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# Numerical Analysis of Sandwiched Composite-FSS Structures

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**Abstract**— A numerical technique to analyze shielding effectiveness of sandwiched FSS-composite structures is proposed. This technique is based on using a dispersive FDTD method in conjunction with a novel periodic boundary condition to model sandwiched FSS-composite elements. Results show that by inserting single or multilayered FSS elements into composite materials, better shielding effectiveness can be achieved.

**Keyword** FDTD, Z-transform, dispersive medium, debye model, periodic structure, periodic boundary condition

## I. INTRODUCTION

Recently, composite materials have been investigated for their potential applications as shielding materials to protect electronics system from electromagnetic pulse or electromagnetic interferences [1]. For composite materials investigated in previous studies, these materials may not have the desired shielding effectiveness in certain frequency band. To enhance the shielding materials at desired frequency band, one technique is to develop multilayered composite material with each layer has different electric and magnetic properties. By properly designing the thickness and electric/magnetic properties of each layer, a wideband composite shielding material can be obtained. However, at low frequency, this stacked structure may not provide the same shielding effectiveness as that at the higher frequency end. To enhance the shielding effectiveness at low frequency end, one possible solution is to introducing additional layer or layers of frequency selective surface (FSS) structure between the interfaces of composite material as shown in Figure 1. In this paper, we refer to this kind of structure as sandwiched composite-FSS structure. Since this layer of FSS structure typically consists of thin layer of metal sheet, this additional layer(s) can be easily fabricated and will not leads to significant increment in the system volume.

To evaluate the shielding performance of the sandwiched structures, numerical procedure is often required. For some canonical FSS structures, it is possible to obtain the polarizability of these FSS structures and then incorporate these information in developing the effectiveness electrical permittivity and permeability. However, if complex FSS

structures are used, it is very difficult to obtain the polarizability of complex FSS structures. To obtain the shielding effectiveness for arbitrary FSS structure, rigorous electromagnetic numerical modeling is required. One of such approach is the finite-difference time-domain (FDTD) method [2]. With appropriate implementation of periodic boundary condition (PBC), this technique can effectively analyze arbitrary sandwiched structures.

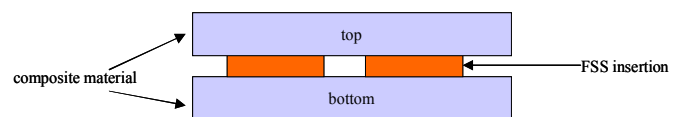


Figure 1. An illustration of a sandwiched FSS-composite structure.

The purpose of this paper is to develop such a technique that can be used towards the shielding effectiveness analysis for arbitrary sandwiched composite-FSS structures. This method is based on the FDTD method. To model the dispersive electrical property of the composite materials, a Z-transform technique is used in the FDTD method. In addition, a simple periodic boundary condition (PBC) is incorporated into the algorithm to model the periodic nature of the FSS insertion. The shielding performance can be obtained using this approach. In addition, with the obtained electric and magnetic field distribution, one can also gain insight information on how to appropriately insert FSS layer(s)..

The remainder of this paper is organized as follows. We shall first describe the Z-transform implementation of dispersive permittivity and permeability [3]. Then, the novel implementation of the PBC for FDTD method is presented. After presenting the numerical results, conclusions are given.

## II. Z-TRANSFORM IMPLEMENTAION OF DISPERSIVE PERMITTIVITY AND PEREABILTY

The composite materials used in this study are given in [4]. The relative permittivity and permeability of the composite

material are estimated using the Maxwell Garnett mixing formula [1]. Figure 2 shows the relative permittivity and permeability of the composite material as a function of frequency. As can be seen from the figure, both relative permittivity and permeability are highly dispersive. To describe such frequency dependent behavior, the Debye model or approximated Debye model are used here.

