horizontal distance from the axis of pile, and β' is damping ratio of the soil. The first exponential term represents the motion decay and a hysteretic damping. The second one describes the phase lag of the motion of the soil away from the vibrating pile.

INTERACTION OF 'RECEIVER' PILE WITH ARRIVING WAVE

Consider now a second pile located at the distance r=s from the first pile. The soil displacement field computed affects the second pile. The first pile induces displacements on the soil, whereas the soil induces displacement on the second pile.
The dynamic equilibrium for the second pile is:

$$m\frac{d^2 w_{11}(z,t)}{dt^2} - E_P A_P \frac{d^2 w_{11}(z,t)}{dz^2} + k_s(w_{21}(z,t) - w_{21}(z,s,t)) = 0 \quad (24)$$

Decomposing Eq. (24) results in:

$$w_{21}(z,t) = w_{21}(z).\exp(i\omega t) \quad (25)$$

After some calculations and determination of particular and general solutions, we have:

$$w_{21}^{r}(z) = \frac{\Lambda_2 \psi z(?)}{2}(-A_{11}\exp(\Lambda_2 z) + B_{11}\exp(-\Lambda_2 z)) + (A_{21}^{'r}\exp(\Lambda_2 z) + B_{21}^{'r}\exp(\Lambda_2 z)) \quad (26)$$

where

$$(?) = \frac{(k + ic\omega)}{(k - m\omega^2 + ic\omega)}) \quad (27)$$

$A_{21}^{'r}, B_{21}^{'r}$ are new integration constant to be determined from the boundary conditions of the receiver pile.

DETERMINATION OF INTERACTION FACTOR

The interaction factor between two identical piles is defined as the response atop the second pile carrying no load its head (hereafter called “receiver pile”), normalized by the corresponding response of the first pile which is loaded with unit, vertical and harmonic load (hereafter called “source pile”). Fig. 3 shows two piles. It is obvious that the receiver pile is subjected to the ground vibrations produced by the vibrating source pile.

It is useful to express the pile-to-pile interaction by the dynamic interaction factor defined by:

$$\alpha = \frac{\text{additional head deflection of second pile caused by first pile}}{\text{head deflection of first pile}} \tag{28}$$

Regarding Fig. 3, the interaction factor of two illustrated cases is different. In case 1, the source pile with length L is fully embedded into the soil (sec. 4.2). However, only a part of receiver pile with length L_5 is embedded into the soil (sec: 6). As seen in Fig. 3, the source pile affects length L_5 of the receiver pile and thus L_4 is not affected. Part L_4 is treated as a free length (sec: 4.1).

The following equations give the integration constants for the source and receiver piles:

$$\begin{aligned}
&for \quad source \quad pile: \\
&1)\, p_1^s(z=0) = p_0 = 1 \\
&2) K_b \times w_{11}^s(z=L) = p_{11}^s(z=L)
\end{aligned} \tag{29}$$

$$\begin{aligned}
&for \quad receiver \quad pile: \\
&1)\, p_1^r(z=0) = p_0 = 0 \\
&2) w_1^r(z=L_4) = w_2^r(z=L_4) \\
&3)\, p_1^r(z=L_4) = p_2^r(z=L_4) \\
&4) K_b \times w_2^r(z=L) = p_2^r(z=L)
\end{aligned} \tag{30}$$

In case 2, the source pile is on the right had side and only L_5 is embedded into the soil (sec. 4.2). Part L_4 is free length and does not affect the receiver pile on the left had side (sec. 4.1). The receiver pile with length of L is fully embedded into the soil (sec. 6).

Eqs. (31) and (32) present the constants for the source and receiver piles for determination of the interaction factor.

$$\begin{aligned}
&for \quad source \quad pile: \\
&1)\, p_1^r(z=0) = p_0 = 1 \\
&2) w_1^r(z=L_4) = w_2^r(z=L_4) \\
&3)\, p_1^r(z=L_4) = p_2^r(z=L_4) \\
&4) K_b \times w_2^r(z=L) = p_2^r(z=L)
\end{aligned} \tag{31}$$

$$\begin{aligned}
&for \quad receiver \quad pile: \\
&1)\, p_1^s(z=0) = p_0 = 0 \\
&2) K_b \times w_{11}^s(z=L) = p_{11}^s(z=L)
\end{aligned} \tag{32}$$