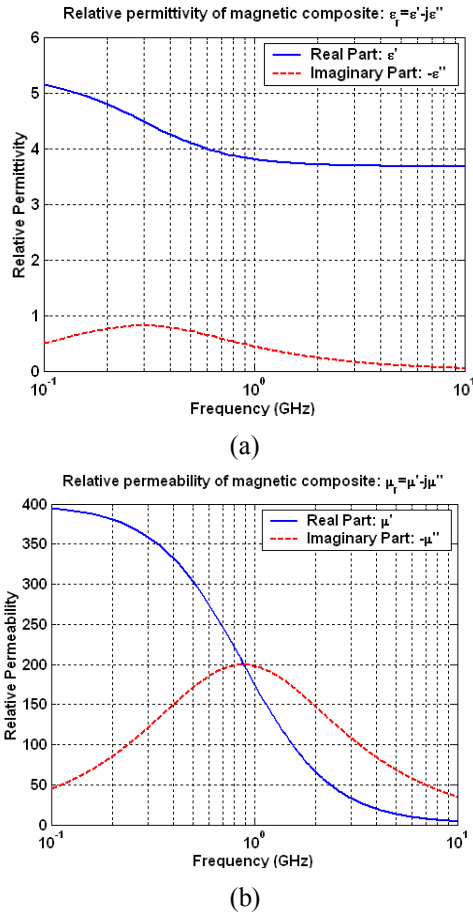


Figure 2. Electrical properties of composite materials. (a) Relative permittivity as a function of frequency (b) Relative permeability as a function of frequency.

Based on the permittivity given in Figure 2(a), the permittivity can be described by

$$\epsilon_r = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + j\omega\tau_e} - j \frac{\sigma_{DC}}{\omega\epsilon_0}, \quad (1)$$

where  $\epsilon_\infty$  corresponds to the relative dielectric constant when the frequency approaches to infinity and  $\epsilon_s$  is the relative dielectric constant when the frequency is equal to zero. The

$\tau_e$  is the electrical relaxation time and  $\sigma_{DC}$  is the conductive loss. The optimal values to represent the permittivity shown in Figure 2(a) are given by  $\epsilon_\infty = 3.7$ ,  $\epsilon_s = 5.2$ ,  $\tau_e = 5.27 \times 10^{-10}$  seconds, and  $\sigma_{DC} = 0.00052$  S/m.

Similarly, the relative permeability of can be best fit by  $\mu_r = \frac{\mu_s}{1 + j\omega\tau_m}$  where  $\mu_s$  is the relative permeability when the frequency is equal to zero. The optimal values for  $\mu_s$  and  $\tau_m$  for this case are given by  $\mu_s = 399.4$  and  $\tau_m = 1.79 \times 10^{-10}$  seconds. It should point out that this expression to describe the frequency dependent permeability cannot be implemented into the FDTD method directly. This is not the standard Debye model either. Modification on this expression will be needed in order to implement this term.

#### A. Implementation of the dispersive permittivity

In frequency domain, the relation between the electric flux and the electric field of the composite materials can be expressed as:

$$D(\omega) = \epsilon_0 \epsilon_r(\omega) E(\omega) = \epsilon_0 \left( \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + j\omega\tau_e} - j \frac{\sigma_{DC}}{\omega\epsilon_0} \right) E(\omega). \quad (2)$$

Performing the Z-transform on the equation above [3], we obtain

$$D(z) = \epsilon_0 \epsilon_r(z) E(z) \cdot \Delta t$$

$$= \epsilon_0 \epsilon_\infty E(z) + \frac{\epsilon_0 (\epsilon_s - \epsilon_\infty) \cdot \Delta t / \tau_e}{1 - e^{-\Delta t / \tau_e} z^{-1}} E(z) - \frac{\sigma_{DC} \cdot \Delta t}{1 - z^{-1}} E(z) \quad (3)$$

Define

$$S(z) = \frac{\epsilon_0 (\epsilon_s - \epsilon_\infty) \cdot \Delta t / \tau_e}{1 - e^{-\Delta t / \tau_e} z^{-1}} E(z) \quad (4)$$

$$= e^{-\Delta t / \tau_e} z^{-1} S(z) + \frac{\epsilon_0 (\epsilon_s - \epsilon_\infty) \cdot \Delta t}{\tau_e} E(z)$$

and

$$I(z) = \frac{\sigma_{DC} \cdot \Delta t}{1 - z^{-1}} E(z) = z^{-1} I(z) + \sigma_{DC} \cdot \Delta t \cdot E(z), \quad (5)$$

we obtain

$$D(z) = \epsilon_0 \epsilon_\infty E(z) + e^{-\Delta t / \tau_e} z^{-1} S(z)$$

$$+ \frac{\epsilon_0 (\epsilon_s - \epsilon_\infty) \cdot \Delta t}{\tau_e} E(z) - z^{-1} I(z) - \sigma_{DC} \cdot \Delta t \cdot E(z) \quad (6)$$

Consequently,  $E(z) = \frac{D(z) - e^{-\Delta t / \tau_e} z^{-1} S(z) + z^{-1} I(z)}{\epsilon_0 \epsilon_\infty + \frac{\epsilon_0 (\epsilon_s - \epsilon_\infty) \cdot \Delta t}{\tau_e} - \sigma_{DC} \cdot \Delta t}$ . (7)

Using this expression, we can update the electric field from the electric flux in each time step during the FDTD simulation. The updating sequence is:

1. Update  $D^n$  using the standard FDTD method
2. Update  $E^n$  using

$$E^n = \frac{D^n - e^{-\Delta t/\tau_e} S^{n-1} + I^{n-1}}{\epsilon_0 \epsilon_\infty + \frac{\epsilon_0 (\epsilon_s - \epsilon_\infty) \cdot \Delta t}{\tau_e} - \sigma_{DC} \cdot \Delta t} \quad (8)$$

3. update  $S^n$  and  $I^n$

$$S^n = e^{-\Delta t/\tau_e} S^{n-1} + \frac{\epsilon_0 (\epsilon_s - \epsilon_\infty) \cdot \Delta t}{\tau_e} E^n \quad (9)$$

$$I^n = I^{n-1} + \sigma_{DC} \cdot \Delta t \cdot E^n$$

Using this procedure, we can obtain the electric field at every time step.

### B. Implementation of the dispersive permeability

Using the relative permeability model developed earlier, the magnetic flux and the magnetic field is related by

$$B(\omega) = \mu_0 \mu_r(\omega) H(\omega) = \mu_0 \frac{\mu_s}{1 + j\omega\tau_m} H(\omega) \quad (10)$$

However, direct implementation of this expression into the FDTD method may lead to instability in the algorithm, since at higher frequency, the relative permeability could be smaller than 1. To guarantee the stability of the algorithm, an additional term  $\mu_0$  is introduced in the expression. For practical implementation, we use

$$B(\omega) = \mu_0 \left( 1 + \frac{\mu_s}{1 + j\omega\tau_m} \right) H(\omega) \quad (11)$$

Introducing an additional term into the expression does not change the relative permeability in the frequency range of our interests (0.1G-10GHz). It leads to stable solution in the FDTD calculation.

Similar to the implementation of dispersive permittivity, we transform the equation above into the Z domain and obtain

$$\begin{aligned} B(z) &= \mu_0 \mu_r(z) H(z) \cdot \Delta t \\ &= \mu_0 H(z) + \frac{\mu_0 \mu_s \cdot \Delta t / \tau_m}{1 - e^{-\Delta t/\tau_m} z^{-1}} H(z) \end{aligned} \quad (12)$$

Define

$$T(z) = \frac{\mu_0 \mu_s \cdot \Delta t / \tau_m}{1 - e^{-\Delta t/\tau_m} z^{-1}} H(z) = e^{-\Delta t/\tau_m} z^{-1} T(z) + \frac{\mu_0 \mu_s \cdot \Delta t}{\tau_m} H(z) \quad (13)$$

and substitute this into (12), we can obtain

$$H(z) = \frac{B(z) - e^{-\Delta t/\tau_m} z^{-1} T(z)}{\mu_0 + \frac{\mu_0 \mu_s \cdot \Delta t}{\tau_m}} \quad (14)$$