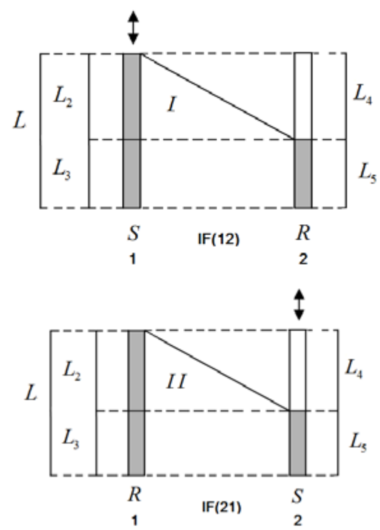
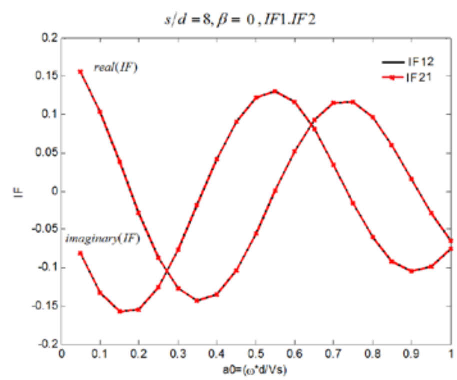


Fig. 3. Definition of interaction factors IF (12) and IF (21)

VERIFICATION

For verification, the interaction factor between two cases is shown in Fig. 3. For case L_4=0 or the ground slope equal to 0 $(\beta = 0)$, IF (12) and IF(21) are identical as seen in Fig. 4. In Fig. 4, the results for s/d=8 and $\beta = 0{,}30$ are also shown.



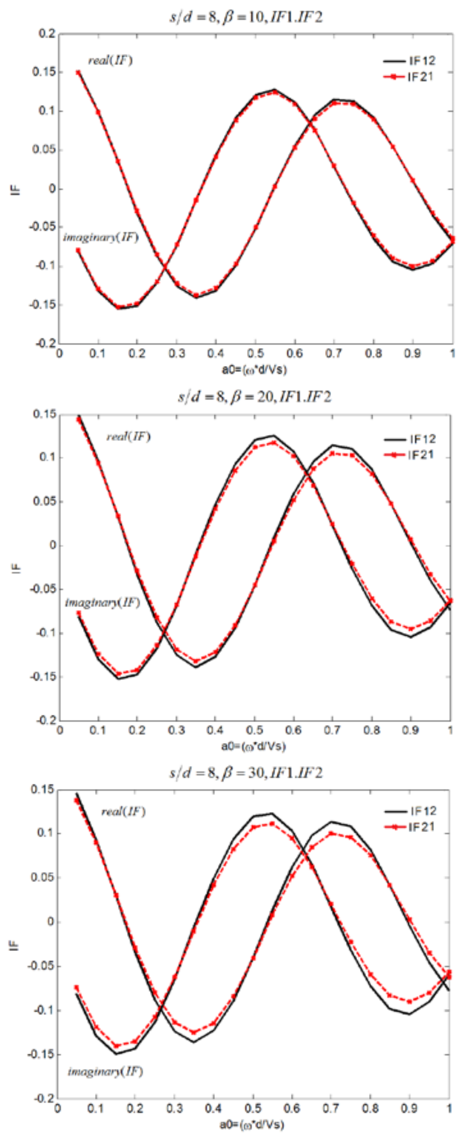


Fig. 4. Comparison between IF (21) and IF (12)
$(E_P/E_S=1000, L/d=40, \nu=0.4, \rho_s/\rho_p=0.7, \beta^{'}=0.05, s/d=8, \beta=0,30)$