The step by step implementation to obtain the magnetic field is:

1. update  $B^n$  using directly from the FDTD method
2. update  $H^n$  using

$$H^n = \frac{B^n - e^{-\Delta t/\tau_m} T^{n-1}}{\mu_0 + \frac{\mu_0 \mu_s \cdot \Delta t}{\tau_m}} \quad (15)$$

3. updating  $T^n$

$$T^n = e^{-\Delta t/\tau_m} T^{n-1} + \frac{\mu_0 \mu_s \cdot \Delta t}{\tau_m} H^n$$

Using the procedure described above, the dispersive permittivity and permeability are now implemented into the FDTD method.

## III. CONSTANT WAVENUMBER TECHNIQUE

To calculate the shielding effectiveness of sandwiched composite structures, a periodic FDTD method is employed here. In this section, we describe a novel technique to implement the PBC in the FDTD method.

For a periodic structure with a periodicity  $px$  along the  $x$  direction, the periodic boundary condition relating the electric and magnetic fields at neighboring cells is given by

$$\begin{aligned} \mathbf{E}(x + p_x, y, z) &= \mathbf{E}(x, y, z) \exp(-jk_x p_x) \\ \mathbf{H}(x + p_x, y, z) &= \mathbf{H}(x, y, z) \exp(-jk_x p_x) \end{aligned} \quad (16)$$

where the horizontal wave number  $k_x$  is defined as  $k_x = k_0 \sin \theta$ .  $k_0 = 2\pi f_0 \sqrt{\epsilon_0 \mu_0}$  is the wave number in the free space and  $\theta$  is the incident angle between the incident wave direction and the normal direction of the composite surface. When this expression is transformed into the time domain, future time data are needed in the updating equation. For example, for the electric field update, we need

$$\mathbf{E}(x = 0, y, z, t) = \mathbf{E}(x = p_x, y, z, t + p_x \sin \theta / C) \quad (17)$$

where  $C$  is the speed of light. Since this equation requires the electric field from future time, this poses a fundamental challenge in formulating PBC in the FDTD method. To circumvent this problem, a constant wave number method was proposed in [2]. If  $k_x$  is a constant for all incident signals, we can simply using the following expression to update the electric and magnetic field at boundaries.

$$\begin{aligned} E(p_x, y, z, t) &= E(0, y, z, t) \exp(-jk_x p_x) \\ H(p_x, y, z, t) &= H(0, y, z, t) \exp(-jk_x p_x) \end{aligned} \quad (18)$$

Since  $\exp(-jk_x p_x)$  is a constant and no future time data are needed in (18).

The implementation of the constant wave number method is similar to the normal incidence method [2]. The standard Yee's scheme is used to update the electric and magnetic fields with time marching. The perfectly matched layers (PML) and the PBC in (18) are used to truncate the computational domain in the vertical and horizontal directions, respectively. It is also important to point out that additional attention needs to be paid to the time domain excitation waveform. A plane wave with a

constant wave number in the  $x$  direction shall be used to excite the structure. As indicated in [5], in order to excite the plane wave successfully, only one transverse field component needs to be specified in the incident plane. Here we launched the  $E_z$  component for incidence and used a Gaussian waveform with linear phase delays in the  $x$  direction as:

$$E_z(x, y_0, z, t) = \exp\left[-\frac{(t-t_0)}{2\sigma^2}\right] \exp(-jk_x x). \quad (19)$$

Electric fields at both sides of the structures are sampled. The shielding effectiveness is defined as

$$SE = 20 \log_{10} \frac{E_{z,average}^i}{E_{z,average}^t}. \quad (20)$$

where  $E_{z,average}^i$  is the averaged value of electric field in the incident plane and  $E_{z,average}^t$  is averaged electric field in the receiving plane.

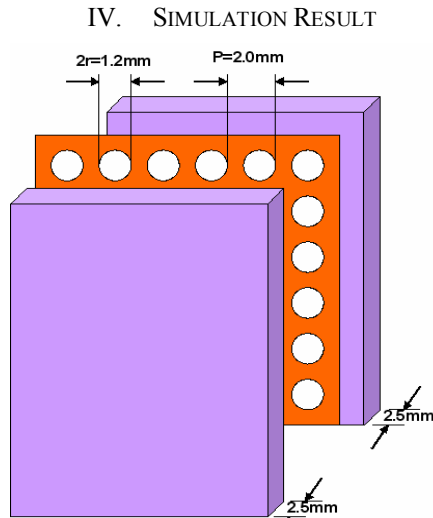


Figure 3. First structure investigated in this study.

The first sandwiched structure studied here is depicted in Figure 3 [4]. An infinite thin perforated metal film is inserted between two composite mater layers with a thickness of 2.5mm. On the metal film, periodic circular array with a 2.0 mm center-to-center spacing in both horizontal and vertical direction is fabricated. Each single circular pattern has a diameter of 1.2 mm.

Figure 4 shows the results of shielding effectiveness using the developed FDTD method. Since this method requires one to use a constant wave number in each simulation, we show here is the shielding effectiveness in a plane as functions of both wave number and the frequency. To extract the shielding effectiveness as a function of incident angle at different

frequencies, one needs to sample this plane on a straight line from the origin as discussed in [5]. Figure 5 shows the shielding effectiveness as a function of frequency for different incident angles. Here, the shielding effectiveness is defined only in terms of the fundamental modes as those used in the FSS transmission coefficient definition. Therefore, it is not a strong function of incident angle. As clearly indicated in the figure, inserting a layer of metallic periodic structure can significantly enhance the shielding effectiveness.

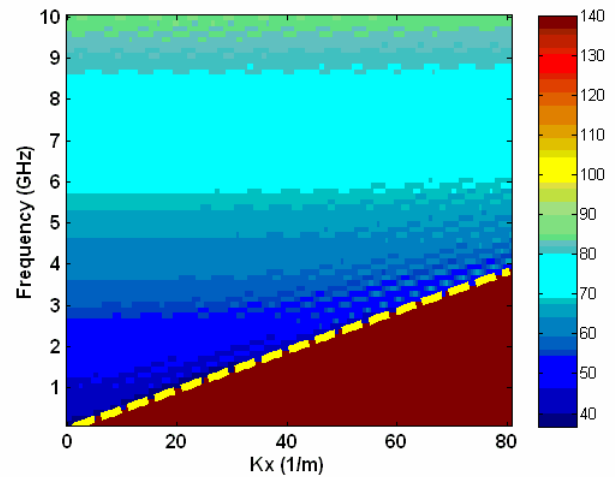


Figure 4. Simulated shielding effectiveness using the constant wave number technique.

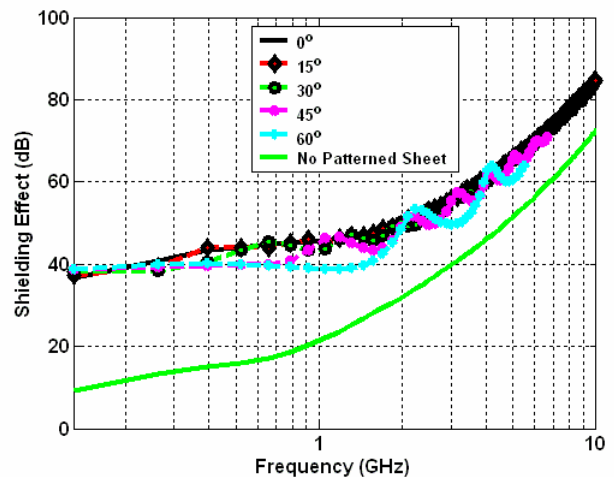


Figure 5. Simulated shielding effectiveness as a function of incident angles.

The second structure considered here is a two-metal layered composite structure as shown in Figure 6. The first metallic layer has circular pattern while the second metallic layer has a cross-dipole pattern. The thickness of the composite material between the two metallic insertions is 2.5mm.