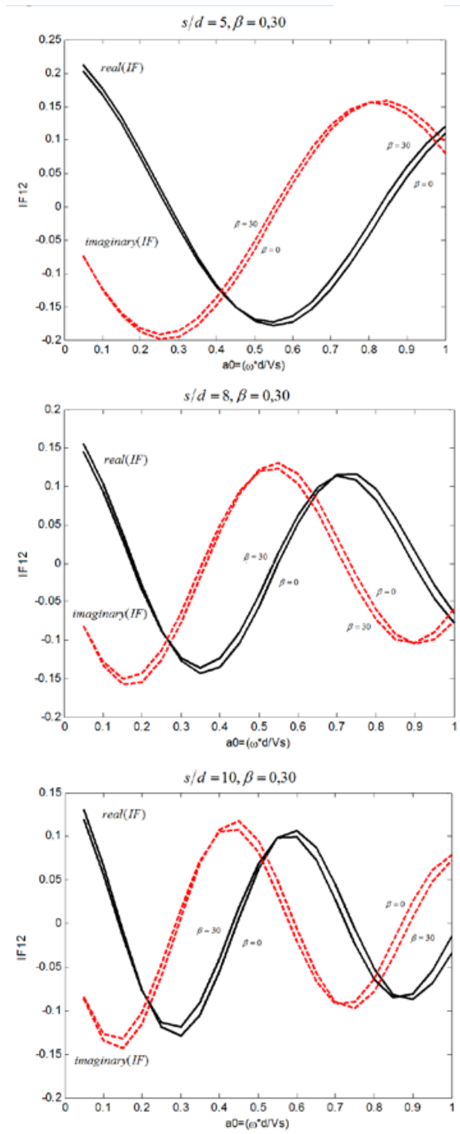
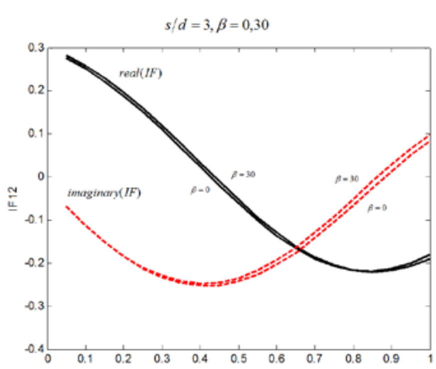


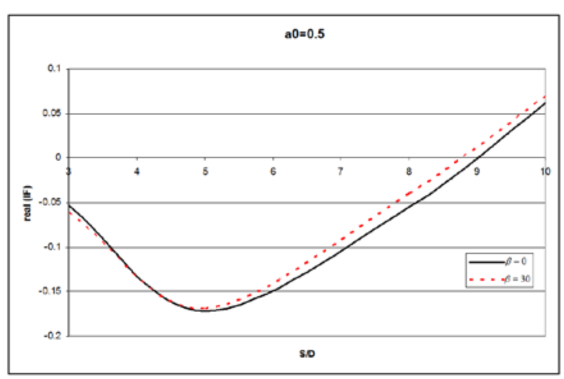
Fig. 5. Variation IF(12) with a_0
$(E_P/E_S=1000, L/d=40, \nu=0.4, \rho_s/\rho_p=0.7, \beta^{'}=0.05, s/d=3,5,8,10, \beta=0,30)$

PARAMETRIC STUDIES

A) Fig.5 shows the variation of IF (12) versus a_o for s/d=3, 5, 8, 10 and $\beta=0,30$.



B) The variation of IF (12) versus s/d ranging 3-10 and for $\beta=0,30$ and $a_0=0.5$ is illustrated in Fig. 6.