Figure 7 and Figure 8 show the shielding effectiveness of this structure as a function of frequency at different incident angles.

As expected, inserting additional layer of metallic FSS structure increases the shielding effectiveness for both high and low frequency regions. However, overall shielding effectiveness could also be affected by the spacing between two metallic insertions as well as patterns of the FSS structures. More research is required in order to fully understand the effective of the metallic insertion on the shielding effectiveness.

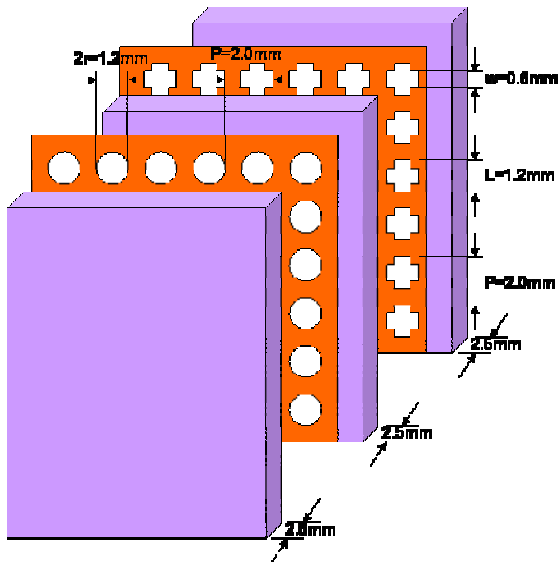


Figure 6. First structure investigated in this study.

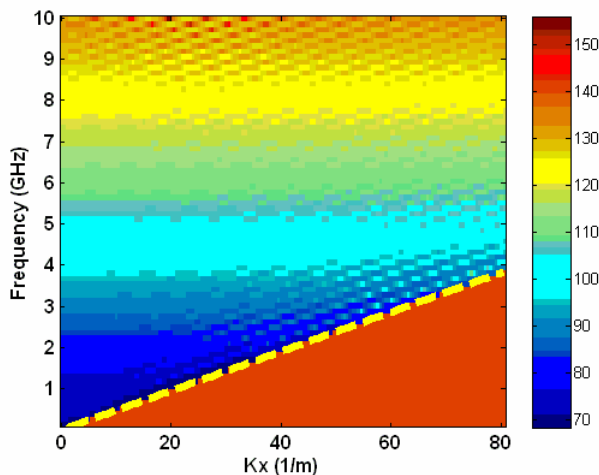


Figure 7. Simulated shielding effectiveness using the constant wave number technique.

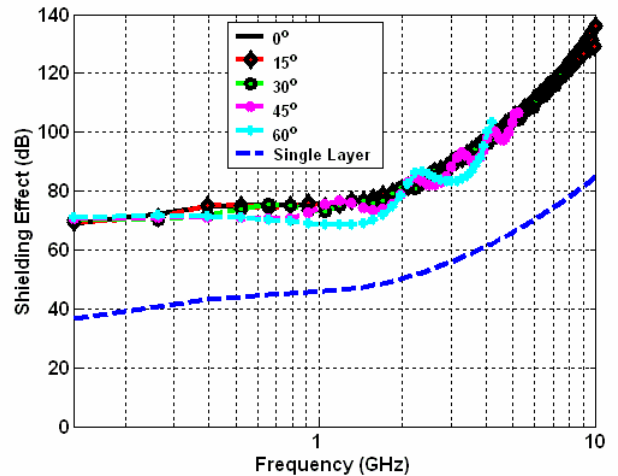


Figure 8. Simulated shielding effectiveness as a function of incident angles.

## V. CONCLUSIONS

An FDTD algorithm to analyze sandwiched composite FSS structure is developed. The algorithm is based on using Z-transform technique to model the dispersive electrical behavior of the composite structure and utilizing periodic boundary condition to truncate the FDTD simulation. Numerical results show that inserting additional metallic FSS structure can improve the shielding effectiveness over composite materials. Increasing the layer of metallic FSS insertions can also lead to improved shielding effectiveness.

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