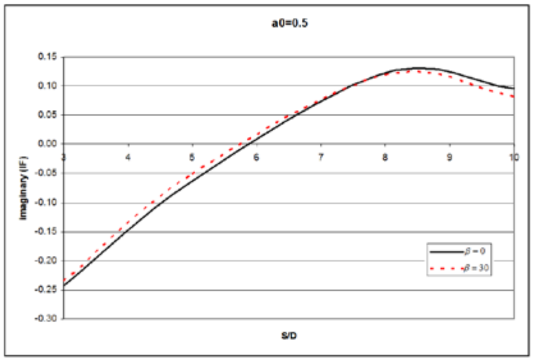
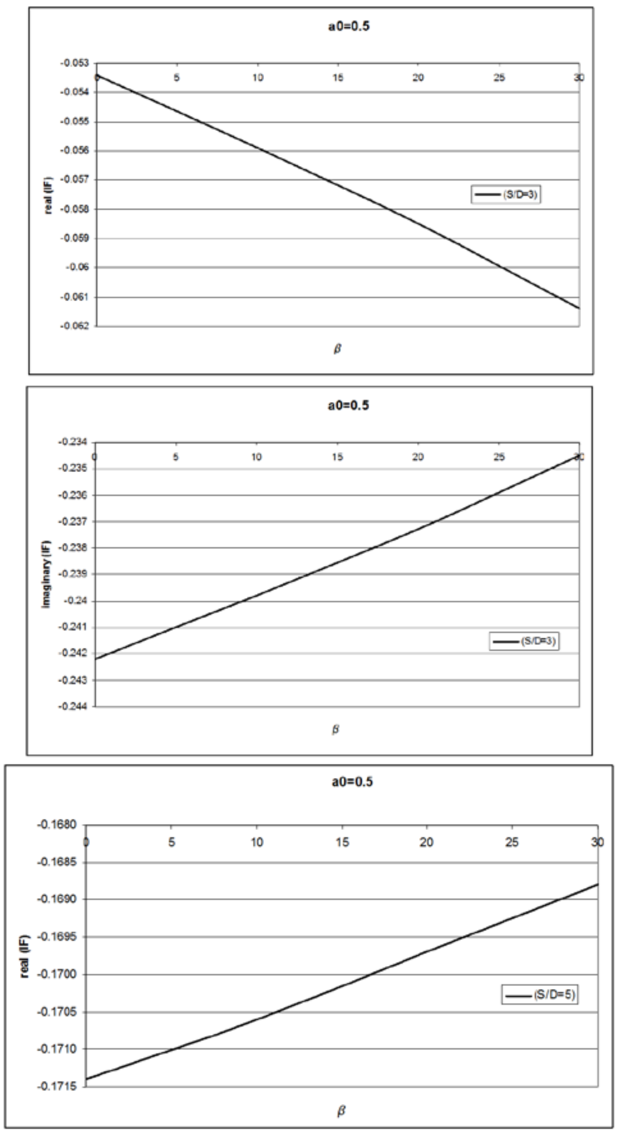


Fig. 6. Variation of IF (21)versus s/d

$(E_P/E_S = 1000, L/d = 40, \nu = 0.4, \rho_s/\rho_p = 0.7, \beta' = 0.05, s/d = 3 to 10, \beta = 0,30)$

C) The results for IF (12) with s/d=3, 5, 8 and $\beta = 0$ *to* 30 and $a_0 = 0.5$ are illustrated in Fig. 7.



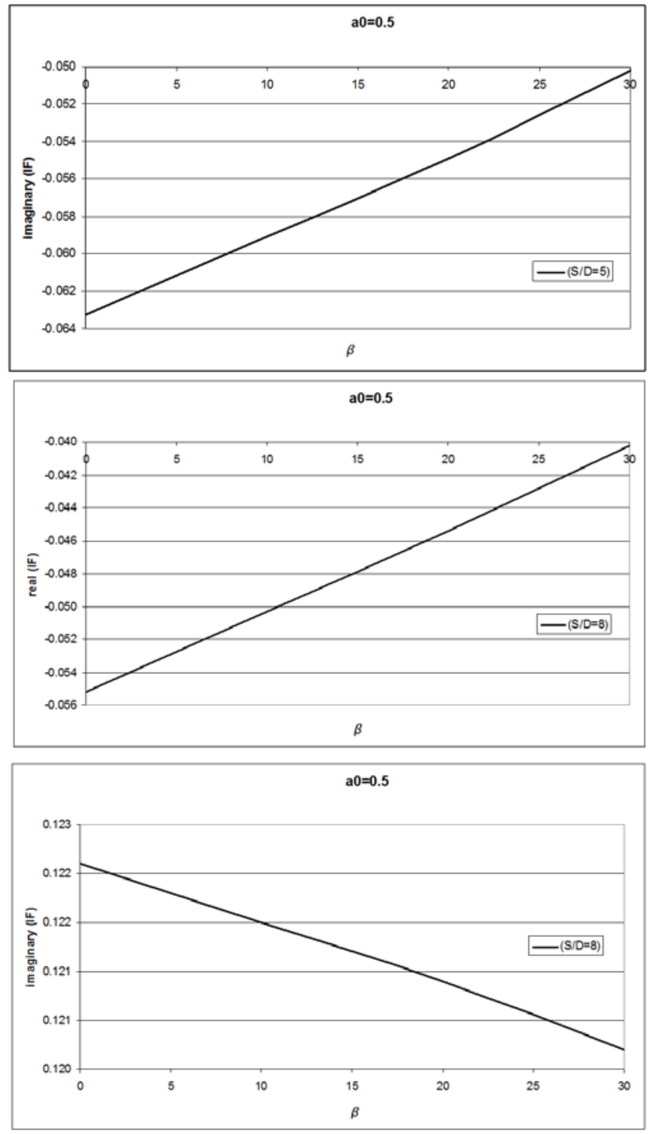


Fig. 7. Variation of IF (21) versus β

$(E_P/E_S = 1000, L/d = 40, \nu = 0.4, \rho_s/\rho_p = 0.7, \beta' = 0.05, s/d = 3, 5, 8, \beta = 0 \text{ to } 30)$

D) The variation of IF (21) versus s/d ranging 3, 5, 8, 10 and $\beta = 0$ and 30 is shown in Fig. 8.

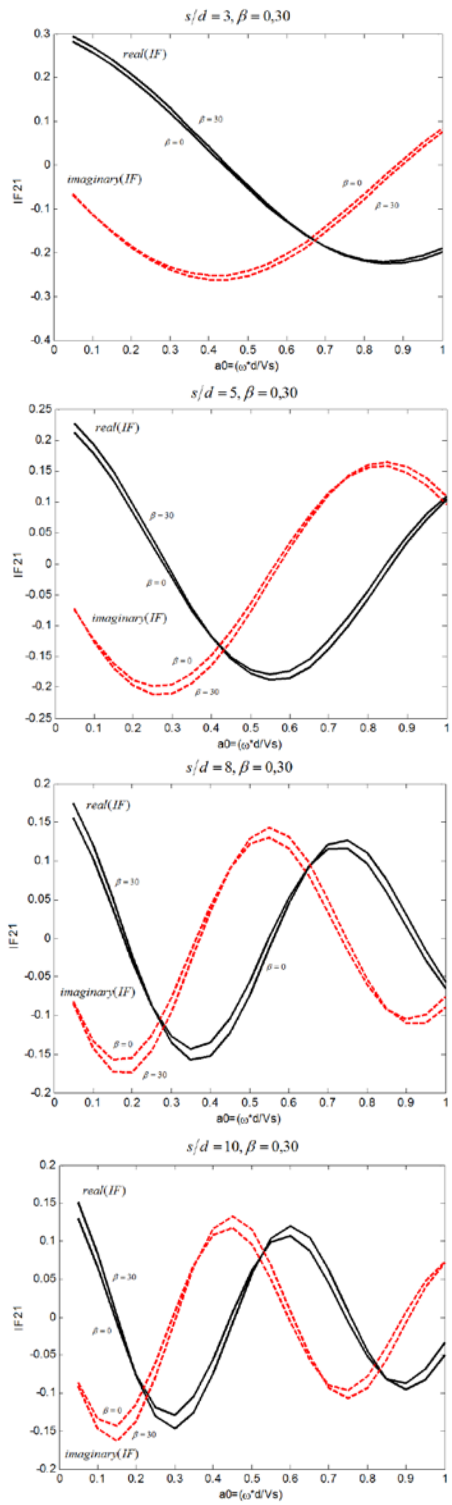


Fig. 8. Variation of IF (21) versus a_0

$(E_P/E_S = 1000, L/d = 40, \nu = 0.4, \rho_s/\rho_p = 0.7, \beta' = 0.05, s/d = 3,5,8,10, \beta = 0,30)$

As seen in above figures, for two piles 1 and 2 of unequal embedded lengths, when pile 1 is loaded and pile 2 is the receiver, the interaction factor is different from the case where pile 2 becomes the source and pile 1 becomes the receiver. For the given example, the difference between the embedment lengths of two piles was small. However, it seems that if the embedded lengths of two piles become more different, the two interactions become more different. As show, for the given example, this difference approaches 20-30%.

CONCLUSIONS

A simple closed-form solution has been presented for calculating dynamic interaction factor for a pair of unequal piles. This method assumes that the pile is located in a sloping ground where the embedded lengths of two piles are different. The soil and piles are assumed to be elastic. An approximate concept based on the interference of cylindrical wave field originating along each pile shaft and spreading radially outward is used to account for the influences of closely spaced piles and the ground slope in pile groups. A comparison between results shows that the interaction factor between two piles with unequal embedded lengths is not identical. In addition, with increasing the ground slope, the interaction between piles for a given pile spacing differs sometimes up to 20-25%.

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